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J. Schou, J. Christensen-Dalsgaard & M.J. Thompson

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THE RESOLVING POWER OF CURRENT HELIOSEISMIC INVERSIONS FOR THE SUN'S INTERNAL ROTATION

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ABSTRACT

The inferred internal solar rotation rate obtained from helioseismic inversion is a non-local average of the true rotation rate; hence some care is required in the interpretation of the results of such inversions. We present kernels describing the weighting in the average, for an inversion that is representative of recent work. Of particular note is the fact that the so-called polar rate is actually an extrapolation from lower latitudes: the current helioseismic data do not permit us to construct an average that is localized at high solar latitude. Nonetheless, our results do not change the basic conclusion that the average radial gradient of the rotation rate in the solar convection zone is small. Finally, we demonstrate that more localized measures of the solar rotation at high latitudes are possible with the more complete data that should become available from new helioseismology projects.

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1. INTRODUCTION

Measurements of the frequencies of normal modes of oscillation of the Sun lead to inferences about the structure and dynamics of the solar interior. In particular, the difference in the frequencies of modes of different azimuthal orders m (known as frequency splittings) have been used to obtain helioseismic determinations of the rotation rate as a function of latitude at fractional radii $0.5 \lesssim r/R \lesssim 0.85$, where r is the radial coordinate and R is the photospheric radius (Brown *et al.* 1989; Christensen-Dalsgaard & Schou 1988; Dziembowski, Goode & Libbrecht 1989; Goode *et al.* 1991; Korzennik *et al.* 1988; Kosovichev 1988; Schou 1991a; Thompson 1990). The principal conclusions that have been drawn thus far from such inversions are (i) that the rotation rate deep in the convection zone exhibits similar latitudinal variation to what is observed at the surface, i.e., there is little radial gradient in the rotation rate through the convective envelope, and (ii) that there is a transition near the base of the convection zone to latitudinally independent rotation in at least the outer part of the radiative interior. The inferred behaviour in the convection zone is contrary to the expectation from numerical simulations (*e.g.*, Gilman & Miller 1986; Glatzmeier 1985a,b) that the rotation rate there would be constant on cylindrical surfaces aligned with the rotation axis. The lack of a significant radial gradient through the bulk of the convection zone and the steep gradient at its base suggest that the dynamo widely believed to be responsible for the Sun's magnetic field is located near the base of the convective envelope (*e.g.*, Gilman, Morrow & DeLuca 1989).

However, the inferred rotation rate at a given point is actually a non-local average of the true rotation rate. The weighting functions appropriate to these averages are known as averaging kernels; by examining them one can readily see the resolution that has been achieved in an inversion. In this letter we analyze an inversion of the currently available rotational splitting data which is representative of other recent inversions. The averaging kernels have substantial extent in latitude; furthermore, it transpires that what is often inferred to be the “polar rotation rate” is in fact essentially an extrapolation from intermediate latitudes.

2. INVERSION OF HELIOSEISMIC SPLITTING DATA

If the Sun were spherically symmetric and not rotating, the angular frequency ω_{nlm} of a mode of radial order n , degree l and azimuthal order m would be independent of m . The first-order effect of rotation at a rate $\Omega(r, \theta)$ is to shift the frequencies of the nonaxisymmetric ($m \neq 0$) modes relative to their axisymmetric counterparts:

$$\omega_{nlm} - \omega_{nl0} = \int_0^R \int_0^\pi K_{nlm}(r, \theta) \Omega(r, \theta) r dr d\theta. \quad (1)$$

Here θ is colatitude; and we assume that the kernels K_{nlm} , which can be obtained from the corresponding eigenfunctions, are known functions. Kernels with respect to $r dr d\theta$ were chosen such that the integration is with respect to the area in the plots of the kernels. Until now, observed values of splittings for individual m have not been available for inversion; instead results of observations have been provided in terms of coefficients a_i ($i \leq 5$) of fits of the form

$$(\omega_{nlm} - \omega_{nl0})/2\pi = L \sum_i a_i(n, l) P_i\left(\frac{m}{L}\right), \quad (2)$$

where P_i is a Legendre polynomial of degree i and $L = \sqrt{l(l+1)}$. To first order, only the coefficients of odd i contain information about the rotation: those coefficients are related to the rotation rate $\Omega(r, \theta)$ through an expression analogous to equation (1), involving kernels $K_{nl}^i(r, \theta)$.

A goal of the inversion is to infer an approximation $\bar{\Omega}(r, \theta)$ to the true rotation rate. In analyses of the coefficients $a_i(n, l)$ ($i = 1, 3, 5$), the latitudinal dependence of $\bar{\Omega}(r, \theta)$ has generally been expanded in θ : here we use

$$\bar{\Omega}(r, \theta) = \bar{\Omega}_0(r) + (x^2 - \frac{1}{5})\bar{\Omega}_1(r) + (x^4 - \frac{2}{3}x^2 + \frac{1}{21})\bar{\Omega}_2(r), \quad (3)$$

where $x = \cos \theta$, and the functions $\bar{\Omega}_0(r)$, $\bar{\Omega}_1(r)$ and $\bar{\Omega}_2(r)$ are determined by a one-dimensional inversion procedure in r (Brown *et al.* 1989; see also Ritzwoller & Lavelle 1991). Such an approach has been dubbed a 1.5-D (as opposed to a full 2-D) inversion (Schou 1991b). Various inversion methods have been employed; the results presented here were obtained with an optimally localized averages inversion for each of the functions $\bar{\Omega}_i$; in turn, analogous to the 1-D procedure described by Christensen-Dalsgaard, Schou &

Thompson (1990). The inversion depends on a trade-off between resolution and error, determined by the value of the trade-off parameter μ defined in that paper; we adopted a value of $\mu = 10^{-2}$ for each $\bar{\Omega}_j$.

As in the one-dimensional case of latitudinally-independent rotation, the resulting solution $\bar{\Omega}$ at any point (r_0, θ_0) is a linear combination of the data, in this case the a_i coefficients. Since these are in turn weighted averages of the true rotation rate $\Omega(r, \theta)$, it follows that the inferred and true rotation rates are related by averaging kernels $\mathcal{K}(r, \theta; r_0, \theta_0)$:

$$\bar{\Omega}(r_0, \theta_0) = \int_0^R \int_0^\pi \mathcal{K}(r, \theta; r_0, \theta_0) \Omega(r, \theta) r dr d\theta, \quad (4)$$

plus an error term propagated from the data errors, but otherwise independent of $\Omega(r, \theta)$ (see Christensen-Dalsgaard *et al.* 1990). The averaging kernels therefore show how the inversion actually averages over the rotation rate in the Sun's interior; of course, they depend upon the mode set, and on the inversion method used and the value of the trade-off parameter chosen for the inversion.

3. RESULTS

We have performed an optimally localized averages inversion of Big Bear splitting data measured in 1986: this set comprises splittings for 780 p modes with degrees $5 \leq l \leq 60$ (Libbrecht 1989). The inferred rotation rates at three different latitudes θ_0 are shown in Figure 1. As with previous inversions, the results indicate that the latitudinal dependence of the rotation observed at the surface persists through much of the convection zone, with a transition to little or no latitudinal variation near the base of the convective envelope (*i.e.*, near $r = 0.7R$). Averaging kernels for these latitudes and three different radii r_0 are shown as contour plots in Figure 2. In all cases the kernels for $\theta_0 = 90^\circ$ and 45° have a local maximum close to the target radius and latitude. These peaks are particularly broad in latitude, however. The kernels corresponding to $\theta_0 = 0^\circ$ are not peaked at the pole: rather, they have positive peaks at roughly the correct radius but at $\theta \simeq 30^\circ$, with a negative contribution from lower latitudes at the same radius. Thus the inferred polar rate is essentially an extrapolation from the rotation rate at lower latitudes. As was the case for the 1-D averaging kernels of Christensen-Dalsgaard & Schou (1988), the kernels

have structure near the surface; this is a result of the lack of shallowly penetrating modes in the set considered, since all the modes have lower turning points deeper than $0.92R$. The kernels at the smallest radius shown ($r_0 = 0.44R$) have substantial large-scale structure far removed from the target radius and latitude.

We have also computed averaging kernels for a 1.5-D regularized least-squares inversion with second-derivative smoothing (cf. Christensen-Dalsgaard *et al.* 1990). The averaging kernels have more structure near the surface than do the kernels illustrated here, and larger regions of negative amplitude in the interior. This is as one would expect from a comparison of the two inversion methods in the 1-D case (Christensen-Dalsgaard *et al.* 1990). However, they are broadly similar to the kernels for the optimally localized averages method: in particular, the two methods have very similar latitudinal resolution.

4. DISCUSSION

We have performed an optimally localized averages inversion for the Sun’s internal rotation rate, as a function of latitude and depth. To illustrate the limited resolution of the inversion we display the corresponding averaging kernels. These show that the results are rather broad averages over radius and latitude of the true rotation rate. The averaging kernels for all target latitudes have a substantial spread in the latitudinal direction. This is a result of the fact that in present data the m -dependence of the frequencies at fixed n and l (which provides information about the latitude-dependence of rotation) is parametrized in terms of an expansion involving very few coefficients; in the dataset that we have analyzed only three terms were retained, corresponding roughly to a latitudinal extent of the averaging kernels of 30° . One might therefore expect that the limited latitudinal resolution would be a general feature of any inversion of the current data, independently of the inversion method employed: indeed we have verified that a regularized least-squares inversion gives very similar results.

Of particular significance is the fact that the inferred “polar” rotation rate is an extrapolation from lower latitudes. This is not entirely surprising. As a function of θ , the mode kernels K_{nlm} are approximately proportional to $[P_l^m(\cos \theta)]^2 \sin \theta$, where $P_l^m(\cos \theta)$ is an associated Legendre function. It follows from the properties of the P_l^m that only modes

of low azimuthal order m sense the rotation rate near the poles, and even for these the kernels have small amplitude near $\theta = 0$. Therefore, even with more complete information on the m -dependence of the frequencies than we have in the current data, it is intrinsically difficult to infer the Sun's internal rotation rate at high solar latitudes.

The inferred rotation curves are broadly consistent with those of previous investigations. We find that rotation depends weakly on radius throughout the convection zone, with a comparatively abrupt transition to apparently latitude-independent rotation at a radius near the base of the convection zone. These qualitative conclusions are not affected by the limited resolution of the inversion, as illustrated by the averaging kernels. In particular, our results are not consistent with rotation being constant on cylindrical surfaces, as has otherwise been indicated by numerical simulations of the dynamics of the convection zone (*e.g.*, Gilman & Miller 1986; Glatzmeier 1985a,b). Such a form of the internal rotation would exhibit significant radial gradients in the rotation rate at low latitudes, contrary to our findings (see also Thompson 1990), and would produce frequency splittings which vary more strongly with l than is observed (Morrow 1988). Nevertheless, while rotation on cylinders can be ruled out, the averaging kernels graphically illustrate the limited resolution with which we can at present determine the actual form of the solar rotation.

Substantial improvement in resolution may be expected from future helioseismic data, particularly when splittings for individual values of m become available. To investigate what may then be achieved, we have computed averaging kernels for this case, using the same mode set as before, but considering also the effect of reducing the errors. The a_i coefficients were converted to individual m splittings using equation (2). Estimated errors on the individual splittings (which enter into the inversion procedure and hence in the computation of the averaging kernels) were obtained by assuming them to be uncorrelated and independent of m for each (n, l) , and such that the quoted errors on the a_i would result from a least-squares fit to individual splittings. This does not precisely correspond to the analysis leading to the original a_i coefficients; but the resulting errors on the individual splittings are approximately consistent with the errors on the a_i 's. The splittings obtained in this way were then inverted using a 2-dimensional regularized least-squares algorithm, as described by Schou (1991b). With strong regularization in latitude the results of the 1.5-dimensional inversion were approximately recovered. However, resolution in latitude can be

much improved by weakening the regularization, as shown in Figure 3. This is achieved at the expense of an increase in error: the error in the solution corresponding to the kernels in Figure 3 is greater by about a factor of two than for the kernels in Figure 2. Nonetheless, the results indicate the improvements in resolution that might be anticipated from more accurate and extensive observations.

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FIGURE CAPTIONS:

Figure 1. The inferred solar rotation rate obtained from a 1.5-D optimally localized averages inversion, displayed at three latitudes: $\theta_0 = 0^\circ$ (pole), $\theta_0 = 45^\circ$ and $\theta_0 = 90^\circ$ (equator). Dashed lines indicate $1\text{-}\sigma$ error bars based on the observers' estimates of the uncertainties in the measured a_i coefficients.

Figure 2. Contour plots of the averaging kernels $R^{-2} \mathcal{K}(r, \theta; r_0, \theta_0)$ for the 1.5-D inversion illustrated in Figure 1, at target radii $r_0 = 0.44R$, $0.65R$ and $0.81R$, and target colatitudes $\theta_0 = 0^\circ$, 45° and 90° . Each plot is in the (r, θ) plane, with $\theta = 0^\circ$ up the page and $\theta = 90^\circ$ to the right. Positive contours are indicated by full lines, negative contours by broken lines; Δ is the value of the separation between contour levels.

Figure 3. Contour plots of averaging kernels $R^{-2} \mathcal{K}(r, \theta; r_0, \theta_0)$ for a 2-D inversion based on individual m -splittings; the mode set is the same as for the kernels shown in Figure 2, but a somewhat larger error magnification has been allowed. Kernels are shown for target radii $r_0 = 0.65R$ and $0.81R$, and target colatitudes $\theta_0 = 0^\circ$, 45° and 90° . See caption to Figure 2.

Fig. 1

Fig. 2

Fig. 3