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Abstract

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SOLAR MODELS WITH ENHANCED ENERGY TRANSPORT IN THE CORE

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ABSTRACT

The discrepancy between the observed and computed flux of solar neutrinos can be eliminated if the temperature gradient in the core of the model, and hence its central temperature, is reduced. This could in principle be accomplished by reducing the core opacity; alternatively, it has been proposed that a fraction of the energy transport in the solar core result from the motion of hypothetical particles (the so-called WIMPs). The resulting changes in solar structure have measurable effects on the frequencies of the solar five-minute oscillations. Here I consider models where the opacity in the solar core has been decreased in a manner which roughly simulates the effects of the WIMPs. The analysis of the models and their frequencies provides insight into the consequences of modifications to the physics of solar models. In particular, it is found that models with the observed neutrino flux are inconsistent with observations of low-degree solar oscillations.

1. INTRODUCTION

The discrepancy between on the one hand the observed values of the capture rate of solar neutrinos in the Homestake ^{37}Cl neutrino detector and the rate of neutrino-electron scattering events in the Kamiokande-II detector and on the other hand the predictions of traditional solar models remains a serious concern (for extensive overviews, see Bahcall & Ulrich 1988; Bahcall 1989). The predicted rate of emission of neutrinos from the highly temperature-sensitive PPIII chain, which dominate neutrino observations based on capture in ^{37}Cl and neutrino-electron scattering, can be reduced by decreasing the temperature in the central part of the solar model. This can be accomplished by decreasing the temperature gradient in the core of the model, either by reducing the core opacity or by introducing alternative means of energy transport, thus reducing the temperature gradient required for the transport by radiation. It has been suggested that a contribution to the energy transport could be come from the presence in the solar core of hypothetical Weakly Interacting Massive Particles (the so-called WIMPs) (*e.g.* Steigman *et al.* 1978; Spergel & Press 1985; Faulkner & Gilliland 1985). Indeed, it is possible, by a suitable choice of parameters for the WIMPs (mass, number density and cross section) to construct models that yield neutrino capture rates in agreement with the observations.

Some indirect support for this idea may come from the evidence for the presence in the Galaxy of invisible mass (so-called dark matter) which could conceivably be supplied by the WIMPs (for an overview, see Kolb & Turner 1990). Nevertheless, given that there is no other independent evidence for the existence of such particles, let alone measurements of their properties, this solution to the neutrino problem remains somewhat speculative. On the other hand, when proposed it was among the more successful “astrophysical” attempts to explain the neutrino discrepancy, in the sense that it was not in obvious conflict with other astrophysical data on the Sun, notably the frequencies of the solar five-minute oscillations. In this the WIMP hypothesis differed from, for example, the suggestion of a generally lower opacity in the solar interior (for example resulting from a lower heavy element abundance in the interior than on the surface: see Joss 1974; Christensen-Dalsgaard, Gough & Morgan 1979), or of substantial core mixing (*e.g.* Schlatzman *et al.* 1981), both of which could be ruled out by early helioseismic data (see, for example, Christensen-Dalsgaard & Gough 1980, 1981; Christensen-Dalsgaard 1984, 1986). In fact, initial analyses of the low-degree frequencies of models with WIMPs (Faulkner, Gough & Vahia 1986; Däppen, Gilliland & Christensen-Dalsgaard 1986) suggested that such models might be closer to the observations than were normal models: specifically, the small separation $\delta\nu_{nl} \equiv \nu_{nl} - \nu_{n-1+l+2}$ (where ν_{nl} is the frequency of a mode with degree l and radial order n) computed for models with WIMPs appeared to be in essentially perfect agreement with observations for $l = 0$ and 1, whereas for normal solar models the computed $\delta\nu_{nl}$

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was slightly too high. This was confirmed by more detailed calculations by Gilliland & Däppen (1988).

It has later been found that this apparent agreement may have been accidental. Gelly *et al.* (1988) made a careful analysis of extensive sets of observations; they found that the measured average frequency separation was in good agreement with normal models. Furthermore, it was found that at least some of the early computations were affected by insufficient numerical precision in the model and frequency calculations. More recent calculations of normal solar models have confirmed the agreement between the computed and observed frequency separations, for a wide range of assumptions about the physics of the calculation (Cox, Guzik & Kidman 1989; Christensen-Dalsgaard & Berthomieu 1991; Christensen-Dalsgaard 1991). On the other hand Cox, Guzik & Raby (1990) found that models where the effects of WIMPs were approximated by an appropriate decrease in the core opacity predicted frequency separations significantly smaller than observed. Recently very precise measurements of the frequency separations were obtained by Elsworth *et al.* (1990); these were in excellent agreement with normal solar models, while obviously inconsistent with models where the neutrino flux had been reduced to the observed value through the inclusion of WIMPs.

Given the importance of the neutrino problem, it may nevertheless be useful to reconsider models with a localized enhancement of the efficiency of energy transport in the core, to investigate whether such models, adjusted to obtain a sufficient reduction of the central temperature to bring the computed neutrino flux into agreement with observation, can be consistent with the observed oscillation frequencies. In particular, it is of interest to analyze in some detail the effects on the structure and the frequencies of the modified energy transport, when compared with recent normal solar models for which the frequency separations $\delta\nu_{nl}$ appear to be in accordance with the observations. A further justification comes from the earlier indications that the inclusion of WIMPs improved the agreement with the helioseismic observations. As an example, I consider models where the opacity in a localized region in the solar core has been reduced, by means of a simple prescription. The intention has not been to give an accurate representation of the effects of WIMPs; indeed, since the energy transport by WIMPs is non-local, a proper treatment of their effect on the solar models requires a full transport calculation (*e.g.* Steigman *et al.* 1978; Spergel & Press 1985; see also Giraud-Héraud *et al.* 1990; Dearborn, Griest and Raffelt 1991; Kaplan *et al.* 1991). Nonetheless, the modifications considered here do in fact lead to sound-speed profiles in the core that are at least superficially similar to those obtained in the detailed WIMP models.

2. PROPERTIES OF THE OSCILLATION FREQUENCIES

To use observed frequencies of solar oscillations to test models without and with enhanced energy transport in the core, it is necessary to isolate aspects of the frequencies which depend on the core structure and are largely insensitive to the uncertainties of other parts of the solar models, particularly the surface layers. This may be done in a systematic way through inverse analyses of the frequencies (*e.g.* Gough 1985; Gough & Kosovichev 1988; Gough 1991; Gough & Thompson 1991). However, the comparison between the observations and models of this nature has traditionally been carried out in terms of frequency separations of low-degree p modes. From asymptotic analysis of the oscillation equations (Tassoul 1980, 1990), or from a careful analysis in terms of rays (Gough 1986), it may be shown that frequencies of such modes approximately satisfy

$$\nu_{nl} \approx \left(n + \frac{l}{2} + \frac{1}{4} + \alpha\right)\Delta\nu - (AL^2 - \delta)\frac{\Delta\nu^2}{\nu_{nl}}. \quad (1)$$

Here

$$\Delta\nu = \left(2 \int_0^R \frac{dr}{c}\right)^{-1}, \quad (2)$$

and

$$A = \frac{1}{4\pi^2\Delta\nu} \left[\frac{c(R)}{R} - \int_0^R \frac{dc}{dr} \frac{dr}{r} \right], \quad (3)$$

c being the adiabatic sound speed, and α and δ are independent of l and are related predominantly to the structure of the outer parts of the model; the integrals are over distance r to the center, and extend from the center to the surface radius R of the model. Also $L^2 = l(l+1)$. A detailed discussion of equation (1) was given by Gough & Novotny (1990). Gabriel (1989) analyzed further corrections to the relation and compared it with computed frequencies of solar oscillation.

From equation (1) it follows that

$$\delta\nu_{nl} \approx -(4l+6)\frac{\Delta\nu}{4\pi^2\nu_{nl}} \int_0^R \frac{dc}{dr} \frac{dr}{r}, \quad (4)$$

where I neglected the small term in the surface sound speed $c(R)$. Thus $\delta\nu_{nl}$ is predominantly determined by conditions in the solar core. Equation (4) shows that the l -dependence of $\delta\nu_{nl}$ is contained in the factor $4l+6$. Hence in the following I shall often consider the scaled separation

$$d_{nl} \equiv \frac{3}{2l+3}\delta\nu_{nl} \approx -\frac{3\Delta\nu}{2\pi^2\nu_{nl}} \int_0^R \frac{dc}{dr} \frac{dr}{r}, \quad (5)$$

thus effectively reducing the separation to $l=0$. It also follows that asymptotically $\delta\nu_{nl}$, and hence d_{nl} , decreases with increasing ν as ν^{-1} .

As an example, Figure 1 shows scaled separations d_{nl} computed for the normal solar model discussed in the following section. It is clear that the computed d_{nl} are in reasonable accordance

with expectations, based on the asymptotic relation (5); however, unlike that relation, d_{nl} shows some dependence on l at fixed frequency, and the decrease with increasing frequency is not quite as rapid as the expected ν^{-1} behavior. In panel a) the computed values are compared with separations for observed frequencies in the compilation by Libbrecht, Woodard & Kaufman (1990) for $l = 0 - 3$; here the frequencies for $l = 0 - 3$ (and hence d_{n0} and d_{n1}) derive predominantly from the whole-disk observations of Jiménez *et al.* (1988), whereas the frequencies for higher degree were obtained from spatially resolved measurements at the Big Bear Solar Observatory. The observed values are fairly similar in magnitude to the computed values but show a striking difference in the variation with ν ; for $l = 0$ and 1 the variation is significantly less steep, whereas there are indications that for $l = 2$ and 3 the ν -dependence is steeper than for the computed d_{nl} . This suggests that the discrepancy between theory and observation is caused predominantly by the modes with $l = 1$ and 2, which enter with different sign in d_{n0} , d_{n1} and in d_{n2} , d_{n3} . Indeed, there are hints in the asymptotic fits to frequency differences by Christensen-Dalsgaard, Gough & Thompson (1988) (*cf.* their Figure 3b) that the observed modes of degree 1 and 2 differ systematically from the behavior of modes with $l = 0$ or of higher degree. In contrast, the separations for $l = 0$ and 1 obtained by Elsworth *et al.* (1990) from whole-disk observations and illustrated in Figure 1b are in excellent agreement with the theoretical results. Finally, Figure 1c shows the computed d_{nl} for $l = 4 - 8$; for these degrees the observed values, based on the frequencies of Libbrecht *et al.* (1990), are so close to the theoretical ones that they would have been barely distinguishable, had they been included in the figure.

It is not clear what causes the systematic difference between the observations by Jiménez *et al.* (1988) on the one hand, and the remaining observations and the normal model on the other. However, it seems reasonable to suspect systematic errors in the Jiménez *et al.* data. In particular we note that these data were obtained from a single site, and hence the frequency determination might be affected by daily sidebands, particularly in view of the fact that the frequency separations $\delta\nu_{nl}$ for $l = 0$ and 1 are close to the separation of 11.57 μ Hz between sidebands. This effect is substantially reduced in the data of Elsworth *et al.* (1990), which used observations from several sites. Also, although the Big Bear observations were from a single site only, there is no comparable confusion between daily sidebands and adjacent modes in data at slightly higher degree; this could explain the apparent absence of systematic shifts of the observed d_{nl} for $l > 3$. Since the changes in the solar core are predominantly reflected in the frequency separations for $l = 0$ and 1, in the following we mainly compare the theoretical results with the observations of Elsworth *et al.*

It is convenient to represent observed or computed frequencies in terms of a limited set of parameters associated with the asymptotic description of the modes. A commonly used procedure

is obtained by fitting the asymptotic expression to the frequencies. In terms of the quantity $x = n + l/2 - n_0$, where n_0 is a suitable reference value, equation (1) can be expanded in the form

$$\nu_{nl} \approx \nu_0 - l^2 D_0 + (\Delta\nu_0 + l^2 \delta_0) x + O(x^2) \quad (6)$$

(Scherrer *et al.* 1983; Christensen-Dalsgaard 1988a), which defines the constants ν_0 , D_0 , $\Delta\nu_0$ and δ_0 . By carrying out a polynomial fit in x to ν_{nl} for each l separately, and subsequently making a linear fit in L to the resulting coefficients, one may determine the coefficients in equation (6). In this way one effectively obtains an average over n and l of $\delta\nu_{nl}$; assuming the l -dependence indicated in equation (4), it may be written as an average over n ,

$$\langle \delta\nu_{nl} \rangle_n \approx (4l + 6) D_0, \quad (7)$$

where, asymptotically

$$D_0 \approx -\frac{1}{4\pi^2(n_0 + 1/4 + \alpha)} \int_0^R \frac{dc}{dr} \frac{dr}{r}. \quad (8)$$

In addition, $\Delta\nu_0$ as determined from the fit provides an estimate of the frequency separation $\Delta\nu$ in equation (1).

It is evident from Figure 1 that the average frequency separation, as given by D_0 , is inadequate to characterize the computed or observed d_{nl} . In particular, the dependence on l in equation (7) assumes that d_{nl} is independent of l , which is clearly not true. To obtain a more detailed comparison of observations and theory Elsworth *et al.* (1990) considered linear least-squares fits of the form

$$d_{nl} = \bar{d}_l + s_l(n - n_1), \quad (9)$$

for a suitable reference order n_1 . Here $\bar{d}_l$ provides an average over n of d_{nl} , whereas s_l characterizes the variation of d_{nl} with radial order or, equivalently, frequency. Note that from the fit in equation (6) one obtains averages over l of $\bar{d}_l$ and s_l given by

$$\langle \bar{d}_l \rangle \approx 6D_0 \quad \langle s_l \rangle \approx -6\delta_0. \quad (10)$$

In Section 4 I consider examples of both types of fit for normal and modified models.

3. MODELS WITH REDUCED CORE OPACITY

To simulate the effect of an additional contribution to the energy transport in the solar core, the opacity κ was artificially reduced in a small temperature interval. Specifically κ , as a function of density ρ and temperature T , was determined from

$$\log \kappa = \log \kappa_0 - A f(\log T), \quad (11)$$

where the unmodified opacity κ_0 was obtained from tables set up using the Los Alamos Opacity Library (Huebner *et al.* 1977), as described by Courtaud *et al.* (1990) and Turk-Chièze (1990); the heavy element abundance was $Z = 0.02$. In equation (11) A is a constant determining the magnitude of the opacity modification, and the function f is defined by $f(z) = \exp[-(y/\Delta \log T)^2]$, where

$$y = \begin{cases} z - \log T_1 & \text{for } z < \log T_1 \\ 0 & \text{for } \log T_1 < z \end{cases} \quad (12)$$

This provides an opacity decrease over a well-defined region in $\log T$, with a continuous transition to zero at lower temperature. In the experiments reported here, $\log T_1 = 7.1$ and $\Delta \log T = 0.04$, where $\log$ is the logarithm to base 10.

The models were computed using the Eggleton, Faulkner & Flannery (1973) equation of state and nuclear reaction parameters from Parker (1986) and Bahcall & Ulrich (1988). All models were calibrated to have the solar radius and luminosity at the assumed solar age of 4.75×10^9 years by adjusting the mixing length parameter and the hydrogen abundance. Two types of models were considered: static models of the present Sun, where the hydrogen profile $X(q)$ was obtained by scaling the profile $X_+(q)$ ($q = m/M$ being the mass fraction) from a reference model by a constant factor (*cf.* Christensen-Dalsgaard 1986); and proper evolutionary models, evolved from a chemically homogeneous initial Zero Age Main Sequence model. The models were compared with an evolution model computed with the same physics, but with no opacity modification. This also provided the reference $X_+(q)$ for the static models. Adiabatic oscillation frequencies were computed for the models, after transferring them to a mesh suitable for resolving the structure of p-mode eigenfunctions. The calculations were carried out as described by Christensen-Dalsgaard (1982, 1991) and Christensen-Dalsgaard & Berthomieu (1991); considerable care was taken to ensure that adequate numerical precision was attained in the computation of the models and the oscillation frequencies.

Some properties of the models are summarized in Table I. The neutrino capture rates for the ^{37}Cl and the ^{71}Ga experiments, in Solar Neutrino Units (SNU), were calculated using the cross sections given by Bahcall (1989). Furthermore, Figure 2 shows the sound speed in the core of models S2 and E2, in a form that can be compared directly with Figure 1 of Gilliland & Däppen (1988). To illustrate the effects of the reduced opacity on the structure of the model, Figure 3 shows differences, at fixed fractional radius r/R , between quantities in the modified Models S2 and E2, and in the normal Model E0. It is immediately apparent that the reduced opacity leads to a decrease in the core temperature, and an increase in the core density. The reduced temperature leads to a reduction in the predicted neutrino capture rates, such that the ^{37}Cl values for Models S2 and E2 are close to being consistent with the observed capture rate of 2.2 ± 0.3 SNU (Bahcall 1989).

As can also be seen from Table I the changes in the core require a change in the initial hydrogen abundance X_0 to obtain solar luminosity; this change in X_0 is probably the dominant cause for the changes in the structure of the outer part of the models, including the increased depth of the convection zone. The changes in Models S1 and E1 are very similar in shape to the corresponding changes in Models S2 and E2, respectively, but approximately half as large; this is also indicated by the results in Table I. This is another instance of the approximately linear response of solar models to even quite substantial changes in the opacity (see also Christensen-Dalsgaard 1988; Christensen-Dalsgaard & Thompson 1991).

A significant difference between Models S2 and E2 is in the behavior of the hydrogen abundance. In Model S2 the change in X is determined by the scale factor applied to $X_+(q)$ to calibrate the model (with some additional contribution, in the differences at fixed r/R , from the change in q). In the more realistic Model E2, on the other hand, the central hydrogen abundance is increased, relative to the normal model, by the reduction in the core temperature and the resulting reduction in the nuclear burning rate at the center. This increase in X at the center also leads to a substantially smaller decrease in the sound speed in the center; another consequence is that the central temperature required to obtain the correct surface luminosity is slightly lower in Model E2 than in Model S2, leading to lower neutrino capture rates. It should be noted, however, that the present evolution calculation overestimates this effect on the core hydrogen abundance, since the opacity decrease is supposed to be the same throughout solar evolution. In contrast, accretion of WIMPs would presumably take place continuously over the solar lifetime, and hence their effect on the structure, and on the evolution of the hydrogen abundance, would come in gradually, leading to a smaller change in X_c in the present Sun. This is confirmed by the results presented by Gilliland *et al.* (1986) (who assumed that the number density of WIMPs increased linearly with time). Also, it is interesting that the sound-speed profiles shown in Figure 1 of Gilliland & Däppen (1988) bear closer resemblance to the profiles for the static Model S2 in Figure 2 than to the profile for the evolution Model E2. There is similar agreement between the sound-speed difference for Model S2 in Figure 3a and the differences obtained by Kaplan *et al.* (1991) between the sound speed in models with WIMPs and in a normal model.

4. OSCILLATION FREQUENCIES FOR THE MODIFIED MODELS

To illustrate the effect on the spectrum of oscillation frequencies of the reduced core opacity Figure 4 shows frequency differences between the static modified Model S2 and the normal model; the differences have been scaled by the inertia ratio Q_{nl} to correct for the fact that high-degree modes involve a smaller fraction of the solar mass, so that their frequencies are easier to perturb

(*e.g.* Christensen-Dalsgaard & Berthomieu 1991). These differences can be understood qualitatively in terms of the difference in sound speed between the two models, shown in Figure 3a (see also Christensen-Dalsgaard, Gough & Pérez Hernández 1988). Modes of degree exceeding 50 are trapped entirely within the convection zone, and their frequency differences are dominated by the negative sound-speed difference very near the surface. When the degree decreases from 50 to 20 the modes begin to penetrate beneath the convection zone, starting at high frequency, and the frequency differences become dominated by the positive sound-speed difference beneath the convection zone. Modes of degree 5 – 10 all penetrate well beneath this region, but do not yet reach the core; these modes are all affected in much the same manner by the positive sound-speed difference beneath the convection zone, and hence the frequency difference is essentially independent of l . Finally, modes of degree less than five penetrate increasingly into the core with decreasing degree. Here the negative sound-speed difference reduces the frequency difference, as does the positive difference in density through the effect on the frequencies of the perturbation in the gravitational potential (*e.g.* Vorontsov 1989; Tassoul 1990). Qualitatively similar results are obtained for the evolution Models E1 and E2.

From Figure 4 it follows that for $4 \leq l \leq 10$ the scaled frequency separations d_{nl} are barely affected by the reduction in the opacity. However, there is a substantial effect on the separations for smaller l . Figure 5 shows d_{nl} for modes with $l \leq 3$ in Models S2 and E2, as well as the observations of Elsworth *et al.* (1990) for $l = 0$ and 1, which were also shown in Figure 1b. Furthermore, to characterize the separations, frequencies of modes with $n + 1/2$ in the range 17 – 29, were fitted in the manner discussed in connection with equation (6), including the term quadratic in x , for degrees between 0 and 3; in this way the fitting coefficients $\Delta\nu_0$ and D_0 were determined. In addition, fits on the form given in equation (9) were made to d_{nl} ; as for Elsworth *et al.* the fit included modes in the range 15 – 27 in n , with $n_1 = 21$. The results for the models discussed here are shown in Table II, as well as the corresponding observed values obtained by Elsworth *et al.* It may also be noted that Gelly *et al.* (1988) obtained $D_0 = 1.52 \pm 0.03 \mu\text{Hz}$ and $\delta_0 = 0.045 \pm 0.004 \mu\text{Hz}$ from analysis of extensive sets of oscillation observations. According to equation (10) this corresponds to $\langle \bar{d} \rangle = 9.12 \pm 0.18 \mu\text{Hz}$, $\langle s \rangle = -0.27 \pm 0.024 \mu\text{Hz}$, in agreement with the averages of the values of Elsworth *et al.* (1990).

Comparison of Figure 1 and Figure 5 shows that d_{nl} has been substantially reduced both for Model S2 and for Model E2, particularly for $l = 0$ and 1. Modes of degree 2 and 3 penetrate to lesser extent into the region where the sound speed and the density is modified, and hence the change in d_{nl} and d_{n3} is substantially smaller. This trend is more evident from Table II, which shows that the decrease in the central opacity leads to a substantial decrease in the separation averaged over n and l , as measured by D_0 , and in $\bar{d}_0$ and $\bar{d}_1$, whereas the change in $\bar{d}_2$ is quite modest. It should be

noticed also that the increase in X at the center of Model E2, and the consequent smaller decrease in the central sound speed in that model compared with Model S2, leads to a smaller change in the frequency separations. The opacity decrease gives rise to an increase in magnitude in s_l in all cases, but the change is moderate for $l = 1$. Furthermore, the change in s_0 is considerably larger, relative to the change in $\bar{d}_0$, for Model S2 than for Model E2. It should also be noticed that the difference between d_{n0} and d_{n1} is increased by the reduction in the core opacity (*cf.* Figure 5), indicating an increase in the departure from the asymptotic behavior; a similar effect was found by Cox *et al.* (1990), who analyzed it in terms of the location of the innermost modes in the eigenfunctions of the modes considered. However, a detailed understanding of the changes in the parameters characterizing the frequency separations requires a careful evaluation of the effects of the perturbation in the gravitational potential and the failure of the asymptotic description due to the rapid spatial variation of the equilibrium structure in the core.

It is evident from Figure 5 and Table II that the computed values of d_{nl} for $l = 0$ and 1 for the models with reduced core opacity are inconsistent with the observations. This is true even for the relatively modest modifications in Models S1 and E1. Also, it may be noted that the observed d_{nl} for $l = 0$ and 1 are quite similar, as is the case for the normal model but not for the modified models. The smaller effects on d_{nl} for $l = 2$ and 3 of the modifications to the model cause no significant change in the comparison between theory and observation, given the available data from Libbrecht *et al.* (1990) shown in Figure 1a.

5. DISCUSSION

As was found in previous investigations, enhancement of the efficiency of the energy transport in the solar core can reduce the central temperature sufficiently to account for the low capture rate of neutrinos in the ^{37}Cl experiment. In fact, the predicted capture rates for Models E2 and S2, where the opacity was reduced by a factor of about 2.5 in the core, are nearly consistent with the observed value. However, it is evident from the results in Figure 5 and Table II that these models are incompatible with the observed oscillation frequencies.

Earlier investigations (Paulkner *et al.* 1986; Däppen *et al.* 1986; Gilliland & Däppen 1988) found that the computed frequency separation was in *better* agreement with observations for models with WIMPs than for normal models, in striking contrast to the results obtained here. One reason for this discrepancy is an improvement in the numerical accuracy of the computed frequencies. Fossat (1985) found indications of significant errors in frequency separations computed by me; this was confirmed by Christensen-Dalsgaard (1986), who localized the problem to insufficient precision in the equilibrium model. In contrast, tests where the number of meshpoints in the calculations

was varied show that the errors from this source in the values presented here for $6D_0$ and for the $\bar{d}_I$ and $\bar{s}_I$ are no bigger than about $0.001 \mu\text{Hz}$ (Christensen-Dalsgaard 1991). In addition, the value obtained here for the observed separation $6D_0$, is slightly larger, by about $0.3 \mu\text{Hz}$, than the value quoted by Däppen *et al.* (1986), which was based on the observations by Claverie *et al.* (1981), Grec, Fossat & Pomerantz (1983) and Duvall & Harvey (1983). Given the substantial improvement in the quality and amount of the observations, this difference can most likely be attributed to random or systematic errors in the earlier datasets.

It might be argued that the present calculation uses an oversimplified equation of state and that the opacity tables are somewhat uncertain. However, within the framework of normal solar models d_{nI} and D_0 depend little on the physics: Christensen-Dalsgaard (1991) found that $6D_0$ varied within a range of only $9.07 - 9.20 \mu\text{Hz}$ amongst models computed with a variety of equations of state, opacity tables and energy generation parameters spanning those usually considered in solar evolution calculations. The effect on the $\bar{d}_I$ was similar. The separations were somewhat more sensitive to the assumed solar age: for a calibrated model with the same physics as the normal Model E0, but with an age of 4.5×10^9 years, $6D_0 = 9.365 \mu\text{Hz}$ (see also Christensen-Dalsgaard 1988b; Gough & Novotny 1990). In fact, Guenther (1989) suggested that the lower age might be more appropriate. However, it is evident that the change in D_0 caused by decreasing the age has no substantial effect on the comparison between the observations and the normal and modified models.

The present calculations give only an approximate representation of the effects of WIMPs on the core structure of the Sun. On the other hand, the properties of the models are quite similar to the consistently computed models of Gilliland *et al.* (1986) and Däppen & Gilliland (1988). Furthermore, the evolution models E1 and E2 probably overestimate the effect of the enhanced energy transport on the core hydrogen abundance; the change in X tends to reduce the changes in core sound speed and density, and hence to reduce the effects on the oscillation frequencies. Even for these models the frequencies are clearly inconsistent with the recent observations by Elsworth *et al.* (1990). Thus it appears very unlikely that models for which the computed neutrino capture rate has been brought into agreement with the observations through the inclusion of WIMPs, or a similar localized contribution to energy transport in the solar core, can be consistent with the helioseismic data.

More detailed information about the solar core can be obtained from inverse analysis of the observed frequencies. It was pointed out by Gough (1990, 1991) that the sound-speed difference between the Sun and normal solar models found from such analyses differs significantly from the difference for models with WIMPs, possibly even suggesting a slightly higher neutrino flux than obtained from normal models. It should also be noted that Dziembowski, Pamyatnykh & Sienkiewicz

(1990) obtained a lower bound on the neutrino flux of models consistent with the seismic data which was substantially larger than the value for normal models. However, this result is sensitive to the rather uncertain details of the inversion very near the solar center; indeed, from a similar but independent analysis Kosovichev, Gough and Thompson (unpublished) obtained a bound on the neutrino flux which was consistent with the observed value, although the corresponding structure implied drastic departures from the assumptions of normal solar models.

The WIMP hypothesis faces additional problems. Measurements of the decay of the Z boson have provided strong constraints on the possible candidates for WIMPs (Krauss 1990). Direct experimental bounds on the properties of WIMPs were obtained by Caldwell *et al.* (1990); as discussed by Dearborn *et al.* (1991) and Kaplan *et al.* (1991) these severely restrict the allowed parameters for WIMPs required to explain the observed solar neutrino flux. Furthermore, there is some evidence that cold dark matter, of which the WIMPs would be an example, have difficulty in accounting for the observed galaxy distribution (Saunders *et al.* 1991).

These lines of evidence clearly further strengthen the case for other solutions to the neutrino problem. Gough (1991) pointed out that since the Sun may be unstable to low-order, low-degree gravity modes, the resulting motion, if reaching a sufficient amplitude, might modify the emitted flux of neutrinos. Alternatively, solutions may be sought in terms of the properties of the neutrinos. Indeed, Bahcall & Bethe (1990) pointed out that with a suitable choice of parameters for the neutrinos matter-induced neutrino oscillations (the so-called the MSW effect; see Wolfenstein 1978; Mikheyev & Smirnov 1985) can explain the observed capture rates in both the ^{37}Cl and the Kamiokande neutrino scattering experiment.

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TABLE I

PROPERTIES OF SOLAR MODELS

Model	A	X_0	α	d_b/R	T_c (10^6 K)	X_c	Neutrinos (SNU) ^{37}Cl ^{71}Ga
E0	0.0	0.6994	2.5167	0.2712	15.685	0.3309	8.89 134.3
S1	0.2	0.7102	2.6566	0.2814	15.012	0.3360	4.87 117.2
S2	0.4	0.7193	2.7947	0.2902	14.410	0.3403	2.96 107.6
E1	0.2	0.7080	2.6220	0.2790	14.954	0.3690	4.59 115.2
E2	0.4	0.7150	2.7200	0.2857	14.313	0.3991	2.67 105.0

NOTES.— E0 is a normal evolved model; S1 and S2 are static, and E1 and E2 are evolved, models with reduced core opacity. A is the amplitude of the reduction in $\log \kappa$ (*cf.* equation (11)), the logarithm being to base 10), X_0 is the envelope hydrogen abundance and α the mixing length parameter required to calibrate the models, d_b is the depth of the convection zone, T_c is the central temperature, and X_c is the central hydrogen abundance. The last two columns give the predicted neutrino capture rates (in Solar Neutrino Units) for a ^{37}Cl and a ^{71}Ga experiment.

TABLE II

FREQUENCY SEPARATIONS AMONGST LOW-DEGREE MODES

Data	$l = 0$			$l = 1$		$l = 2$	
$\Delta\nu_0$ (μHz)	$6D_0$ (μHz)	$\bar{d}_0$ (μHz)	s_0 (μHz)	$\bar{d}_1$ (μHz)	s_1 (μHz)	$\bar{d}_2$ (μHz)	s_2 (μHz)
E0	135.43	9.169	9.250	-0.303	9.681	-0.255	10.493 -0.277
S1	135.44	8.023	7.833	-0.360	8.765	-0.275	9.970 -0.306
S2	135.44	7.034	6.635	-0.414	7.956	-0.295	9.550 -0.333
E1	135.47	8.395	8.222	-0.319	9.111	-0.262	10.085 -0.292
E2	135.49	7.697	7.331	-0.344	8.575	-0.271	9.748 -0.309

Obs. 9.00 ± 0.06 -0.29 ± 0.03 9.36 ± 0.06 -0.28 ± 0.04

NOTES.—Parameters for the models listed in Table I. $\Delta\nu_0$ and D_0 result from the fit (6) to the asymptotic expression (1), including modes of degree 0 to 3 and with $n + 1/2l$ between 17 and 29. A corresponding fit to the observed frequencies quoted by Libbrecht *et al.* (1990) gives $\Delta\nu_0 = 135.51 \mu\text{Hz}$, $6D_0 = 9.09 \mu\text{Hz}$. The remaining columns give the results of linear least-squares fits to the scaled frequency separations d_n (*cf.* equation (9)), including modes with n in the range 15–27, and taking $n_l = 21$. The bottom row gives the results of a corresponding fit by Elsworth *et al.* (1990) to observed frequency separations.

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Figure captions:

Figure 1. Scaled separations d_n based on computed frequencies for a normal solar model, and on observed frequencies of low-degree modes, compiled by Libbrecht *et al.* (1990). In panel a) are shown computed separations for modes of degree $l = 0$ (—), $l = 1$ (-----), $l = 2$ (-----), and $l = 3$ (-----), all with heavy lines. The thin lines show observed data, based on the compilation by Libbrecht *et al.* (1990), using the same line-styles; in addition the observed points have been indicated by the following symbols: $l = 0$ (+), $l = 1$ (*), $l = 2$ (□), and $l = 3$ (×). Panel b) repeats the computed values for $l = 0$ and 1, together with the observations by Elsworth *et al.* (1990), using the same line styles and symbols as in panel a); the error bars indicate the standard deviation of the observed values. Finally, panel c) shows computed separations for $l = 4$ (·····), $l = 5$ (—), $l = 6$ (-----), $l = 7$ (-----), and $l = 8$ (-----).

Figure 2. The logarithm to base 10 of the adiabatic sound speed c , in cgs units, in the cores of the normal model E0 (—), the modified static model S2 (-----) and the modified evolution model E2 (-----). The abscissa is fractional radius r/R .

Figure 3. Differences, at fixed fractional radius r/R , between the modified static model S2 and the normal model E0 (panel a), and the modified evolution model E2 and the normal model E0 (panel b), in the sense (modified model) – (normal model). The variables shown are $\delta \ln c$ (—); $\delta \ln \rho$ (-----); ρ being density; $\delta \ln T$ (-----); T being temperature; and δX (-----), where X is the hydrogen abundance by mass.

Figure 4. Scaled frequency differences (see text) between the modified static model S2 and the normal model E0, for selected values of the degree l . Points corresponding to the same value of l have been connected by lines, according to the following convention: $l = 0 - 5$ (—); $l = 10, 20, 30, 40, 50$ (-----); and $l = 70 - 1100$ (-----). Some of the curves have been labelled by the value of l .

Figure 5. Scaled frequency separations d_n for low-degree modes in the modified models, and from the observed frequencies of low-degree modes obtained by Elsworth *et al.* (1990). Panel a) shows results for the static Model S2, whereas panel b) shows results for the evolution Model E2. The line styles and symbols are the same as in Figure 1.

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 2

Fig. 3a

Fig. 3b

Fig. 4

Fig. 5a

Fig. 5b