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THE STRONG-CP PROBLEM AND TARGET-SPACE MODULAR INVARIANCE IN 4-D STRINGS

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ABSTRACT

We consider the problem of strong CP violation in the context of four-dimensional strings. We remark that the action of target-space modular invariance on the massless fermions (a chiral $U(1)$ Kähler rotation) plays the role of a Peccei-Quinn symmetry in a large class of four-dimensional strings. The would-be axions are provided by massless scalar fields with vanishing potential and couplings to massive coloured particles. An invisible axion mechanism of the type discussed by Kim and Shifman, Vainshtein and Zakharov is at work, guaranteeing the smallness of the QCD θ -parameter. In addition, by studying a class of supersymmetry (susy)-breaking effective actions invariant under target-space modular invariance, we show that the dynamical minima are such that the infinite tower of massive string excitations does not give any contribution to the strong θ -parameter. Moreover these vacua lead to real susy-breaking soft terms. This would provide an explanation for the smallness of the electric dipole moment of the neutron which tends to be two to three orders of magnitude too large in generic supersymmetric models.

The origin of CP-violation in particle interactions is still not properly understood. We know that due to the existence of instantons QCD has a periodic free parameter θ . Due to the Adler-Bell-Jackiw anomaly, it turns out that the phase of the determinant of the mass matrix of coloured fermions also behaves as a θ term so that θ always appears in the Lagrangian through the (redefinition-invariant) combination

$$\bar{\theta} = \theta + T(R) \arg \det M \quad (1)$$

where M is the mass matrix of coloured fermions and $T(R)$ is the quadratic Casimir. The $\bar{\theta} F \tilde{F}$ interactions induce CP-violating transitions like contributions to the electric dipole moment of the neutron (EDMn). In order to be in agreement with experiment one needs $\bar{\theta} \leq 10^{-9}$. This constitutes the strong-CP problem.

In supersymmetric models the problem is in some sense worse, since there are new contributions to the EDMn. There are new complex phases associated to the soft supersymmetry-breaking parameters like the gaugino masses M and trilinear scalar terms A . One-loop diagrams induce contributions to the EDMn about $10^2 - 10^3$ times bigger than the experimental limits. Thus even if one finds a mechanism to relax the value of $\bar{\theta}$ down to zero (e.g. an invisible axion model) we are still left with too large contributions to the EDMn.

Up to now the implications of string theory for the problem of strong-CP have not been analyzed in much detail (see however [1]). In principle the situation looks even more difficult to control since now the determinant in eq.(1) refers to the infinite tower of coloured fermions in the string spectrum.

String interactions by themselves are CP-conserving. It is the structure of the vacuum which may or may not violate CP. From this point of view one may say that *in string theory the violation of CP is necessarily spontaneous*. Complex vacuum expectation values (vev's) of massless scalars in the theory (moduli) may lead to CP violation. As remarked in [2] for generic (complex) values of the complex structure moduli fields of a given four-dimensional heterotic string compactification CP is broken. The strong-CP problem persists in string models. Although the theory contains fields with axion-like couplings to $F \tilde{F}$ (the moduli and the axionic partner of the dilaton [1]) they cannot be used as invisible axions since their couplings are outside the cosmological limits, and/or they obtain large

masses due to non-perturbative string effects together with the contribution of world-sheet instantons [1]. Alternatively, one might think about the existence of some *global* $U(1)$ Peccei-Quinn symmetry in four-dimensional string models. This is however not possible [3] in strings; any $U(1)$ symmetry is necessarily local, and hence it is not obvious how an invisible axion type of mechanism could be implemented in strings.

In this letter we argue that target-space modular symmetries may provide the required axionic symmetries. In particular the chiral $U(1)$ Kähler rotations associated to modular invariance can play the role of a Peccei-Quinn symmetry in large classes of semirealistic four-dimensional strings. We show that any scalar field with flat potential whose vev gives masses to coloured particles gets axion-like couplings. The existence of these types of fields is generic in string models and leads to an invisible-axion solution to the strong CP problem. As we remarked above, a vanishing θ -term is not enough to get a small electric dipole moment of the neutron in a supersymmetric theory, since susy-breaking soft terms are generically complex and give extra one-loop contributions. We argue that an effective Lagrangian approach to the non-perturbative string dynamics may yield interesting information concerning the CP structure of string vacua. In particular we find that in a class of supersymmetry-breaking effective actions consistent with target-space modular invariance, the action is minimized for CP-conserving values of the overall modulus. This leads to real susy-breaking soft terms, in agreement with bounds obtained from the EDMn.

It has been recently pointed out [4-8] that target space duality symmetries [9-12] could play an important role in constraining the possible form of the non-perturbative dynamics leading to e.g., supersymmetry breaking. In particular, a large class of four-dimensional heterotic strings [13-16] have a symmetry called target-space modular invariance [17-21]. In all supersymmetric compactifications there is a complex field T whose real part is related to the overall compactification radius R as $T = R^2 + iB$, B being an axion-like field. Under the above symmetry one has

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad a, b, c, d \in \mathbf{Z}, \quad ad - bc = 1. \quad (2)$$

The target space modular transformations contain the well-known duality trans-

formation $R \rightarrow 1/R$ as well as discrete shifts for the axionic background B . These actions generate the infinite discrete group $SL(2, \mathbf{Z})$. The different massless fields have different transformation properties (“weights”) under modular transformations and the overall effective action should be invariant. This symmetry has been shown to be preserved at any finite order of perturbation theory [22] in four-dimensional strings of the orbifold type. Since asymmetric orbifolds [23] may be described as symmetric orbifolds in the presence of background fields (“discrete torsion” [24]) one expects to have similar types of symmetries in asymmetric orbifolds and related constructions [14],[15],[16]. In general, for non-flat compactifications ((2,2) Calabi-Yau compactifications) there also exist invariances under generalized target space duality transformations acting as discrete reparametrizations on the moduli fields. For example, in the model studied in [25], [26] with a single Kähler class (complex structure) modulus field, the duality group is an infinite discrete subgroup of $SL(2, \mathbf{R})$ which includes the discrete shift invariance ($T \rightarrow T + i$) as well as an $\mathbf{Z}_5$ symmetry which relates large to small radii of the compact six-space. Recently it was discussed [27] how one can construct automorphic functions (the generalization of the Dedekind function) for general Calabi-Yau target space duality groups. However here we will restrict ourselves to the case of the target space modular invariance $SL(2, \mathbf{Z})$ relevant for the orbifold type of four-dimensional strings. We expect however that the main physical conclusions will also apply to other theories involving other types of duality symmetries.

Although duality is only shown to be a symmetry order by order in perturbation theory, one expects that non-perturbative effects respect these discrete symmetries (which may, however, be broken spontaneously). Adopting this viewpoint, one must require that the effective Lagrangian of the massless fields including both perturbative and non-perturbative (e.g. gaugino condensation) effects should be modular invariant. This requires [4],[5], [8] that any superpotential in the theory must be a modular form of weight -3

$$W \rightarrow e^{i\alpha(a,b,c,d)} (icT + d)^{-3} W, \quad (3)$$

where $\alpha(a, b, c, d)$ is a field-independent phase. Properties like this are rather non-trivial and substantially constrain the possible non-perturbative dynamics,

as we will discuss below. In fact, modular transformations induce a particular Kähler transformation

$$\begin{aligned} K(\phi, \phi^*) &\longrightarrow K(\phi, \phi^*) + F(\phi) + F^*(\phi^*), \\ W(\phi) &\longrightarrow e^{-F(\phi)} W(\phi) \end{aligned} \quad (4)$$

in which the “gauge” function F is modulus dependent and given by

$$F = \log(icT + d)^3. \quad (5)$$

In order that the $N = 1$ supergravity is Kähler (and modular) invariant one has to perform certain field (modulus) dependent chiral rotations on gauginos λ^a and matter fermions χ_i (with canonical kinetic energy) [28]:

$$\begin{aligned} \lambda^a &\longrightarrow e^{-\frac{1}{4}(F-F^*)} \lambda^a = \left(\frac{(icT + d)}{(ict + d)^*} \right)^{-\frac{3}{4}} \lambda^a, \\ \chi_i &\longrightarrow e^{\frac{1}{4}(1+\frac{2}{3}n_i)(F-F^*)} \chi_i = \left(\frac{(icT + d)}{(ict + d)^*} \right)^{\frac{3}{4}(1+\frac{2}{3}n_i)} \chi_i, \end{aligned} \quad (6)$$

where n_i is the modular weight of the matter fields.

Although the classical action is invariant under these transformations, the above chiral rotations are obviously anomalous. Thus there seems to be a target-space modular invariance anomaly.

To be specific, consider now an orbifold-like four-dimensional string containing in its massless spectrum singlet chiral multiplets N_i with the following properties:

- i) Their scalar potentials have flat directions in $\langle N_i \rangle$.
- ii) They have trilinear couplings W_Q to coloured chiral superfields Q_j and $\bar{Q}_k$

$$W_Q = \lambda_{ijk}(T) N_i Q_j \bar{Q}_k. \quad (7)$$

The fields N_i , Q_j and $\bar{Q}_k$ have modular weights n_i , n_j , n_k , respectively. (In the untwisted orbifold sector the matter fields have generically modular weight $n = -1$.) Then, in order for W_Q to have modular weight -3 , $\lambda_{ijk}(T)$ must be a

modular function of weight $-3-n_i-n_j-n_k$. (Thus for the untwisted matter fields, λ_{ijk} has modular weight zero and generically is just a T -independent constant.) In this situation from eqs.(1) one gets the following contribution to the θ -parameter

$$\theta_{i \text{ quark}} = 2T(R) \log \det \left(\frac{W_{jk}}{W_{jk}^*} \right) = 2T(R) \log \det \left(\frac{\lambda_{ijk}(T) N_i}{\lambda_{ijk}^*(T^*) N_i^*} \right). \quad (8)$$

where $T(R)$ is the quadratic Casimir of the coloured fermions. Notice that under a modular transformation one has

$$\theta_{i \text{ quark}} \longrightarrow \theta_{i \text{ quark}} + 2T(R) (-3 - n_j - n_k) \log((icT + d)/(icT + d)^*). \quad (9)$$

Thus we see that under a modular transformation the phases of the N_i singlets are shifted, indicating their potential axionic character. The associated Peccei-Quinn symmetry is nothing but the Kähler $U(1)$ rotation given by eq.(6).

Let us consider now the contribution of the infinite number of massive fields with T -dependent chiral masses $M(T)$ (Kaluza-Klein and winding modes) to the θ -parameter. As discussed in [29],[5], [30],[6],[8], [31], the heavy string modes provide for a local counterterm $\theta_{\text{heavies}} (F\tilde{F})$ with

$$\theta_{\text{heavies}} = b' \log \left(\frac{\eta(T)}{\eta(T^*)} \right)^2. \quad (10)$$

A nice understanding of this formula arises when computing the target space free energy [27] for a closed string compactified on a two-dimensional torus. Here the T -dependent mass formula for the momentum and winding modes has the following form

$$M_{m,n}^2(T, T^*) = \frac{|M_{m,n}(T)|^2}{(T + T^*)} = \frac{|m + inT|^2}{(T + T^*)} \quad (11)$$

where m is a momentum and n a winding number. In an effective field theory which incorporates all these massive states the non-holomorphic factor $(T + T^*)$ is due to the non-canonical kinetic energy of the massive states where the

chiral masses $M_{m,n}(T) = m + inT$ are just the second derivatives of the massive superpotential (see [27] for details). Then the target space free energy looks like

$$\begin{aligned} F &= \log \det M_{m,n}^2(T, T^*) = \sum_{m,n} \log \frac{|m + inT|^2}{(T + T^*)} \\ &= \log[|\eta(T)|^4 (T + T^*)]. \end{aligned} \quad (12)$$

Thus we recognize that θ_{heavies} in eq.(10) is indeed nothing other than the “standard” θ -parameter for the infinite number of massive momentum and winding states:

$$\theta_{\text{heavies}} = b' \arg \det M_{m,n}(T). \quad (13)$$

It is important to realize that the contribution of the the infinitely many massive fields exactly cancels the potential modular anomaly of the light quarks discussed before. Specifically, one can check that the variation in equation (9) is precisely cancelled by a corresponding piece coming from the variation of eq.(10). Indeed, for the class of string vacua considered here the b' coefficient in (10) has been recently computed [31], [32]. Thus e.g., for orbifolds with gauge group $E_8 \times E_6 \times U(1)^2$ one has

$$b' = -3C_G + \sum_R T(R)(3 + 2n_R) \quad (14)$$

where the sum runs over the light chiral fields in the string spectrum and n_R is again the modular weight of the matter fields. There is one term for each light coloured particle, and the reader can easily check that the variation (9) (plus the contribution of the gauginos) is precisely cancelled by a corresponding term from (10). This mechanism is the Kähler analogue [32], [33] of the Green-Schwarz mechanism for cancelling anomalies. It is not surprising that the heavy string modes restore modular invariance at the quantum level since after all target-space duality is a symmetry which relates light to heavy string modes.

Notice that in the field theory limit ($\text{Re}T \rightarrow \infty$ [34]) eq.(10) gives rise to an axionic coupling of $\text{Im}T$ (sometimes called “model-dependent” axion [1]). The present analysis allows us to understand why such an axionic coupling was present

to start with: the contribution of massive modes to the mixed modular-QCD anomaly is non-vanishing and leads to a coupling of $\text{Im}T$ to $F\tilde{F}$. This field cannot play the physical role of an axion since world-sheet instantons (and non-perturbative string effects) give it a large mass (see below). What we would like to stress is that although $\text{Im}T$ cannot be used as an invisible axion, there are generically in the spectrum other fields (the singlets N_i) which do also couple to the mixed modular-QCD anomaly and have also axion-like couplings.

As we mentioned above, equation (8) tells us that the phase of the flat potential scalars N_i giving masses to coloured matter fields have axion-like couplings to the $SU(3)$ gauge bosons. The important point here is that these kinds of colour singlet scalars are ubiquitous in four-dimensional strings. In the orbifold examples of ref. [35] some of the singlet fields with flat directions (S_1, S_2, S_3) possess trilinear couplings to colour triplet pairs ($D_j, \bar{D}_j$). A linear combination of all these flat potential scalars will thus behave as a standard (invisible) axion. This axion will be of the “heavy quark” type discussed by Kim [36] and Shifman, Vainshtein and Zakharov [37] about ten years ago. Similar flat direction singlets coupling to heavy colour triplets exist in other string models [38]. One of the interesting aspects of the present approach is that the action on fermions of *target space modular invariance plays the role of Peccei – Quinn symmetry* here. This action has the character of a local R-symmetry (the $U(1)$ rotation associated to Kähler invariance). In this sense the mechanism is quite model-independent and is built into the models from the beginning. Let us remark that it is this same symmetry which is the origin of the axion-like coupling of $\text{Im}T$ (for large $\text{Re}T$). The interesting point is that the same symmetry induces *other* axion-like couplings not subject to the avatars of $\text{Im}T$.

Two comments concerning this mechanism are in order. First, it is important to check that the linear combination of singlets with axion couplings to QCD does not have similar couplings to other gauge factors condensing at a high scale. If this were the case the would-be axion would be given a large mass by the non-perturbative effects of the high scale condensation. However this possibility is very unlikely since it would require that precisely the same linear combination of singlets couples both to $SU(3)$ and the other group. This means that the structure of matter fields in both groups should be correlated in an unexpected

(certainly non-generic) manner. Notice this is an important difference with other attempts to use string particles as axions [1] which tend to couple universally to all the gauge groups.

The second point concerns the effects of supersymmetry-breaking. Eventually susy-breaking will take place and the value of the modulus T will be dynamically determined. The counterterm in eq.(10) will be “frozen”, and indeed this is what makes $\text{Im}T$ useless as a potential axion. On the other hand the axionic couplings of N_i phases eq.(8) will remain. The question is whether the dynamics of supersymmetry breaking (e.g. gaugino condensation) can generate a potential for the would-be axion (ruining the invisible axion mechanism) or not. One can argue that indeed that is not the case and the would-be axion will in general survive the susy-breaking effects. We know that upon supersymmetry breaking two types of soft terms will appear in the scalar potential: a) mass terms of the form $m^2(N_i N_i^*)$ and b) soft trilinear scalar terms of the form $\lambda_{ijk} A N_i \tilde{Q}_j \tilde{Q}_k + h.c..$ The phases of the N_i fields will not get any mass from any of these terms, since coloured scalars by assumption do not get any vev. Thus the invisible axion will be light enough so that it is the QCD dynamics which will govern it and drive it to a zero vev.

Since an invisible axion mechanism is at work, the θ -parameter is relaxed to zero independently of its original “bare” value. In the absence of such a mechanism one could have thought of the following alternative. One expects four main contributions to the θ -parameter in a string model

$$\theta_{\text{string}} = \text{Im}S + \theta_{\text{quarks}} + \theta_{\text{gluino}} + \theta_{\text{heavies}}. \quad (15)$$

where S is the four-dimensional complex dilaton scalar field. In the one-modulus simplified situation we are discussing, θ_{heavies} corresponds to the infinite tower of coloured string excitations and it is given by eq.(10).^{*} Before supersymmetry

^{*} Note that it is possible to redefine the tree level S -field like $S \rightarrow S' = S + b' \log \eta(T)^2$ (see [32]). If the coefficients b' for all gauge group factors are identical, i.e. there is only a universal T -dependence of θ_{heavies} as it is the case for orbifolds with no $N = 2$ subsectors [29] like the $\mathbf{Z}_3$, $\mathbf{Z}_7$ orbifolds, eq.(15) has no T -dependence at all for every gauge group. (This is consistent with recent string computations of [39].) However the redefined field S' (corresponding to the one-loop transformed of the antisymmetric tensor field plus dilaton (linear multiplet) [32]) transforms non-trivially under target space duality transformations and cancels in this way the target space modular anomaly.

breaking this is in fact the only contribution, since those are the only massive fields in the theory. Once supersymmetry is broken a gluino mass (generically complex) will appear and give rise to θ_{gluino} . Finally, $SU(2) \times U(1)$ breaking will be induced and the standard contribution to θ coming from a complex quark mass matrix will appear. It is conceivable (as we will argue below) that upon supersymmetry breaking the physical minimum is obtained for $\text{Im}T = 0$. This leads to $\theta_{\text{heavies}} = 0$. To the extent that the susy-breaking dynamics is governed by the modulus T (i.e. F_T), one would expect also a real gluino mass and $\theta_{\text{gluino}} = 0$. We would just be left with the ordinary contribution from quarks. Since the Kobayashi-Maskawa phases should be present, one expects in principle a non-vanishing argument of the determinant of quark mass matrices. Thus the strong-CP problem will still persist. However, at least we would have achieved a vanishing contribution to θ from the heavy sectors of the theory. Furthermore, there are classes of field-theory models [40] in which $\arg \det(m_q) = 0$ at the tree level in a natural way and if a string model with the required properties could be built we would have achieved a complete cancellation of the three contributions to the θ parameter.

The possibility mentioned above of a susy-breaking mechanism leading to $\text{Im}T = 0$ is anyway interesting from another point of view. The soft susy-breaking terms generated might be real for $\text{Im}T = 0$. In this case the second CP-violation problem in supersymmetric models (too large an EDMn from one-loop graphs involving soft terms) would also be solved. Let us recall again that this problem (although much milder) is present even if an axion relaxes θ to zero. To find out if the minima correspond to real or complex moduli T , one needs to make some assumptions about the non-perturbative effects. We will parametrize those effects through the inclusion of a non-perturbative superpotential $W = W(S, T)$ which is a function of the modulus T and the dilaton S . Modular invariance requires (cf. eq.(3)) W to be a modular form of weight -3. If on physical grounds we require the absence of zeroes and singularities at finite T one is left with generic non-perturbative superpotentials of the form

$$W = \frac{\Omega(S)}{\eta(T)^6} \quad (16)$$

where $\Omega(S)$ parametrizes the dilaton dependence. Since the dilaton is inert under

modular invariance transformations its dynamics are not constrained. We will assume for the moment that the expectation value of S is real and will discuss this point later on. It is very remarkable that the usual gaugino condensation approach to supersymmetry breaking leads (when combined with modular invariance arguments) precisely to this type of superpotentials [5]. Independently of the form of Ω the general expression for the scalar potential is

$$V = \frac{1}{S_R T_R^3 |\eta(T)|^{12}} \{ |S_R \Omega_S - \Omega|^2 + 3 |\Omega|^2 \left(\frac{T_R^2}{4\pi^2} |\hat{G}_2|^2 - 1 \right) \} \quad (17)$$

where $S_R = 2\text{Re}S$, $T_R = 2\text{Re}T$ and $\Omega_S = \partial\Omega/\partial S$. Here the function $\hat{G}_2$ is the weight-two Eisenstein function $\hat{G}_2 = G_2 - 2\pi/T_R$ (see ref.[5] for more details). The potential has extrema for $S_R \Omega_S - \Omega = 0$ and, as a consequence of modular invariance, is a periodic function of $\text{Im}T$. The shape of the potential is such that the minima occur at $\text{Im}T = 0$ modulo integers [5]. Since one can always restrict oneself to a fundamental region the minima will always occur at real T . One can also check that supersymmetry is spontaneously broken at the minima.

In some sense $\text{Im}T$ behaves very much as a Peccei-Quinn axion. Before world-sheet instanton effects are taken into account there is a continuous PQ symmetry under which $\text{Im}T \rightarrow \text{Im}T + c$, $c \in \mathbf{R}$. World-sheet instanton effects break the continuous PQ symmetry to the discrete subgroup $\text{Im}T \rightarrow \text{Im}T + n$, $n \in \mathbf{Z}$, and generate, together with the non-perturbative string effects, a periodic axion potential V which is minimized for integer $\text{Im}T$. At the same time the axion gets a mass. Of course this is somewhat different compared to the standard axion for which the QCD instanton effects generate the axion potential. Thus in the present case $\text{Im}T$ gains a huge mass of the order of the susy-breaking scale, and hence it cannot be used as a standard (invisible) Peccei-Quinn axion. The above result just guarantees that there is no spontaneous CP-violation originating from the modulus T . In general, a generic string compactification has several moduli fields analogous to T which parametrize different deformations of the compactification variety. The general duality symmetries in these more general cases are very complex and have not up to now been studied in any detail. They will however contain as a subgroup the modular invariance of the overall modulus T (at least in orbifold-like constructions). Moreover the imaginary part of the additional

moduli are also known to have axion-like properties and hence one expects the above conclusions to remain valid in the multi-moduli case.

One may include charged matter fields in the above analysis. The Kähler potential of the latter is known (at least for small fields) for the orbifold strings [41],[21]. Once supersymmetry is broken along the lines described above, soft supersymmetry breaking terms are generated. These include masses for charged scalar field and other soft terms proportional to the superpotential (like the “A” term). For example, consider a $\mathbf{Z}_3$ type of orbifold. The Kähler potential including matter fields ϕ looks like (at lowest order in ϕ)

$$K = -\log(S + S^*) - 3\log(T + T^*) + \alpha_n(T + T^*)^{-n}\phi\phi^* + \dots \quad (18)$$

where $\alpha_n \geq 0$ and $-n = 1, 2, 3$ respectively for untwisted, twisted, and twisted moduli fields. K has the correct modular properties since those fields have modular weights $-1, -2$ and -3 . Including a trilinear matter coupling one has

$$W(S, T, \phi) = \frac{\Omega(S)}{\eta(T)^6} + w_3(T)\phi^3 \quad (19)$$

where $w_3(T)$ is a moduli-dependent coupling constant which is a modular form of weight $3(n - 1)$. Computing the scalar potential one finds soft trilinear terms of the form $(m_{3/2}A\phi^{*3} + h.c.)$ with (see ref. [8])

$$A = \frac{1}{(S_R T_R^3)^{1/2}} \frac{\hat{G}_2}{2\pi} (T_R^2 w_3 T + T_R (3n - 3) w_3). \quad (20)$$

This vanishes for untwisted matter ($w_3 T = 0$, $n = 1$) but it does not for twisted fields. Still, at the physical minimum this quantity is always real. In fact, as shown in ref.[8], the soft susy-breaking $(\text{mass})^2$ terms for the matter scalars are in general negative, so that the real physical minimum will also have vevs for those scalars and also reduced gauge symmetry. One may worry that in this physical minimum the value of $\text{Im}T$ could be shifted towards non-vanishing values. In fact the approximate Kähler potential in eq.(18) does not depend on $\text{Im}T$ and hence the minimization conditions for that variable will not be essentially affected by the matter fields as long as the driving source for supersymmetry-breaking is gaugino condensation (and not an induced large vev for W_{matter}).

The same will happen for the susy-breaking gaugino masses; they will have no complex phases. This is good news since generic sources of supersymmetry breaking lead to complex phases, leading in turn to contributions to the EDMn which are two to three orders of magnitude too large (unless a very heavy spectrum of susy masses is assumed). Thus the same symmetry (target-space modular invariance) would be the origin of both the vanishing θ_{heavies} value at the Planck mass and the absence of complex phases in the soft susy-breaking terms.

Let us now briefly comment about the phase of the dilaton complex field S . As we have discussed, the generic presence of invisible axions in four-dimensional strings relaxes the θ parameter to zero, regardless of its initial value. Thus an imaginary value for S is anyway harmless as long as the θ parameter is concerned. On the other hand, a complex value for S (and thus for $\Omega(S)$) may be dangerous for the phases of the susy-breaking soft terms. This is of course a less severe problem than the one associated to the θ -parameter but still one would like to understand the small size of the phases of the soft terms. As we mentioned above, since S is inert under modular transformations we are unable to obtain any constraint on that field from that symmetry. Simple models lead to real vevs for S . Thus, for example, in the gaugino condensation approach followed in refs. [42],[5] one has $\Omega(S) = c + e^{\alpha S}$ and the potential is minimized for real S if c is real (which is a natural assumption if the breaking of CP in strings is supposed to be spontaneous and not explicit). In the latter approach the possible origin of the constant c is uncertain so that a more natural source of a hierarchy would be desirable. It would be interesting to study under what conditions alternative models may yield real $\langle S \rangle$. An interesting possibility is the existence of an additional modular invariance type of symmetry $SL(2, \mathbf{Z})$ with the field S as its modulus [43]. In this case one could use exactly the same steps as with T to show the reality of S at the minimum (S would look very much like a modulus of an additional compactified complex torus).

From the above discussion the following picture of the CP violation structure of four-dimensional string models emerges. Target-space modular invariance (or its generalizations) leads to real values for the moduli of the string. This is due to the periodic dependence on $\text{Im}T$ inherent to the modular group. It follows that the vacuum contribution of the massive string excitations to the QCD θ

parameter vanishes. At this level all the Yukawa couplings are real and the θ parameter vanishes. The susy-breaking soft terms are also real, in agreement with the experimental bounds on the EDMn. The standard CP-violating couplings appear through effective Yukawa couplings involving complex scalars N_i . These are charged fields (singlets with respect to the standard model interactions) with flat potentials which are generic and abundant in this type of theories. In most models they have non-renormalizable Yukawa couplings to ordinary quark, lepton and Higgs superfields. If these singlets get large (in general complex) vevs, effective trilinear superpotentials inducing standard renormalizable Yukawa couplings are obtained. Those effective Yukawa couplings will generically be complex and the standard Kobayashi-Maskawa phases will appear. A subset of these singlet fields N_i will typically give masses to extra coloured particles in the theory. In this situation we have shown that a linear combination of these singlets will necessarily have axion-like couplings to QCD. The result is an invisible axion mechanism of the type discussed by Kim [36] and by Shifman, Vainshtein and Zakharov about ten years ago [37]. It is quite attractive that the action of target-space modular invariance on fermions (a chiral Kähler rotation) provides us with the required built-in Peccei-Quinn type of symmetry.

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