Cosmological Lower Bound on the EDM of the Electron.

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Abstract

Multi-Higgs extensions of the standard model provide a natural mechanism for electroweak baryogenesis as well as a substantially big contribution to the dipole electric moments of the elementary particles. In the model with only two Higgs doublets both phenomena originate from one and the same source, namely CP-violation in the neutral scalar sector. We present an estimate of the cosmological lower bound on the dipole moment of the electron, which appears to be numerically close to the experimental upper bound.

Introduction.

It was realized recently that CP violation in the Higgs sector of extended versions of the electroweak theory can give a large contribution to the electric dipole moments of the neutron [1, 2] and the electron [3]. A good example is a minimally extended version of the SM with two complex Higgs doublet fields. The Higgs potential here allows CP-violating terms, while softly broken discrete symmetry forces one scalar (ϕ_2) to couple only to down-type quarks and leptons and the other (ϕ_1) to couple to the up-type quarks. In this model, Barr and Zee [3] have found that the maximal Higgs boson contribution to the EDM of the electron ($d_e \simeq 10^{-26}$ e · cm) is of the same order of magnitude as its present experimental bound ($d_e = (-2.7 \pm 8.3) \times 10^{-27}$ e · cm [4]). In contrast, the KM model of CP violation tends to give a very tiny value for the electric dipole moment of the electron ($< 10^{-36}$ e · cm [5]). However, the magnitude of CP violation is an arbitrary parameter of the theory so that it is impossible to predict d_e with the use of any available particle physics information.

At the same time it has become clear that the baryonic asymmetry of the universe (BAU) can be a natural consequence of electroweak theory [6], in particular of its multi-Higgs extensions [7, 8]. The key point of the scenario is the high rate of the anomalous fermion number non-conservation at non-zero temperatures [9] and CP violation in the Higgs sector. If we presume that the scalar content of the theory consists of two Higgs doublets, then the Higgs part of CP violation is described by only one parameter. Therefore, electroweak BAU and neutral Higgs exchange contribution to the d_e vanish simultaneously as we turn off the CP-violating part of the Higgs potential. In other words, we can relate the baryonic asymmetry of the Universe to the dipole electric moments of the electron and the neutron. Hopefully, the magnitudes and signs of d_e and d_n can be calculated.¹

The aim of the present paper is an estimate of the scalar CP violation parameter from cosmology and present arguments in favour of the existence of a lower bound on the Higgs-mediated electron EDM coming from the observed value of the BAU. Our results can also be applied to the EDM of the neutron. However, we shall not discuss it here, due to large uncertainties contained in the calculation of d_n because of the strong dynamics [2]. The paper is organized as follows. First we will briefly describe CP violation in the model and elaborate in more detail the mechanism of the dynamical baryon number generation during the electroweak phase transition. Then we will show how the magnitude of the CP violation λ_{CP} can be related both to the BAU produced in the EW phase transition and the EDM of the electron d_e .

¹Electroweak baryogenesis is possible also in the standard theory with one Higgs doublet, provided there is spontaneous breaking of parity in the hot temperature phase [6, 10]. In this case there is a connection between the sign of the BAU and CP violation in the K^0 decays.

CP violation in the model and electroweak baryogenesis.

We choose the scalar potential in the form:

$$\begin{aligned} V(\varphi_1, \varphi_2) = & \lambda_1((\varphi_1^\dagger \varphi_1) - v_1^2)^2 + \lambda_2((\varphi_2^\dagger \varphi_2) - v_2^2)^2 + \\ & \lambda_3(\varphi_1^\dagger \varphi_1 + \varphi_2^\dagger \varphi_2 - v_1^2 - v_2^2)^2 + \lambda_4((\varphi_2^\dagger \varphi_2)(\varphi_1^\dagger \varphi_1) - (\varphi_2^\dagger \varphi_1)(\varphi_1^\dagger \varphi_2)) \\ & + \lambda_5(\text{Re}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \cos \xi_0)^2 + \lambda_6(\text{Im}(\varphi_1^\dagger \varphi_2) - v_1 v_2 \sin \xi_0)^2 \end{aligned} \quad (1)$$

from which it is apparent that both fields φ_1 and φ_2 acquire non-zero expectation values in the true vacuum: $\langle \varphi_1 \rangle > 0$; $\langle \varphi_2 \rangle > 0$; $v_2 \exp i\xi_0$ (see [11] for discussion). With some special choices of the parameters the suitably defined CP operation leaves the Lagrangian invariant. These are (a) λ_5 or $\lambda_6 = 0$, (b) $\xi_0 = \pi k/2$ or $\lambda_5 = \lambda_6$. In the first case, however, CP is spontaneously broken by the vacuum expectation values of the Higgs fields carrying a non-trivial relative phase. In case (b) CP will not be violated either explicitly or spontaneously.

The fermionic asymmetry generated during the first-order electroweak phase transition is proportional to the expectation value of the gauge topological number

$$\frac{\alpha_W}{8\pi} \int_0^\infty d^4 x F F^d \quad (2)$$

because of the anomaly equation. Due to the fact that this quantity is P- and CP-odd the effective action for the bosonic fields has to break both discrete symmetries. The CP breaking is provided by the Higgs potential while the parity odd piece appears when we integrate out the fermionic degrees of freedom. It was found to be [8]

$$\Delta S_F = -i \frac{7}{4} \zeta(3) \left(\frac{m_t}{\pi T} \right)^2 \frac{g_W^2}{16\pi^2 v_1^2} (\mathcal{D}_1 \varphi_1^\dagger \sigma^\mu \mathcal{D}_0 \varphi_1 + \mathcal{D}_0 \varphi_1^\dagger \sigma^\mu \mathcal{D}_1 \varphi_1) \epsilon^{ijk} F_{jk}^a. \quad (3)$$

Here m_t is the mass of the top quark, ζ is the Riemannian ζ -function and σ^μ are the Pauli matrices.

For simplicity we shall consider a spinodial decomposition phase transition. In this case the scalar field uniformly rolls down towards to the true vacuum starting from zero in the restored phase. We are expecting the same results for the phase transition going through the bubbles nucleation (as we think happened in the real world [8]). For the spatially uniform scalar fields term (3) acts as a chemical potential for the Chern-Simons number [8],

$$S_F = \mu N_{CS}, \quad \mu = -i \frac{7}{4} \zeta(3) \left(\frac{m_t}{\pi T} \right)^2 \frac{2}{v_1^2} \mathcal{O}(\phi_1), \quad \mathcal{O}(\phi_1) = (\phi_1^\dagger \mathcal{D}_0 \phi_1 - (\mathcal{D}_0 \phi_1)^\dagger \phi_1). \quad (4)$$

The number density of fermions created during the transition will be just given by

$$n_F = n_f \int_0^\infty dt \Gamma_{F\bar{F}}(t) \mu(t), \quad (5)$$

where Γ_{sph} is the time-dependent rate of the sphaleron transitions², and $n_f = 3$ is the number of fermionic generations. Now we shall estimate the maximal number of baryons which can be created for a fixed magnitude of CP violation $\lambda_{CP} = (\lambda_3 - \lambda_6) \sin 2\xi_0$, which naturally appears in n_B as well as in the dipole electric moment. The basic idea is to convert time integration to the integration via the W mass appearing in the rate. Let us consider first the dependence of the rate on the vacuum expectation value of the Higgs field. In the symmetry broken phase the rate is given by the expression

$$\Gamma_1 \simeq T^{(\frac{\alpha_W}{4\pi})^4} N_{tr} N_{rad} \left(\frac{2M_W(T)}{\alpha_W T} \right)^7 \exp\left(-\frac{E_{sph}(T)}{T}\right), \quad (6)$$

where factors $N_{tr} \simeq 26$, $N_{rad} \simeq 5.3 \cdot 10^3$ are the zero mode normalization [15], and $E_{sph}(T)$ is the sphaleron mass at non-zero T . In the symmetry restored phase the rate is equal to [16, 15]

$$\Gamma_2 = \kappa (\alpha_W T)^4, \quad (7)$$

where the coefficient $0.1 < \kappa < 1$ was found in numerical simulations [17]. It is clear from physical reasons that the rate of the sphaleron transitions should be a decreasing function of M_W , because the height of the barrier separating different topological vacua increases with the mass of W . The natural way to interpolate between two regions is to use (6) for some $M_W > M_{crit}$ and (7) for the opposite case. M_{crit} is the solution of the equation $\Gamma_1 = \Gamma_2$, $M_{crit} = 7\alpha_W T$. For this estimate we used an expression for the lower bound for the sphaleron mass in a two Higgs doublet model $E_{sph} > 3M_W/\alpha_W$ [13, 18]. Let us go now to the chemical potential $\mu(t)$. We can write

$$\phi_1 = w_1 \exp(i\gamma + i\theta/2), \quad \phi_2 = w_2 \exp(i\gamma - i\theta/2), \quad (8)$$

where γ is a gauge degree of freedom which can be rotated away, and θ is a relative phase between ϕ_1 and ϕ_2 . The Universe must be neutral with respect to all gauge charges, so that

$$\mathcal{O}(\phi_1) + \mathcal{O}(\phi_2) = 0. \quad (9)$$

This gives the connection between time derivatives $\dot{\gamma}$ and $\dot{\theta}$,

$$2\dot{\gamma} = -\frac{w_1^2 - w_2^2}{w_1^2 + w_2^2} \dot{\theta}. \quad (10)$$

Hence, the operator $\mathcal{O}(\phi_i)$ can be expressed entirely through the relative phase θ ,

$$\mathcal{O}(\phi_1) = \frac{1}{2} w^2 \sin^2 2\alpha \dot{\theta}, \quad (11)$$

where we have defined $w^2 = w_1^2 + w_2^2$, $\tan \alpha = w_1/w_2$. Now we should relate the relative phase to w entering the decay rate. This should be done only for sufficiently small w ,

²The scalar field changes during the transition, so that the rate does. After the phase transition B-violating reactions must be out of thermal equilibrium. This requirement puts some constraints on the Higgs mass spectrum of the theory [6, 12, 13, 14].

because the sphaleron rate falls rapidly at large w . Phase transition occurs at the moment when the curvature of the scalar potential nearly vanishes in some particular direction. At this time the mass matrix for the Higgs fields has a zero eigenvalue,

$$m_1^2(T_c) m_2^2(T_c) = v_1^2 v_2^2 (\lambda_3^2 \cos^2 \xi + \lambda_6^2 \sin^2 \xi). \quad (12)$$

In the first approximation, the fields ϕ_1 and ϕ_2 change in the direction of the steepest descent of the potential near the origin. For small w this direction is given by

$$\theta = -\arctan\left(\frac{\lambda_3 \sin \xi}{\lambda_6 \cos \xi}\right) + \frac{w^2}{4v_1 v_2} \lambda_{CP} \sin 2\alpha, \quad \tan \alpha = \frac{w_1}{w_2} = \frac{m_1^2(T)}{m_2^2(T)}, \quad (13)$$

with $m_i(T_c)$ being the temperature-dependent scalar masses at the moment of the phase transition (see the calculation in [13]). It is not difficult to see that eq.(13) provides actually a good approximation to the solution of the real time evolution equation for the Higgs fields rolling down towards the true vacuum. Now we can perform the integration in (5). Note that the region $M_W < M_{crit}$ gives the main contribution. Finally we get for the baryonic asymmetry

$$\Delta = \frac{45}{2\pi^2 N_{eff}} \kappa v_1 \alpha_W^6 \sin^3 2\alpha \lambda_{CP} \frac{m_1^2 T_c^2}{v_1^3 v_2}, \quad (14)$$

with N_{eff} being the number of effectively massless degrees of freedom. From here we can determine λ_{CP} by comparing the result with the observable baryonic asymmetry $4 \times 10^{-11} < \Delta < 10^{-10}$. If we take $m_t = 200 \text{ GeV}$, $T_c \sim 100 \text{ GeV}$ (this is the maximal possible temperature of the electroweak phase transition, at which B-violation is frozen out after the transition [6, 12, 13, 14]) and an experimental limit on the ratio $v_1/v_2 > 0.3$ [19] we can find the minimal CP-violation parameter at which the electroweak baryogenesis is possible. Taking into account all uncertainties, it is

$$|\lambda_{CP}^{min}| = 0.01. \quad (15)$$

Electric dipole moment of the electron.

On the other hand, the Higgs-mediated EDM of the electron which comes from two-loop diagrams including the neutral Higgs line, was shown [3] to be proportional to the imaginary part of the neutral Higgs boson propagator:

$$d_e \simeq 1.3 \cdot 10^{-26} \cdot \text{Im}(Z_0) e \cdot \text{cm}, \quad (16)$$

$$\frac{<\varphi_1^0 \cdot (\varphi_2^0)^*>}{v_1 v_2} \equiv i\sqrt{2} G_F \sum_{m_i} \frac{Z_{0i} k}{q^2 - m_i^2} \rightarrow \frac{i\sqrt{2} G_F}{q^2 - m_0^2} Z_0,$$

where we denote the smallest mass in the Higgs spectrum by m_0 . Given the bounds on the CP violating parameter in the scalar potential λ_{CP} , we are able to re-express the latter formula in terms of the λ_{CP} :

$$|\text{Im}(Z_0)| = \frac{2}{\alpha_W \pi} \left(\frac{m_W}{\Delta m_H} \right)^2 |\lambda_{CP}|, \quad (17)$$

Here we denote by Δm_H the difference between the squared masses of the scalar Higgs particle m_H^2 and the pseudoscalar m_A^2 . In fact, it is a rather more complicated combination of the Higgs masses which stands in the denominator of (17) (details of the calculation will be given in a forthcoming paper). Using the lower bound for λ_{CP} (15) we get:

$$|d_e| > 3 \times 10^{-27} \left(\frac{m_W}{\Delta m_H} \right)^2 e \text{ cm.} \quad (18)$$

Typically, the mass of the Higgs boson should be sufficiently small in order to generate asymmetry [6, 12, 13, 14]. For the two Higgs doublet case the maximal mass is about 120 GeV [13, 14]. It would be natural to expect that other masses have the same order of magnitude. Hence a reasonable estimate for $(\Delta m_H)^2$ would be $(120 \text{ GeV})^2$ so that $|d_e| > 10^{-27} \text{ e cm}$. This number is very close to the present experimental limit $d_e = (-2.5 \pm 8.5) \times 10^{-27} [4]$. Moreover, in all our estimates we have used the most profitable numbers from the point of view of the baryonic asymmetry (maximal possible sphaleron rate, the best mixing angle α , the smallest possible ratio v_1/v_2 , etc). So we consider eq. (18) as a really conservative lower bound. This provides a hope that in future experiments the non-zero dipole moment can be discovered. In the case of negative results of the experiments electroweak baryogenesis in a two-doublet model will be ruled out.

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