Chapter 9

Asymmetries at the Z pole: The Quark and Lepton Quantum Numbers

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The impressive progress on the knowledge of lepton and quark electroweak couplings over the LEP and SLC decade is reviewed. The experimental methods for measuring the forward-backward asymmetry of charged-fermion pair-production are described, for different fermion species. The precise measurements of the left-right asymmetry and of tau polarisation at the Z resonance are also reminded. After discussing the determination of the Weinberg electroweak mixing angle, lepton and quark couplings are extracted by combining asymmetry and polarisation measurements with measurements of partial decay widths of the Z boson, performed at LEP in the same years.

1. Introduction

The present knowledge of quark and lepton electroweak neutral couplings is largely based on the data collected in e^+e^- collisions at the Z pole, between 1989 and 1998, by LEP and SLC. In that years, measurements of lepton couplings improved by two order of magnitudes with respect to previous experiments, based on neutrino scattering, and individual quark couplings were measured for the first time. The jump in precision was possible because parity violation in Z boson production and decay has a direct consequence on experimental data, yielding measurable asymmetries that can be used to determine the the Weinberg electroweak mixing angle $\sin^2 \theta_W$ and, by including measurements of the Z partial widths, the couplings themselves.

Before describing the legacy measurements performed by ALEPH,¹ DELPHI,² L3,³ OPAL,⁴ SLD⁵ and their interpretations, in next Section definitions and properties of the basic asymmetries sensitive to the electroweak mixing angle and neutral fermion couplings are briefly recalled.

2. Asymmetries and Polarisations at the Z pole

The differential cross section for the process $e^+e^- \rightarrow f^+f^-$ via exchange of a Z boson can be written as

$$\begin{aligned} \frac{d\sigma}{d\Omega} = B(s) \times \{ & [1 + \cos^2 \theta] [(g_{Le}^2 + g_{Re}^2)(g_{Lf}^2 + g_{Rf}^2)] \\ & + 2 \cos \theta [(g_{Le}^2 - g_{Re}^2)(g_{Lf}^2 - g_{Rf}^2)] \}, \end{aligned} \quad (1)$$

where $g_{L(R)e}$ and $g_{L(R)f}$ are the left (right) couplings for the initial and final state, respectively, θ is the scattering angle of the final-state fermion with respect to the initial-state fermion direction (the electron in this case) and $B(s)$ is a *Breit-Wigner* term angular-independent. The coefficient of the cosine in Eq. (1) would vanish if the initial or final-state left and right couplings were identical, as in the electromagnetic case. Parity violation in Z production and decay leads to a non-zero forward-backward asymmetry, A_{FB} , defined as:

$$A_{FB} = \frac{\int_{\theta=0}^{\theta=\pi/2} \frac{d\sigma}{d\cos\theta} - \int_{\theta=\pi/2}^{\theta=\pi} \frac{d\sigma}{d\cos\theta}}{\int_{\theta=0}^{\theta=\pi/2} \frac{d\sigma}{d\cos\theta} + \int_{\theta=\pi/2}^{\theta=\pi} \frac{d\sigma}{d\cos\theta}}. \quad (2)$$

Integration over the scattering angle gives

$$A_{FB} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f \quad (3)$$

where

$$\mathcal{A}_e = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2} = \frac{2 g_{Ve}/g_{Ae}}{1 + (g_{Ve}/g_{Ae})^2}, \quad (4)$$

and similarly

$$\mathcal{A}_f = \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} = \frac{2 g_{Vf}/g_{Af}}{1 + (g_{Vf}/g_{Af})^2}. \quad (5)$$

In Eqs. (4) and (5) vector and axial couplings are introduced ($g_{Ve} = g_{Le} + g_{Re}$, $g_{Ae} = g_{Le} - g_{Re}$ and similarly for f). One can see that $\mathcal{A}_e$ depends on the ratio between vector (g_{Ve}) and axial vector (g_{Ae}) coupling constants of the electron, which are in turn related to the *effective* electroweak mixing angle, defined as^{6,7}

$$\sin^2 \theta_{\text{eff}}^\ell \equiv \frac{1}{4} \left(1 - \frac{g_{V\ell}}{g_{A\ell}} \right). \quad (6)$$

If polarised beams are available, or if polarisation can be measured in the final state, other useful asymmetries can be defined. As an example, if a polarised electron beam collides with unpolarised positrons at a centre-of-mass energy equal to m_Z , the total cross section is much bigger if left-handed polarisation is used. The relative difference between the two cross sections (σ_L and σ_R) is the left-right asymmetry

(A_{LR}), which is related to the right-handed (g_{Le}) and left-handed (g_{Re}) electron couplings by

$$A_{LR} = \frac{\sigma_{e_L^-} - \sigma_{e_R^-}}{\sigma_{e_L^-} + \sigma_{e_R^-}} = \frac{g_{Le}^2 - g_{Re}^2}{g_{Le}^2 + g_{Re}^2} \equiv \mathcal{A}_e. \quad (7)$$

A measurement of A_{LR} provides direct access to the asymmetry between the initial-state couplings, $\mathcal{A}_e$, which represents also the net polarisation acquired by the Z boson along the direction of the electron beam in case of collisions of unpolarised beams. In this case the Z polarisation, and therefore $\mathcal{A}_e$, can be measured by analysing the polarisation of the fermion emitted by the Z boson since angular momentum conservation relates the two quantities. In practice at the Z pole this is possible only if the emitted fermion is a tau lepton, by measuring the tau polarisation in $e^+e^- \rightarrow Z \rightarrow \tau\bar{\tau}$.

Polarised beams are also useful to provide a direct measurement of the asymmetry between final state couplings, $\mathcal{A}_f$, by measuring the polarised forward-backward asymmetry, defined as

$$A_{FB}^{pol}(f) = \frac{(\sigma_{e_L^- f_F} - \sigma_{e_R^- f_F}) - (\sigma_{e_L^- f_B} - \sigma_{e_R^- f_B})}{\sigma_{e_L^- f_F} + \sigma_{e_R^- f_F} + \sigma_{e_L^- f_B} + \sigma_{e_R^- f_B}}, \quad (8)$$

where f_F and f_B indicate forward and backward outgoing fermions, respectively. At the Z peak

$$A_{FB}^{pol}(f) = \frac{3}{4} \frac{g_{Lf}^2 - g_{Rf}^2}{g_{Lf}^2 + g_{Rf}^2} \equiv \frac{3}{4} \mathcal{A}_f \quad (9)$$

showing that Eq. (8) is only dependent on the final state couplings, as anticipated.

3. Forward-Backward Asymmetries

The forward-backward asymmetry at the Z pole (Eq. (3)) has been measured at LEP for individual lepton species (e, μ , τ) and for heavy quarks (c and b). Measurements were also performed inclusively for hadrons and, though with much lower precision, for s and u, d quarks. The data were collected by ALEPH, DELPHI, L3 and OPAL in the years 1986–1995 (LEP1 phase) and correspond to an integrated luminosity of 150 pb^{-1} per experiment, yielding a total of more than 15 millions hadronic Z decays and 1.7 millions leptonic Z decays.

Lepton and quark forward-backward asymmetries have several conceptual and experimental differences. Assuming lepton universality the ratios between vector ($g_{V\ell}$) and axial vector ($g_{A\ell}$) coupling of the Z to charged leptons are equal, therefore asymmetries involving leptons provide a direct determination of the effective mixing angle (Eq. (6)) using the relation $A_{FB} = \frac{3}{4} \mathcal{A}_e \mathcal{A}_f = \frac{3}{4} \mathcal{A}_\ell^2$. The quark forward-backward asymmetries depends on $\mathcal{A}_f = \frac{2g_{Vq}/g_{Aq}}{1+(g_{Vq}/g_{Aq})^2}$ where the subscript q indicates the quark flavour. The ratio of quark couplings can be expressed in terms of

$\sin^2 \theta_{\text{eff}}^\ell$ and non-universal corrections as⁸

$$\frac{g_{Vq}}{g_{Aq}} = 1 - \frac{2Q_q}{I_{3L,q}} (\sin^2 \theta_{\text{eff}}^\ell + C_q), \quad (10)$$

where Q_q is the electric charge and $I_{3L,q}$ is the third component of the weak isospin. The residual vertex correction C_q can be computed assuming the Standard Theory. For $udsc$ quarks it is small and has very little dependence on the parameters of the model, while for b it depends on the top mass because of additional $Z \rightarrow b\bar{b}$ vertex corrections ($C_b = +0.0014$). In case of quarks $\mathcal{A}_f$ is large and weakly dependent on $\sin^2 \theta_{\text{eff}}^\ell$ leaving most of the dependence on the weak mixing angle to $\mathcal{A}_e$. Therefore for quarks A_{FB} is essentially linearly dependent on $\sin^2 \theta_{\text{eff}}^\ell$, while for leptons it shows a quadratic dependence. The consequence of this behaviour is shown in Table 1 where the magnitude of A_{FB} and its sensitivity to $\sin^2 \theta_{\text{eff}}^\ell$ is given for leptons, for u -type and d -type quarks, showing that the latter can provide the most precise measurements of the electroweak mixing angle, within the Standard Theory.

Most measurements of forward–backward asymmetries have been determined by fitting the data to the differential angular distribution

$$\frac{dN}{d\cos\theta} = C(\cos\theta) \cdot \left(1 + \cos^2\theta + \frac{8}{3} A_{FB} \cos\theta \right) \quad (11)$$

where θ is the scattering angle of the fermion in the centre-of-mass system and $C(\cos\theta)$ is an acceptance function modifying the differential cross section (Eq. (1)). Experimentally the A_{FB} measurement requires the identification of the fermion in the final state, i.e. a measurement of its charge. Again, lepton and hadron asymmetries are different in this respect:

- in leptonic Z decays typically the selection of two fermions of opposite charge is required, automatically defining the final-state fermion.
- in hadronic Z decays at least one fermion must be tagged and its charge measured, as for the measurement of heavy quark asymmetries described in Section 3.2.

Table 1. Magnitude of the forward–backward asymmetry and its sensitivity on $\sin^2 \theta_{\text{eff}}^\ell$ for various fermion species at the pole of the Z . The value of 0.2316 is used for $\sin^2 \theta_{\text{eff}}^\ell$. For comparison the last line gives the magnitude and the sensitivity for A_{LR} .

	A_{FB}	$\frac{\partial A_{FB}}{\partial \sin^2 \theta_{\text{eff}}^\ell}$
Leptons	.02	−1.7
u and c quarks	.07	−4.0
d , s and b quarks	.10	−5.6
A_{LR}	.15	−7.8

In both cases it can be shown that the acceptance function is symmetric, provided the selection efficiency does not depend on the fermion charge.

Forward-backward asymmetries depend on the centre-of-mass energy, because of the interference between photon and Z exchange diagrams. Near the Z peak the asymmetry depends on the electric charge of the final fermion and on its axial coupling and has very little dependence on other electroweak parameters. The functional dependence can be approximated as

$$A_{FB}^f(s) \simeq A_{FB}^f(m_Z^2) + \frac{(s - m_Z^2)}{s} \frac{3\pi\alpha(s)}{\sqrt{2}G_F m_Z^2} \frac{2Q_e Q_f g_{Ae} g_{Af}}{(g_{Ve}^2 + g_{Ae}^2)(g_{Vf}^2 + g_{Af}^2)}, \quad (12)$$

where the running fine structure constant ($\alpha(s)$), the Fermi constant (G_F) and the electric charges (Q_e and Q_f) have been introduced. The dependence is maximal for leptons ($\Delta A_{FB}^\ell / \Delta E_{CM} \simeq 0.00009/\text{MeV}$), while down-type quarks show the smallest energy dependence. This effect is corrected for using the precise determination of the LEP beam energy,⁹ by extrapolating the measured asymmetry to m_Z . All forward-backward asymmetries have also to be corrected for the effect of initial state radiation, for imaginary parts of the couplings (in particular for $\text{Im}(\Delta\alpha)$), for the effect of pure photon exchange and the presence of box diagrams. Specific corrections are also applied for final state photon radiation (leptons) and gluon emission (hadrons). The uncertainty of these corrections is, in all cases, much smaller than the total error, dominated by the statistical uncertainty for all measurements. The corrected asymmetry for a fermion f is indicated as $A_{FB}^0(f)$ in next Sections.

3.1. Lepton forward-backward asymmetries

At LEP the forward-backward asymmetry of $e^+e^- \rightarrow \ell^+\ell^-(\gamma)$ events was determined by fitting the data to Eq. (11); the angle θ was defined by the scattering angle of the final-state negative lepton. For tau leptons the direction was given by the sum of the momenta of charge particles associated to the tau decays; the tau charge was measured in the same way.

In the case of e^+e^- final state, the t -channel photon-exchange process induces an important asymmetric correction and requires a careful treatment. The contribution of this process was taken into account by subtracting it from the measured angular distribution. Semi-analytical calculations incorporating leading-log photonic corrections, first-order non-log terms and first-order weak corrections¹⁰ were used for this correction. The t -channel influence was reduced by analysing the data in a restricted angular region, typically in the $-0.9 \leq \cos\theta \leq +0.7$ range. (Within this range the t -channel contributes 12% to the total cross section, therefore calculations with 1% precision yielded an uncertainty of 0.1%.)

The asymmetries $A_{FB}(\ell)$ ($\ell = e, \mu$ and τ) measured at LEP¹¹⁻¹⁴ were extracted with a fit to the measured $A_{FB}(s)$ using data collected near the Z peak and at the off-peak points used to measure the Z lineshape. The fitting formula took into account

the energy dependence of the asymmetry and the fit was done simultaneously with the lineshape data to account for the effect of the energy uncertainty. In the simultaneous fit of the Z lineshape and $A_{FB}^\ell(s)$, the axial couplings were essentially determined by the lineshape and then used to transport the off-peak measurements of $A_{FB}^\ell(s)$ to $\sqrt{s} = m_Z$.

The measurement of $A_{FB}^0(\ell)$ was a rather straightforward measurement with low systematic uncertainties. For the μ and τ channels the systematic uncertainties were related to the applied corrections, to the presence of background and to possible detector asymmetries. Typical systematic errors quoted by the LEP experiments were of the order of $\Delta A_{FB} = 0.0005\text{--}0.001$ for muons and $\Delta A_{FB} = 0.001\text{--}0.003$ for taus, depending on the experiment. For electrons, the theoretical uncertainty introduced in the treatment of the t-channel terms (≈ 0.0014) had to be taken into account increasing the typical error to $\Delta A_{FB} \approx 0.002$. The uncertainty on the centre-of-mass energies gives a contribution of $\Delta A_{FB}^0(\ell) = 0.0004$, comparable to the experimental systematics. These last two uncertainties were common to the four experiments and had to be treated in a correlated way when averaging the measurements.

The combination of the results of the four LEP experiments gave

$$A_{FB}^0(e) = 0.0145 \pm 0.0025, \quad (13)$$

$$A_{FB}^0(\mu) = 0.0169 \pm 0.0013, \quad (14)$$

$$A_{FB}^0(\tau) = 0.0188 \pm 0.0017. \quad (15)$$

The three measurement can be combined assuming lepton universality, giving

$$A_{FB}^0(\ell) = 0.0171 \pm 0.0010, \quad (16)$$

$$\sin^2 \theta_{\text{eff}}^\ell = 0.23099 \pm 0.00053. \quad (17)$$

The dependence of the asymmetries on the centre-of-mass energy, $A_{FB}^\ell(s)$, is consistent with the expected value and sign of the lepton axial couplings and it is shown in Fig. 1.

3.2. Heavy quark asymmetries

Forward-backward asymmetries for b and c quarks were determined at LEP with three different techniques. The first method (*lepton tagging*) was based on the presence of a lepton in a jet as a tag for $Z \rightarrow b\bar{b}$ or $Z \rightarrow c\bar{c}$ events. Lepton kinematics was used to discriminate the different lepton sources, on a statistical basis. With this method the charge of the lepton can be used to define the scattered fermion. The second method (*lifetime tagging*) relied on the selection of $Z \rightarrow b\bar{b}$ events using the properties of long-lived B hadrons, followed by the use of a jet-charge measurement. The third method was conceptually similar to the first one, with lepton

Fig. 1. Measurement of the forward-backward asymmetries for the three lepton species at various centre-of-mass energies, as measured by the ALEPH experiment.¹¹

tagging replaced by *D meson tagging* and was solely used for the measurement of the c asymmetry. Here only the first two methods are briefly described, but results from D meson tagging are used in the LEP heavy quark asymmetry combination mentioned at the end of this Section.

3.2.1. Lepton tagging

In order to illustrate the main aspects of the lepton tagging method it is useful to consider hadronic Z decays selected by requiring leptons at high- $p_\perp$ with respect to the axis of their associated jet. A proper $p_\perp$ cut can provide a high-purity sample of $Z \rightarrow b\bar{b}$ events, with enhanced $b \rightarrow \ell^-$ content. Such a sample can be used to measure the b asymmetry by fitting the polar angle distribution of the thrust axis to the functional form provided by Eq. (11). The thrust axis is conventionally defined as oriented towards the hemisphere containing the lepton if this is negatively charged, towards the other hemisphere otherwise. Semileptonic $b \rightarrow \ell^-$ decays carry the correct charge correlation between quark and tagging lepton, yielding a forward-backward asymmetry in the oriented thrust axis polar angle distribution. The observed asymmetry (A_{FB}^{obs}) can be related to the $Z \rightarrow b\bar{b}$ forward-backward asymmetry (A_{FB}^b) in terms of the contributions of the different components of the selected sample:

$$A_{FB}^{obs} = (1 - 2\bar{\chi})(f_{r.s.}^b - f_{w.s.}^b)A_{FB}^b + f_{bkg}^b \eta_{bkg}^b A_{FB}^b - f^{c \rightarrow \ell^+} A_{FB}^c - f_{bkg}^c \eta_{bkg}^c A_{FB}^c + f^{uds} \overline{A_{FB}^{uds}}, \quad (18)$$

where $f_{r.s.}^b$ and $f_{w.s.}^b$ are the fractions of prompt leptons from b decays with right/wrong charge correlation between the lepton charge and the b quark charge, dominated by $b \rightarrow \ell^-$ and $b \rightarrow c \rightarrow \ell^+$ decays, respectively, and the term $(1 - 2\bar{\chi})$ is introduced to correct for neutral b meson mixing ($\bar{\chi} \approx 0.13$). The charm asymmetry is indicated by A_{FB}^c and the fraction of prompt leptons from charm decays by $f^{c \rightarrow \ell^+}$. The factors η_{bkg}^b and η_{bkg}^c describe the (small) correlation between the charge of fake and non-prompt leptons and the charge of the primary quark in b and c events, and the last term accounts for the contribution of light quark background.

High- $p_\perp$ leptons can be selected in both hemispheres, and the counting of same sign and opposite sign pairs ($N_{o.s.}$, $N_{s.s.}$) gives the possibility to measure from the data the charge correlation in b events $\mathcal{P}_b = (1 - 2\bar{\chi})(f_{r.s.}^b - f_{w.s.}^b)$, once the small contribution from charm and light quark events has been subtracted:

$$f_{o.s.} = \frac{N_{o.s.}}{N_{o.s.} + N_{s.s.}} = \mathcal{P}_b^2 + (1 - \mathcal{P}_b)^2 + c \text{ and uds corrections.} \quad (19)$$

The analysis of the dilepton sample to evaluate $\mathcal{P}_b$ from the data, lowers considerably the dependence of the measurement upon the knowledge of semileptonic b decays (rates and kinematic properties), as well as b meson mixing. It should be also noted that since the measurement makes use of both forward negative leptons and backward positive leptons to tag forward b quarks, the detector acceptance has nearly no effect on the extracted value of the asymmetry: a sizeable effect could arise only in case of inefficiencies that are both forward-backward asymmetric and different for positive and negative leptons, which was very unlikely for LEP detectors.

The extraction of A_{FB}^b from high- $p_\perp$ leptons, using Eq. (18), depends on the evaluation and on the modelling of the charm component, and on the assumed value of A_{FB}^c . At LEP¹⁵⁻¹⁸ the whole $p_\perp$ spectrum of lepton candidates was studied, providing a simultaneous measurement of A_{FB}^b and A_{FB}^c .

3.2.2. Inclusive measurements

Heavy quark tagging methods based on lifetime have high performance, but they do not provide information about the quark charge. Inclusive methods have been developed to estimate the charge of the b quark, to complement lifetime tags for the measurement of forward-backward asymmetries.

The jet charge is usually defined as

$$Q_h = \frac{\sum_i q_i p_{\parallel i}^k}{\sum_i p_{\parallel i}^k}, \quad (20)$$

where $p_{\parallel i}$ is the momentum of a particle parallel to the thrust axis, and the sum runs over all charged particles in a hemisphere. The parameter k can be tuned to obtain high sensitivity to the quark charge, while keeping low correlation between the charge of the two hemispheres (values used at LEP were between 0.3 and 1).

In a pure sample of b events, the forward-backward asymmetry is proportional to the mean charge flow between the two hemispheres

$$Q_{FB}^b \equiv \langle Q_F^b - Q_B^b \rangle = \delta_b A_{FB}^b \quad (21)$$

where δ_b is a parameter called charge separation. At parton level δ_q (the charge separation for a generic quark q) is equal to twice the quark charge, but hadronization and decays lower its value, diluting the measured charge flow. A precise determination of the forward-backward asymmetry requires an evaluation of δ_q with the lowest possible uncertainty. The advantage of high-purity single-flavour samples, that in practice can be obtained for b quarks only, lies on the possibility of measuring δ_q from the data, lowering considerably the use of theoretical assumptions in the evaluation of this parameter, and therefore lowering its uncertainty. A sketch illustrating the technique used at LEP to measure the charge separation is presented in Fig. 2.

In an asymmetry analysis, pure b samples cannot be selected and contributions of the other flavours have to be taken into account; for instance, the charge flow can be written as

$$Q_{FB} = f_b \delta_b A_{FB}^b + f_c \delta_c A_{FB}^c + f_{uds} \delta_{uds} A_{FB}^{uds}, \quad (22)$$

where the f_b , f_c , f_{uds} are the fraction of b, c and light quark events in the selected sample, and light quark have been treated as a single class.

In the initial LEP measurements, the sample composition as well as the charm and light quark charge separations was estimated with the simulation, δ_b was extracted from the data and the b asymmetry derived from the observed charge

Fig. 2. Sketch showing the distributions of charge flow and total charge: the difference in width between the two distribution is related to the charge separation.

flow. In the final measurements^{19–22} sophisticated methods using double tagging techniques were employed and most parameters were determined in situ from data.

3.2.3. Heavy quark asymmetries: Combined results and QCD corrections

The LEP measurements of b and c forward–backward asymmetries using lepton tagging, lifetime tagging and D mesons tagging^{15–25} were combined to merge the experimental information in an optimal way.²⁶ As mentioned earlier, the extraction of the effective electroweak mixing angle and of the quark couplings requires the evaluation of the corrected b and c asymmetries $A_{FB}^0(b)$ and $A_{FB}^0(c)$, from the measured asymmetries. Heavy quark asymmetries are affected by radiative effects due to strong interactions related to virtual vertex and gluon bremsstrahlung diagrams, which modify the angular distribution of the fermions emitted in the final state. The emission of a hard gluon, for example, may scatter both b and $\bar{b}$ in the same hemisphere (forward or backward): in such events the original electroweak asymmetry is destroyed. The effect of such radiative effects is to lower the experimentally observed asymmetry by a few percent. Detailed calculation based on perturbative QCD, including second-order corrections for massless quarks and quark mass effects at first-order, were used.²⁷ In practice experimental cuts reduce considerably

the QCD corrections.²⁸ For instance momentum cuts, which are applied in lepton tagging, select events with reduced gluon radiation. Furthermore in some cases the effect of hard gluon radiation is automatically incorporated by analysis procedure. This is the case for the inclusive measurements based on jet charge techniques, because the b charge separation, measured with data, is an effective parameter that includes the QCD smearing.

The final LEP averages for the b and c forward–backward asymmetries at the Z pole are²⁹

$$A_{FB}^0(b) = 0.0992 \pm 0.0016,$$

$$A_{FB}^0(c) = 0.0707 \pm 0.0035.$$

There is a +15% correlation between the two results. Both results are dominated by the statistical uncertainties. In particular, for the b asymmetry, the systematic uncertainties related to the QCD corrections is a factor three lower than the statistical error.

The dependence of the b and c asymmetries on the centre-of-mass energy, $A_{FB}^b(s)$ and $A_{FB}^c(s)$, is regulated by the quark electric charge and its axial coupling (Eq. (12)). Their observed energy dependence is shown in Fig. 3 and compared with the ST prediction. The different slope for b and c quarks is due to the absolute value of their electric charge, that is twice larger for up-type quarks. The asymmetry is increasing in both cases because the two quark types have opposite sign (and same absolute value) for the axial couplings.

Fig. 3. Measurement of the b and c forward–backward asymmetries as a function of the centre-of-mass energy.²⁹ The ST expectation for the two quark types is shown. The value of $\sin^2 \theta_{\text{eff}}^\ell$ given in Section 6 is used to normalise the vertical scale for the ST prediction.

4. Asymmetries with Polarised Beams

4.1. Measurement of the left-right asymmetry (A_{LR})

The measurement of A_{LR} requires the availability of longitudinally polarised beams. At SLC longitudinal polarisation of the electron beam was achieved by a circularly polarised laser source hitting a GaAs photocathode,³⁰ allowing SLC to be operated with an electron beam polarisation of about 75%. Equation (7), modified to take into account the average beam polarisation ($\mathcal{P}_e$), reads

$$A_{LR} = \frac{1}{\mathcal{P}_e} \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}. \quad (23)$$

The main experimental issue, for a precise measurement of A_{LR} , is an accurate determination of the beam polarisation. This need could be overcome if both electron and positron beams were independently polarised, a scheme originally proposed for LEP³¹ that never went into operation, because it would have required the installation of spin rotators. The standard SLC operating cycle consisted of two close electron bunches, the first of which was polarised, while the other was used to produce unpolarised positrons. The sign of the electron polarisation was randomly chosen, so that the measurement was not affected by time variations of the apparatus efficiency.

The SLD experiment monitored the longitudinal SLC-beam polarisation with a polarimeter based on Compton scattering of electrons by circularly polarised laser light taking place after the interaction point. The Compton cross sections for spin-parallel ($j=3/2$) and spin anti-parallel ($j=1/2$) interactions are different, and this difference is a function of the normalized scattered-photon energy fraction (x). The difference can be written, in terms of x , as

$$\frac{d\sigma^{3/2}}{dx} - \frac{d\sigma^{1/2}}{dx} = \frac{d\sigma}{dx} (1 - \mathcal{P}_\gamma \mathcal{P}_e A(x)) \quad (24)$$

where $A(x)$ is the Compton asymmetry function.³² The laser beam polarisation ($\mathcal{P}_\gamma$, typically 99.8%) was continuously monitored. The statistical accuracy on $\mathcal{P}_e$ was of $\pm 1\%$ every three minutes. The relative systematic uncertainties^{33,34} in the polarisation measurement are summarized in Table 2. The depolarisation of the electron beam during the e^+e^- collision was checked by measuring the polarisation with and without beam collisions and was found to be negligible. The total contribution of the systematic uncertainty on the beam polarisation to the measurement of A_{LR} was 0.52 % (high-statistics 1997/8 run); this is the main source of systematic uncertainty in the left-right asymmetry measurement.

The asymmetry of the left-right rates was measured with a simple event selection, since A_{LR} does not depend on the final state as long as this is an s-channel process. Care must be taken in rejecting Bhabha scattering events, because of the t-channel contribution to $e^+e^- \rightarrow e^+e^-$. The total sample comprised approximately 537000 Z decays and was mostly made of hadronic events,

Table 2. Relative (%) systematic uncertainties on the electron beam polarisation at SLD in two data-taking periods.^{33,34} The last entry represents the uncertainty on the difference between the measured polarisation and the polarisation at the interaction point (IP).

Source of uncertainty	1994/5 (%)	1997/8 (%)
Laser polarisation	0.2	0.1
Detector Linearity	0.5	0.2
Detector Calibration	0.29	0.40
Electronic Noise	0.20	0.20
Transport from polarimeter to SLD IP	0.17	0.15

with a small tau contributions ($\sim 0.3\%$). The events produced with left-handed (N_L) and right-handed (N_R) polarisation were counted and their asymmetry $A_m = (N_L - N_R)/(N_L + N_R) \sim 0.12$ was measured. The measured asymmetry is related to A_{LR} by the following expression

$$A_{LR} = \frac{A_m}{\langle \mathcal{P}_e \rangle} + \frac{1}{\langle \mathcal{P}_e \rangle} \left[f_b(A_m - A_b) - A_L + A_m^2 A_{\mathcal{P}} - E_{cm} \frac{\sigma'(E_{cm})}{\sigma(E_{cm})} A_E - A_\epsilon + \langle \mathcal{P}_e \rangle \mathcal{P}_p \right] \quad (25)$$

where a number of small corrections, listed below, are incorporated. In Eq. (25) A_X indicates the left-right asymmetry of X , defined as $A_X \equiv \frac{X_L - X_R}{X_L + X_R}$. The first term in the square bracket represents the correction for the background: f_b is the background fraction and A_b the background left-right asymmetry. The second term represents the asymmetry of the integrated luminosity, while the third term takes into account the asymmetry in the beam left and right absolute polarisations. The fourth term corrects for the different centre-of-mass energies when left or right beams are used: $\sigma'(E_{cm})$ is the derivative of the cross section with respect to E_{cm} . The fifth term represents the left-right asymmetry in selection efficiency: it is totally negligible for an apparatus with symmetric acceptance in polar angle. Finally, the last term corrects for possible longitudinal polarisation of the positron beam. This was measured with a dedicated experiment based on Möller scattering and found negligible. The sum of the corrections in the square brackets of Eq. 25 gives $[+0.16 \pm 0.07]\%$ for the 1997/8 high luminosity run.

Also for the left-right asymmetry there is a dependence on the centre-of-mass energy because of the $Z\text{-}\gamma$ s-channel interference:

$$A_{LR}(s) = \mathcal{A}_e + 0.00002 \Delta E (\text{MeV}) + 0.00005 \quad (26)$$

where ΔE is the difference between m_Z and the actual centre-of-mass energy, while the constant term accounts for the correction due to the imaginary part of $\Delta\alpha$. In order to apply the correction and compute the asymmetry at the Z pole the

centre-of-mass energy of the experiment must be precisely known. SLC employed two energy spectrometers (one for the electron and one for the positron beam) calibrated, through an energy scan, to the precise measurement of m_Z at LEP. The measured average offset was -46 MeV and the total centre-of-mass energy uncertainty 29 MeV. The measured left–right asymmetry was also corrected for the effect of initial state radiation (the most sizeable QED correction, which lowers the asymmetry as expected from Eq. (26)), for the effect of pure photon exchange (which slightly dilutes the asymmetry) and for other higher order QED/electroweak effects (as the already mentioned imaginary part of $\Delta\alpha$). The total correction (including the centre-of-mass energy offset) was 0.00358 ± 0.00058 , the error being essentially due to the uncertainty on the beam energy. When this uncertainty is added in quadrature to the uncertainty on the electron beam polarisation and the uncertainty on the corrections of Eq. (25) a total systematic error on A_{LR} of 0.64% is obtained.

The final result,³⁴ including the statistical errors, was

$$A_{LR} = 0.15138 \pm 0.00216, \quad (27)$$

$$\sin^2 \theta_{\text{eff}}^\ell = 0.23097 \pm .00027 \quad (28)$$

giving the most precise measurement of the electroweak mixing angle.

4.2. Heavy quark asymmetries with polarised beams

The quark asymmetries discussed in Section 3.2, based on measurements employing unpolarised beams, are probing the product of initial and final state couplings, $\mathcal{A}_e \mathcal{A}_q$. On the other hand the polarised forward–backward asymmetry ($A_{FB}^{\text{pol}}(q)$), defined by Eq. (8), is solely dependent on $\mathcal{A}_q$. The polarised forward–backward asymmetry of b and c quarks have been measured by SLD³⁵ using flavour tagging methods very similar to the ones used for the unpolarised case. Inclusive samples of $Z \rightarrow b\bar{b}$ events selected thanks to the long b lifetime provided a precise determination of $\mathcal{A}_b$ using jet-charge techniques. Semileptonic b and c decays gave a simultaneous determination of $\mathcal{A}_b, \mathcal{A}_c$ through the analysis of inclusive lepton spectra; D mesons were used to measure $\mathcal{A}_c$. Measurements were corrected for QCD effects, which are similar to the unpolarised case.

The combination of SLD results gave²⁹

$$\mathcal{A}_b = 0.923 \pm 0.020,$$

$$\mathcal{A}_c = 0.670 \pm 0.027$$

with a small correlation between $\mathcal{A}_b$ and $\mathcal{A}_c$ (11%). The measurements are consistent with the ST predictions of $\mathcal{A}_b = 0.935$ and $\mathcal{A}_c = 0.668$, respectively. These predictions have a small uncertainty (≈ 0.001 for b quarks) because the quark $\mathcal{A}_q$ parameters are only weakly dependent on $\sin^2 \theta_{\text{eff}}^\ell$.

5. Measurement of the tau Polarisation in Z Decays

As mentioned in Section 2, in e^+e^- collisions the Z boson acquires a net polarisation equivalent to $\mathcal{A}_e$, which can be measured if the polarisation of the outgoing fermions is analysed. Tau leptons in $Z \rightarrow \tau^+\tau^-$ events can be used as polarimeters by measuring statistically the properties of their decay products.

The helicity of the two taus from Z decay are nearly 100% anti-correlated, except for very small $\mathcal{O}(\frac{m_f^2}{m_Z^2})$ corrections. It is convenient to measure the τ polarisation as a function of the angle (θ) between the τ^- and the electron beam. The definition of the tau polarisation for any $\cos\theta$ bin is given by

$$P_\tau = \frac{\sigma_R - \sigma_L}{\sigma_R + \sigma_L}, \quad (29)$$

where σ_R is the cross section to produce a right-handed τ^- and σ_L is the cross section to produce a left-handed τ^- . It can be shown that

$$P_\tau(\cos\theta) = \frac{A_{pol}(1 + \cos^2\theta) + \frac{8}{3}A_{pol}^{FB} \cos\theta}{(1 + \cos^2\theta) + \frac{8}{3}A_{FB} \cos\theta} \quad (30)$$

where A_{pol} is the total τ^- polarisation (integrated over $\cos\theta$), A_{FB} indicates the forward-backward asymmetry of the tau pairs and $A_{pol}^{FB} = \frac{\sigma_{F,R} - \sigma_{F,L} + \sigma_{B,L} - \sigma_{B,R}}{\sigma_{tot}}$ is the forward-backward polarisation asymmetry. The total polarisation and the polarisation asymmetry are related to the couplings as

$$A_{pol}^{FB} = -\frac{3}{4}\mathcal{A}_e, \quad (31)$$

$$A_{pol} = -\mathcal{A}_f. \quad (32)$$

The LEP experiments determined $\mathcal{A}_e$ and $\mathcal{A}_\tau$ simultaneously by measuring the $P_\tau(\cos\theta)$ distribution and fitting it to the functional dependence given by Eq. (30). This procedure gives better total error than measurements integrated over the hemispheres by giving more weight to $\cos\theta$ bins with higher sensitivity.

The polarisation of the τ is measured exploiting the parity violation of its weak decay,³⁶ that is mediated by a pure V-A current. Five tau decay channels, amounting to a branching ratio of about 90% have been used ($\tau \rightarrow \pi\nu$, $\tau \rightarrow \rho\nu$, $\tau \rightarrow a_1\nu$, $\tau \rightarrow e\nu\bar{\nu}$, $\tau \rightarrow \mu\nu\bar{\nu}$). Tau decays to charged kaons, having relatively low branching ratio, were included in the corresponding pion channels.

The principle of the polarisation analysis is easily understood by taking the simplest channel, $\tau \rightarrow \pi\nu$. The tau decay in this channel, for the two helicity cases, is sketched in Fig. 4. Because of the left-handedness of the neutrino, in case of decays of right-handed taus, the pion is boosted in the direction of the tau. The opposite is true for decays of left-handed taus. It follows that the energy of the pion discriminates between the two parent-tau helicity states. The tau differential decay

Fig. 4. The principle of the tau polarisation analysis taking the $\tau \rightarrow \pi \nu$ channel as an example.

width, given in term of the scaled pion energy $x_\pi = \frac{E_\pi}{E_{\text{beam}}}$ is

$$\frac{1}{\Gamma} \frac{d\Gamma}{dx_\pi} = 1 + P_\tau(2x_\pi - 1) \quad (33)$$

as can be shown by boosting into the laboratory frame the rest-frame decay angular distribution of a spin 1/2 particle decaying into two particles of spin 1/2 and spin 0, respectively.

The measurement of the polarisation used two sets of reference decay distributions, one for $P_\tau = -1$ and one for $P_\tau = 1$, obtained applying the $\tau \rightarrow \pi \nu$ selection cuts to simulated data. The tau polarisation could be extracted by performing a binned maximum likelihood fit of the measured distributions to the sum of the corresponding simulated distributions normalized by the coefficients $\frac{N}{2}(1 + P_\tau)$ and $\frac{N}{2}(1 - P_\tau)$, where N is the number of events. Background events, mostly coming from cross-contamination from other τ decays passing the $\tau \rightarrow \pi \nu$ selection, were included in the simulated data. Similar procedures were used for the other channels.

The angular dependence of the tau polarisation measured at LEP^{37–40} can be seen in Fig. 5. The LEP combined fit²⁹ gives

$$\mathcal{A}_e = -\frac{4}{3}A_{pol}^{FB} = 0.1498 \pm 0.0049, \quad (34)$$

$$\mathcal{A}_\tau = -A_{pol} = 0.1439 \pm 0.0043. \quad (35)$$

The correlation between $\mathcal{A}_e$ and $\mathcal{A}_\tau$ is small (+1.2%). To the measurements of A_{pol} and $\frac{4}{3}A_{pol}^{FB}$ a small correction is applied to take into account the difference between the centre-of-mass energy and the Z pole, the effects of the photon exchange, the $Z-\gamma$ interference and initial and final state radiation. The correction amounts to $\sim +0.005$ in both cases and its uncertainty (~ 0.0002) is small because of the

Fig. 5. Angular distribution of the tau polarisation measured by the four LEP experiments.²⁹ The solid and dashed lines represents the result of fits without and with the assumption of lepton universality, respectively.

precise knowledge of the beam energy at LEP. Assuming lepton universality the two measurements can be combined, giving

$$\mathcal{A}_\ell = 0.1465 \pm 0.0033.$$

The uncertainty on $\mathcal{A}_\ell$ was statistically dominated, the systematic component is equal to 0.0015. The corresponding value of the effective weak mixing angle is

$$\sin^2 \theta_{\text{eff}}^\ell = 0.23159 \pm 0.00041.$$

The $\mathcal{A}_e$ and $\mathcal{A}_\tau$ measurements from the four LEP Collaborations are shown in Fig. 6 and compared to the $\mathcal{A}_e$ measurement of SLD.

6. Interpretations

6.1. The determinations of $\sin^2 \theta_{\text{eff}}^\ell$

The measurements of the asymmetries presented in the previous Sections can be interpreted as a measurement of $\sin^2 \theta_{\text{eff}}^\ell$. For the leptonic forward–backward asymmetries, for the measurements of $\mathcal{A}_e$ and $\mathcal{A}_\tau$ from tau polarisation, and for the measurement of A_{LR} the interpretation requires the only assumption of lepton universality. The derivation of $\sin^2 \theta_{\text{eff}}^\ell$ from hadronic measurements requires the

Fig. 6. Comparison of the $\mathcal{A}_e$ and $\mathcal{A}_\tau$ measurements by the ALEPH (A), DELPHI (D), L3 (L) and OPAL (O) experiments. The ellipses give the standard error countours (corresponding to 39% CL for a two-dimension gaussian). The combination of the four experiments is represented by a star (central value) and a thicker ellipse. The horizontal band indicates the $\mathcal{A}_e$ measurement by SLD (S) from the left–right asymmetry; the allowed range is given by plus and minus one standard deviation with respect to the central value. (Courtesy of André Rougé.)

knowledge of the $\mathcal{A}_q$ terms that, as already discussed, have only a mild dependence on $\sin^2 \theta_{\text{eff}}^\ell$ in the Standard Theory (ST). For this class of measurements the validity of the ST for the $\mathcal{A}_q$ terms is assumed; this assumption is corroborated by the direct measurements of $\mathcal{A}_b, \mathcal{A}_c$ using polarised beams, which agree with the ST.

A compilation of the various results is shown in Fig. 7, where the dependence of $\sin^2 \theta_{\text{eff}}^\ell$ on the Higgs boson mass is also indicated. The six results shown in the figure are obtained, respectively, from the lepton forward–backward asymmetry, the tau polarisation, the left–right asymmetry, the b forward–backward asymmetry, the c forward–backward asymmetry and measurements of charge asymmetry using all quark flavours. The average of the six measurements gives:

$$\sin^2 \theta_{\text{eff}}^\ell = 0.23153 \pm 0.00016$$

with a χ^2 of 11.8 for five degrees of freedom corresponding to a confidence level of 3.7%. This confidence level is relatively low, because the most precise determinations, based on A_{LR} and on the b asymmetry are about 3σ apart. From the experimental point of view A_{LR} and the combination of b asymmetry measurements are both dominated by statistical errors, with accurate studies of the much lower systematic uncertainties. In particular the b asymmetry was measured by the four LEP experiments with two very different methods (Section 3.2) yielding largely uncorrelated results; the χ^2/dof of the eight-measurement combination was 0.55 showing agreement among experiments and methods. On the other hand a

Fig. 7. The determinations of $\sin^2 \theta_{\text{eff}}^\ell$ from the measurements described in this chapter and their average.²⁹ The measurements are, starting from the top, the lepton forward–backward asymmetry, the tau polarisation, the left–right asymmetry, the b forward–backward asymmetry, the c forward–backward asymmetry and the jet charge asymmetry using all quark flavours. The results are compared to the ST prediction, as a function of the Higgs boson mass. The uncertainty due to $\alpha(m_Z^2)$ on the ST predictions is indicated by a band. The effect of varying the top mass within the range indicated in the figure is added as two extra side bands.

departure of the b couplings from their ST expectation seems to be excluded by the precise measurements of $\mathcal{A}_b$ and R_b . Therefore in the $\sin^2 \theta_{\text{eff}}^\ell$ average and in the subsequent extraction of the couplings this discrepancy is assumed to be related to a statistical fluctuation.

Recently new determinations of $\sin^2 \theta_{\text{eff}}^\ell$ have been published by experiments at hadron colliders, Tevatron and LHC, based on lepton-pair production in $p\bar{p}/pp$ collisions (Drell–Yan events).⁴¹ The electroweak mixing angle is determined by the forward–backward asymmetry in the centre-of-mass, approximated by the Collins–Soper frame,⁴² of dileptons with invariant mass close to the Z mass. The measurement requires an assumption on the direction of the incoming quark/antiquark, therefore it depends on the knowledge of the parton distribution functions (PDFs). Precisions comparable to the less precise measurements in Fig. 7 have been obtained.

6.2. Extraction of neutral current couplings

The couplings of the neutral current to leptons ($\ell = e, \mu, \tau$) can be determined using three ingredients:

- the Z leptonic partial widths,^{11–14}
- the $\mathcal{A}_\ell$ parameters as determined by the leptonic asymmetries,
- the energy dependence of the leptonic forward–backward asymmetries.

The partial width of the decay $Z \rightarrow \ell^+ \ell^-$, gives the sum of the squares of the couplings, following the relationship

$$\Gamma_{Z \rightarrow \ell^+ \ell^-} = \frac{\sqrt{2} G_F m_Z^3}{12\pi} [g_{V\ell}^2 + g_{A\ell}^2], \quad (36)$$

while the ratio of the vector and axial couplings is given by the leptonic measurements of $\mathcal{A}_\ell$ (Eq. (4)), i.e. by the measurement of A_{LR} , of the tau polarisation and of the leptonic forward–backward asymmetries. The energy dependence of the asymmetries (Eq. (12)) fixes the value of the axial couplings, up to a common sign. This last ingredient is required, since the widths and asymmetries do not change if $g_{V\ell}$ and $g_{A\ell}$ replace each other, as can be seen from Eqs. (36) and (4).

The measured vector and axial couplings to electron, muon and tau are compared in Fig. 8 to test the hypothesis of lepton universality. The measurements are in agreement and lepton universality is tested to less than 0.1% for axial couplings

Fig. 8. The vector and axial couplings of the neutral current to electrons, muons and taus.²⁹ The 68% CL allowed regions are shown with dashed, dotted and dash-dotted lines, respectively. The combination of the measurements from the three lepton species, assuming lepton universality, is also shown (full line). The shaded area shows the ST prediction, within the allowed values for the top and Higgs boson masses. The uncertainty on $\alpha(m_Z)$ is indicated by the small arrow.

and to a few percent for the smaller vector couplings:

$$\begin{aligned} \frac{g_{V\mu}}{g_{Ve}} &= 0.961 \pm 0.063 & \frac{g_{A\mu}}{g_{Ae}} &= 1.0002 \pm 0.00064 \\ \frac{g_{V\tau}}{g_{Ve}} &= 0.958 \pm 0.030 & \frac{g_{A\tau}}{g_{Ae}} &= 1.0019 \pm 0.00073. \end{aligned}$$

The b and c quark couplings can be extracted with the same procedure adopted for the lepton case, by using the measurements of R_b ,⁴³ and R_c ,⁴⁴ the values of $\mathcal{A}_b$ and $\mathcal{A}_c$ determined by the polarised heavy quark asymmetries, and the energy dependence of the b and c forward-backward asymmetries. With this method the axial (vector) b couplings can be tested to a precision of approximately 2% (3%). Similar precisions are obtained with the tests of the c couplings (the bounds in this case are somewhat weaker mainly because of the larger uncertainty on the measured value of R_c). All couplings are found to agree with the ST.

The $\mathcal{A}_q$ parameters for b and c quarks can also be evaluated from the unpolarised b and c asymmetries using Eq. (2) and the value of $\mathcal{A}_e$ derived from $A_{FB}^0(\ell)$, from the tau polarisation and from A_{LR}^0 . This interpretation of the heavy quark unpolarised asymmetries is bound to lead, however, to a rather low value of $\mathcal{A}_b$ (0.881 ± 0.017 , compared to the ST expectation of 0.935) because of the 3σ discrepancy between $A_{FB}^0(b)$ and A_{LR}^0 already mentioned in the discussion concerning the determination of $\sin^2 \theta_{\text{eff}}^\ell$. As a consequence a rather high (low) value of the axial (vector) b couplings is obtained and the agreement with the ST is marginal for both couplings.²⁹ It must be stressed, however, that this discrepancy is totally correlated with the one seen in Fig. 7.

The measurements of the vector and axial couplings for various fermion species are depicted in Fig. 9. The regions allowed by the experimental measurements at 68% CL are shown. The precision obtained for the lepton measurements is

Fig. 9. The vector and axial couplings of the neutral current to various fermion species.²⁹ The regions allowed by the experimental measurements (68% CL) are shown.

impressive: with the scale used in this figure the three measurements are represented by three superimposed dots. Considerable precision is obtained also for the heavy quark couplings. Constraints are obtained on the couplings of lighter quarks by measurements of forward-backward asymmetries using kaons⁴⁵ and high-momentum stable particles.⁴⁶ As the large uncertainties of these measurements do not allow the study of the energy dependence, the contours indicating the allowed regions are symmetric with respect to the line $g_{Vf} = g_{Af}$. The constraints on neutrino couplings are computed from the measurement of the Z invisible width,^{11–14} assuming three neutrino families with identical neutral couplings. In this case the experimentally allowed region is represented by a very thin ring.

7. Summary and Outlook

In the years 1989–1998 the precision on the Weinberg electroweak mixing angle from lepton and quark asymmetries reached an impressive relative uncertainty of 0.07%. Such a precision is not easily attainable at hadron colliders, because it requires an accurate knowledge of the initial state. Eventually it could be reached at LHC depending on the progress in understanding parton distribution functions.

The measurements of lepton neutral couplings reminded in these pages improved by two order of magnitudes the tests of neutral-current lepton-universality available before the start of LEP and SLC, based on νe and $\nu\mu$ scattering. The values of $g_{V\ell}$ and $g_{A\ell}$ compared with ST predictions (Fig. 9) clearly indicated a low mass for the Higgs boson, as confirmed in 2012 by the direct observation at LHC.^{47,48}

Quark neutral couplings were also measured in the same decade. Heavy quark couplings (c and b) were determined at the level of a few percent, however the ratio of vector to axial b couplings, when extracted by comparing the measurement of the b forward-backward asymmetry to leptonic asymmetries measurements, shows a deviation of about 3σ with respect to the ST expectation. Whether this discrepancy can be ascribed to a statistical fluctuation, or to other effects, will be determined by future experiments.

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