Chapter 10

The W Boson Mass Measurement

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The measurement of the W boson mass has been growing in importance as its precision has improved, along with the precision of other electroweak observables and the top quark mass. Over the last decade, the measurement of the W boson mass has been led at hadron colliders. Combined with the precise measurement of the top quark mass at hadron colliders, the W boson mass helped to pin down the mass of the Standard Model Higgs boson through its induced radiative correction on the W boson mass. With the discovery of the Higgs boson and the measurement of its mass, the electroweak sector of the Standard Model is over-constrained. Increasing the precision of the W boson mass probes new physics at the TeV-scale. We summarize an extensive Tevatron (1984–2011) program to measure the W boson mass at the CDF and DØ experiments. We highlight the recent Tevatron measurements and prospects for the final Tevatron measurements.

1. Introduction

A series of fixed target and collider experiments have motivated and confirmed both the matter content of the Standard Theory (ST) in terms of fermion multiplets¹ as well as the gauge transformations under the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group.¹ In the area of electroweak-symmetry breaking, the predicted Higgs mechanism³ has been spectacularly confirmed recently by the observation of the Higgs boson at the LHC.² In weak interaction physics, the direct measurements of the W boson mass and width have steadily improved in precision. Measurements of the effective Weinberg angle $\sin^2 \theta_{\text{eff}}^\ell$ at hadron colliders are approaching the precision formerly achieved at LEP and SLD.

The direct measurement of the Higgs boson mass² has provided the last missing parameter defining the electroweak sector in the ST. As a result, M_W and $\sin^2 \theta_{\text{eff}}^\ell$ can now be predicted at loop-level in terms of other known quantities in the ST, as well as in extensions of the ST.⁴ Therefore, the ST and its extensions can be

stringently tested by precise measurements of M_W and $\sin^2 \theta_{\text{eff}}^\ell$. New physics can cause observable departures from their Standard Theory values.

2. History of the W Mass Measurement

After the W and Z bosons were discovered⁵ by the UA1 and UA2 experiments at the $Spp\bar{S}$ at CERN, these experiments made the initial measurements of their mass. The CDF experiment used the Tevatron Run 0 data, and both the CDF and DØ experiments used the Tevatron Run 1 data^{6,7} to perform more precise measurements of the W boson mass. The electron-positron collider LEP II increased its energy above the Z -boson pole and started producing W boson pairs concurrently with the Tevatron Run 1. The scan of cross section as a function of collider center-of-mass energy yielded the first LEP II measurements of M_W at threshold. With further increase in collider energy, higher statistics and precision were obtained when the semi-leptonic and all-hadronic decay channels of the WW system were reconstructed for the M_W measurement.

The CDF and DØ measurements of M_W from the Tevatron Run 1 were combined to yield^{6–8}

$$M_W = 80454 \pm 59 \text{ MeV}. \quad (1)$$

The final combined result⁹ from ALEPH,¹⁰ DELPHI,¹¹ L3¹² and OPAL¹³ experiments at LEP II was

$$M_W = 80376 \pm 33 \text{ MeV}. \quad (2)$$

A recent review¹⁴ of the Tevatron Physics program provides a summary of other electroweak physics measurements.

3. Theoretical Considerations of M_W

The loop-level expression for the predicted M_W in terms of other known quantities is¹⁵

$$M_W^2 \left(1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi\alpha}{\sqrt{2}G_F} \frac{1}{1 - \Delta r}, \quad (3)$$

where Δr represents the loop-induced radiative corrections. The tree-level relation in the ST is obtained by setting $\Delta r = 0$. α is the electromagnetic coupling and G_F is the Fermi constant extracted from the muon decay lifetime. The coupling of the W and Z bosons to the Higgs field's vacuum expectation value v determines their tree-level masses. In the ST, the contributions to Δr are dominated by (i) the running of the electromagnetic coupling due to light-quark loops, (ii) the contribution from the top (t) and bottom (b) quark loop in the W boson propagator, and (iii) the loops in the W boson propagator involving Higgs bosons. The top and bottom quarks have large Yukawa couplings to the Higgs field, and the difference between their couplings

is also large. This difference severely breaks the “custodial” $SU(2)$ symmetry which connects the tree-level relationship between the W and Z boson masses to the weak mixing angle. As a result, the $t\bar{b}$ loop makes an additional contribution to splitting between the W and Z boson masses. Furthermore, the mixing between the T_3 generator of $SU(2)_L$ and the $U(1)_Y$ generator caused by $\sin^2 \theta_{\text{eff}}^\ell \neq 0$ causes a difference between the WWh and ZZh couplings. Because of this difference, Higgs boson loops also contribute to the splitting between the W and Z boson masses. If a smaller value of $\sin^2 \theta_{\text{eff}}^\ell$ had occurred in nature, an interesting consequence would have been that the M_W measurement would be less sensitive to the Higgs boson mass m_H .

New physics that contributes to the precision electroweak observables primarily through loop corrections to the gauge-boson self-energies, i.e. through propagator corrections, can be generally described by the S, T, U oblique parameters.¹⁶ These parameters relate to the gauge-boson self-energies $\Pi(Q^2)_{VV'}$ (where Q^2 is the renormalization scale) as follows: the slope ($\Pi'_{VV'}$) of $\Pi_{VV'}$ with respect to Q^2 is given by S , the difference of $\Pi(0)_{WW}$ and $\Pi(0)_{ZZ}$ is given by T , and the difference of slopes Π'_{WW} and Π'_{ZZ} is given by U . Propagator effects of new physics that violate the custodial $SU(2)$ symmetry are captured by T and U . The intercept and/or the slope of Π are more easily affected by new physics contributions than a *difference* in the slopes for the W and Z boson propagators. Since new physics contributions to U tend to be of higher order than contributions to S and T , it is common to work in the $U = 0$ approximation for simplicity.

These oblique parameters are defined to be zero in the ST. The radiative corrections to M_W and $\sin^2 \theta_{\text{eff}}^\ell$ due to new physics can be written as

$$\begin{aligned}\Delta r &\approx \Delta r^{SM} + \frac{\alpha}{2s_W^2} S + \frac{\alpha c_W^2}{s_W^2} T + \frac{s_W^2 - c_W^2}{4s_W^4} U, \\ \Delta \sin^2 \theta_{\text{eff}}^\ell &= \Delta \sin^2 \theta_{\text{eff}}^{\ell, SM} + \frac{\alpha}{4(c_W^2 - s_W^2)} S + \frac{\alpha s_W^2 c_W^2}{c_W^2 - s_W^2} T.\end{aligned}\quad (4)$$

Measurements of Δr and $\Delta \sin^2 \theta_{\text{eff}}^\ell$ impose a two-dimensional constraint on new physics in the ST plane, because the coefficients of S and T are different in Eqn. 4. Constraints in the ST plane from the data are shown in Fig. 1. Improved electroweak measurements can guide the search for new physics and complement direct searches, as illustrated by the ST variation from two models of new physics shown in Figs. 1 and 2 respectively.

4. Tevatron M_W Measurements from Run 2

The CDF and DØ experiments at Fermilab’s Tevatron $p\bar{p}$ collider have produced four measurements of M_W from the Run 2 ($\sqrt{s} = 1.96$ TeV) data. The two

Fig. 1. (a) Illustration of the constrained region of ST parameter space from measurements, reproduced with permission from Ref. 17, and (b) illustration of the constrained region of ST parameter space from measurements, compared to a range of predictions from Littlest Higgs models. Figure (b) is reproduced with permission from Ref. 18.

Fig. 2. Illustration of the constrained region of ST parameter space from measurements, compared to a range of predictions from warped extra-dimensional models without (a) and with (b) an additional custodial symmetry introduced. Figures reproduced with permission from Ref. 18.

measurements from CDF,^{19,20} using 200 pb^{-1} and 2.2 fb^{-1} , respectively, of integrated luminosity, are $M_W = 80413 \pm 48 \text{ MeV}$ and $M_W = 80387 \pm 19 \text{ MeV}$. The second measurement subsumed the former result as it was inclusive of the data used for the first measurement. The two measurements^{21,22} from the DØ experiment used consecutive, independent datasets corresponding to 1 fb^{-1} and 4.3 fb^{-1} , respectively, of integrated luminosity to measure $M_W = 80401 \pm 43 \text{ MeV}$ and $M_W = 80367 \pm 26 \text{ MeV}$. The combined Tevatron result²³ is

$$M_W = 80387 \pm 16 \text{ MeV}. \quad (5)$$

Hadron colliders are now the source of the most precise measurements of M_W , having significantly surpassed the precision achieved by LEP II.

5. Techniques for M_W Measurement at Hadron Colliders

The key elements of the experimental technique to measure M_W have not changed over the last two decades.²⁴ During this time, the simulation of W boson production and decay, the detector response and resolution, and the detector calibrations have become increasingly more accurate. Subtle effects have steadily been understood and included in the calibration of the data or in the simulation.

Inclusive production of W bosons, describable by quark-antiquark annihilation at leading order (LO) in QCD, is by far the largest source of W bosons at the Tevatron. This sample provides the basis for the M_W measurement. The two-body decay of W bosons to quark-antiquark pairs, which have the largest branching ratio, is fully reconstructible as an invariant mass peak. However, the hadronic decay of the W boson is not usable for the mass measurement for two reasons. Firstly, the systematic uncertainties on jet energy calibration are simply too large to allow competitive measurements of M_W . Secondly, the QCD dijet background is very large and the dijet mass peak from W boson decay sits above an enormous QCD background. It is difficult to trigger on this mode with high acceptance due to the background rates. Hence the use of the hadronic decay mode for the M_W measurement is practically impossible. However, electron and muon momenta can be precisely measured and accurately calibrated, and these leptonic decays can be triggered and identified with high efficiency and small backgrounds. Thus, the M_W measurement has always been performed with the leptonic decay modes.

The invariant mass cannot be reconstructed due to the undetectable neutrino in the two-body leptonic decay of the W boson. Many of the complications and systematic uncertainties associated with the M_W measurement are related to the presence of the neutrino. The observable momenta in these events are the 3-momentum of the lepton and the transverse momentum (p_T) of the hadronic “recoil” particles which balance the p_T of the boson. The longitudinal component of the momentum of these recoiling particles is not discernable because of the unknown energy-momentum flowing down the beampipe. Compared to electron-positron colliders where the final-state 4-momentum is fully reconstructible, it is a disadvantage at hadron colliders that the most of the energy associated with the interactions of the spectator partons is flowing at very small angles to the beam, outside the detector acceptance.

Hadron collider experiments exploit the characteristic feature called Jacobian edge in the p_T distribution of the charged lepton. This feature is present in any two-body decay, where the p_T distribution rises up to $p_T \sim M_W/2$ and falls rapidly past this value. The Jacobian edge can be understood in the rest frame of the W boson, where in the limit of zero intrinsic width, a rest-frame decay angle of 90° with respect to the beam axis leads to the largest possible p_T of the leptons. Since the edge occurs at half the mass of the decaying particle, its location provides sensitivity to the W boson mass.

The lepton p_T distribution is affected by the angular distribution of the boson decay in its rest frame, and by the p_T -boost of the boson. The effect of $p_T(W)$ has been taken into account using two approaches. In one approach, the lepton p_T distribution is fitted using simulated templates and the simulation includes a theoretical calculation of the boson kinematics, including the boson p_T spectrum. The theoretical model is constrained using the $p_T(Z)$ measurement where the $Z \rightarrow \ell\ell$ kinematics can be measured well. Thus, in this approach the theoretical model is only used to relate the p_T spectra of the W and Z bosons.

In the second approach, $\vec{p}_T(W)$ is measured in each event and used in conjunction with $\vec{p}_T(\ell)$. Typically, the hadronic activity recoiling against the W boson is small and diffuse without the presence of hard jets. The quantity $\vec{u}_T$ is defined as the inclusive vector sum of transverse energies over all calorimeter towers (excluding towers containing energy deposits from the charged lepton), and $\vec{p}_T(W) \equiv -\vec{u}_T$. The $\vec{u}_T$ measurement is biased by the non-linear response and resolution of the detector, as well as by underlying event energy (from spectator parton interactions) and additional $p\bar{p}$ collisions. Since $\vec{u}_T$ cannot be reliably corrected for these detector effects, they have to be carefully included in the simulation. The neutrino p_T vector $\vec{p}_T(\nu) \equiv -\vec{p}_T(\ell) - \vec{u}_T$ is inferred by imposing transverse momentum balance.

A quantity that combines the information in $\vec{p}_T(\ell)$ and $\vec{u}_T$ is the transverse mass m_T , which is analogous to the invariant mass but computed using only the transverse quantities; $m_T = \sqrt{2p_T^\ell p_T^\nu (1 - \cos \Delta\phi)}$, where $\Delta\phi$ is the azimuthal opening angle between the two leptons. The Jacobian edge is also present in the m_T distribution. In fact, while the Jacobian edge in the $p_T(\ell)$ distribution is smeared by $p_T(W)$, the edge in the m_T distribution is not affected to first order in $p_T(W)/M_W$ because the measurement of $p_T(W)$ is incorporated into m_T . The disadvantage of this approach is the systematic uncertainty associated with the $\vec{u}_T$ measurement. The distributions of m_T , $p_T(\ell)$ and $p_T(\nu)$ are all used to extract (correlated) measurements of M_W , providing cross-checks since they have different systematic uncertainties.

5.1. Lepton momentum and energy calibration

The precision on the charged lepton's energy/momentum calibration is the single most important aspect of the M_W measurement. The electron energy is measured using the uranium-liquid argon (U-LAr) sampling calorimeter of the DØ experiment,²² and its direction is obtained from the scintillator fiber tracker. The track momentum resolution was considered to be inadequate for this measurement and therefore the muon channel was not used.

The absolute energy scale is set by calibrating the measured $Z \rightarrow ee$ boson mass to the world-average value.²⁵ In order to extrapolate the calibration from the Z boson mass to the W boson mass, non-linearity in the response has to be accounted for. One of the issues in the calibration of the energy response is understanding the

sources of non-proportional response. The energy lost by the electron in the passive material upstream of the calorimeter is not proportional to its energy, hence it causes the calorimeter response to be non-proportional to the original electron's energy. This has been studied in detail in the $D\bar{O}$ analysis using the longitudinal segmentation of the calorimeter. The U-LAr electromagnetic calorimeter provides readout in four longitudinal segments, with the first two samples corresponding to ≈ 2 radiation lengths (X_0) each, and the third (fourth) sample corresponding to 7 (10) X_0 , all at normal incidence. The amount of material traversed upstream of the calorimeter affects the electron shower development, which is reflected in the fractional energy deposition in the first three longitudinal samples of the electromagnetic calorimeter. These longitudinal energy fractions are measured in $Z \rightarrow ee$ data (shown in Fig. 3) and reproduced in the GEANT-based²⁶ detector simulation by incorporating an additional passive layer. These studies are performed in bins of electron pseudo-rapidity, and cross-checked with the $W \rightarrow e\nu$ data. Non-linear effects are further constrained by studying the variation of the measured Z boson mass with electron energy. The energy response for electrons is characterized by a scale factor α and an offset β , with results shown in Fig. 3.

The dependence of the calibration on pseudo-rapidity and instantaneous luminosity is also studied carefully. The electron energy response depends on the instantaneous luminosity for two reasons. Firstly, it affects the underlying event energy deposited in the electron cone. Secondly, the particle flux through the calorimeter depends on the instantaneous luminosity, which in turn affects the average current through the signal boards and the high-voltage lost across them. The high voltage affects the response in the liquid Argon gaps. The electron energy resolution is also studied carefully and its dependence on pseudo-rapidity and other factors is simulated.

Fig. 3. The amount of uninstrumented material in front of the EM calorimeter of the $D\bar{O}$ experiment, in units of radiation lengths of copper (a), and the fit results for the electron energy scale α and offset β (b). Both results are extracted from fits to the $D\bar{O}$ data. Different bins of instantaneous luminosity are shown as the different ellipses in (b) with consistent results. Instantaneous luminosity is shown in units of $36 \times 10^{30} \text{ cm}^{-2} \text{ s}^{-1}$. Figures reproduced with permission from Ref. 22.

The strategy^{19,27} for the lepton momentum calibration on the CDF experiment is to understand the central drift chamber²⁸ and the solenoid magnetic field²⁹ from first principles. One advantage of this strategy is that both electron and muon tracks can be used, roughly doubling the available statistics. More importantly, a number of additional systematic cross-checks become available to verify the robustness of the result. The CDF electromagnetic (EM) sampling calorimeter^{30,31} uses lead absorber and plastic scintillator with relatively coarse transverse granularity compared to DØ, and with no longitudinal segmentation. There are $\approx 19\%$ radiation lengths of material upstream of the drift chamber, in which bremsstrahlung radiation occurs. Since bremsstrahlung photons are coalesced with the electron shower in the calorimeter, the energy resolution of the calorimeter cluster is better than the track-based measurement. The W and Z boson kinematics in the electron channel are reconstructed using the calorimeter cluster energy and the direction of the track. The tracker momentum calibration is transferred to the EM calorimeter by fitting the peak of $E_{\text{cal}}/p_{\text{track}}$ near unity. Electrons from $W \rightarrow e\nu$ and $Z \rightarrow ee$ decays are used for this purpose.

The CDF strategy provides the opportunity to make independent measurements of the $Z \rightarrow ee$ and $Z \rightarrow \mu\mu$ masses based on the calibrations of the tracker and the EM calorimeter. Comparing these Z boson mass measurements to the world-average value provides confirmation of the calibration strategy. The Z boson mass measurements are subsequently used as additional calibration points in order to exploit the full power of the data. Though not competitive with the LEP measurements, the CDF measurements of the Z boson mass are the most precise at hadron colliders.

The first step in the calibration of the tracker is to derive precise wire-by-wire alignment constants for the drift chamber (which has $\approx 30,000$ wires). This is done using cosmic ray tracks recorded *in-situ* with collider data. CDF developed the technique to fit both sides of the cosmic ray trajectory to a single helix by creating a special reconstruction algorithm³² for this purpose. This two-sided helix fit brings in unique constraints which are not available from collider tracks originating at the beamline. Due to these constraints, information³³ on internal deformations of the drift chamber (relative rotations of radial layers, and relative twists of the cylinder end plates) can be extracted from the hit residuals with respect to this fit.

Additional information on the gravitational and electrostatic deflections of the wires between the end plates is obtained by comparing the track parameters of the diametrically opposite segments of the same cosmic ray track, because in the absence of any biases these parameters should match. The goal is to minimize the biases in curvature and polar angle measurements and provide a highly linear tracker response in curvature by studying these effects in detail. After using these alignment constants for track reconstruction, the $\langle E_{\text{cal}}/p_{\text{track}} \rangle$ are compared between positrons and electrons and final tweaks to track parameters are applied to equalize them.

Fig. 4. The fractional momentum scale correction extracted from $J/\psi \rightarrow \mu\mu$ data on CDF, shown as a function of the average p_T^{-1} of the muons (a), and the distribution of $E_{\text{cal}}/p_{\text{track}}$ used to calibrate the EM calorimeter on CDF, overlaid with the best-fit simulation (b). Figures reproduced with permission from Ref. 27.

In the CDF simulation, which is customized for the M_W measurement, particles are propagated through a high-granularity spatial grid of passive material towards the EM calorimeter. The grid is extracted from a detailed GEANT geometry which incorporates the silicon sensors, support and readout structures, the beampipe and the drift chamber's internal construction. During the particle propagation, the Landau-distributed ionization energy loss, bremsstrahlung (including detailed estimation of the Landau-Pomeranchuk-Migdal³⁴ suppression of soft photon bremsstrahlung), Compton scattering and e^+e^- conversion of photons, as well as multiple scattering^{35–37} are simulated. Hits are deposited on the drift chamber wires according to measured efficiencies and resolutions, and used to fit tracks in the same manner as the collider data. The absolute momentum scale, the total amount of passive material and magnetic field non-uniformity are measured using fits to the $J/\psi \rightarrow \mu\mu$ and $\Upsilon \rightarrow \mu\mu$ mass peaks, including the variation as a function of muon momentum (see Fig. 4) and polar angle.

A detailed GEANT4 simulation³⁸ of the lead-scintillator sampling calorimeter geometry is performed to understand the longitudinal shower development of electrons and photons. Low-energy nonlinearity due to absorption of soft shower particles, and high-energy non-linearity and non-gaussian resolution due to longitudinal shower leakage, are calculated. The sampling term in EM calorimeter resolution is also estimated from this simulation. These predictions are parameterized³⁸ and incorporated into the custom M_W simulation. The calorimeter thickness is tuned in pseudo-rapidity bins using the rate of events with low values of $E_{\text{cal}}/p_{\text{track}}$, while the radiative material upstream of the calorimeter is tuned using rate of events with high values of $E_{\text{cal}}/p_{\text{track}}$. The $E_{\text{cal}}/p_{\text{track}}$ -based energy calibration (see Fig. 4) is repeated in bins of E_{cal} to measure the residual non-linearity.

Validation of these procedures is obtained by fitting for the $Z \rightarrow ee$ mass using sub-samples of radiative and non-radiative electrons. The fits are performed separately using either the calorimeter energies or the track momenta for reconstructing

the invariant mass. The consistency of all these fits gives confidence in the analysis procedures.

5.2. Hadronic recoil simulation

A first-principles simulation of the hadronic recoil vector $\vec{u}_T$ is hindered by its soft and diffuse nature, which also causes the calorimeter response to the boson $\vec{p}_T$ to be significantly less than one. Some of the causes are that soft particles with $p_T < 400 \text{ MeV}$ curl up in the magnetic field, soft photons may be absorbed in the upstream material, and neutron interactions and hadronic showers are more difficult to simulate than EM showers. Furthermore, the underlying event and additional $p\bar{p}$ collisions produce hadronic energy flow which is uncorrelated with the hard scatter and which degrade the resolution of the $\vec{u}_T$ measurement. The conclusion is that applying corrections to the measured $\vec{u}_T$ to account for these effects is not an effective method. Therefore, the response and resolution effects are modeled in the custom simulation.

A parametric description of the hadronic recoil response and resolution is extracted from the $Z \rightarrow \ell\ell$ data and beam-crossing triggers. The p_T -balance between $\vec{p}_T(\ell\ell)$ and $\vec{u}_T$ in Z boson events as a function of $p_T(\ell\ell)$ is used to tune the parameterization of the boson p_T response and resolution in the custom simulation. Information for the modeling of the underlying event and additional $p\bar{p}$ collisions is extracted from data events triggered randomly on beam crossings and on inelastic $p\bar{p}$ collisions (minimum-bias events).

Another important effect is that certain calorimeter towers receive large energy deposits from the charged lepton(s). This energy contamination masks the hadronic energy deposition in the same towers, therefore these towers have to be omitted from the calculation of $\vec{u}_T$. The hadronic energy that is lost in these towers creates a bias in the measured $\vec{u}_T$, and this bias $\Delta u_{||}$ is parallel to the direction of the lepton. In the calculation of m_T , the component $u_{||}$ of $\vec{u}_T$ that is parallel to the lepton enters linearly, while the perpendicular component $u_{\perp}$ enters at higher order, thus any bias in $u_{||}$ directly biases m_T . Therefore $\Delta u_{||}$ and its dependence on event kinematics are measured in $W \rightarrow \ell\nu$ data and parameterized with care. Figure 5 shows the $u_{||}$ distribution from CDF and the W boson mass measurements in sub-samples separated by $u_{||}$ from $D\bar{O}$.

5.3. Backgrounds

Backgrounds in the $W \rightarrow \ell\nu$ sample arise from three sources; (i) Z boson events in which a lepton is lost and most of its energy is undetected or misreconstructed, causing large missing E_T to be inferred, (ii) irreducible background from $W \rightarrow \tau\nu \rightarrow \ell\nu\bar{\nu}\nu$, and (iii) QCD jets being misidentified as leptons. Most of the backgrounds are small (of $\mathcal{O}(1\%)$ or less) except for the $Z \rightarrow \mu\mu$ background in the CDF analysis. The background fractions and kinematic shapes have to be

Fig. 5. The distribution of $u_{||}$ from CDF overlaying data and simulation (a), and W boson mass fits from two sub-samples of $D\bar{O}$ data binned in $u_{||}$ (b). Figures reproduced with permission from Refs. 27 and 22 respectively.

determined to a precision commensurate with the other systematic uncertainties in the analysis.

An electron can be lost in the gap between the central and endcap calorimeters or between the azimuthal modules in CDF central EM calorimeter, where the calorimeter is poorly instrumented. If the associated track is also not reconstructed, a $Z \rightarrow ee$ event mimics a $W \rightarrow e\nu$ event. This background is determined from the data by $D\bar{O}$ and from a combination of simulation and data by CDF. If a muon from a $Z \rightarrow \mu\mu$ decay is emitted outside the pseudo-rapidity acceptance of the CDF central drift chamber (which extends up to $|\eta| \approx 1$), the event will mimic a $W \rightarrow \mu\nu$ candidate event. This background is estimated from the CDF simulation as it is geometrical in origin. The Z boson background has a bigger effect on the W mass fit than a monotonically falling background distribution because of the Jacobian edge in the Z boson distribution, which biases the fitted M_W upwards. The $W \rightarrow \tau\nu$ background is estimated from simulation including the effect of the τ polarization.

A combination of fragmentation and reconstruction effects can cause jets to be mis-identified as electrons. As these effects are rare and difficult to simulate, data-driven techniques are used to estimate the rates and distributions of such backgrounds. Multijet production where a jet fragments to an isolated, high- p_T $\pi^0 \rightarrow \gamma\gamma$, followed by an asymmetric $\gamma \rightarrow ee$ conversion in the detector material, is the typical source of mis-identified electrons. Mismeasurement of other jets can lead to the inference of sufficient missing E_T to satisfy the $W \rightarrow e\nu$ selection. A background-enriched sample, obtained by inverting or loosening the electron identification criteria or by relaxing the missing E_T requirement, is used to obtain the shapes of the background kinematic distributions. The background normalization is obtained by fitting the distribution of electron identification variables, or the

distribution of missing E_T , using corresponding templates of pure signal (from simulation) and background.

The rate for misidentification of jets as muons due to punch-through is small at CDF. However, another source of background arises due to $\pi/K \rightarrow \mu$ decays-in-flight (DIF). A low p_T meson can be mis-reconstructed as a high- p_T muon if it decays within the tracking volume of the drift chamber, due to the helical fit of a kinked trajectory. This background source of $W \rightarrow \mu\nu$ candidates arises from DIF in minimum-bias events. Poor track-fit χ^2 , large impact parameter, and the “seagull” — like pattern of hits are characteristics of DIF tracks that differentiate them from prompt muons. The DIF background has a hard spectrum of reconstructed p_T since the fitted track-curvature is approximately random and uniformly distributed.

5.4. Production and decay model

Fitting the data distributions to extract M_W requires a good theoretical understanding of W boson production and decay. The relevant aspects are (i) the parton distribution functions (PDFs) which determine the longitudinal momentum distribution of the W boson, (ii) the effect on the lepton p_T spectra from the transverse momentum of the W boson, (iii) the effect on the lepton p_T spectra and the correlation between the lepton and boson p_T due to the decay angular distribution of the W boson, and (iv) the kinematic distributions of the radiative photons which share energy with the charged lepton.

For a detector with full acceptance, the transverse momentum distributions of the decay leptons would be insensitive to the longitudinal momentum of the W boson. However, if only central leptons are used in the analysis, the acceptance for a given lepton p_T depends on the longitudinal boost. The fitted M_W therefore depends on the modeling of the longitudinal momentum distribution of the W boson, which is determined by the PDF choice. The PDFs provided by the global fitting groups have associated parametrizations of uncertainties. The uncertainty in the extracted M_W is obtained by reweighting the simulated events by the ratio of varied PDFs. The PDF error sets provided by the CTEQ³⁹ and MSTW⁴⁰ groups are used to quote the uncertainty in M_W . For the central value, DØ has used the CTEQ6.1 PDF set while CDF has used the CTEQ6.6 set. CDF has cross-checked that the central values obtained from the CTEQ6.6 and MSTW2008 PDF sets are consistent within the quoted uncertainty.

The relevant range of the boson p_T is $P_T(W) < 25$ GeV, where the spectrum is strongly affected by QCD parton showering and non-perturbative physics. The RESBOS⁴³ program includes NLO and the dominant NNLO amplitudes, resummation of parton showers and a non-perturbative form factor that depends on the beam-energy and Q^2 . The dilepton p_T spectrum (in the case of CDF) and the distribution of the azimuthal opening angle in $Z \rightarrow \ell\ell$ events (in the case of DØ) have been used to tune the parameters of the non-perturbative form factor. The

perturbative effects have a small impact on the M_W analysis since they are more important at high $p_T(W)$ and those events are not used for the measurement. The Q^2 -dependence has a small effect because it controls the difference between the $p_T(Z)$ and $p_T(W)$ spectra. Both the perturbative effects and the Q^2 -dependence have been obtained from global fits⁴¹ to data. Recently, the CDF analysis has fitted the $p_T(Z)$ spectrum with both the non-perturbative parameter and α_S as a second parameter.

The RESBOS program also calculates the decay angular distribution at NLO and partial-NNLO level. Higher-order effects have negligible impact at the current levels of precision. As the precision of future measurements improves, a complete NNLO calculation would be desirable.

CDF and DØ incorporate the rates and distributions of final-state radiative (FSR) photons using PHOTOS⁴² interfaced to RESBOS. CDF has calibrated PHOTOS-FSR against the more complete HORACE⁴⁴ calculation, which is an exact NLO electroweak calculation interfaced to a leading-logarithm photon shower. HORACE also corrects photons in the multi-photon shower for the difference between the leading-logarithm calculation and the NLO calculation. In the future, an exact NNLO electroweak calculation may be required.

5.5. Results

The measurements from CDF and DØ were summarized in Sec. 2. The uncertainties in these measurements are shown in Table 1. The Tevatron (world) average of $M_W = 80387(80385) \pm 16(15)$ MeV²³ is about 1.6σ above the ST prediction: $M_W = 80358 \pm 8$ MeV.¹⁷ The comparison puts stringent bounds on new physics. The inputs for the ST prediction are the Z -pole measurements from LEP and SLD, the top quark mass from Tevatron and LHC experiments, the Higgs boson mass from LHC, and a recent determination of the hadronic vacuum polarization contribution to $\alpha_{EM}(M_Z^2)$.

Table 1. Uncertainties in units of MeV on the combined result (m_T fit) on M_W from CDF (DØ) using 2.2 (4.3) fb⁻¹ of integrated luminosity. “na” denotes the uncertainty is not individually tabulated.

Source	CDF Uncertainty	DØ Uncertainty
Lepton energy scale and resolution	7	17
Recoil energy scale and resolution	6	5
Lepton tower removal	2	na
Backgrounds	3	2
PDFs	10	11
$p_T(W)$ model	5	2
Photon radiation	4	7
Statistical	12	13
Total	19	26

CDF and DØ have analysed a quarter and a half, respectively, of their full Run 2 dataset, and the full datasets are being analyzed. The PDF uncertainty is the dominant source of uncertainty and it is correlated between the two experiments. The PDFs can be constrained by the W boson charge asymmetry and the Z boson rapidity distributions. Including PDF constraints from the Tevatron and the LHC can help obtain a combined Tevatron measurement of M_W with a total uncertainty of 10 MeV.

6. Summary and Conclusions

The Tevatron experiments have published a variety of measurements using W and Z bosons which tested higher-order QCD calculations and the electroweak sector of the ST. The precision electroweak measurement program at the Tevatron has included M_W , Γ_W and the Z boson forward–backward asymmetry A_{FB} . In particular, the impact of the M_W measurement on the electroweak sector has been equivalent to the asymmetry measurements from LEP and SLD. Further improvement in the precision of M_W is anticipated with a goal of 10 MeV, and these will be legacy measurements from the Tevatron.

Beyond the Tevatron, the LHC promises nearly unlimited samples of W and Z bosons for the M_W measurement with vanishing statistical error. The challenge at the LHC will be to overcome the issues associated with various sources of systematic uncertainty. As we have learnt from the Tevatron experience, we can be optimistic that improvements in analysis techniques over time will continue to overcome the challenges.

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References

1. S. Glashow, *Nucl. Phys.* **22**, 579 (1961); A. Salam and J. C. Ward, *Phys. Lett.* **13**, 168 (1964); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967).
2. G. Aad *et al.* (The ATLAS Collaboration), *Phys. Lett. B* **716**, 1 (2012); S. Chatrchyan *et al.* (The CMS Collaboration), *Phys. Lett. B* **716**, 30 (2012).
3. P. W. Anderson, *Phys. Rev.* **130**, 439 (1960); F. Englert and R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964); P. W. Higgs, *Phys. Rev. Lett.* **13**, 508 (1964); G. S. Guralnik, C. R. Hagen, and T. W. B. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964).
4. M. Ciuchini, E. Franco, S. Mishima, and L. Silvestrini, *J. High Energy Phys.* **08**, 106 (2013).
5. G. Arnison *et al.* (UA1 Collaboration), *Phys. Lett. B* **122**, 103 (1983); UA2 Collaboration (M. Banner *et al.*), *Phys. Lett. B* **122**, 476 (1983); UA1 Collaboration (G. Arnison *et al.*), *Phys. Lett. B* **126**, 398 (1983); UA2 Collaboration (P. Bagnaia *et al.*), *Phys. Lett. B* **129**, 130 (1983).
6. CDF Collaboration (T. Affolder *et al.*), *Phys. Rev. D* **64**, 052001 (2001).
7. D0 Collaboration (V. M. Abazov *et al.*), *Phys. Rev. D* **66**, 012001 (2002); D0 Collaboration (B. Abbott *et al.*), *Phys. Rev. D* **62**, 092006 (2000); D0 Collaboration (B. Abbott *et al.*), *Phys. Rev. D* **58**, 092003 (1998).
8. D0 Collaboration (V. M. Abazov *et al.*) (CDF Collaboration, The D0 Collaboration, Tevatron Electroweak Working Group), *Phys. Rev. D* **70**, 092008 (2004).
9. ALEPH, CDF, D0, DELPHI, L3, OPAL, and SLD Collaborations, the LEP Electroweak Working Group, the Tevatron Electroweak Working Group, and the SLD Electroweak and Heavy Flavour Groups, CERN Report No. CERN-PH-EP-2010-095, and Fermilab Report No. FERMILAB-TM-2480-PPD, 2010.
10. ALEPH Collaboration (S. Schael *et al.*), *Eur. Phys. Jour. C* **47**, 309 (2006).
11. DELPHI Collaboration (J. Abdallah *et al.*), *Eur. Phys. Jour. C* **55**, 1 (2008).
12. L3 Collaboration (P. Achard *et al.*), *Eur. Phys. Jour. C* **45**, 569 (2006).
13. OPAL Collaboration (G. Abbiendi *et al.*), *Eur. Phys. Jour. C* **45**, 307 (2006).
14. D. Bandurin *et al.*, *Int. J. Mod. Phys. A* **30**, 1541001 (2015).
15. A. Sirlin, *Phys. Rev. D* **22**, 971 (1980).
16. M. E. Peskin and T. Takeuchi, *Phys. Rev. Lett.* **65**, 964 (1990); M. Golden and L. Randall, *Nucl. Phys. B* **361**, 3 (1990); B. Holdom and J. Terning, *Phys. Lett. B* **247**, 88 (1990); M. E. Peskin and T. Takeuchi, *Phys. Rev. D* **46**, 381 (1992); G. Altarelli and R. Barbieri, *Phys. Lett. B* **253**, 161 (1991); G. Altarelli, R. Barbieri and S. Jadach, *Nucl. Phys. B* **369**, 3 (1992) [Erratum-ibid. B **376**, 444 (1992)].
17. Gfitter Group Collaboration (M. Baak *et al.*), *Eur. Phys. Jour. C* **74**, 3046 (2014).
18. M. Baak, M. Goebel, J. Haller, A. Hoecker, D. Kennedy, K. Mönig, M. Schott, and J. Stelzer (The Gfitter Group), *Eur. Phys. Jour. C* **72**, 2003 (2012).
19. CDF Collaboration (T. Aaltonen *et al.*), *Phys. Rev. Lett.* **99**, 151801 (2007); CDF Collaboration (T. Aaltonen *et al.*), *Phys. Rev. D* **77**, 112001 (2008).
20. CDF Collaboration (T. Aaltonen *et al.*), *Phys. Rev. Lett.* **108**, 151803 (2012).
21. D0 Collaboration (V. M. Abazov *et al.*), *Phys. Rev. Lett.* **103**, 141801 (2009).
22. D0 Collaboration (V. M. Abazov *et al.*), *Phys. Rev. D* **89**, 012005 (2014).
23. CDF and D0 Collaborations (T. Aaltonen *et al.*), *Phys. Rev. D* **88**, 052018 (2013).
24. A. V. Kotwal and J. Stark, *Ann. Rev. Nucl. Part. Sci.* **58**, 147 (2008).
25. Particle Data Group (K. Nakamura *et al.*), *J. Phys. G* **37**, 075021 (2010).
26. R. Brun and F. Carminati, CERN Program Library Long Writeup, W5013, 1993 (unpublished), version 3.15.
27. CDF Collaboration (T. Aaltonen *et al.*), *Phys. Rev. D* **89**, 072003 (2014).

28. T. Affolder *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **526**, 249 (2004).
29. H. Minemura *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **238**, 18 (1985).
30. L. Balka *et al.*, *Nucl. Instrum. Methods Phys. Res. Sect. A* **267**, 272 (1988).
31. K. Yasuoka, S. Mikamo, T. Kamon, and A. Yamashita, *Nucl. Instrum. Meth. Phys. Res. Sect. A* **267**, 315 (1987).
32. A. V. Kotwal, H. K. Gerberich and C. Hays, *Nucl. Instrum. Methods Phys. Res. Sect. A* **506**, 110 (2003).
33. A. V. Kotwal and C. Hays, *Nucl. Instrum. Methods Phys. Res. Sect. A* **762**, 85 (2014).
34. A. B. Migdal, *Phys. Rev* **103**, 1811 (1956); L. D. Landau and I. J. Pomeranchuk, *Dokl. Akad. Nauk. SSSR* **92**, 535 (1953); **92**, 735 (1953).
35. H. Bichsel, *Rev. Mod. Phys.* **60**, 663 (1988).
36. G. R. Lynch and O. I. Dahl, *Nucl. Instrum. Methods Phys. Res. Sect. B* **58**, 6 (1991).
37. MuScat Collaboration (D. Attwood *et al.*), *Nucl. Instrum. Methods Phys. Res. Sect. B* **251**, 41 (2006).
38. A. V. Kotwal and C. Hays, *Nucl. Instrum. Methods Phys. Res. Sect. A* **729**, 25 (2013).
39. P. Nadolsky, H. Lai, Q. Cao, J. Huston, J. Pumplin, D. Stump, W. Tung, and C.-P. Yuan, *Phys. Rev. D* **78**, 013004 (2008); J. Pumplin, D. Stump, J. Huston, H. Lai, P. Nadolsky, and W. Tung, *J. High Energy Phys.* **0207**, 012 (2002).
40. A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, *Eur. Phys. J. C* **63**, 189 (2009).
41. F. Landry, R. Brock, P. Nadolsky, and C.-P. Yuan, *Phys. Rev. D* **67**, 073016 (2003).
42. P. Golonka and Z. Was, *Eur. Phys. J. C* **45**, 97 (2006).
43. F. Landry, R. Brock, P. M. Nadolsky, and C.-P. Yuan, *Phys. Rev. D* **67**, 073016 (2003); C. Balazs and C.-P. Yuan, *Phys. Rev. D* **56**, 5558 (1997); G. A. Ladinsky and C.-P. Yuan, *Phys. Rev. D* **50**, 4239 (1994).
44. C. M. Carloni Calame, G. Montagna, O. Nicrosini, and A. Vicini, *J. High Energy Phys.* **10** 109 (2007).
45. CDF Collaboration (T. Aaltonen *et al.*), *Phys. Rev. Lett.* **100**, 071801 (2008).
46. D0 Collaboration (V. M. Abazov *et al.*), *Phys. Rev. Lett.* **103**, 231802 (2009).