Chapter 12

Global Fits of the Electroweak Standard Theory: Past, Present and Future

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The last decades have seen tremendous progress in the experimental techniques for measuring key observables of the Standard Theory (ST) as well as in theoretical calculations that has led to highly precise predictions of these observables. Global electroweak fits of the ST compare the precision measurements of electroweak observables from lepton and hadron colliders at CERN and elsewhere with accurate theoretical predictions of the ST calculated at multi-loop level.

For a long time, global fits have been used to assess the validity of the ST and to constrain indirectly (by exploiting contributions from quantum loops) the remaining free ST parameters, like the masses of the top quark and Higgs boson before their direct discovery. With the discovery of the Higgs boson at the Large Hadron Collider (LHC), the electroweak sector of the ST is now complete and all fundamental ST parameters are known. Hence the global fits are a powerful tool to probe the internal consistency of the ST, to predict ST parameters with high precision, and to constrain theories describing physics beyond the ST.

In this chapter we review the global fits of the electroweak sector of the ST from an experimentalist's perspective. We briefly recall the most important achievements from the past (mainly driven by the precise measurements of Z pole observables), discuss the present situation after the accurate measurements of the top quark and Higgs boson masses, and present prospects of the fits as expected from new measurements at the LHC and future lepton colliders.

1. Introduction

Formulated during the 60s and 70s of the last century, the Standard Theory (ST) of particle physics describes the elementary particles and their electromagnetic, weak and strong interactions as a relativistic quantum field theory using a $U(1)_Y \times SU(2)_L \times SU(3)_C$ gauge theory.^{1–5} The past few decades have seen tremendous progress in the experimental techniques for measuring key observables of the ST and at the same time theoretical progress which led to highly precise

theoretical calculations of these observables in the ST. A dedicated world-wide effort to study the electroweak sector of the ST with precise measurements, at lepton colliders (LEP, SLC) around the Z pole and beyond during the 90s,^{6,7} then the measurements of the top quark mass with increasing precision at hadron colliders (Tevatron, LHC),⁸ and finally the discovery of the Higgs boson at the LHC^{9,10} with the determination of its mass,¹¹ has turned this research area into a science of high precision.

The experimental and theoretical progress of the last decades make global fits of the electroweak sector an ideal concept for stringent tests of the ST validity and its internal consistency. Including the influence of loop effects in the calculations, physics can even be studied at much higher energy scales than available in the centre-of-mass energies of the existing particle colliders. Important achievements of the fits following this approach include the correct prediction of the masses of the top quark and the Higgs boson before their direct discovery.

Beginning with the description of the high-precision measurements of Z pole observables, during the last decades global fits have demonstrated impressively the consistency of the ST with the experimental data. After the direct measurements of the top quark and Higgs boson masses all electroweak ST parameters are measured experimentally and the influence of the loop corrections is fixed in the ST. As a result, global fits can indirectly predict key observables, like the mass of the W boson or the effective weak mixing angle, with a precision exceeding the direct experimental measurements. Hence such fits, in combination with precise measurements, form a powerful tool for global assessments of the accuracy of ST calculations.

At the same time multiple theoretical extensions of the ST have been proposed in the literature to solve known shortcomings of the ST, like e.g. the hierarchy problem or the existence of dark matter. These theories describing physics beyond the ST (BST) often lead to changes in the theoretical prediction of precision observables compared to the ST, e.g. from the effect of new hypothetical heavy particles to be considered in the calculation of the radiative corrections of the observables. In global fits these small changes can be exploited using the concept of oblique parameters which parametrise the deviation of the experimental data from the radiative corrections as calculated in the ST.^{12,13} A comparison of the determined values of these parameters with their prediction in various BST models — normally dependent on new parameters — allows one to constrain the parameters of the new theory, and sometimes the BST theory can be excluded entirely.

In this document we review global fits of the electroweak sector of the ST from an experimentalist's point of view. The document is organised as follows. The next section presents the ingredients to the fits, including a short recap of the theoretical ST calculations of observables and their experimental measurements available today. In Section 3 we discuss important milestones in the history of electroweak fits, while Section 4 gives an overview of the current status of the fit after the Higgs discovery. Section 5 summarises the constraints on BST physics derived from the

fits. In Section 6 the perspectives of the fits at future colliders, like an additional run at the LHC or lepton colliders, are discussed.

2. Ingredients of Electroweak Fits

There are two ingredients needed for global fits: accurate measurements and precise calculations. In global fits of the electroweak sector of the ST, both types have evolved together and influenced each other over the last decades: the calculations have adapted to what the experiments can measure and the measurements have adapted to what can be calculated best. The measurements as well as the calculations entering the fits have been reviewed in detail in dedicated chapters of this book. Here only a brief summary is presented.

Because of their clean experimental and theoretical environment, observables from experiments at the e^+e^- colliders LEP and SLC could be measured during the 90s with very high precision. For these electroweak precision measurements around the Z resonance the concept of pseudo-observables has been adapted.¹⁴ In this approach a pseudo-observable is an observable in a world without initial and final state radiation and with only a Z boson exchanged in the process $e^+e^- \rightarrow f\bar{f}$. Typical pseudo-observables are the mass and width of the Z boson, the forward-backward asymmetry for a produced fermion or the polarisation of final state τ leptons. Since the experiments measure the observables including radiation and photon exchange after unavoidable experimental cuts, corrections must be applied which are, however, model independent in the sense that they only depend on QED effects and do not influence the radiative corrections that may contain new physics. The latter appear in the predictions of the pseudo-observables. Due to precise second-order calculations including exponentiation the uncertainties of the QED corrections are not relevant for the final precision.¹⁵ Close to the Z pole only radiative corrections affecting the Z propagator are of interest while the contributions from weak box diagrams are numerically irrelevant.¹⁴

For these reasons electroweak corrections can be parametrised with only three parameters: $\Delta\rho_f$ normalising the absolute Z fermion couplings, $\Delta\kappa_f$ correcting the weak mixing angle obtained from the ratio of the vector to the axial-vector coupling, normally written as $\sin^2\theta_{\text{eff}}^f = (1 + \Delta\kappa_f)\sin^2\theta$, and Δr correcting the relation between the fine structure constant α , the Fermi constant in muon decays G_F , the mass of the Z boson M_Z and the mass of the W boson M_W . A dependence of these parameters on the fermion type f is introduced by vertex corrections. These are independent of most BST effects apart from the case of the b quark, where corrections containing a top quark can be important.

2.1. Experimental measurements

The coupling sector of the electroweak model is given by three parameters, so the three most precisely measured ones are used for the predictions.

These are¹⁶: α known to a relative precision of $\Delta\alpha/\alpha = 3 \cdot 10^{-10}$, G_F known to $\Delta G_F/G_F = 5 \cdot 10^{-7}$ and M_Z measured precisely in the scan of the Z resonance in e^+e^- collisions.

From measurements at CERN’s LEP collider and SLC two classes of observables are used: the total and partial decay widths of the Z boson and asymmetries measured at the Z pole.⁶ The total and partial widths are sensitive to the absolute size of the vector and axial vector couplings of the Z ($\propto (g_V^2 + g_A^2)$), while the asymmetries are sensitive to the ratio of these couplings (g_V/g_A) and thus to the effective weak mixing angle. The ratio of the Z partial decay widths to hadrons and leptons is also sensitive to the strong coupling constant $\alpha_s(M_Z^2)$. In addition, the fits use the mass and width of the W boson, both measured at LEP2 and the Tevatron.¹⁷ For the determination of the loop corrections in principle the masses of all fermions and of the Higgs boson are needed. However, in practice only the mass of the top quark m_t , measured at the Tevatron and the LHC,⁸ and the mass of the Higgs boson M_H , measured at the LHC,^{11,18,19} are relevant. All relevant input observables that enter the electroweak fits are summarised in Table 1. The exact definitions for the Z observables and $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ can be found in Ref. 6.

Table 1. Observables used in electroweak fits.

Observable	Description	Collider
<i>Z</i> resonance scan:		
M_Z	Mass of the Z boson	LEP
Γ_Z	Width of the Z boson	LEP
σ_0^{had}	Hadronic pole cross section	LEP
R_ℓ^0	Ratio of hadronic and leptonic partial width	LEP
<i>Z</i> asymmetries:		
$A_{\text{FB}}^{0,1}$	Forward–backward asymmetry for leptons	LEP
$\mathcal{A}_\ell(\text{LEP})$	Asymmetry parameter from the τ -polarisation	LEP
$\mathcal{A}_\ell(\text{SLD})$	Asymmetry parameter from the left–right asymmetry	SLC
$\sin^2 \theta_{\text{eff}}^\ell(Q_{FB})$	Weak mixing angle from inclusive quark asymmetries	LEP
$A_{\text{FB}}^{0,b}$	Forward–backward asymmetry for b quarks	LEP
$A_{\text{FB}}^{0,c}$	Forward–backward asymmetry for c quarks	LEP
$\mathcal{A}_b$	Asymmetry parameter for b quarks	SLC
$\mathcal{A}_c$	Asymmetry parameter for c quarks	SLC
Other inputs:		
R_b^0/R_c^0	Ratio of the Z partial width to b/c quarks to its hadronic width	LEP, SLC
M_W	Mass of the W boson	LEP, TEV
Γ_W	Width of the W boson	LEP, TEV
m_t	Mass of the top quark	TEV, LHC
M_H	Mass of the Higgs boson	LHC
$\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$	Hadronic contribution to $\alpha(M_Z^2)$	

2.2. Theoretical predictions

In general, calculations at two-loop order precision are available for all pseudo-observables today. The effective mixing angle $\sin^2\theta_{\text{eff}}^f$ is known up to two-loop order with leading three- and four-loop terms.^{20–22} The W mass is calculated to the same order with the addition of four-loop QCD corrections.^{23–26} For the Z partial decay widths also the full two-loop corrections are available apart from closed fermion loops.²⁷ Final state QED and QCD radiation is included via radiator functions known to $\mathcal{O}(\alpha_s^4)$ for massless final-state quarks, $\mathcal{O}(\alpha_s^3)$ for massive quarks,^{28–30} and $\mathcal{O}(\alpha^2)$ for contributions with closed fermion loops.³¹ The width of the W boson has only been calculated at one-loop order precision only,³² but the experimental precision is much worse than for the other observables.

3. Important Milestones of the Electroweak Fit

Global fits of the electroweak sector of the ST are possible if the input parameters are over-constrained with a precision better than effects from new particles at the 1-loop level. As mentioned earlier the electroweak sector of the ST is determined by three parameters and the three most precise ones, α , G_F and M_Z were chosen. α and G_F were known with good precision already for a long time. With the start of data taking at LEP the value of M_Z was quickly measured with a much better precision than all electroweak observables (apart from α and G_F). All other observables could then be predicted using these three parameters plus the loop corrections from the new parameters of interest.

Already at the Singapore conference in 1990 the mass of the top quark could be predicted from a global fit to be $m_t = 139 \pm 32 \pm 20 \text{ GeV}$, where the first error is experimental and the second quantifies the impact of the unknown Higgs boson mass.³³ This fit used the very first data obtained at LEP as well as the ratio of the masses M_W/M_Z measured at CERN's UA2 experiment and the Tevatron and a value of the weak mixing angle $\sin^2\theta$ measured in lepton nucleon scattering experiments. At that time the LEP data already allowed to set a direct lower limit of 44 GeV on the Higgs boson mass. The above-mentioned uncertainty on m_t from the unknown Higgs boson mass was evaluated by varying M_H in the fits between the lower LEP limit and 1 TeV.

From 1993 onwards the LEP electroweak working group (LEPEW) published official fits every year.³⁴ For these fits the ST predictions for the central values of the (pseudo-)observables were obtained from calculations with the ZFITTER program.³⁵ The results were always cross-checked with the TOPAZ program,³⁶ giving identical results.

A major confirmation of the concept of global electroweak fits was obtained in 1995 when the top quark was discovered by CDF³⁷ and D0³⁸ with a mass of $m_t = 176 \pm 8(\text{stat.}) \pm 10(\text{sys.}) \text{ GeV}$ (CDF) and $m_t = 199_{-21}^{+19}(\text{stat.}) \pm 22(\text{syst.}) \text{ GeV}$ (D0). The prediction from electroweak fits at that time was $m_t = 178 \pm 8(\text{exp.}) \pm$

Fig. 1. (a) Prediction of the top quark mass obtained from electroweak fits^{33,34,39–41} compared with the direct measurements⁴² as a function of time. (b) Prediction of the Higgs boson mass obtained from electroweak fits^{34,41,42} compared with the direct measurements^{9–11} as a function of time.

19(Higgs) GeV, beautifully confirming the ST at the loop level. From 1995 onwards the data were also precise enough to allow simultaneous fits of m_t and M_H . Figure 1(a) compares the prediction of the mass of the top quark obtained in global fits with the results of the direct measurements as a function of time. Good agreement is always seen. Starting from 2009, more direct information on the value of the mass of the Higgs boson — initially from searches at the Tevatron and CERN’s LHC, later from the direct mass measurements at LHC — have allowed the global fits to determine the mass of the top quark with much better precision, as indicated by the dark band in Fig. 1(a) (cf. Section 4).

Once a reasonably precise top-mass measurement was available from CDF and D0, indirect limits on the mass of the Higgs boson could also be obtained from electroweak fits. The Higgs-mass prediction obtained in these fits as a function of time is shown in Fig. 1(b). From the beginning the fit results have shown nice agreement with the value of $M_H \approx 125$ GeV later found by CERN’s ATLAS and CMS,^{9,10} again confirming the ST at the loop-level. Figure 2(a) shows the $\Delta\chi^2$ profile of the last fit obtained by the LEP electroweak working group³⁴ containing all LEP data and an M_W value from the Tevatron close to the present precision.⁷ The minimum is located around 100 GeV indicating the preference of a light Higgs. The absolute value of χ^2 of the fits has always been good, with a fit probability better than 10%. A detailed description of the fit and its results together with the data measured at LEP and the SLD can be found in Ref. 6.

Since a long time a slight tension is determined between $\sin^2 \theta_{\text{eff}}^{\ell}$ obtained from the left–right asymmetry using polarised beams at SLD, A_{LR} , and the forward–backward asymmetry for b quarks measured at LEP, A_{FB}^b , with A_{LR} showing a preference for small M_H values, while the measured value of A_{FB}^b indicates larger Higgs masses. Many studies have been performed trying to clarify the situation. However the ST, including the precise measurements of the mass of the top quark and the Higgs boson, predicts a value almost exactly in the middle of the two measurements,

Fig. 2. (a) $\Delta\chi^2$ profile of the global fit as a function of M_H as obtained by the LEP Electroweak Working Group.^{7,34} (b) Comparison of the fit results with the direct measurements in units of the experimental uncertainty.^{41,46}

supporting the interpretation that the tension is most likely a statistical fluctuation.

Over the years other groups have performed electroweak fits as well. The Particle Data Group (PDG) regularly performs a fit working strictly in the $\overline{\text{MS}}$ scheme.¹⁶ The Gitter group has written a fit package in C++ trying to include always the latest experimental data and theoretical predictions.⁴¹ The groups obtained very similar results, demonstrating that the treatment of data and ST predictions is in general very well understood.

The LEPEW also tried to include electroweak measurements from lower energies like atomic parity violation,⁴³ Møller scattering⁴⁴ and neutrino–nucleon scattering.⁴⁵ These measurements contribute little to the precision, however in the case of neutrino–nucleon scattering large ununderstood systematic uncertainties contribute significantly; the low energy data are no longer used for the central results.

4. Current Status After the Higgs Discovery

The discovery of a new boson in 2012 by the ATLAS⁹ and CMS¹⁰ collaborations at the LHC and the subsequent confirmation that this boson has a spin of zero and that its properties are consistent with the properties of the ST Higgs boson^{19,47–49} constitutes one of the greatest triumphs of the ST. A mass value around 125 GeV is in striking agreement with the indirect ST prediction obtained using the tool of global

electroweak fits, which have always predicted a light Higgs by exploiting the contributions from quantum loops to electroweak precision observables (cf. Fig. 1(b)).

Assuming the new boson discovered by ATLAS and CMS is indeed the long sought ST Higgs boson, and interpreting its precise mass measurement as a measurement of M_H in the ST, has a dramatic impact on the scope and the interpretation of global fits of the electroweak sector of the ST. For the first time all fundamental parameters of the ST are known and all electroweak observables can be predicted at the loop level allowing stringent consistency tests. For observables like the top quark mass, the W boson mass and the weak mixing angle the knowledge of the mass of the Higgs boson leads to an enormous reduction of the uncertainty of the prediction of the observable^a based on two-loop calculations in the ST. These more precise predictions can be confronted with the direct measurements and often the fit results are more precise. Furthermore models beyond the ST can be tested using the approach of oblique parameters without the additional uncertainty from the Higgs boson mass.

Due to the weak logarithmic dependence of the radiative corrections on M_H the results of the electroweak fit are insensitive to the exact combination method of the ATLAS and CMS mass measurements. In Ref. 46 a weighted average of $M_H = 125.14 \pm 0.24$ GeV was used which agrees within 1.3σ with the prediction of the fit. Consequently, including the M_H measurement in the list of input observables leads only to a small change of the goodness-of-fit compared to earlier results with global p -values around 0.20.⁴⁶ In Fig. 2(b) a comparison of the fit results with the direct measurements in units of the experimental uncertainty is shown for each observable. These pull values demonstrate impressively that the ST prediction is able to describe all input measurements consistently: no observable shows a pull value exceeding 3σ . The long-standing tension between leptonic and hadronic asymmetries is clearly visible (cf. Section 3).

The dark-grey region of Fig. 3(a) indicates the currently allowed regions at 68% and 95% CL in the $(M_W$ vs. m_t) plane of the fit using the Higgs boson mass as measured by ATLAS and CMS.^{11,18,19,b} For comparison the ST prediction for various values of M_H is also indicated as thin solid and dotted lines. The uncertainties on other input parameters, like e.g. $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$, and uncertainties in the theoretical calculation, e.g. on M_W from missing higher orders, widen the allowed region around the thin line of $M_H = 125.14$ GeV such that the dark-grey area is obtained. The allowed region of a fit without using the M_H measurement is indicated in the figure by the light-grey regions. From a comparison of the dark-grey and light-grey regions it can be seen that the additional M_H information drastically reduces the

^aThe top quark mass cannot be predicted directly but only indirectly using the consistency of loop effects.

^bUsing the private Higgs-mass average instead of the official value, which was not yet available at the time of the fits, results in no visible difference.

Fig. 3. (a) Allowed regions in the $(M_W \text{ vs. } m_t)$ plane for fits including (dark) and excluding (light) the M_H measurement.⁴⁶ For comparison the direct M_W and m_t measurements are shown as vertical and horizontal bands and ellipses. The corresponding direct measurements are excluded from the fits. (b) Allowed regions in the $(M_W \text{ vs. } \sin^2 \theta_{\text{eff}}^l)$ plane with the same notation as used in the left figure.⁴⁶

allowed values of the ST parameters m_t and M_W . The direct and indirect results on M_W and m_t are in agreement within their uncertainties which constitutes an important consistency test of the ST after the Higgs discovery. Similarly, in Fig. 3(b) the allowed regions of the fit are shown in the $(M_W \text{ vs. } \sin^2 \theta_{\text{eff}}^l)$ plane. Again the impact of the new M_H measurement is clearly visible.

Including the new M_H measurements in the prediction of the W mass in the electroweak fit (i.e. the direct M_W measurement is excluded) leads to:⁴⁶ $M_W = 80.358 \pm 0.008 \text{ GeV}$, which exceeds the precision of the world average of the direct measurements with an uncertainty of 15 MeV.¹⁷ For the indirect determination of the effective leptonic weak mixing angle $\sin^2 \theta_{\text{eff}}^l$, the fit obtains its minimum at $\sin^2 \theta_{\text{eff}}^l = 0.23149 \pm 0.00007$, which is again more precise than the average of the LEP/SLD measurements⁶ with an uncertainty of 0.00016. Since the top quark mass enters the electroweak fit only in loop corrections the precision of the fit prediction is worse ($\pm 2.4 \text{ GeV}$) than the direct measurement (currently $\pm 0.76 \text{ GeV}$ ⁸). These values demonstrate impressively the precision in the indirect determination of ST parameters that can be obtained using the tools of global fits after the Higgs discovery.

5. Constraints on Physics Beyond the ST

As explained the precision observables on the Z pole and M_W can be parametrised on loop level by three parameters: $\Delta\rho$, $\Delta\kappa$ and Δr , where the three parameters are largely correlated in terms of the input parameters.⁵⁰ In 1990, two similar parametrisations have been developed to take out these correlations: the STU parametrisation¹² and the ε parameters.¹³ In both parameter sets one parameter (T , ε_1) absorbs the large isospin violating corrections proportional to m_t^2 . Another parameter (S , ε_3) takes the rest of the corrections to $\sin^2 \theta_{\text{eff}}^l$ and the third parameter

Fig. 4. (a) Allowed regions in the $(\epsilon_3$ vs. ϵ_1) plane as obtained from the LEP/SLC data in 1998 compared to prediction from the ST (grey sickle) and from technicolour models.^{52,53} (b) Current experimental constraints on the oblique parameters S and T (with $U = 0$) using all observables (dark-grey) compared to the ST prediction (black dot).¹⁶ Individual experimental constraints are illustrated by dashed and dotted ellipses.

the remainder in M_W . The ϵ parameters are defined such that they are zero on Born level and typical deviations are of the order of α , while the STU parameters are zero in the ST with assumed values of m_t and M_H , and typical deviations are of the order of one. An additional parameter can be defined to absorb the vertex corrections to R_b .

These parametrisations allow the experiments to publish their data in a simple but reasonably model independent way by publishing the experimental results on the STU or ϵ parameters as obtained from a global fit. Theorists can then easily interpret them by comparing the experimental values with the theoretical predictions obtained e.g. in models of physics beyond the ST. In the past decades this concept was heavily used to constrain various BST models. In Ref. 51 a collection of example results are given.

A first application of this idea using the ϵ parameters was an analysis which compared the experimental results with technicolour (TC) predictions.⁵² Figure 4(a) shows the $(\epsilon_3$ vs. ϵ_1) plane as measured (meaning obtained in a global fit) in 1998, compared to the prediction in the ST and the technicolour predictions using a simple copy of QCD at higher scales. While the ST prediction was in agreement with the experimental results, this family of TC models could clearly be excluded at this early stage.

In Fig. 4(b) the current experimental situation in the $(T$ vs. $S)$ plane is compared to the ST prediction whose uncertainty is much reduced by the availability of the M_H measurement. The influence of the various experimental observables is illustrated by the dotted and dashed ellipses. The values obtained currently for

$U = 0^c$ show a strong correlation between S and T and are compatible with zero within uncertainties.^{16,46} Hence, the data are in impressive agreement with the ST prediction ($S = T = 0$). If only one of the three parameters is allowed to vary, this parameter would deviate at the 1.5σ level, reflecting the slight deviation in M_W .¹⁶

Most models tested in Ref. 51 are excluded by now by the discovery of the Higgs boson and the measurement of its properties. In recent analyses the STU parameters have been analysed together with the Higgs coupling measurements and they improve significantly limits on universal extra dimensions⁵⁴ or on the littlest Higgs model.⁵⁵

6. Perspectives of the Electroweak Fit

Also in the coming years the interplay between precision experiments and precision theory can be used to probe the ST with unprecedented accuracy and constrain physics models beyond the ST. From experiments at the LHC and future colliders it is expected that some of the key observables entering the global ST fits and featuring a strong sensitivity to electroweak loop effects will be measured with increased experimental precision. At the LHC the datasets to be collected in the future can be used to reduce the systematic uncertainties of crucial measurements (e.g. the energy calibration of the detectors). The clean experimental environment of e^+e^- collider will allow for highly precise measurements with often small systematic and theoretical uncertainties. On the theoretical front multi-loop corrections to these observables should become available and the accuracy of the determination of the hadronic contribution to the fine-structure constant $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ will be increased. In Ref. 46 a detailed study of the prospects of the global ST fit for the LHC with 300 fb^{-1} and ILC/GigaZ is presented. The interested reader is referred to the original literature. Here only the most important aspects relevant for the next two decades of particle physics are presented.

Among the key observables where experimental progress can be expected at the above-mentioned facilities are the W boson mass, the top quark mass and the effective weak mixing angle. The precision on the Higgs mass itself is of minor importance for the fit due to its weak logarithmic dependence in the loop calculations. For the W boson mass a combination of the LEP, Tevatron and LHC (with 300 fb^{-1}) could⁵⁶ optimistically lead to a total precision of 8 MeV, while at the ILC/GigaZ a precision of 5 MeV is assumed from cross-section measurements around the WW production threshold.⁵⁷ The top mass measurements at the LHC with 300 fb^{-1} could reach a final experimental precision of 0.6 GeV. It is essential that the current theoretical uncertainties due to colour reconnection effects and the

^cThis is a reasonable choice since the contribution to U is negligible in most BST models.

mass definition are understood by then. At the moment an additional theory uncertainty of 0.5 GeV is assumed on the interpretation of the measured top quark mass. Scans of the $t\bar{t}$ production threshold at ILC should yield a precision of only 30 MeV where an additional theoretical uncertainty of about 100 MeV is conservatively taken into account.^{57,58} While the LHC is unlikely to be able to improve the precision of any electroweak observable related to the Z boson, the measurements of the left–right asymmetry A_{LR} of leptonic and hadronic Z decays at the ILC/GigaZ are expected to yield a precision of $1.3 \cdot 10^{-5}$ for $\sin^2 \theta_{\text{eff}}^\ell$ which represents an improvement of the present world average by more than a factor of 10.⁵⁷ The uncertainty on $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ is expected to reduce by roughly a factor of two.⁵⁹ Since theoretical progress is difficult to quantify exactly it has been assumed in Ref. 46 that all theoretical uncertainties will be reduced by a common factor of four, which in many cases will require ambitious three-loop electroweak calculations.

With these expected improvements the prospects of global fits can be estimated and compared to the current precision. In these studies the central values of the observables are adjusted to values compatible with a Higgs boson mass of $M_H \simeq 125$ GeV to allow a fair comparison. With the above-mentioned improvements on M_W and m_t the LHC can improve the indirect constraint on M_H from presently (with $M_H \simeq 125$ GeV) $_{-27}^{+33}$ GeV to $_{-18}^{+21}$ GeV. An even more substantial improvement is expected for the ILC/GigaZ with an expected M_H uncertainty of about ± 7 GeV.⁴⁶ The expected improvements from LHC and ILC/GigaZ on both the fit prediction and the direct measurements in the $(M_W \text{ vs. } \sin^2 \theta_{\text{eff}}^\ell)$ plane are illustrated in Fig. 5(a). The improved indirect fit precision expected in the LHC scenario from theoretical improvements and improvements in $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ and m_t would by far exceed the current precision of the direct measurements, which cannot be improved at the LHC significantly. A precise $\sin^2 \theta_{\text{eff}}^\ell$ measurement from the ILC would then be needed to confront the indirect fit determination with a direct measurement matching (or even exceeding) in precision. These studies at ILC/GigaZ

Fig. 5. (a) Indirect fit constraints in the M_W vs. $\sin^2 \theta_{\text{eff}}^\ell$ plane for the present and extrapolated future scenarios compared to the direct measurements.⁴⁶ (b) Present and expected results on the oblique parameters S and T with U fixed to zero.⁴⁶

will represent tests of the ST using global fits with unprecedented precision. At a future circular collider with about 100 km circumference, which is currently studied at CERN, also the uncertainties on the Z mass and width can be improved substantially. If beam polarisation can be established and the systematics can be brought under control there is also a potential to improve the measurement of the weak mixing angle by an order of magnitude.⁶⁰

The expected future constraints on S and T for a fixed value of $U = 0$ are shown in Fig. 5(b) for the LHC only and for LHC + ILC. Again the fit assuming current uncertainties of the observables is shown for comparison. The shift in the position of the ellipses between the present data and future scenarios is caused by different central values used for the electroweak observables in these scenarios to obtain $M_H \simeq 125$ GeV. The future scenarios are by construction centred at $S = T = 0$. The ST prediction with uncertainties is also indicated. Only a minor improvement is expected for the LHC, while an improvement by a factor of three to four is expected with future lepton colliders. With these studies the e^+e^- colliders provide excellent indirect sensitivity to physics effects beyond the ST. The error on the top quark mass will be the limiting factor until a more precise value will become available from the threshold scan at a lepton collider. The error on the Higgs boson mass is irrelevant in any scenario using the present precision.

If the constraint $U = 0$ in Fig. 5(b) is dropped, the parameters S and T are only determined by $\sin^2 \theta_{\text{eff}}^\ell$ and the leptonic partial decay width of the Z boson Γ_l which is determined from a scan of the Z resonance. However, improvements in Γ_l are difficult to predict due to the unclear situation in the determination of the beam energy at a linear accelerator and due to beamstrahlung effects. Without further improvements in Γ_l the $(T \text{ vs. } S)$ ellipse in Fig. 5(b) will get rather narrow in one direction due to the $\sin^2 \theta_{\text{eff}}^\ell$ constraint but will remain very long in the other. At a future circular collider the beam energy will be measured precisely with resonant depolarisation similar to LEP⁶¹ and beamstrahlung will be much weaker so that here significant progress on Γ_l can be expected⁶² allowing a precise S and T measurement without the $U = 0$ constraint.

7. Conclusion

During the last decades global fits of the electroweak sector of the Standard Theory have been a crucial tool for highly accurate tests of the model's consistency and for indirect predictions of unmeasured model parameters. Among the greatest achievements in the past are certainly the correct prediction of the mass of the top quark and more recently the mass of the Higgs boson before their actual direct discovery.

This impressive success is based on the intense interplay of precision measurements in collider experiments at CERN and elsewhere and highly accurate theoretical calculations taking into account quantum loop effects which give access to

physics at energy scales which are much higher than the energies directly available in the experimental facilities. The high precision of measurements of electroweak observables at the Z pole obtained at CERN's LEP collider, together with key measurements at the SLC and at hadron colliders are of crucial importance for these studies and allow tests of the theory with unprecedented accuracy.

With even more accurate measurements at future colliders and theoretical progress matching this precision, and e.g. including high-precision Higgs coupling measurements and predictions, the results of the electroweak fits will remain interesting for many years to come. Deeper insights into the fundamental building blocks of nature and their interactions will most certainly be obtained, and the effects of potential new physics phenomena may possibly be discovered.

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