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STRING THRESHOLD CORRECTIONS AND FLIPPED SU(5)

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ABSTRACT

We revise previous calculations of the effective unification scale m_{SU} at which the extrapolated low-energy gauge couplings should appear to become equal, and we show explicitly how to calculate m_{SU} in the fermionic construction of four-dimensional strings. In the case of the flipped SU(5) GUT derived from string, the SU(5) and U(1) couplings defined in the DR scheme become equal to g_{SU} at

$$m_{\text{SU}} \simeq 1.76 \times g_{\text{SU}} \times 10^{18} \text{ GeV}$$

This scale is significantly larger than m_{GUT} , the scale at which the low-energy SU(3) and SU(2) couplings become equal if extrapolated using the renormalization group equations of the minimal supersymmetric extension of the Standard Model. The existence of an intermediate SU(5) $\times$ U(1) phase could have an observable effect on the calculated value of $\sin^2 \theta_W$.

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In order for string theorists to convince the rest of the physics community that they have found the Theory of Everything (TOE), they need to find some experimental confirmation. A possible strategy for obtaining this confirmation is the derivation of a semblance of the Standard Model from string theory, and the verification of some distinctive string-theoretic prediction. Several realistic models have been derived from string [1], of which flipped SU(5) [2] is closest to our hearts. Possible predictions of string models include qualitatively new phenomena beyond the Standard Model, such as fractionally-charged particles [3], or new candidates for dark matter [4], or a spectacular large internal dimension at a few TeV [5], or quantitative predictions for some parameters of the Standard Model. The latter can be compared with predictions from supersymmetric GUTs for gauge couplings, the grand unification scale and quark masses.

In a conventional field-theoretical GUT, the experimental values of two gauge couplings, e.g., the fine structure constant α_{em} and the strong coupling α_3 , can be used to calculate the grand unification scale m_{GUT} and the common value $\alpha_3 = \alpha_2 = \alpha_1 = \alpha_{GUT}$ of the Standard Model gauge couplings at m_{GUT} . These can then be used to predict the ratio of α_2 and α_1 at present energies, namely $\sin^2\theta_W$ [6]. This prediction is unsuccessful if just the Standard Model particles are included in the perturbative two-loop renormalization group equations [7] below m_{GUT} , but agrees with LEP data to within ± 0.002 if the minimal supersymmetric extension of the Standard Model is used with sparticles weighing about 1 TeV [8], as is now well known [9]. The corresponding value of $m_{GUT} \simeq (1 \text{ to } 2) \times 10^{16}$ GeV, and $\alpha_{GUT} \simeq 1/24$.

In principle, a four-dimensional string model should be able to predict m_{GUT} and α_{GUT} . In practice, the string technology currently available [10, 11] enables one to calculate a string unification scale m_{SU} , at which all the extrapolated gauge couplings should appear to become equal. It is a necessary consistency condition on the GUT philosophy that $m_{GUT} \leq m_{SU}$, but it could well be that this inequality is strict, since m_{GUT} could be a dynamically-generated intermediate mass scale [2]. In this case there would be a range of scales $m_{GUT} \leq \mu \leq m_{SU}$ over which the effective gauge group was a product of the observable-sector GUT group with other factors. To determine whether there is such a GUT range of energies, it is clearly necessary to compare the value of m_{GUT} estimated on the basis of low-energy data with the calculation of m_{SU} in any specific string-derived model.

In this paper, we revise previous formulae for m_{SU} [10, 11], and show explicitly how to calculate m_{SU} in the fermionic construction of four-dimensional string models [12], using as example the flipped SU(5) model derived from string [2]. We find that in this model

$$m_{SU} \simeq 1.76 \times g_{SU} \times 10^{18} \text{ GeV} \quad (1)$$

which is significantly higher than the value of m_{GUT} estimated by extrapolation from low-energy data using the perturbative two-loop renormalization group equations for the minimal supersymmetric extension of the Standard Model [8, 9]. The version of flipped SU(5) derived from string contains, however, additional particles which could alter the predicted value of m_{GUT} . We present a simple one-loop calculation which indicates that there could

be a significant effect on the calculated value of $\sin^2\theta_W$ if there were even a short range of scales μ where the effective four-dimensional observable-sector gauge group was $SU(5) \times U(1)$.

In any field theory derived from a four-dimensional string model, the coupling constants g_i of the gauge group factors G_i are determined in terms of the string coupling g and the integer level k_i of the corresponding Kac-Moody algebra:

$$g_i^2 = \frac{g^2}{k_i} \quad (2)$$

before renormalization. The simplest way to unify all interactions is to take all the levels k_i equal to unity. This also guarantees the absence of “exotic” light representations in the spectrum: only triplets of $SU(3)$ and/or doublets of $SU(2)$ are allowed [12, 13]. Then, after renormalization, the gauge couplings g_i become equal at some string unification scale that is calculable in terms of the Planck mass $M_P \simeq 1.2 \times 10^{19}$ GeV, and threshold corrections [10, 11] that sum the effects of all massive string states weighing $\gtrsim (\alpha')^{-1/2} = gM_P/\sqrt{8\pi}$. At the one-loop level, these string corrections to the gauge couplings can be written in the form:

$$\frac{16\pi^2}{g_i^2(\mu)} = k_i \frac{16\pi^2}{g^2} + b_i \ln \frac{M^2}{\mu^2} + \Delta_i \quad (3)$$

where μ is a field-theoretical renormalization scale, the b_i are the field-theoretical one-loop β -function coefficients, and the Δ_i are the threshold corrections. The β -functions can be obtained from infrared divergences in string theory, with μ playing the rôle of an infrared cutoff, and the precise value of M depends on the renormalization scheme chosen.

When dealing, as here, with a field theory that is approximately supersymmetric, it is most convenient to use a renormalization scheme that respects supersymmetry. The scheme most often used in testing GUTs and in extracting $\alpha_3(m_Z)$ from experiment is the $\overline{MS}$ scheme [14], but this is not supersymmetric. However, it is closely related to the $\overline{DR}$ scheme [15], differing only in the treatment of traces of Dirac γ -matrices. Accordingly, we evaluate the gauge couplings appearing in (3) in the $\overline{DR}$ -scheme. This is not the scheme most immediate in string theory, which has an effective Pauli-Villars (PV) regularization scheme. String threshold corrections were considered in [11], but an apparently incorrect expression was given for M calculated in the PV scheme, which was also incorrectly identified with the $\overline{MS}$ scheme.

We follow [11] in defining the threshold corrections Δ_i by the following integrals over the modulus $\tau \equiv \tau_1 + i\tau_2$ of the world-sheet torus

$$\Delta_i = \int_{\Gamma} \frac{d^2\tau}{\tau_2} (B_i(\tau, \bar{\tau}) - b_i) \quad (4)$$

where the integral is restricted to the fundamental domain $\Gamma \equiv \{\tau : \tau_2 > 0, |\tau_1| < 1/2, |\tau| > 1\}$, and $B_i(\tau, \bar{\tau})$ is related to the string partition function $Z \equiv \text{Tr} q^H \bar{q}^{\bar{H}}$ by insertions of the square of the helicity operator Q_s , and of the i 'th gauge group generator Q_i :

$$B_i(\tau, \bar{\tau}) \equiv -\text{Tr} Q_s^2 Q_i^2 q^H \bar{q}^{\bar{H}} \quad (5)$$

where $q \equiv e^{i\pi\tau}$ and $H(\bar{H})$ is the string Hamiltonian for left-(right-) movers. In the limit $\tau_2 \rightarrow \infty$, B_i gives the β -function coefficient b_i , and the integral in (4) converges. The dependence of the threshold corrections Δ_i on the moduli parameters of the internal space has been calculated recently for symmetric orbifold compactifications of the ten-dimensional heterotic superstring [16]. With this definition of the Δ_i , we find that for gauge couplings defined by a PV scheme ⁵

$$M_{PV} = \left(\frac{2e^{1-\gamma} 3^{-3/2}}{\pi\alpha'} \right)^{\frac{1}{2}} \simeq 1.03 \times g \times 10^{18} \text{ GeV} \quad (6)$$

which is a factor of two larger than M defined in [11]. Converting now to the $\overline{DR}$ scheme, we find that

$$M_{\overline{DR}} = M_{PV} \quad (7)$$

using the arguments given in ref.[15].

In the fermionic construction of four-dimensional strings [12] the moduli parameters are fixed, and the partition function is written as a sum over all boundary conditions for the world-sheet fermions that define the model:

$$Z \equiv \frac{1}{|\eta(\tau)|^4} \sum_{\underline{a}, \underline{b}} c\left(\frac{\underline{a}}{\underline{b}}\right) Z\left[\frac{\underline{a}}{\underline{b}}\right] \quad (8)$$

The Dedekind functions $\eta(\tau)$ arise from the two transverse space co-ordinates, $\underline{a}$ and $\underline{b}$ are boundary condition vectors (component values 1 or 0 denote periodic or antiperiodic world-sheet fermions), the coefficients $c(\frac{\underline{a}}{\underline{b}})$ are phases which determine the GSO projections, and $Z[\frac{\underline{a}}{\underline{b}}]$ is a product of the contributions of all the individual world-sheet fermions. For example, a single complex fermion yields

$$Z\left[\frac{\underline{a}}{\underline{b}}\right] = \frac{\theta\left(\frac{\underline{a}}{\underline{b}}\right)}{\eta(\tau)} = \frac{1}{\eta(\tau)} \sum_{n=-\infty}^{\infty} e^{i\pi\tau(n+\frac{\underline{a}}{2})^2 + i\pi\tau b n} \quad (9)$$

in units where the string tension $(\alpha')^{-1} = 2$.

The eigenvalues of the helicity operator Q_s in the trace (5) are given by the charges corresponding to the two-dimensional $U(1)$ current $J_s = i\psi_3\psi_4$, where $\psi_{3,4}$ are the left-moving fermionic superpartners of the two transverse space co-ordinates. Therefore, the insertion of Q_s^2 in (5) implies that the θ -function in the contribution (9) of the fermionic space-time co-ordinates is replaced by:

$$\theta\left(\frac{\underline{a}_\psi}{\underline{b}_\psi}\right) \rightarrow \theta'\left(\frac{\underline{a}_\psi}{\underline{b}_\psi}\right) \equiv \sum_{n=-\infty}^{\infty} \left(n + \frac{\underline{a}_\psi}{2}\right)^2 e^{i\pi\tau(n+\frac{\underline{a}_\psi}{2})^2 + i\pi\tau \underline{b}_\psi n} = \frac{1}{i\pi} \frac{d}{d\tau} \theta\left(\frac{\underline{a}_\psi}{\underline{b}_\psi}\right) \quad (10)$$

⁵ $\gamma \simeq 0.57722$ is the Euler-Mascheroni constant.

Similarly, a Cartan generator of the gauge group can be expressed as a linear combination of charges corresponding to right-moving U(1) currents of the form $J_I \equiv \bar{\psi}_I \bar{\psi}_I^*$ for some complex two-dimensional fermions $\bar{\psi}_I$. The insertion of Q_I^2 in (5) then implies that $\bar{\theta}(\frac{a_I}{b_I}) \rightarrow \bar{\theta}'(\frac{a_I}{b_I})$, while the insertion of $Q_I Q_J$ for $I \neq J$ implies that one has to replace

$$\bar{\theta}(\frac{a_I}{b_I}) \rightarrow \hat{\theta}(\frac{a_I}{b_I}) \equiv \sum_{n=-\infty}^{\infty} (n + \frac{a_I}{2}) e^{i\pi\tau(n + \frac{a_I}{2})^2 + i\pi b_I n} \quad (11)$$

and similarly for $\bar{\theta}(\frac{a_J}{b_J})$.

The terms of the sum in the partition function (8) can be separated into three classes depending on the number $N = 4, 2$ or 1 of supersymmetries that are left unbroken by the boundary conditions $(\underline{a}, \underline{b})$. The $N = 4$ boundary conditions transform 8 left-moving fermions in the same way, and their contribution to (5) is proportional to

$$\sum_{s_1, s_2 = \{0,1\}} (-1)^{s_1+s_2} \theta'(\frac{s_1}{s_2}) \theta^3(\frac{s_1}{s_2}) \sim \frac{d}{d\tau} \left(\sum_{s_1, s_2} (-1)^{s_1+s_2} \theta^4(\frac{s_1}{s_2}) \right) \quad (12)$$

which is zero because of a well-known θ -function identity. On the other hand, the $N = 1$ boundary conditions leave two left-moving fermions simultaneously periodic, so their contribution to (5) is proportional to the odd θ -function $\theta(\frac{1}{1})$ (or $\theta'(\frac{1}{1})$), which vanishes identically. As a result, only the terms which preserve exactly $N = 2$ supersymmetries can make non-vanishing contributions to the threshold corrections. This was already observed in [16] for the moduli dependence of the threshold corrections, but there could in principle have been constant corrections to which the $N = 1$ sectors also contributed, as in the case of the $\mathbf{Z}_3$ orbifold [11]. However, the above analysis shows that such $N = 1$ contributions are absent in the fermionic constructions. The above analysis is general; now we focus on the specific fermionic model of interest to us.

The flipped $SU(5) \times U(1)$ gauge group is generated [2] by five complex two-dimensional fermions $\bar{\psi}^I (I = 1, \dots, 5)$ which all have identical boundary conditions. Notice that at least one boundary condition vector must transform the $\bar{\psi}^I$ with a non-trivial phase, otherwise the remaining gauge group is $SO(10)$. A natural string basis for the $U(5)$ Cartan generators comprises the charges Q^I of the two-dimensional U(1) currents $J^I \equiv \bar{\psi}^I \bar{\psi}^{*I}$, in which the corresponding couplings are manifestly equal at the tree level. By a simple change of basis, one can define the four traceless $SU(5)$ Cartan generators: $\frac{1}{\sqrt{2}}(Q_1 - Q_2)$, $\frac{1}{\sqrt{6}}(Q_1 + Q_2 - 2Q_3)$, $\frac{1}{\sqrt{12}}(Q_1 + Q_2 + Q_3 - 3Q_4)$ and $\frac{1}{\sqrt{20}}(Q_1 + Q_2 + Q_3 + Q_4 - 4Q_5)$, and the orthogonal U(1) generator $\frac{1}{\sqrt{5}}(Q_1 + Q_2 + Q_3 + Q_4 + Q_5)$. Since the ψ^I all have identical boundary conditions, one finds

$$Q_{SU(5)}^2 = Q_1^2 - Q_1 Q_2, \quad Q_{U(1)}^2 = Q_1^2 + 4Q_1 Q_2 \quad (13)$$

and hence

$$\delta Q^2 \equiv Q_{U(1)}^2 - Q_{SU(5)}^2 = 5Q_1 Q_2 \quad (14)$$

The difference between the $U(1)$ and $SU(5)$ threshold corrections, which determines the string unification scale m_{SU} at which the $\overline{DR}$ $U(1)$ and $SU(5)$ couplings become equal, is then given by (4-11), where the sum in (8) runs over the $N = 2$ sectors.

Let us now consider in more detail the function $\hat{\theta}(\frac{a}{b})$ of equation (11). Simple inspection of its form reveals that if $a = 0$ it vanishes for $b = 0$ or 1 , whilst if $a = 1$ it vanishes for $b = 0$. Furthermore, in the case $a = b = 1$ one starts with the product of 5 odd θ -functions $\bar{\theta}(\frac{1}{1})$ corresponding to the 5 complex fermions $\bar{\psi}^I$, out of which two are replaced by $\hat{\theta}(\frac{1}{1})$ according to Eq. (14), so the result is zero again. Hence a non-vanishing contribution can arise only if the a_I and/or b_I correspond to complex fermions with general twists that are neither periodic nor antiperiodic.

The revamped flipped $SU(5)$ string model [2] is defined by a set of 8 basis vectors of boundary conditions for the world-sheet fermions:

$$B = \{S, \zeta, b_1, b_2, b_3, b_4, b_5, \alpha\} \quad (15)$$

The vector S leaves periodic exactly 8 left-moving real fermions and plays the rôle of a supersymmetry generator: when added to a sector it gives its superpartner. The vector ζ leaves periodic exactly 8 right-moving complex fermions and generates an E_8 gauge group factor. All the vectors (15) denote fermions which are either periodic or antiperiodic, except α , which has components with values $1/2$ denoting fermions that are twisted by a phase $-i$, used to break $SO(10)$ to $SU(5) \times U(1)$. The basis vectors

$$B_4 \equiv \{S, \zeta, \mathbf{1} = b_1 + b_2 + b_3 + \zeta, b'_4 = b_1 + b_4, b'_5 = b_2 + b_5, \alpha\} \quad (16)$$

define an $N = 4$ space-time supersymmetric model with an “observable” $U(8) \times U(1)^6$ gauge group and a “hidden” $O(10) \times SU(4)$ gauge group. Finally, the vectors b_1 and b_2 play the rôle of a $\mathbf{Z}_2 \times \mathbf{Z}_2$ twist corresponding to an asymmetric orbifold, which reduces to $N = 1$ supersymmetry and breaks the “observable” group down to $U(5) \times U(1)^4$. In the massless (light) spectrum one finds; (a) from the untwisted sector: 3 pairs of $\underline{5} + \bar{\underline{5}}$ Higgses and 11 $SU(5) \times U(1)$ singlets, 5 of which are also neutral under all the other $U(1)$ ’s; (b) from the twisted sectors: 4 generations, one anti-generation, one pair of $\underline{5} + \bar{\underline{5}}$ Higgses, 14 $SU(5) \times U(1)$ singlets, and some “hidden” matter fields: 5 fields that are in $(\underline{10}, \underline{1}) + (\underline{1}, \underline{6})$ representations of $SO(10) \times SU(4)$ which are $SU(5) \times U(1)$ singlets, and 6 fields in $(\underline{1}, \underline{4}) + (\underline{1}, \bar{\underline{4}})$, which are $SU(5)$ singlets but have non-zero $U(1)$ charges implying electric charges $\pm 1/2$.

Using Eqs. (4,5,13,14) and applying the rules (10,11), we find that the difference between the $U(1)$ and $SU(5)$ threshold corrections is given by

$$\delta\Delta \equiv \Delta_1 - \Delta_5 = \int_{\Gamma} \frac{d^2\tau}{\tau_2} \left\{ \frac{5}{\tau_2^2} |\eta(\tau)|^{-4} \hat{Z}(\tau) - (b_1 - b_5) \right\} \quad (17)$$

where $\hat{Z}(\tau)$ is the modular-invariant function

$$\hat{Z}(\tau) \equiv \tau_2^2 \sum_{\underline{a}, \underline{b}} c(\frac{\underline{a}}{\underline{b}}) \frac{\theta'(\frac{a_\psi}{b_\psi})}{\eta} \frac{\hat{\theta}^2(\frac{a_I}{b_I})}{\bar{\eta}^2} Z[\frac{\underline{a}}{\underline{b}}]_{\text{rest}} \quad (18)$$

where $Z[\frac{a}{b}]_{\text{rest}}$ is a product of individual fermion contributions as given in Eq.(9). The sum in (18) runs over all possible pairs of boundary condition vectors $(\underline{a}, \underline{b})$ generated by the basis B (15). The above analysis shows that non-vanishing contributions can be provided only by pairs which preserve $N = 2$ space-time supersymmetry, and have a_I and/or b_I entries equal to $1/2$. Thus, up to a modular transformation, these pairs can be written as $\underline{a} = \underline{a}' + (b_1 \text{ or } b_2 \text{ or } b_1 + b_2)$ and $\underline{b} = \underline{b}'$, where $(\underline{a}', \underline{b}')$ is generated by the $N = 4$ basis B_4 (16) with the restriction that at least one of the two must contain the vector $\alpha : b_1, b_2$ and $b_1 + b_2$ generate the three $N = 2$ subsectors of the underlying $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold.

Direct inspection of the explicit forms of the vectors [2] reveals that only a few terms survive in the sum (18). The reason is that most pairs leave some fermions periodic simultaneously, yielding a contribution proportional to the odd θ -function $\theta(\frac{1}{1})$ which vanishes identically. It turns out that $\hat{Z}(\tau)$ is given by the modular orbit:

$$\begin{aligned} \hat{Z}(\tau) = & f(\tau) + f(\tau + 1) + f(\tau + 2) + f(\tau + 3) \\ & + f(-\frac{1}{\tau}) + f(-\frac{1}{\tau} + 1) + f(-\frac{1}{\tau} + 2) + f(-\frac{1}{\tau} + 3) \\ & + f(-\frac{1}{\tau + 1}) + f(-\frac{1}{\tau + 1} + 1) + f(-\frac{1}{\tau + 1} + 2) + f(-\frac{1}{\tau + 1} + 3) \end{aligned} \quad (19)$$

where

$$f(\tau) \equiv 2 \times 2\tau_2^2 \left\{ \sum_X (Z[\frac{X}{0}] + Z[\frac{X+S}{0}]) + (Z[\frac{\alpha_1 + b_1 + b_4}{b_4 + b_5}] + Z[\frac{\alpha + b_1 + b_4}{b_4 + b_5 + S}]) \right\} \quad (20)$$

with X running over the 12 combinations $\alpha + (b_1, b_2, b_1 + b_2) + (0, b'_4, b'_5, b'_4 + b'_5)$. The two factors of 2 in (20) account for the two terms obtained by replacing α by $-\alpha$ or $\alpha + \zeta$, respectively. Using Eq.(9) together with the explicit forms of the vectors and the values of the GSO projection coefficients given in [2]

$$c(\frac{X}{0}) = (-1)^{X_\psi}, \quad c(\frac{\alpha + b_1 + b_4}{b_4 + b_5}) = -c(\frac{\alpha + b_1 + b_4}{S + b_4 + b_5}) = -1 \quad (21)$$

we find

$$\begin{aligned} f(\tau) = & -\frac{1}{2^8} \frac{\tau_2^2}{\eta^{10} \bar{\eta}^{22}} \theta(\frac{0}{0}) \bar{\theta}^2(\frac{0}{0}) \bar{\theta}^3(\frac{1}{0}) \hat{\theta}(\frac{1}{0}) \\ & \times \left\{ [\theta^2(\frac{1}{0}) \frac{1}{i\pi} \frac{d}{d\tau} \theta^2(\frac{0}{0}) - \theta^2(\frac{0}{0}) \frac{1}{i\pi} \frac{d}{d\tau} \theta^2(\frac{1}{0})] \bar{\theta}^{10}(\frac{1}{0}) \right. \\ & \quad \times [5|\theta^4(\frac{1}{0}) \theta^6(\frac{0}{0})| + 6|\theta^6(\frac{1}{0}) \theta^4(\frac{0}{0})| + |\theta^8(\frac{1}{0}) \theta^2(\frac{0}{0})|] \\ & - [\theta^2(\frac{0}{1}) \frac{1}{i\pi} \frac{d}{d\tau} \theta^2(\frac{0}{0}) - \theta^2(\frac{0}{0}) \frac{1}{i\pi} \frac{d}{d\tau} \theta^2(\frac{0}{1})] \bar{\theta}^8(\frac{1}{0}) \bar{\theta}^2(\frac{1}{1}) \\ & \quad \left. \times |\theta^2(\frac{0}{0}) \theta^4(\frac{0}{1}) \theta^4(\frac{1}{0})| \right\} \end{aligned} \quad (22)$$

A normalization factor of 2^{-9} has been included in Eq.(22) to count seven $\mathbf{Z}_2$ twists and one $\mathbf{Z}_4$ twist as per the basis elements (15).

Before going on to evaluate (22), we remind the reader that one linear combination of the U(1) factors is anomalous [2], and that the model must choose non-zero v.e.v.'s for some of the scalar fields (twisted moduli) so as to cancel the anomalous U(1) D-term contribution to the vacuum energy [17]. This corresponds to a shift in the string vacuum, and one could ask whether this shift could alter the estimate (17-22) of $\delta\Delta$, and hence of m_{SU} . However, we recall that this type of threshold calculation includes the effects of all string states weighing $\gtrsim gM_{\text{P}}/\sqrt{8\pi}$, and indeed yields $m_{\text{SU}} = \mathcal{O}(gM_{\text{P}}/\sqrt{8\pi})$, whereas the (extra) masses acquired from shifting the string vacuum are of higher order in g . Hence these states only affect the value of m_{SU} at higher order.

The expression (17) has been evaluated using a double gaussian integration. We have chosen the order of integration to be $\int d\tau_2 \int d\tau_1$ for two reasons. First, because the τ_1 interval is finite (using the variable $z = \exp(-\frac{\pi}{2}\tau_2)$ would also give a finite interval, but would give rise to problems in the cancellation of the divergent pieces in $\frac{1}{z}$). Secondly, because for a given value of τ_2 the theta functions can be calculated using truncated sums up to some integer N checking that the ratio of the contributions of the N and $N+1$ terms is smaller than ϵ . We varied ϵ from 10^{-5} to 10^{-8} without a significant change in our result. To keep control over the integration, we made separate integrations on the different terms of (19), except that we had to consider the combinations $f(-\frac{1}{\tau} + 1) + f(-\frac{1}{\tau} + 3)$ and $f(-\frac{1}{\tau+1} + 1) + f(-\frac{1}{\tau+1} + 3)$, in order to cancel the $\exp(\pi\tau_2)$ divergences. For $\tau_2 > 15$, the cancellation of this dominant behaviour and of the constant term is not excellent, even using 64-bit words. The final result is

$$\delta\Delta = 24.13 \quad (23)$$

The one-loop β -function coefficients are given by:

$$\begin{aligned} b_5 &= -15 + 2N_g + N_h \\ b_1 &= 2N_g + N_h + \frac{5}{4}N_4 \end{aligned} \quad (24)$$

where N_g is the number of generations and antigerations, N_h is the number of $\underline{5} + \bar{\underline{5}}$ Higgs pairs, and N_4 is the number of hidden SU(4) pairs $\underline{4} + \bar{\underline{4}}$ which carry electric charges $\pm 1/2$. Using $N_g = 5, N_h = 4, N_4 = 6$ as in the flipped SU(5) model [2], we find

$$b_5 = -1, \quad b_1 = \frac{43}{2}; \quad \delta b = \frac{45}{2} \quad (25)$$

The constant (23) can be written as $\delta\Delta = \delta b \ln(1.71)^2$, and provides a big increase of about 70% in the unification mass M given in Eq.(6) to be used in Eq.(3), leading to the advertised result (1) for the effective superunification scale m_{SU} .

We observe that, in the above analysis, m_{SU} is largely determined by the moduli dependence which was exhibited in ref.[16]:

$$\Delta_i = \sum_{\alpha=1}^3 \left[-\hat{b}_i^\alpha \text{Re} \ln \{ \text{Im} T_\alpha \eta^4(T_\alpha) \} + c_i^\alpha \right] \quad (26)$$

where the index α runs over the three $N = 2$ sectors of the $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold, with $\hat{b}_i^\alpha$ and T_α the corresponding $N = 2$ β -function coefficients and moduli fields, respectively. It turns out [18] that the possible non-moduli corrections (c_i^α) to (26) are small in our fermionic model, as was also found in the $\mathbf{Z}_3$ model treated in ref.[11]. In fact, considering a common modulus field $T = i$ and using the relation [5] $\sum_{\alpha=1}^3 \hat{b}_i^\alpha = b_i$, one finds [18] that the value of $\delta\Delta$ is very close to its numerical value (23). This leads us to believe that any free fermionic model related to a $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold will have a value of m_{SU} similar to (1). Moreover [18], we note that the values of the moduli in free fermionic models correspond to minima of the moduli-dependent factors in (26). Hence other models with three $N = 2$ sectors are likely to have larger values of m_{SU} in general.

We now discuss some implications of the result (1) for phenomenological tests of the flipped SU(5) GUT derived from string [2]. The difference between the one-loop SU(5) and U(1) β -functions seen in Eq.(25) implies that the ratio of the SU(5) and U(1) coupling deviates rapidly from unity at scales below m_{SU} . This in turn implies that the effective value of $\sin^2\theta_W$ is reduced below the symmetry value of 3/8:

$$\sin^2\theta_W = \frac{15}{16 + 24(\alpha_5/\alpha_1)} \quad (27)$$

To gauge the possible significance of this reduction, we can use Eq.(3) to find

$$\frac{\alpha_5}{\alpha_1} \simeq 1 + \frac{45}{8\pi} L \alpha_5 \quad (28)$$

where $L \equiv \ln(m_{\text{SU}}^2/\mu^2)$, so that

$$\sin^2\theta_W(\mu) \simeq \frac{3}{8 + \frac{27}{\pi} L \alpha_5} \quad (29)$$

The LEP value of $\sin^2\theta_W$ certainly deviates by less than 5% from the prediction of a minimal supersymmetric GUT [8, 9]. Eq.(29) gives a 5% reduction in $\sin^2\theta_W(\mu)$ when

$$L = \frac{8\pi}{\alpha_5} \frac{0.05}{27} \simeq 1.12 \quad (30)$$

(where we have taken $\alpha_5 \simeq 1/24$), corresponding to $\mu \simeq 0.57 m_{\text{SU}} \simeq 7.1 \times 10^{17}$ GeV.

This value of μ is significantly larger than the value of $m_{\text{GUT}} \sim (1 \text{ to } 2) \times 10^{16}$ GeV found [8, 9] in the minimal supersymmetric extension of the Standard Model, calculated using the conventional $\overline{MS}$ prescription, according to which Dirac γ -algebra is done in n dimensions, whilst $\text{Tr} \mathbf{1} = 4$. In fact, as pointed out above, it would be better to use the $\overline{DR}$ scheme [15] in supersymmetric GUT calculations, since in this scheme the Dirac γ -algebra is performed in 4 dimensions, which has the effect of preserving supersymmetry in loop calculations. The values obtained for physical quantities such as the masses of heavy gauge bosons X,Y should be independent of the regularization scheme used, apart from higher-order effects. However, the scale m_{GUT} at which $\alpha_3 = \alpha_2 = \alpha_5$ could in principle vary by

a factor of $\mathcal{O}(1)$ in different regularization schemes. In the case at hand, we know that in field theory [19]

$$\frac{16\pi^2}{g_i^2(\mu)|_{\overline{DR}}} = \frac{16\pi^2}{g_i^2(\mu)|_{\overline{MS}}} - \frac{1}{3}C_{A_i}, \quad (31)$$

where C_{A_i} is the second-rank Casimir of the i 'th gauge group. Making the corrections (31) to the standard one-loop expression

$$\frac{\pi}{6} \left(\frac{3}{5\alpha_{em}} - \frac{8}{5\alpha_3} \right) = \ln \frac{m_{\text{GUT}}}{m_Z} \quad (32)$$

we find the following correction to the calculated value of m_{GUT} :

$$m_{\text{GUT}}|_{\overline{DR}} = 1.07 m_{\text{GUT}}|_{\overline{MS}}. \quad (33)$$

This change in the value of m_{GUT} is smaller than the uncertainty in $\overline{MS}$ calculations induced by our ignorance of $\Lambda_{\overline{MS}}$. We conclude, in the absence of a complete two-loop calculation, that the range of values of m_{GUT} calculated in the $\overline{DR}$ scheme would not differ significantly from the range calculated in the $\overline{MS}$ scheme.

This is also the typical scale at which $\alpha_3 = \alpha_2 = \alpha_1$ in the minimal field-theoretical variant of flipped SU(5) [20]. However, string-derived flipped SU(5) models have additional particles which have masses below m_{GUT} , and could increase the calculated value of m_{GUT} when included in the two-loop renormalization group equations. Moreover, there is a natural mechanism in string-derived flipped SU(5) models for obtaining $m_{\text{GUT}} = \mathcal{O}(g_{\text{SU}} m_{\text{SU}})$. The $\underline{10}$ and $\overline{\underline{10}}$ Higgses that break SU(5) could participate in the cancellation of the anomalous U(1) D-term, since they have non-zero U(1) charges. In this case, the natural order of magnitude of the $\underline{10}$ and $\overline{\underline{10}}$ v.e.v.'s $V, \bar{V}$ would be

$$V = \bar{V} = \mathcal{O}(1) \sqrt{\frac{180}{192\pi^2}} g_{\text{SU}}^2 \frac{M_{\text{P}}}{\sqrt{8\pi}} \quad (34)$$

which is numerically about 4×10^{17} GeV for $g_{\text{SU}}^2/4\pi \simeq 1/24$.

This is a very promising new scenario which merits further investigation. A full calculation of the experimental value of $\sin^2\theta_W$ in this type of flipped SU(5) model would involve using the two-loop renormalization group equations, including the Yukawa couplings as well as the gauge couplings. It would also require detailed knowledge of the masses of all the extra particles that the model contains beyond those in the minimal supersymmetric extension of the Standard Model. As already noted, many of these may have masses below m_{GUT} , so that the minimal supersymmetric GUT calculation of m_{GUT} does not necessarily apply. A detailed exploration of the circumstances under which the flipped SU(5) model can accommodate the LEP data would take us beyond the scope of this paper. However, we have demonstrated the importance of knowing precisely the string unification scale m_{SU} , and the possibility of a detectable deviation of $\sin^2\theta_W$ from the minimal supersymmetric GUT prediction.

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