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Operator Phases in BF-Type Topological Field Theories: II

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Abstract

The one-loop off-shell effective action is re-examined for the case of BF theories in three-dimensions. Within the context of the Vilkovisky-DeWitt reparametrization invariant framework, it is shown how the choice of an acceptable field space metric is crucial for obtaining gauge invariant results, even when this metric is field independent. It is found that the phase contribution to the 1-loop effective action is proportional to the pure Chern-Simons term. Possible dependence of these results on the choice of field metric are briefly discussed.

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1 Introduction

In a previous publication [1], we computed the phase contribution to the one-loop off-shell effective action for a three-dimensional topological field theory with classical action $\int B \wedge F_A$. The background field method was employed, and although covariance with respect to the background fields was maintained throughout, the resulting phase was of the form

$$\int B \wedge F_A + B \wedge B \wedge B ;$$

as such, it violated one of the classical symmetries. A likely source for this anomalous behaviour was suggested as being due to an unusual feature in the geometry of the space of fields.

In the present paper, we show that when an acceptable field space metric is chosen, and taken account of in the functional measure, the results for the phase are altered; the anomaly disappears, and agreement is obtained with the results of ref. [2]. The computation entails an evaluation of the η -function of a first order operator which appears in the one-loop background field expansion. However, since the η -function is sensitive to the signature of this operator, caution must prevail when dealing with field space metrics which, although field independent, alter this signature. Indeed, this is precisely the situation which unfolds in the present instance, due to the unusual field space metric.

Within the geometrical approach to the effective action, as advocated by Vilkovisky-DeWitt [3, 4], one should consider a reparametrization invariant definition of the effective action. This involves taking due account of the metric G on the field configuration space, such that, for example, the functional measure is weighted with the usual $\sqrt{\det G}$ factor. However, in many cases G is field-independent, and it can be shown that the $\sqrt{\det G}$ factor has zero effect on the real part of the effective action. In the case at hand, we find that the η -function is indeed affected by the $\sqrt{\det G}$ factor, despite its field independence; this is due to the fact that the η -function is sensitive to the signature, as mentioned above.

The outline of this article is as follows: we continue in the following section with a brief review of our previous results. It is then shown how to

incorporate the field space metric into the calculation of the η -function; in addition to the pure BF theory, we also consider a coupled system of BF and Chern-Simons theory. We conclude with a discussion of the possible dependence of the results on the initial choice of field metric.

2 Review

Let us begin by specifying the classical action of interest [5, 6, 7]; we have

$$S_c = \int d^3x \operatorname{tr} \epsilon^{\alpha\beta\gamma} B_\alpha F_{\beta\gamma} . \quad (1)$$

Here, $F_{\alpha\beta} = \partial_\alpha A_\beta - \partial_\beta A_\alpha + [A_\alpha, A_\beta]$ is the curvature of the gauge connection A_α , $B = B_\alpha^a T^a dx^\alpha$ is a 1-form in the adjoint representation of the gauge group, and all traces shall be written in the fundamental representation. We are taking the theory to be defined on $\mathbf{R}^3$, where the momentum space calculational procedure being used is valid. We are using the conventions where the structure constants are real and completely antisymmetric, with $[T^a, T^b] = f^{abc} T^c$. For the fundamental representation of $SU(n)$, the matrices T^a are skew-hermitian and we take $\operatorname{tr} T^a T^b = -\frac{1}{2} \delta^{ab}$, while for the quadratic Casimir we have $f^{acd} f^{bcd} = c_v \delta^{ab}$.

It is straightforward to establish the local symmetries of the above action, namely

$$\begin{aligned} \delta A_\alpha &= D_\alpha \omega , \\ \delta B_\alpha &= D_\alpha \theta + [B_\alpha, \omega] . \end{aligned} \quad (2)$$

Now, for the purposes of a one-loop background field calculation, we shall decompose the fields into a background plus a quantum part as follows:

$$A \rightarrow A + A^q , B \rightarrow B + B^q , \quad (3)$$

where we are taking arbitrary background configurations which are not necessarily on-shell. The classical symmetry then takes the form

$$\begin{aligned} \delta A_\alpha &= D_\alpha \omega , \\ \delta A_\alpha^q &= [A_\alpha^q, \omega] , \\ \delta B_\alpha &= D_\alpha \theta + [B_\alpha, \omega] , \\ \delta B_\alpha^q &= [B_\alpha^q, \omega] + [A_\alpha^q, \theta] , \end{aligned} \quad (4)$$

where it is important to note that the covariant derivative here is with respect to the background field. One should also note that the quantum fields transform linearly as vectors. Our aim now is to quantize the theory, and we wish to find a set of gauge fixing conditions which are covariant (i.e. they transform as vectors) with respect to (4). One such set is given by

$$\begin{aligned} G_\phi &\equiv D \cdot A^q = 0 \ , \\ G_\pi &\equiv D \cdot B^q + [B_\alpha, A^{q\alpha}] = 0 \ , \end{aligned} \quad (5)$$

and indeed it can be checked that they transform as

$$\begin{aligned} \delta G_\phi &= [G_\phi, \omega] \ , \\ \delta G_\pi &= [G_\pi, \omega] + [G_\phi, \theta] \ . \end{aligned} \quad (6)$$

The part of the action which is quadratic in the quantum fields (bearing in mind that the ghosts and multipliers are taken to be purely quantum) is given by

$$\begin{aligned} S_q^{(2)} &= \int d^3x \operatorname{tr} \{ \epsilon^{\alpha\beta\gamma} (2B_\alpha^q D_\beta A_\gamma^q + B_\alpha [A_\beta^q, A_\gamma^q]) \\ &\quad + 2\phi D \cdot A^q + 2\pi (D \cdot B^q + [B_\alpha, A^{q\alpha}]) \} + \text{ghosts} \ . \end{aligned} \quad (7)$$

We are concerned here with the first order operator $H_{ab}^{\alpha\beta}$ connecting the gauge fields and multipliers; specifically this is given by

$$\begin{aligned} S_q^{(2)} &= \frac{1}{2} \int d^3x \begin{pmatrix} B_\alpha^q & A_\alpha^q & \phi & \pi \end{pmatrix}^a \\ &\times \begin{pmatrix} 0 & -\epsilon^{\alpha\gamma\beta} D_\gamma^{ab} & 0 & D_\alpha^{ab} \\ -\epsilon^{\alpha\gamma\beta} D_\gamma^{ab} & -\epsilon^{\alpha\gamma\beta} f^{acb} B_\gamma^c & D_\alpha^{ab} & f^{acb} B_\alpha^c \\ 0 & -D_\beta^{ab} & 0 & 0 \\ -D_\beta^{ab} & -f^{acb} B_\beta^c & 0 & 0 \end{pmatrix} \begin{pmatrix} B_\beta^q \\ A_\beta^q \\ \phi \\ \pi \end{pmatrix}^b \ . \end{aligned} \quad (8)$$

Eqn. (8) can be seen to define a rank-2 symmetric object H_{ij} , which is pinned between the fields Φ^i and Φ^j .

The main object of interest is therefore the path integral

$$Z = \int d\Phi \ e^{i\Phi^i H_{ij} \Phi^j} \ , \quad (9)$$

where $\Phi^i \equiv (B_\alpha^{qa}(x), A_\alpha^{qa}(x), \phi^a(x), \pi^a(x))$, $d\Phi$ is the measure on the space of quantum fields, and H_{ij} , defined via (8), depends on the background fields of A and B .

To regularize this expression we define the ζ and η functions of the operator H via its eigenvalues λ_n :

$$\begin{aligned}\zeta_H(s) &= \sum_n |\lambda_n|^{-s} , \\ \eta_H(s) &= \sum_n (\text{sign} \lambda_n) |\lambda_n|^{-s} .\end{aligned}\tag{10}$$

It then follows that

$$Z_{reg} = e^{\frac{1}{2}\zeta'_H(0) + \frac{i\pi}{4}\eta_H(0)} .\tag{11}$$

It is the η -function evaluated at $s = 0$ which is of primary concern here, as it represents the possibility of a non-zero phase contribution to the partition function; recall that the theory is defined via a partition function $Z = \int dA dB e^{iS_c}$. To examine this η -function in more detail, it is most convenient to begin with the following integral representation [8]

$$\eta_H(s) = \frac{1}{\Gamma(\frac{s+1}{2})} \int_0^\infty dt t^{(s-1)/2} \text{tr}[H e^{-H^2 t}] .\tag{12}$$

It is important to point out that $\eta_H(0)$ depends on the sign (but not the scale) of H . In order to proceed with the calculation, one decomposes H as $H_0 + H_1$, where H_0 is independent of the background fields, and H_1 contains the interaction terms, see for example [9]. The functional traces can be evaluated in momentum space with the definition

$$\begin{aligned}Tr[\mathcal{O}] &= \int \frac{d^n p}{(2\pi)^n} \langle p | Tr' \mathcal{O} | p \rangle \\ &= \int \frac{d^n p}{(2\pi)^n} d^n x d^n y \langle p | x \rangle \langle x | Tr' \mathcal{O} | y \rangle \langle y | p \rangle .\end{aligned}\tag{13}$$

Here, the prime indicates a trace over any other indices carried by the operator $\mathcal{O}$. Our conventions are such that $\langle p | x \rangle = e^{-ipx}$ and $A(p) = \int d^n x e^{-ipx} A(x)$. For details of how to proceed with the calculation, we refer to [1].

3 The Field Space Metric

In order to deal with an off-shell effective action which is independent of the background gauge choice and parametrizations of the field variables, one must resort to the Vilkovisky-DeWitt program [3, 4]. The initial step in this procedure is to construct an acceptable metric on the field configuration space. In our case, this is the space of gauge fields and multipliers B, A, ϕ, π . Let us consider first the gauge fields: the transformations (2), written in condensed notation [3, 4] are

$$\delta\phi^i = K_\alpha^i \epsilon^\alpha . \quad (14)$$

Here, the field ϕ^i labels the classical fields A and B , the symmetry generators are denoted by K_α^i , and the infinitesimal gauge parameters are $\epsilon^\alpha = (\omega, \theta)$. More explicitly, we have

$$\begin{aligned} K_{\omega^b(y)}^{A_a^a(x)} &= D_{\alpha x}^{ab} \delta(x-y) , K_{\theta^b(y)}^{A_a^a(x)} = 0 , \\ K_{\omega^b(y)}^{B_a^a(x)} &= f^{acb} B_\alpha^c(x) \delta(x-y) , K_{\theta^b(y)}^{B_a^a(x)} = D_{\alpha x}^{ab} \delta(x-y) . \end{aligned} \quad (15)$$

Now, the notion of an acceptable field metric is defined by the requirement that it admits the gauge generators as Killing vectors [3, 4], that is

$$0 = G_{ik} \partial_j K_\alpha^k + G_{jk} \partial_i K_\alpha^k + (\partial_k G_{ij}) K_\alpha^k , \quad (16)$$

for all α . In the present instance, it is straightforward to establish that a constant, field independent, metric which satisfies (16) does exist, the most general acceptable metric being

$$\begin{aligned} G_{A_\alpha^a(x) A_\beta^b(y)} &= \sigma \delta_{\alpha\beta}^{ab} \delta(x-y) \\ G_{A_\alpha^a(x) B_\beta^b(y)} &= \lambda \delta_{\alpha\beta}^{ab} \delta(x-y) , \lambda \neq 0 \\ G_{B_\alpha^a(x) B_\beta^b(y)} &= 0 , \end{aligned} \quad (17)$$

where σ and λ are constants. It is interesting to note that a diagonal metric (i.e. $G_{AA} = G_{BB} = 1$) is not allowed. In order to proceed further, we also require a suitable metric on the multiplier space, as these fields appear in our basic integral (9). Eqn. (6) defines the transformations of the gauge fixing

conditions; in tandem with the requirement that $\phi G_\phi + \pi G_\pi$ be invariant with respect to (4), we find that the multiplier fields transform as:

$$\begin{aligned}\delta\phi &= [\phi, \omega] , \\ \delta\pi &= [\pi, \omega] + [\phi, \theta] .\end{aligned}\tag{18}$$

The above requirement ensures that the path integral is being performed in a manner which is background field invariant. Finally, an acceptable form for the multiplier metric is determined by requiring that the inner product in the multiplier space be invariant with respect to (18), i.e. $\delta(G_{\alpha\beta}\lambda^\alpha\lambda^\beta) = 0$, where $\lambda^\alpha \equiv (\phi, \pi)$. The most general solution is:

$$\begin{aligned}G_{\phi^a(x)\phi^b(y)} &= 0 \\ G_{\phi^a(x)\pi^b(y)} &= \lambda'\delta^{ab}\delta(x-y) , \lambda' \neq 0 \\ G_{\pi^a(x)\pi^b(y)} &= \sigma'\delta^{ab}\delta(x-y) ,\end{aligned}\tag{19}$$

where λ' and σ' are constants. The complete field space metric is therefore

$$G_{ij} = \begin{pmatrix} 0 & \lambda\delta_{\alpha\beta} & 0 & 0 \\ \lambda\delta_{\alpha\beta} & \sigma\delta_{\alpha\beta} & 0 & 0 \\ 0 & 0 & 0 & \lambda' \\ 0 & 0 & \lambda' & \sigma' \end{pmatrix} \delta^{ab}\delta(x-y) .\tag{20}$$

Armed with this metric, we can now return to eqn. (9). In order to ensure reparametrization invariance of the effective action, the measure should be $d\Phi = \sqrt{\det \tilde{G}} \prod_i d\Phi^i$. However, in addition the Vilkovisky-DeWitt procedure requires that H_{ij} be replaced by a bona fide rank-2 symmetric tensor on the space of fields, namely $H_{ij} \rightarrow H_{ij} - \Gamma_{ij}^k S_{,k}$. Here, Γ_{ij}^k is the connection constructed from the metric on the space of gauge inequivalent gauge fields, for details of its construction, see [3, 4]. In general, one must take account of the correction term proportional to the connection symbol. However, there is a gauge, which is termed the Landau-DeWitt gauge, in which the correction term is proportional to the Christoffel symbol only. It is defined by

$$G_{ij}K_\alpha^i\phi_q^j = 0 ,\tag{21}$$

for all α . For a field independent metric, the Christoffel symbol vanishes, and hence the Vilkovisky-DeWitt correction vanishes. We shall call this the “no

VD" gauge. A simple check will reveal that the "no VD" gauge is precisely the one we have chosen, namely, (5). Thus, it only remains to take account of the $\sqrt{\det G}$ term in the measure. One can see that it has the effect of raising an index on H , i.e.

$$\det^{-1/2}[H_{ij}] \rightarrow \det^{-1/2}[G^{ik}H_{kj}] = \det^{-1/2}[H^i_j] . \quad (22)$$

Indeed, it is $\det[H^i_j]$ which is a reparametrization invariant object, and not $\det[H_{ij}]$. The $\sqrt{\det G}$ factor in the measure is designed precisely to ensure the former, see for example [10]. We see at this point that if the field metric is diagonal, i.e. $G_{ij} = \delta_{ij}$, then there is no difference between up and down indices. As we have seen, however, the acceptable field metric in the present case is not diagonal, and hence care must be taken in the positioning of indices, especially when considering an object such as the η -function. The metric (20) differs in signature from a diagonal one, and hence its inclusion in (22) is crucial for obtaining gauge invariant results.

Before proceeding with the actual calculation, we can address the question of the scale factors present in the metric (20). The line element is given by

$$ds^2 = G_{ij}d\Phi^i d\Phi^j = \sigma(dA^q)^2 + 2\lambda dA^q dB^q + \sigma'(d\pi)^2 + 2\lambda' d\phi d\pi . \quad (23)$$

We can clearly re-scale the fields as follows:

$$A^q \rightarrow \frac{A^q}{\sqrt{\lambda}} , B^q \rightarrow \frac{B^q}{\sqrt{\lambda}} , \pi \rightarrow \frac{\pi}{\sqrt{\lambda'}} , \phi \rightarrow \frac{\phi}{\sqrt{\lambda'}} . \quad (24)$$

In this way, the metric (20) takes the form:

$$G_{ij} = \begin{pmatrix} 0 & \delta_{\alpha\beta} & 0 & 0 \\ \delta_{\alpha\beta} & \frac{\sigma}{\lambda}\delta_{\alpha\beta} & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & \frac{\sigma'}{\lambda'} \end{pmatrix} \delta^{ab}\delta(x-y) . \quad (25)$$

Our task now is to compute the η -function for the operator

$$\begin{aligned}
& (G^{ik}H_{kj})_{\alpha\beta}^{ab}(x,y) \\
&= \begin{pmatrix} -\epsilon^{\alpha\gamma\beta}D_{\gamma x}^{ab} & \epsilon^{\alpha\gamma\beta}(\frac{\sigma}{\lambda}D_{\gamma x}^{ab} - f^{acb}B_{\gamma x}^c) & D_{\alpha x}^{ab} & -\frac{\sigma}{\lambda}D_{\alpha x}^{ab} + f^{acb}B_{\alpha x}^c \\ 0 & -\epsilon^{\alpha\gamma\beta}D_{\gamma x}^{ab} & 0 & D_{\alpha x}^{ab} \\ -D_{\beta x}^{ab} & \frac{\sigma'}{\lambda'}D_{\beta x}^{ab} - f^{acb}B_{\beta x}^c & 0 & 0 \\ 0 & -D_{\beta x}^{ab} & 0 & 0 \end{pmatrix} \delta(x-y) .
\end{aligned} \tag{26}$$

This is obtained by combining the inverse of (25) with (8). We should mention that the signs of λ and λ' are required to be the same for convergence reasons. This can be seen by first constructing H_j^i with the un-scaled metric G^{ik} , and then considering $(H_0^2)^i_j$. One finds that the (3,3) and (4,4) components have the structure $-\frac{1}{\lambda\lambda'}\partial^2$. This shows that the sign of λ and λ' must be the same in order to provide an exponentially decaying convergence factor in the definition of η . We shall assume that $\text{sign}(\lambda) = \text{sign}(\lambda') = 1$. The alternative case, $\text{sign}(\lambda) = -1$, corresponds to changing the sign of H_j^i ; this is equivalent to beginning with a partition function weighted by e^{-iS_c} .

The details of the calculation are similar to those presented in [1]; however, there is one extra feature which is, perhaps, worth a mention. The “free” (i.e. background field independent) part of H squares to

$$(H_0^2)_{\alpha\beta}^{ab} = -\partial^2 I_{\alpha\beta}^{ab} + X_{\alpha\beta}^{ab} , \tag{27}$$

where I is the unit matrix in the field space, and $X^2 = 0$. We then have

$$e^{-H_0^2 t} = e^{\partial^2 t} [I - tX] , \tag{28}$$

and it can be shown that the X terms have zero effect on the calculation. Without further ado, we present the results: in the expansion in powers of the interaction H_1 , we find only two terms contribute to $\eta(0)$, namely

$$\begin{aligned}
H_1\{H_0, H_1\} &\rightarrow \frac{2c_v}{\pi^2} \int d^3x \text{tr} \epsilon^{\alpha\beta\gamma} A_\alpha \partial_\beta A_\gamma , \\
H_1^3 &\rightarrow \frac{4c_v}{3\pi^2} \int d^3x \text{tr} \epsilon^{\alpha\beta\gamma} A_\alpha A_\beta A_\gamma , \\
H_1\{H_0, H_1\}^2 &\rightarrow 0 .
\end{aligned} \tag{29}$$

Thus, our final result for the one-loop phase contribution to the pure BF theory is

$$\eta_H(s=0) = \frac{2c_v}{\pi^2} \int d^3x \operatorname{tr} \epsilon^{\alpha\beta\gamma} (A_\alpha \partial_\beta A_\gamma + \frac{2}{3} A_\alpha A_\beta A_\gamma) , \quad (30)$$

which is in fact twice the η -function of the pure Chern-Simons theory [11]. This result is in agreement with that obtained in [2], where a Pauli-Villars type regularization scheme was used. It should be noted that the result (30) is independent of the scale parameters $\frac{\sigma}{\lambda}$ and $\frac{\sigma'}{\lambda'}$. We shall return to a discussion of this point presently.

It turns out that no further work is required to treat the coupled system defined by the classical action:

$$S_c = \int d^3x \operatorname{tr} \epsilon^{\alpha\beta\gamma} B_\alpha F_{\beta\gamma} + \frac{k}{4\pi} \int d^3x \operatorname{tr} \epsilon^{\alpha\beta\gamma} (A_\alpha \partial_\beta A_\gamma + \frac{2}{3} A_\alpha A_\beta A_\gamma) , \quad (31)$$

where k is the usual integer-quantized Chern-Simons coupling.

The point here is that the symmetries of (31) are the same as those of pure BF theory; hence, the acceptable field metric and the “no VD” gauge are identical. The addition of the Chern-Simons term in (31) simply alters the original H , eqn. (8), by one term, namely, $-\epsilon^{\alpha\gamma\beta} D_\gamma^{ab}$ in the $(2,2)$ position. However, upon forming the operator H_j^i , one sees that this term has its position shifted to the $(1,2)$ component. As such, it plays a similar role to the $(1,2)$ component in (26) which is proportional to $\frac{\sigma}{\lambda}$. As we have mentioned above, $\eta(0)$ is independent of the scale factor $\frac{\sigma}{\lambda}$; thus the additional Chern-Simons term has no effect on the evaluation of $\eta(0)$. We can thus conclude that the phase contribution to the one-loop off-shell effective action for the coupled system is identical to that for pure BF , namely eqn. (30). Again, this agrees with ref. [2].

4 Discussion

The above calculation demonstrates clearly that in order to achieve a reparametrization invariant effective action one must take care to incorporate the $\sqrt{\det G}$ factor which is present in the functional measure. In many circumstances,

the gauge generators allow for a field independent Killing metric, and as such this factor has no effect on the real part of the effective action. The subtlety in the present case is that since we are dealing with a theory which is first order in derivatives, one encounters a determinant which possesses a non-zero phase. The inclusion of the $\sqrt{\det G}$ factor in the field space volume is then crucial for obtaining gauge invariant results. Although a Killing metric exists which is field independent, its presence in the functional measure has a significant effect on the imaginary part of the effective action. In [1], a standard background field calculation was performed which yielded an anomalous result. It is now clear that the inclusion of the $\sqrt{\det G}$ factor restores gauge invariance.

The question arises as to the dependence of the results on the initial choice of field metric. A class of Killing metrics for the theory at hand is given by (25), in which two scale parameters are present. Thus, it appears that there is some freedom in the choice of metric, as represented by the different values one may choose for these parameters. However, as mentioned above, it is straightforward to check that the final result (30) is independent of their values. Thus, although there is some freedom in the choice of metric, the resulting η -function is unchanged.

The framework of Vilkovisky-DeWitt claims to provide a unique off-shell effective action. However, clearly there are some choices involved, most notably the initial choice of field space metric, see [12]. For a given metric, the Vilkovisky-DeWitt program does indeed assign meaning to a unique off-shell effective action (for that given metric). However, for a given classical theory, there is no a priori reason why different choices of acceptable Killing metrics should yield identical effective actions. It may very well transpire that a classical theory which possesses different Killing metrics gives rise to inequivalent quantum theories. In the case under study, however, we have seen that the η -function contribution to the effective action is independent of the representative chosen from the class of metrics which we have studied.

There may be theories in which one can establish that the Vilkovisky-DeWitt effective action is independent of the choice of field space metric. In such a case, the effective action would then correspond to a topological invariant on some appropriate field space. Promising in this regard are topological field theories; in some cases one can establish that the partition function of

a topological field theory is a topological invariant of some underlying space upon which the original theory is built, see for example [13]. It would be interesting to establish these results within a Vilkovisky-DeWitt context; work in this direction is in progress.

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