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SYMMETRIES OF COSMOLOGICAL SUPERSTRING VACUA

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Abstract

The concept of discrete scale-factor-duality transformations is extended to a full, continuous $O(d, d)$ group. The action of the group transforms "cosmological" solutions of the low-energy, string-modified Einstein equations (including non-trivial dilaton and anti-symmetric tensor fields) into other solutions, thus representing the natural generalization of Narain's construction to time-dependent, not-necessarily-compact manifolds. Possible connections with $2D$ black holes are also pointed out.

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1. INTRODUCTION

In this short note we shall develop and extend the concept [1] of scale factor duality (SFD) as a symmetry group of classical string motions, of the low-energy string effective action and of the corresponding equations of motion.

Let us recall a few characteristics of SFD which make it quite distinct from the more conventional R -duality [2] [3]:

1. SFD does not rely on compactification of target space.
2. Even for a compact target space, SFD relates a physically expanding to a physically contracting dimension. By contrast, time-dependent R -duality relates, e.g. an expanding circle to one which apparently contracts, but actually describes, in dual terms, the same physical expansion.
3. While R -duality could presumably be a true symmetry, i.e. one relating two theories with the same spectrum and S -matrix, SFD is more likely [1] to connect different time-dependent vacua of string theory. Thus SFD appears rather as a generalization of Narain's construction [4] [5] of inequivalent static compactifications to a time-dependent and not necessarily compact target space.

Our results will make the connection to Narain's work even more compelling. Indeed, we shall be able to show that the low-energy effective action possesses, for time-dependent metric, torsion, and dilaton background fields $(G_{\mu\nu}, B_{\mu\nu}, \phi; \mu, \nu = 0, 1, 2, \dots, d)$, a full continuous $O(d, d)$ symmetry group whose action transforms "cosmological" solutions of the equations of motion into other (generally inequivalent) solutions.

In Sect. 2 we shall give a simple derivation of the $O(d, d)$ symmetry of the low-energy effective action, for time-dependent $G_{\mu\nu}, B_{\mu\nu}$ and ϕ and obtain the corresponding equations of motion. In Sect. 3 we elucidate the meaning of the symmetry as well as its relation to Narain's work and we comment, finally, on the possible relevance of our results for two-dimensional black holes.

and the field Φ defined as:

$$\Phi = \phi - \ln \sqrt{\det G}. \quad (7)$$

In this notation the action (1) takes its final form:

$$S = \int dt \, e^{-\Phi} \left\{ \Lambda + (\dot{\Phi})^2 + \frac{1}{8} \text{Tr} \left[\dot{M} \eta \, \dot{M} \eta \right] \right\}, \quad (8)$$

where

$$\eta = \begin{pmatrix} \mathbf{0} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix} \quad (9)$$

is an $O(d, d)$ metric in non-diagonal form and $\mathbf{1}$ stands for the d -dimensional unit matrix.

The action (8) is manifestly invariant under the global $O(d, d)$ group acting as:

$$\Phi \rightarrow \Phi, \quad M \rightarrow \Omega M \Omega^T \quad (10)$$

$$\Omega^T \eta \Omega = \eta. \quad (11)$$

The occurrence of $O(d, d)$ as a way to relate different string vacua immediately suggests a relation with Narain's work [4]. The discussion of this relation is deferred to Sect. 3.

Let us recover here instead the particular cases discussed in ref. [1]. For $B = 0$ and a diagonal G , the Z_2^d group of ref. [1] is simply given in terms of a generic d -dimensional projector Π by the choice:

$$\Omega = \begin{pmatrix} \Pi & \mathbf{1} - \Pi \\ \mathbf{1} - \Pi & \Pi \end{pmatrix}, \quad \Pi^2 = \Pi. \quad (12)$$

Notice that, for a non-diagonal G , this subgroup of $O(d, d)$ generates in general a non-vanishing "torsion" B .

We close this section by writing down the equations of motion that follow from the original action once the fields are restricted to the type specified in eq. (3). Generally speaking, by just varying the reduced action (8), one might lose some of the equations. In our case, one can check that the only "missing" condition is the one that follows from reintroducing G_{00} in the action and from setting to zero the corresponding variation. This gives directly the "zero energy" condition

$$(\dot{\Phi})^2 + \frac{1}{8} \text{Tr} \left[\dot{M} \eta \, \dot{M} \eta \right] - V = 0, \quad (13)$$

and is equivalent to a change in M :

$$M \rightarrow M = \Omega^T M \Omega. \quad (20)$$

This is precisely the change of M given in (10) (see e.g. [1], [10] for a discussion of the need to transform the dilaton ϕ too). Indeed, any M can be obtained by the action of the group acting, e.g. on $M = 1$. M itself belongs to $O(d, d)$, but, unlike the most general Ω , M is symmetric. The subgroup of $O(d, d)$ that leaves a given M invariant is an $O(d) \times O(d)$, exactly as in Narain's case [4] where, for constant G and B , inequivalent theories (different M s) are labelled by points in the coset $O(d, d)/(O(d) \times O(d))$.

For time-dependent solutions the situation is somewhat different since one also has to specify the first derivative of M at $t = 0$. A counting of parameters now shows that $O(d, d)$ is not large enough to connect all possible solutions. $M(0)$ and $\dot{M}(0)$ contain indeed $2d^2$ parameters, while an $O(d, d)$ transformation contains $2d^2 - d$ parameters. That means that there are still d invariants to be specified, in order to characterize a class of solutions. It is not hard to prove that a convenient set of invariants is provided by

$$I_p = \text{Tr}(\eta \dot{M})^{2p}, \quad p = 1, \dots, d. \quad (21)$$

M s which correspond to different invariants should obviously yield inequivalent solutions but one could ask whether M 's which are related by $O(d, d)$ rotations correspond to the same theory as naively suggested by the fact that they are related through the "canonical" transformation (18).

The same question could be asked, of course, in the constant-background case. Consider, for instance, the case of a single compactified dimension, a circle of radius R , and $B = 0$. The general "canonical" transformation would seem to be:

$$X' \rightarrow aP, \quad P \rightarrow a^{-1}X' \quad (22)$$

which would lead to the (incorrect) symmetry under $G \rightarrow a^{-2}G^{-1}$. The problem with such a transformation is that it does not preserve the spectrum of (the zero modes of) X' and P (it is not unitary) for arbitrary values of a . Only if we take $a = R^2$, is the spectrum left unchanged and we can really perform the necessary change of variable in the functional integral. We thus end up with just the usual, discrete Z_2 symmetry [2],[7]:

$$G \rightarrow R^{-4}G^{-1}. \quad (23)$$

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