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Magnitude of luminosity cross sections with lost fermions at LEP

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Abstract

In luminosity measurements at LEP, possible contributions arise from final states in which one or both of the electrons escapes detection. Analytical estimates for the two most important such processes are derived, and found to be reassuringly small. This is corroborated by Monte Carlo simulation.

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1 Introduction

Predictions for the small-angle cross section in Bhabha scattering, which is used for the determination of the luminosity in e^+e^- collisions, are slowly reaching a level of accuracy that is comparable to what is required for precision phenomenology. Partly this is due to detailed analytical calculations such as [1], and partly to a careful use of gradually improved Monte Carlo simulations such as [2]. A common feature of these studies is that usually both the e^+ and the e^- are required to scatter over an appreciable angle so as to end up in the luminosity monitor: this scattering is then dressed with QED corrections in the form of virtual photons and real bremsstrahlung. Since these monitors do not distinguish between charged and neutral energy depositions, the possibility arises that it is actually the bremsstrahlung that triggers the detector, with one of the electrons (or both) ending up outside of the acceptance. Because of the forward peaks in Bhabha scattering, this can *in principle* give a non-negligible contribution to the cross section. In view of the required high precision such an alternative can only be disregarded if one has at least a semi-quantitative estimate of its cross section, and finds this to be small enough. At present this is done by extrapolating results obtained from the standard Monte Carlo programs – a somewhat clumsy and (due to statistical errors) not too accurate method. In this paper we shall instead derive some semi-analytical estimates, together with corresponding Monte Carlo results from small dedicated programs designed to produce these event types only.

In this paper we consider two possibilities: the process

$$e^+(p_1) e^-(p_2) \rightarrow e^+(q_1) e^-(q_2) \gamma(k) , \quad (1)$$

where, say, the e^- is undetected and its rôle is taken over by the photon; and the process

$$e^+(p_1) e^-(p_2) \rightarrow e^+(q_1) e^-(q_2) \gamma(k_1) \gamma(k_2) , \quad (2)$$

where both fermions can escape detection. Since all other possibilities can be viewed as ‘standard’ radiative corrections to these two processes (and hence give cross sections that are again smaller) we only have to discuss these two.

We shall use a simplified model for the luminosity acceptance: a particle is taken to be detected when its scattering angle with respect to the beams is between a minimum $\theta_{\min}$ and a maximum $\theta_{\max}$, typical values being $\theta_{\min} = 50$ mrad and $\theta_{\max} = 150$ mrad. Additional cuts such as minimum-energy requirements will only reduce a cross section so that for the moment they can be left out. Note that under these cuts the cross section σ_0 for the Born process $e^+e^- \rightarrow e^+e^-$ is about

$$\sigma_0 = \frac{16\pi\alpha^2}{s} \left(\frac{1}{\theta_{\min}^2} - \frac{1}{\theta_{\max}^2} \right) , \quad (3)$$

where s is the total invariant mass squared. We shall derive approximate cross sections for the processes (1) and (2) under the above cuts. It is important to realize that what we are interested in here is not so much the total cross section but rather that part of it that cannot easily be treated by the standard analytical or Monte Carlo calculations. We may therefore safely assume that the photons are *not* collinear with the fermions: events in which they are collinear are already treated by existing programs.

2 The $e\gamma$ final state

We start with the process (1) where we assume the e^- to be lost, i.e. emitted at less than $\theta_{\min}$ away from the beam, while the e^+ and the photon are detected. If we make the (very good) approximations of neglecting the s -channel diagrams and the interference between diagrams with bremsstrahlung from the electron and those with bremsstrahlung from the positron, the result for T , the matrix element squared, summed and averaged over spins, is

$$T = e^6 (s^2 + s'^2 + u^2 + u'^2) \left(\frac{-1}{(p_1 k)(q_1 k)t'} + \frac{-1}{(p_2 k)(q_2 k)t} \right), \quad (4)$$

where $s = (p_1 + p_2)^2$, $s' = (q_1 + q_2)^2$, $t = (p_1 - q_1)^2$, $t' = (p_2 - q_2)^2$, $u = (p_1 - q_2)^2$, and $u' = (p_2 - q_1)^2$. Note that we have also neglected the electron mass m_e : this is justified since the bremsstrahlung photon, being required to be detected, cannot be collinear with any of the fermions (as already remarked above). The only invariant in the denominator that can become small is therefore t' , if we assume that it is the e^- that goes undetected: we can therefore also drop the second term in Eq.(4). To a relative accuracy of order $\mathcal{O}(m_e^2/s)$ we can approximate

$$t' = -\frac{s}{2}x(1 - c_2 + \delta) \quad \text{with} \quad \delta = \frac{2m_e^2}{s} \left(\frac{1-x}{x} \right)^2, \quad (5)$$

where c_2 is the cosine of the e^- scattering angle, and x its energy in units of the beam energy. Because of the peaking of the cross section at $c_2 = 1$, and the fact that the e^+ and the γ must be very nearly back-to-back, we can make the further approximations

$$\begin{aligned} k^0/p_1^0 &\sim 1 - x, & s' &\sim xs, & u &\sim -xs, \\ u' &\sim -s, & (p_1 k) &\sim s(1-x)/2, \end{aligned} \quad (6)$$

so that the expression for T reduces to

$$T = \frac{16e^6}{s} \frac{1+x^2}{x(1-x)^2} \frac{1}{1-c_2+\delta}. \quad (7)$$

Under the same assumptions the phase space factor can be written as

$$\begin{aligned} d\Phi &= \delta^4(p_1 + p_2 - q_1 - q_2 - k) d^4 q_1 \delta(q_1^2) d^4 q_2 \delta(q_2^2) d^4 k \delta(k^2) \\ &= \frac{s}{64} \frac{x}{(1-x)} dx d\Omega_1 d\Omega_2, \end{aligned} \quad (8)$$

where $\Omega_{1,2}$ are the $q_{1,2}$ solid angles. Performing the Ω_2 integral and the azimuthal integral for q_1 , we obtain the following cross section:

$$d\sigma = \frac{\alpha^3}{s} \left\{ \log \left(\frac{Q_{\min}^2}{m_e^2} \right) + 2 \log(x) - 2 \log(1-x) \right\} \frac{1+x^2}{(1-x)^3} dx dc_1, \quad (9)$$

where c_1 is the cosine of the q_1 polar angle, and $Q_{\min}^2 = s(1 - \cos \theta_{\min})/2$. The integration over x and c_1 is not completely trivial. Under the assumption that $\vec{q}_2$ points along $\vec{p}_2$, x is related to the e^+ and γ polar scattering angles θ_1 and θ_γ by $x \sim 1 - \theta_1/\theta_\gamma$: as x increases and hence k^0 decreases, the photon angle has to increase in order to balance the e^+ transverse momentum¹. Since the photon angle is at most $\theta_{\max}$ we have integration boundaries

$$\theta_{\min} < \theta_1 < (1-x)\theta_{\max} \quad , \quad 0 < x < 1 - \theta_{\min}/\theta_{\max} \equiv 1-r \quad . \quad (10)$$

We thus arrive at

$$\begin{aligned} \frac{d\sigma}{dx} = & \frac{\alpha^3}{2s} \left\{ \log \left(\frac{Q_{\min}^2}{m_e^2} \right) + 2 \log(x) - 2 \log(1-x) \right\} \\ & \left\{ \theta_{\max}^2 \frac{1+x^2}{1-x} - \theta_{\min}^2 \frac{1+x^2}{(1-x)^3} \right\} \quad . \end{aligned} \quad (11)$$

With all these approximations the accuracy of this result is of course limited. It is therefore reasonable to concentrate only on the leading term, with $\log Q_{\min}^2/m_e^2$. Accounting also for the possibility that it is the e^+ which is lost instead of the e^- , we arrive at the final approximate result

$$\sigma \sim \frac{\alpha^3}{s} \theta_{\max}^2 \log \left(\frac{Q_{\min}^2}{m_e^2} \right) \left\{ (2-r^2) \log \left(\frac{1}{r} \right) - \frac{1}{2}(1-r)(5-3r) \right\} \quad . \quad (12)$$

With the values $\theta_{\min} = 50$ mrad, $\theta_{\max} = 150$ mrad and $\sqrt{s} = 91.17$ GeV we find $\sigma_0 \sim 1.14 \cdot 10^{-4}$ GeV⁻² and $\sigma \sim 1.31 \cdot 10^{-11}$ GeV⁻², i.e. a truly small contribution of about $0.6 \cdot 10^{-9}$ %. It is actually quite simple to implement the above formulae in a small Monte Carlo program [3]. This yields a cross section which, for essentially all $\theta_{\min}$ and $\theta_{\max}$, is somewhat smaller due to an exact treatment of the kinematics and the matrix element, as well as more realistic cuts on the photon momentum. Some results are gathered in table 1. We conclude that this contribution to the luminosity cross section is negligible. The obvious reason is that whereas the Born cross section contains a factor $1/t^2$ leading to the $1/\theta_{\min}^2$ behaviour of σ_0 , the radiative matrix element goes only with a single power of $1/t'$ (note that under our cuts we have $t/t' \gg 1$ typically) which leads to a logarithmic term. The relative contribution is therefore at least suppressed by $\alpha\theta_{\min}^2/\pi$ times one relatively large logarithm: for a typical luminosity monitor the other logarithm, $\log(1/r)$, is always of order unity. Indeed from table 1 we see that the relative contribution becomes infinitesimally small at small angles.

¹This holds for small angles: for arbitrary angles it reads $x = 1 - \tan(\theta_1/2)/\tan(\theta_\gamma/2)$.

$\theta_{\min}(\text{ mrad})$	$\theta_{\max}(\text{ mrad})$	$\sigma_0(\text{ GeV}^{-2})$	$\sigma_{\text{anal}}(\text{ GeV}^{-2})$	$\sigma_{\text{MC}}(\text{ GeV}^{-2})$
50	150	$1.15 \cdot 10^{-4}$	$1.31 \cdot 10^{-11}$	$1.19 \cdot 10^{-11}$
15	50	$1.30 \cdot 10^{-3}$	$1.45 \cdot 10^{-12}$	$1.31 \cdot 10^{-12}$
5	15	$1.15 \cdot 10^{-2}$	$9.52 \cdot 10^{-14}$	$8.25 \cdot 10^{-14}$
1	5	$3.09 \cdot 10^{-1}$	$1.46 \cdot 10^{-14}$	$1.41 \cdot 10^{-14}$

Table 1: Cross sections for the $e\gamma$ final state from both the analytic expression and the Monte Carlo, for various angular ranges.

3 The $\gamma\gamma$ final state

We now turn to process (2). Due to the complications of 4-body kinematics our discussion will be even more qualitative than before, and our analytic estimate more crude. In what follows we shall be careful always to *overestimate* the cross section, however only by factors that are of order unity (between 1 and 4, say). We consider only those Feynman graphs where photon 1 is emitted by the positron line, and photon 2 by the electron line. The expression for T now reads

$$\begin{aligned}
T &= \frac{2e^8}{z^2} \frac{D}{(p_1 k_1)(q_1 k_1)(p_2 k_2)(q_2 k_2)} , \\
D &= (2(p_1 Q)(p_2 Q) + z(p_1 p_2))^2 + (2(p_1 Q)(q_2 Q) + z(p_1 q_2))^2 + \\
&\quad (2(q_1 Q)(q_2 Q) + z(q_1 p_2))^2 + (2(q_1 Q)(q_2 Q) + z(q_1 q_2))^2 + \\
&\quad z^2 \left((p_1 p_2)^2 + (p_1 q_2)^2 + (q_1 p_2)^2 + (q_1 q_2)^2 \right) , \tag{13}
\end{aligned}$$

where $Q = p_1 - q_1 - k_1 = -(p_2 - q_2 - k_2)$ and $z = Q^2$: again, here we have neglected m_e . In contrast to the single-bremsstrahlung case here a double pole reappears: this stronger singularity would suggest a larger contribution. Note the absence of any simple factorization: however, for $k_{1,2}^0 \rightarrow 0$ the standard infrared limit is recovered. Also note that in our description no symmetry factor for the photons must be used. It is helpful to introduce $R_1 = q_1 + k_1$ and $R_2 = q_2 + k_2$, and $s_{1,2} = R_{1,2}^2$. Typically we shall have of course that s_1 and s_2 are small compared to s . Using $c = \cos(\vec{p}_1, \vec{R}_1)$ we can write

$$z = \frac{1}{2} \left(s - s_1 - s_2 - c \left[(s - s_1 - s_2)^2 - 4s_1 s_2 \right]^{\frac{1}{2}} \right) . \tag{14}$$

Its minimum (for $c = 1$) is about $s_1 s_2 / s$, a good deal smaller than either s_1 or s_2 : in particular it is very well possible to have $Q^2 > (p_1 - q_1)^2$, a situation known in the jargon as the Compton peak, and relevant in multiperipheral-scattering type processes for instance at HERA [4]. Furthermore, we have the following inequality:

$$|(q_1 Q)| = |(p_1 q_1) - (k_1 q_1)|$$

$$\begin{aligned}
&< \max(|(p_1 q_1)|, |(q_1 k_1)|) \\
&< \max(|(p_1 R_1)|, |(q_1 k_1)|) \\
&= \max(\tfrac{1}{2}(s_1 + z), \tfrac{1}{2}s_1) = \tfrac{1}{2}(s_1 + z) .
\end{aligned} \tag{15}$$

We first (over)estimate D by the following expression:

$$D = 3sz^2 + 2(s_1 + z)^2(s_2 + z)^2 , \tag{16}$$

where we have used Eq.(15) and the fact that $(a + b)^2$ can be overestimated by $2(a^2 + b^2)$.

We proceed by writing the phase space integration element in a suitable form:

$$\begin{aligned}
d\Phi &\equiv \delta^4(p_1 + p_2 - q_1 - q_2 - k_1 - k_2) d^4 q_1 \delta(q_1^2) d^4 q_2 \delta(q_2^2) d^4 k_1 \delta(k_1^2) d^4 k_2 \delta(k_2^2) \\
&= \frac{ds_1 ds_2 dz d(p_1 k_1) d(p_2 k_2) d\phi d\phi_1 d\phi_2}{16s(s_1 + z)(s_2 + z)} ,
\end{aligned} \tag{17}$$

where ϕ is the azimuthal angle of $\vec{R}_1$ around the beams, $\phi_{1,2}$ those of $\vec{k}_{1,2}$ around $\vec{p}_{1,2}$ in the $R_{1,2}$ rest frames, and we have traded the various polar angles for invariant products. The differential cross section then reads, after trivial integration over the azimuthal angles:

$$d\sigma = \frac{2\alpha^4}{\pi s^2} \frac{3s^2 z^2 + 2(s_1 + z)^2(s_2 + z)^2}{z^2 s_1 s_2 (s_1 + z)(s_2 + z)} ds_1 ds_2 dz d\log(p_1 k_1) d\log(p_2 k_2) . \tag{18}$$

In this expression the first term, $3s^2 z^2$, gives the order of magnitude of the cross section (if z goes to its minimum $s_1 s_2 / s$, both terms go as $s_1^2 s_2^2 / s^2$). We therefore only keep this term, thereby (counterintuitively) cancelling also the z^2 in the denominator.

We now turn our attention to the phase space boundaries we have to impose. We are explicitly not interested in the ‘usual’ type of event where the bremsstrahlung photons are more or less collinear with the fermions that scatter over some appreciable angle. Those events therefore have typically $z > s_{1,2}$. We therefore choose the constraint $z < s_{1,2}$: if we include a factor 2 in the cross section we may impose the ordering $z < s_2 < s_1$. Constraints on the $p_i k_i$ integrations are obtained as follows. The photons need to have scattering angles of at least $\theta_{\min}$ to be observed: in addition they must have a certain energy $E_0 = x_0 \sqrt{s}/2$, with x_0 typically 0.5 or so (note that if the photon itself has less energy but happens to be accompanied by an outgoing fermion such that the *pair* has enough energy, this is again not an event of the type we are dealing with here). Hence $(p_i k_i)$ must be at least $s_0 \equiv s x_0 (1 - \cos \theta_{\min})$. On the other hand we must have $(p_i q_i) > 0$ so that $(p_i k_i) < (s_i + z)/2$. After the above considerations the cross section reduces to

$$d\sigma = \frac{12\alpha^4}{\pi} \frac{1}{s_1 s_2 (s_1 + z)(s_2 + z)} \log\left(\frac{s_1 + z}{2s_0}\right) \log\left(\frac{s_2 + z}{2s_0}\right) dz ds_2 ds_1 . \tag{19}$$

We now use the fact that $s_i + z > 2s_0$ so (with $z < s_i$) certainly $s_i > s_0$, and we replace $\log((s_i + z)/2s_0)$ by $\log(s_i/s_0)$ (again overestimating the cross section). The z integration can now be easily performed: approximating its lower limit by $z = 0$ (or assuming $s_{1,2} \ll s$ we find

$$\int \frac{dz}{(s_1 + z)(s_2 + z)} = \frac{1}{s_1 - s_2} \log \left(\frac{2s_1}{s_1 + s_2} \right) < \frac{1}{s_1} , \quad (20)$$

where again the inequality is always good to better than a factor 2. After the s_2 integral we then have

$$d\sigma = \frac{6\alpha^4}{\pi} \frac{1}{s_1^2} \log^3 \left(\frac{s_1}{s_0} \right) ds_1 , \quad (21)$$

and the total cross section reads

$$\begin{aligned} \sigma &= \frac{6\alpha^4}{\pi} \left[-\frac{1}{s_1} \left(\log^3 \left(\frac{s_1}{s_0} \right) + 2 \log^2 \left(\frac{s_1}{s_0} \right) + 6 \log \left(\frac{s_1}{s_0} \right) + 6 \right) \right]_{s_1=s_0}^{s_1=s} \\ &= \frac{36\alpha^4}{\pi s_0} \left(1 - \mathcal{O} \left(\frac{s_0}{6s} \log^3 \frac{s}{s_0} \right) \right) . \end{aligned} \quad (22)$$

Here we have cavalierly assumed that the upper limit on s_1 is s . As the major contribution comes from low s_1 values this influences only the subleading terms which are suppressed by a factor s_0/s .

Some remarks are in order here. In the first place the cross section goes as $1/(1 - \cos \theta_{\min})$ just like the Born cross section, in contrast to the $e\gamma$ case considered before. Although not containing any large logarithms, this (two-loop) contribution is therefore larger: using the same parameters $\sqrt{s} = 91.17$ GeV, $\theta_{\min} = 50$ mrad, $\theta_{\max} = 150$ mrad, and $x_0 = 0.5$, we find $\sigma = 2.4 \cdot 10^{-8} \text{ GeV}^{-2}$, or a correction of $2 \cdot 10^{-4}$. In the second place, performing the above integrations for the second, neglected term in Eq.(18) leads to a contribution $(\alpha^4/\pi s) \log^4(s/s_0)$ plus subleading terms, which has more logarithms but no factor $1/s_0$.

It must be clear, what with all the above overestimates, that the exact cross section must be a good deal smaller than the one given in Eq.(22) although we expect it to be qualitatively similar. Again a small Monte Carlo exercise [5] leads, for the above parameters, to a cross section of $2.5 \cdot 10^{-10} \text{ GeV}^{-2}$, some two orders of magnitude less (even without vetoing on events where the fermions accidentally are also detected). More results are gathered in table 2. This factor is due to the succession of overestimates made (about a factor 10 in total, most of it coming from our approximation for D and the z integral of Eq.(20)), plus the crudity of our modelling of the phase space boundaries: imposing the exact angular and energy cuts on the photons rejects about 85% of the events. However since our analytic overestimate already gives us a negligible correction to the Born luminosity cross section this need not worry us.

$\theta_{\min}(\text{ mrad})$	$\theta_{\max}(\text{ mrad})$	$\sigma_0(\text{ GeV}^{-2})$	$\sigma_{\text{anal}}(\text{ GeV}^{-2})$	$\sigma_{\text{MC}}(\text{ GeV}^{-2})$
50	150	$1.15 \cdot 10^{-4}$	$2.46 \cdot 10^{-8}$	$2.45 \cdot 10^{-10}$
15	50	$1.30 \cdot 10^{-3}$	$2.77 \cdot 10^{-7}$	$3.38 \cdot 10^{-9}$
5	15	$1.15 \cdot 10^{-2}$	$2.50 \cdot 10^{-6}$	$3.00 \cdot 10^{-8}$
1	5	$3.09 \cdot 10^{-1}$	$6.26 \cdot 10^{-5}$	$1.01 \cdot 10^{-6}$

Table 2: Cross sections for the $\gamma\gamma$ final state from both the analytic expression and the Monte Carlo, for various angular ranges.

We conclude that for a typical luminosity monitor setup those events where one or both of the fermions escape detection and their rôle is taken over by bremsstrahlung photons will not contribute appreciably to the total cross section. It appears that it is not necessary to include in Bhabha event generators a dedicated treatment of these regions of phase space: but it may be advisable to make simulation programs expressly cut out these regions in order to avoid unwanted rare events with large weights.

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