CERN-TH-6161/91

CP VIOLATION WITH SELF-TAGGING B_d MODES

Isard DUNIETZ
Theoretical Physics Division
CERN, CH 1211 Geneva 23

ABSTRACT

Gronau and Wyler (GW) extract the weak phase γ and remove the dependence on final state phases by measuring six charged B meson rates, which are grouped into two triangles. This note applies the GW method to self-tagging B_d modes. Self-tagging eliminates the need for time-dependent measurements. Of interest are the six rates, $\Gamma(B_d \rightarrow D^0 K^{*0}) = \Gamma(\bar{B}_d \rightarrow \bar{D}^0 \bar{K}^{*0})$, $\Gamma(B_d \rightarrow \bar{D}^0 K^{*0}) = \Gamma(\bar{B}_d \rightarrow D^0 \bar{K}^{*0})$, $\Gamma(B_d \rightarrow D_{CP}^0 K^{*0})$ and $\Gamma(\bar{B}_d \rightarrow D_{CP}^0 \bar{K}^{*0})$, where D_{CP}^0 denotes a CP eigenmode of a D^0 or $\bar{D}^0$. Here the K^{*0} must be seen in its $K^+\pi^-$ mode, which tags the beauty flavour. An additional handle exists to resolve an ambiguity in $|\sin \gamma|$, which is accomplished by isospin symmetry from B_u and B_d data. Preliminary results of feasibility study are given.

CERN-TH-6161/91

July 1991

CP violation in neutral B^0 modes yields information about the angles of the Cabibbo Kobayashi Maskawa (CKM) unitarity triangle. Whereas the angles α and β are probed by the decays $B_d \rightarrow \psi K_S$ and $\pi^+ \pi^-$ respectively, the last angle γ is probed by the B_s decay $B_s \rightarrow \rho^0 K_S$.¹ Not only is the B_s meson harder to produce than the B_d one, but it is expected to oscillate rapidly in time. Time-dependent measurements are thus necessary to extract CP violating effects in the $B_s - \bar{B}_s$ system. Even if the technological challenges of time-dependent measurements of $B_s \rightarrow \rho^0 K_S$ were to be overcome, and a CP violating asymmetry extracted, it would still not be clear that $\sin 2\gamma$ had been observed, because the mode $B_s \rightarrow \rho^0 K_S$ could have large contributions with different weak CKM phases.

An alternative method for measuring this angle γ was recently advocated by Gronau and Wyler (GW).² It is based on the observation that the mode $B \rightarrow D^0 K$ interferes with $B \rightarrow \bar{D}^0 K$, when D^0 is seen in a CP eigenmode. CP violation can occur.³ The extraction of the weak phase γ is made possible by removing the final state phase difference, because the modulus of each individual amplitude can be determined separately. GW measure the angle γ with some discrete ambiguity from six charged B decay modes, which are grouped into two triangles. In an earlier paper by Gronau and London, time-dependent measurements of analogous B_d modes extract, among others, the angle γ .⁴

This note extracts γ by applying those ideas to neutral B_d modes, where time-dependent measurements are not required, because no tagging is necessary—that is, no separation of a B_d from its antiparticle $\bar{B}_d$.⁵ They are not required because the modes are self-tagging. That simplifies the experimental analysis considerably for the B_d decays.

The success of this approach rests on the possibility of accurately measuring the angles of the two triangles that are constructed out of the six rates. It is, therefore, helpful when the two interfering amplitudes are of comparable size. Whereas the two interfering amplitudes for the charged B

modes could be quite dissimilar in moduli, the B_d ones could have similar magnitudes. How that might come about will be shown below. Thus, the optimal place to look for such interference phenomena might be the ones advocated in this note. As in all such discussions, experiment will ultimately determine the best modes. For the sake of completeness, this note reviews several of the thought-provoking ideas discussed by GW as they relate to the B_d modes.

The B_d modes of interest are $B_d \rightarrow D_{1(2)}^0 K^{*0}$ and $\bar{B}_d \rightarrow D_{1(2)}^0 \bar{K}^{*0}$, where $D_{1(2)}^0$ is the CP even (odd) state, $D_{1(2)}^0 \equiv (D^0 + (-) \bar{D}^0) / \sqrt{2}$. The K^{*0} must be seen in its charged mode, $K^+ \pi^-$, which occurs $2/3$ of the time. It is the sign of this charged kaon which tags the flavour of the neutral B_d . This self-tagging eliminates the need for time-dependent measurements. The state D_1^0 can be identified by its CP even eigenmodes, $\pi^+ \pi^-$, $K^+ K^-$, etc., and D_2^0 by its CP odd ones, $K_S \pi^0$, $K_S \rho^0$, $K_S \omega$, $K_S \phi$, etc..⁶ In the $D^0 - \bar{D}^0$ system, the CKM model predicts tiny CP violation effects that are neglected here.

Two interfering amplitudes contribute to the process $B_d \rightarrow D_{1(2)}^0 K^{*0}$, with different weak phases, and with probably different final state phases,

$$\begin{aligned} A(B_d \rightarrow D_{1(2)}^0 K^{*0}) &= \frac{1}{\sqrt{2}} (A(B_d \rightarrow D^0 K^{*0}) + (-) A(B_d \rightarrow \bar{D}^0 K^{*0})) \\ &= \frac{1}{\sqrt{2}} (|A| e^{i\gamma} e^{i\delta} + (-) |B| e^{i\tau}) \end{aligned} \quad (1)$$

Here $|A|$ and $|B|$ are the magnitudes, δ and τ the final state phases, and γ and 0 the weak phases for the amplitudes $B_d \rightarrow D^0 K^{*0}$ and $B_d \rightarrow \bar{D}^0 K^{*0}$, respectively. The CP conjugated process leaves the final state phases unchanged and complex conjugates the weak phases,

$$\begin{aligned} A(\bar{B}_d \rightarrow D_{1(2)}^0 \bar{K}^{*0}) &= \frac{1}{\sqrt{2}} (A(\bar{B}_d \rightarrow \bar{D}^0 \bar{K}^{*0}) + (-) A(\bar{B}_d \rightarrow D^0 \bar{K}^{*0})) \\ &= \frac{1}{\sqrt{2}} (|A| e^{-i\gamma} e^{i\delta} + (-) |B| e^{i\tau}) \end{aligned} \quad (2)$$

The final state phases probably differ because the final state from $B_d \rightarrow D^0 K^{*0}$ has pure isospin 1, whereas that from $B_d \rightarrow \bar{D}^0 K^{*0}$ is a superposition of

isospin 0 and 1.^{3,2#refBS and GW} They differ in their charm quantum numbers as well. The rates differ in general and CP violation occurs,

$$|A(B_d \rightarrow D_1^0 K^{*0})|^2 - |A(\bar{B}_d \rightarrow D_1^0 \bar{K}^{*0})|^2 = -2 |A(B_d \rightarrow D^0 K^{*0})| |A(\bar{B}_d \rightarrow \bar{D}^0 K^{*0})| \sin \Delta \sin \gamma. \quad (3)$$

Here the final state phase difference is denoted by $\Delta \equiv \delta - \tau$.

The weak phase γ is simple to obtain.^{2#ref GW} One measures the following six rates. The first two are likely to show CP violation,

$$\Gamma(B_d \rightarrow D_1^0 K^{*0}) \neq \Gamma(\bar{B}_d \rightarrow D_1^0 \bar{K}^{*0}).$$

The four remaining rates are grouped into two pairs of equal magnitudes, $\Gamma(B_d \rightarrow D^0 K^{*0}) = \Gamma(\bar{B}_d \rightarrow \bar{D}^0 \bar{K}^{*0})$, $\Gamma(B_d \rightarrow \bar{D}^0 K^{*0}) = \Gamma(\bar{B}_d \rightarrow D^0 \bar{K}^{*0})$.

The flavour of the D^0 and $\bar{D}^0$ is obtained by the charge of the decay kaon or lepton. In the D^0 - $\bar{D}^0$ system, the CKM model predicts negligible mixing effects which we disregard. Measurement of the six rates determines the sides of the two triangles (see Figure 1). The angle from $A(\bar{B}_d \rightarrow \bar{D}^0 \bar{K}^{*0})$ to $A(B_d \rightarrow D^0 K^{*0})$ is 2γ . To increase statistics, the whole analysis ought to be repeated with D_2^0 .

Measuring the weak phase γ is not as simple as that, for it involves a discrete ambiguity.^{2#refGW} The cosine theorem for planar triangles determines unambiguously $\cos(\Delta + \gamma)$ and $\cos(\Delta - \gamma)$ from the B_d and $\bar{B}_d$ triangles, respectively. A trivial trigonometric exercise reveals that $\sin^2 \gamma$ satisfies a quadratic equation with coefficients that depend on the observables, $\cos(\Delta + \gamma)$ and $\cos(\Delta - \gamma)$. In general, a two-fold ambiguity in $|\sin \gamma|$ occurs. It arises because the final state phase difference Δ might be confused with the weak phase γ . The additional sign ambiguity in $\sin \gamma$ is removed by using the known CP violation constraint of the neutral kaon system--that is, the ϵ parameter.^{2#refGW} That constraint allows only the range,^{1#refBDGN}

$$0.1 \lesssim \sin \gamma \leq 1. \quad (4)$$

The quantity $\sin \gamma$ is never zero, always positive and larger than about 0.1.

The remaining ambiguity in the magnitude is trickier to remove than the sign one. GW suggest looking at a variety of processes which would be likely to remove the ambiguity. This note increases the number of usable processes for its removal. In addition, it explores another consistency check--namely, that isospin symmetry relates B_d to B^- modes and allows the determination of the difference, $\Delta(B^-) - \Delta(B_d)$. Here Δ denotes the final state phase difference. In the following, we elaborate on how to eliminate the ambiguity.

GW recommend studying the exclusive processes in the charged modes, $B^\pm \rightarrow D^0 (\bar{D}^0, D_{1(2)}^0) X^\pm$, where $X^\pm$ is $K^{*\pm}, K^{*\pm}, K^+\pi^-\pi^-,$ etc.. Whereas the weak phase is common to all such decay processes, the final state phase differences Δ differ from process to process. The ambiguity could thus be eliminated by studying many different processes. This same idea can be applied to the exclusive processes of the $B_d \rightarrow D^0 (\bar{D}^0, D_{1(2)}^0) X^0$, where X^0 is any mode with flavour content $\bar{s}d$, as long as its net strangeness can be unambiguously deduced.⁷ This net strangeness tags the beauty flavour. Thus X^0 stands for $K^+\pi^-, K^+\pi^-\pi^0,$ etc.. Were we to use $X^0 = K_S^0\pi^0, K_S^0,$ or any other mode without a definite strange quantum number, we would have to resort to time-dependent measurements in order to remove the B_d - $\bar{B}_d$ mixing effects.⁸ A further increase in usable processes is obtained by replacing the $D^0 (\bar{D}^0)$ by a $D^{*0} (\bar{D}^{*0})$ or any other $\bar{c}u$ ($\bar{c}u$) resonances, as long as those resonances can be seen in a CP eigenmode.^{6#refBS}

More can be learnt about the final state phases from a combined analysis of isospin related B_u and B_d processes. Consider, for instance, the modes $B^\pm \rightarrow D^0 (\bar{D}^0, D_{1(2)}^0) K^{*\pm}$, with its final state phase difference $\Delta(B^-)$, and $B_d^- \rightarrow D^0 (\bar{D}^0, D_{1(2)}^0) \bar{K}^{*0}$, with its final state phase difference $\Delta(B_d)$. The difference $\Delta(B^-) - \Delta(B_d)$ can be determined within some discrete ambiguity, and could be used as an additional consistency check for the elimination of the ambiguity in $|\sin \gamma|$.

In greater detail, isospin symmetry creates a triangle relation between the three processes,

$$A(B^- \rightarrow D^- \bar{K}^{*0}) - A(B^- \rightarrow \bar{D}^0 K^{*-}) = \sqrt{2} A(\bar{B}_d \rightarrow \bar{D}^0 \bar{K}^{*0}). \quad (5)$$

Because a weak phase is common to the above three processes and cancels in this triangle relation, the angles of the triangle yield information only about the final state phases. A different weak phase is common to the other three processes, which form a different triangle,

$$A(\bar{B}_d \rightarrow D^+ K^{*-}) + A(\bar{B}_d \rightarrow D^0 \bar{K}^{*0}) = -\sqrt{2} A(B^- \rightarrow D^0 K^{*-}). \quad (6)$$

The study of those additional two triangles (see Figure 2) reveals the difference of Δ for the charged B relative to the neutral B_d processes, $\Delta(B^-) - \Delta(\bar{B}_d)$, and aids in eliminating the ambiguity in magnitude. The respective charge conjugated processes of eqs. (5)-(6) form congruent triangles rotated by overall phases. Including them doubles statistics.

Comments:

- 1) As mentioned in the opening remarks, the success of this approach rests on the possibility of accurately measuring the angles of the two triangles of Figure 1. The two interfering amplitudes in the charged B modes could be dissimilar in magnitude,

$$\left| \frac{A(B^- \rightarrow \bar{D}^0 X^-)}{A(B^- \rightarrow D^0 X^-)} \right| \ll 1. \quad (7)$$

Not only is a CKM suppression factor at work, but also the numerator is colour suppressed, whereas the denominator has a colour allowed contribution which interferes with a small colour suppressed one. Using the BSW model,⁹

$$\left| \frac{A(B^- \rightarrow \bar{D}^0 X^-)}{A(B^- \rightarrow D^0 X^-)} \right| \sim \left| \frac{V_{ub}}{V_{cb}} \frac{a_2}{a_1} \right| = \left| \frac{V_{ub}}{V_{cb}} \frac{1}{0.22} \frac{a_2}{a_1} \right| \sim O\left(\frac{1}{10}\right). \quad (8)$$

However, rescattering contributions, such as $B^- \rightarrow \rho^0 D_s^- \leftrightarrow \bar{D}^0 K^{*-}$ for $X^- = K^{*-}$, might increase this ratio.

For the neutral modes, the situation is more hopeful. The same CKM suppression factor operates but, in contrast to the charged B decays, both

interfering amplitudes are colour suppressed. Within the BSW model no further suppression takes place other than the CKM one,

$$\left| \frac{A(B_d \rightarrow D^0 X^0)}{A(B_d \rightarrow \bar{D}^0 X^0)} \right| \sim \left| \frac{V_{ub}}{V_{cb}} \frac{1}{V_{cb}} \frac{1}{s_c} \right| \sim O\left(\frac{1}{2}\right). \quad (9)$$

Thus, the optimal places to look for such interference phenomena might be the ones advocated in this note. Again, rescattering effects may upset this prediction and the best modes will be determined through experiment.

- 2) In general, a rate asymmetry occurs. The two triangles of Figure 1 will not be congruent and the angle γ will be extracted. However, even when $\Delta = 0$ and no asymmetry occurs, the phase γ can be measured by drawing the two triangles of Figure 1.^{2, #refGW} It is interesting that a weak phase can be extracted even when no rate asymmetry occurs.

Suppose that the two triangles of Figure 1 are exactly congruent and non-trivial. The phase from $A(B_d \rightarrow \bar{D}^0 K^{*0})$ to $A(B_d \rightarrow D^0 K^{*0})$ must be γ . It cannot be Δ , because then $\sin \gamma = 0$ (see Eq. 3), contradicting the ϵ -constraint (see Eq. 4), and there is no ambiguity in magnitude.

In practice, exact congruence will never be shown. It is possible that $\sin \gamma$ is at its lower bound and that the angle of the triangle is mainly due to a final state phase difference Δ . The triangles might then turn out to be congruent to existing experimental accuracy. It is clear, however, that if one of the two solutions for $|\sin \gamma|$ were found to be smaller than its lower bound obtained from the ϵ -constraint, then the other solution would have to be $|\sin \gamma|$. A detailed study is underway. Among other topics, it tries to determine how accurate the congruence must be in order to achieve a desired $\delta \cos \gamma$.¹⁰

To develop a preliminary feel for the sensitivity of this method, suppose that $\Delta = 0$. Figure 3 plots the accuracy on $\cos \gamma$ for

- (a) a hadronic collider experiment with $5 \times 10^{11} B_d$ mesons, and

(b) an $\Upsilon(4S)$ experiment with superb detection efficiencies and $5 \times 10^7 B_d$ mesons.

We took as branching rates $B(B_d \rightarrow \bar{D}^0 K^{*0}) = 4 \times 10^{-5}$, $B(B_d \rightarrow D^0 K^{*0}) = 10^{-5}$, and fixed N^0 as the number of produced B_d . When $\cos \gamma$ is chosen, the number of produced $B_d \rightarrow D_1^0 K^{*0}$ [$N^0(D_1^0 K^{*0})$] is obtained from the cosine theorem.

The Table lists the visible fractions (f) and detection efficiencies (ϵ) for the various final state particles. The particle $\bar{D}^0$ is seen in its flavour tagging $K^+ \pi^-$, $K^+ \pi^- \pi^+ \pi^-$ modes, and the K^{*0} in its flavour tagging $K^+ \pi^-$ one. The D_1^0 is seen in its $K^+ K^-$, $\pi^+ \pi^-$ modes in a collider or fixed target experiment. In contrast, many more CP eigenmodes may be used in an $\Upsilon(4S)$ experiment with a visible fraction of 5%. Because this is a cursory feasibility study, we grouped them all as CP even eigenstates. In a detailed analysis careful attention must be paid to their CP parities, individual detection efficiencies and visible fractions.

The observed number of $B_d \rightarrow D_1^0 K^{*0}$ is then

$$N(D_1^0 K^{*0}) = N^0(D_1^0 K^{*0}) f(D_1^0) \epsilon(D_1^0) f(K^{*0}) \epsilon(K^{*0}).$$

Similar relations hold for the other B_d modes. The error on $\cos \gamma$ is obtained by combining in quadrature the statistical error on the observed number of relevant B_d decays with reasonable estimates of systematic errors on detection efficiencies listed in the Table. Figure 3 shows the dependence of $\delta \cos \gamma$ on $\cos \gamma$ for the two experiments.

The mode $B_s \rightarrow \rho^0 K_S$ is quite complementary to the modes advocated here. Whereas our modes could not rule out $\gamma = 0$ when $\cos \gamma \approx 1$, the $B_s \rightarrow \rho^0 K_S$ asymmetry could, because $\sin 2\gamma = 2 \sin \gamma \cos \gamma \gtrsim 0.2$ is large. Vice versa, for a value of $\sin 2\gamma \approx 0$ which yields no asymmetry in $B_s \rightarrow \rho^0 K_S$, the modes here would show striking effects.

3) The value of Δ might be independently obtained by using $SU(3)$ related processes. Perhaps theory will develop so that they can be predicted from first principles, such as lattice calculations.

Conclusion

A crucial test of the CKM model is whether the unitarity triangle closes, $\alpha + \beta + \gamma = 0$. In contrast to α and β , the angle γ is notoriously difficult to measure. The usual wisdom is to use $B_s \rightarrow \rho^0 K_S$. This note advocates extracting γ from six self-tagging B_d rates,

$$\bar{B}_d \rightarrow D^0 (\bar{D}^0, D_{1(2)}^0) K^{*0}.$$

Because this extraction does not involve $B^0 - \bar{B}^0$ mixing, the rates need not be time-dependent. In contrast, this mixing causes the CP asymmetries for the classic modes, $B_d \rightarrow \psi K_S$, $\pi^+ \pi^-$ and $B_s \rightarrow \rho^0 K_S$.

Although it is likely that the interfering amplitudes of those B_d modes have comparable magnitudes, the charged B ones used by GW might be quite dissimilar in size. If that were the case, the B_d modes advocated here would be superior to the charged ones.

Isospin relations, when applied to both B_d and B_u modes, yield information on the final state phase difference, $\Delta(B^-) - \Delta(B_d)$. This is a consistency check, which might aid in removing the ambiguity in $|\sin \gamma|$. Preliminary results of a feasibility study were also shown.

Acknowledgements

I thank Boris Kayser for discussions, which clarified some of the issues in this report. I am grateful to J. Bijnens, J.-R. Cudell, D. Denegri, J. Incandela, Y. Lemoigne, M. Lusignoli, M. Gronau, M. Masetti, A. Pich, and B. Stech for useful discussions.

References

- 1 For a definition of those angles, see SLAC report, 1989, edited by D. Hitlin, SLAC-353.
- 2 M. Gronau and D. Wyler, Technion Report, Technion-PH-91-14, 1991, to be published in Phys. Lett. B.
- 3 I. I. Bigi and A. I. Sanda, Phys. Lett. B 211, 213 (1988).
- 4 M. Gronau and D. London, Phys. Lett. B 253, 483 (1991), emphasize that the angles α and β can be extracted, but what those modes really measure is γ and β . The angle α is deduced by assuming that the triangle closes, $\alpha + \beta + \gamma = \pi$.
- 5 That tagging is not required for such modes was known to Bigi and Sanda, Ref. 3, see also Y. Koide, Phys. Rev. D40, 1685 (1989), and I. Dunietz and J. Rosner, CERN-TH.5899/90, 1990 (unpublished). However, the measurement of γ by removing the dependence on final state phases was not discussed in Ref. 3.
- 6 A general discussion on how to obtain the CP parity from CP eigenmodes can be found in I. Dunietz and A. Snyder, Phys. Rev. D43, 1593 (1991), see especially the longer version, SLAC report, SLAC-PUB-5234, 1990 (unpublished).
- 7 For instance, the mode $K^+ \pi^- \pi^0$ can be used, whereas the mode $K_S^0 K^- \pi^+$ cannot, because it sometimes incorrectly tags the beauty flavour.
- 8 M. Gronau and D. London, Ref. 4, have discussed those modes in light of time-dependent measurements.
- 9 M. Bauer, B. Stech and M. Wirbel, Z. Phys. C34, 103 (1987).
- 10 I. Dunietz, in preparation.

Table

The observable fraction (f) and detection efficiency (ϵ) for various particles. Values are quoted for a hadronic collider experiment and in brackets for an ideal $\Upsilon(4S)$ one.

particle	f	ϵ
$D^0, \bar{D}^0$	0.115 (0.115)	0.1 ± 0.005 (1.00 ± 0.05)
D_1^0	5.7×10^{-3} (5×10^{-2})	0.10 ± 0.01 (1.0 ± 0.1)
$K^{*0}, \bar{K}^{*0}$	2/3 (2/3)	0.1 ± 0.005 (1.00 ± 0.05)

Figure Captions

Figure 1. Six processes arranged into two triangles. The angle from $A(\bar{B}_d \rightarrow \bar{D}^0 \bar{K}^{*0})$ to $A(\bar{B}_d \rightarrow D^0 \bar{K}^{*0})$ is 2γ .

Figure 2. Isospin relations that inform us about the final state phases.

(a) Processes governed by $b \rightarrow u + \bar{c}s$. (b) Processes governed by $b \rightarrow c + \bar{u}s$.

Figure 3. For $\Delta = 0$, feasibility study on the measurement of $\cos \gamma$. Visible fractions and detection efficiencies come from the Table. Lower curve assumes only statistical errors, whereas upper curve also uses the errors on the ϵ given in the Table.

(a) A collider experiment with 5×10^{11} B_d mesons.

(b) An $\Upsilon(4S)$ experiment with 5×10^7 B_d mesons.

Figure 1

Figure 2

Figure 3