CERN-TH-6168/91
CTP-TAMU-46/91
ACT-42

THRESHOLD CORRECTIONS AND MODULAR INVARIANCE IN FREE FERMIONIC STRINGS

S. Kalara^{1,2}, J.L. Lopez^{1,2*} and D.V. Nanopoulos^{1,2,3}

1. Center for Theoretical Physics, Department of Physics,
Texas A&M University, College Station, TX 77843 -- 4242, U.S.A.
2. Astroparticle Physics Group, Houston Advanced Research Center (HARC)
The Woodlands, TX 77381, U.S.A.
3. Theoretical Physics Division, CH -- 1211 Geneva 23.

ABSTRACT

We examine the modular invariant properties of the low-energy effective action of free fermionic models. This entails a detailed comparison between the so-called string and super-gravity bases, since only in the latter are the modular invariant properties evident, whereas the S -matrix calculations are performed in the former. We then adjoin these results to the Kähler anomaly method for obtaining the untwisted moduli dependence of the string threshold corrections to the gauge couplings. Finally we apply our results to the flipped $SU(5)$ string model and determine the common unification scale for all the gauge couplings in the model.

CERN-TH-6168/91
CTP-TAMU-46/91
ACT-42
June 1991

* Supported in part by an ICSC-World Laboratory Scholarship

1. Introduction

String derived models of particle interactions are endowed with an extraordinarily rich symmetry structure [1]. Compared to ordinary grand unified theories, the string derived effective low-energy action has far more predictive power and leads to penetrating insight into the connection between seemingly unrelated sectors of the theory [2,3].

Although the task of constructing the low-energy effective string action which manifests all the symmetries of the original string theory is by no means straightforward, over the past few years great strides have been made in this direction [4]. One of the genuinely stringy symmetries which has attracted a great deal of attention is the so-called “modular symmetry”, a generalization of the basic duality symmetry of string models [5,6]. In a large class of models, such as orbifold [6,7] and Calabi-Yau compactifications [8], this is an infinite-dimensional discrete symmetry. The existence of such symmetry has been known for quite some time [5]. However, its phenomenological implications have not been explored to their fullest extent [9,10,11].

One of the coveted predictions of string theory is the unification of all gauge couplings at some string scale M , that is $g_a(M) = g/\sqrt{k_a}$, where k_a is the level of the Kac-Moody algebra representing the gauge group G_a [12]. This relation holds for all hidden and/or observable gauge groups. From the low-energy point of view, this implies that the fine structure constant α_e , the strong coupling constant α_3 , and the weak mixing angle $\sin^2 \theta_W$, are all calculable from the knowledge of M and g . In theory this scenario is realized by extrapolating $g_a(M)$ down to the electroweak scale using the standard renormalization group analysis [13]. However, in practice any precise calculation of the low-energy observables requires the inclusion of the effect of the infinite number of massive string modes (the so-called “threshold effects”) in the renormalization group equations [14,15].

Recent attempts to calculate explicitly these effects have led to some very interesting general results. Among these is the connection between the threshold corrections and modular invariance [16,17,18,19]. In this paper we first review the status of the traditional calculation of threshold corrections. We then examine the modular invariance properties of the low-energy effective action obtained through the S-matrix approach, especially in models derived in the free fermionic formulation of superstring theory [21]. This entails a detailed comparison between

the so-called string and supergravity bases, since only in the latter are the modular invariant properties evident, whereas the S-matrix calculations are performed in the former. We then adjoin these results to the Kähler anomaly method for obtaining the untwisted moduli dependence of the threshold corrections. We finally apply our results to the flipped $SU(5)$ string model and determine the common unification scale for all the gauge couplings in the model.

2. Threshold corrections: N=2 method

In the one-loop approximation, the renormalized gauge coupling at a scale μ is given by [15]

$$\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g^2} + b_a \ln \frac{M^2}{\mu^2} + \Delta_a, \quad (1)$$

where b_a are the β -function coefficients, and Δ_a are the threshold corrections to the inverse gauge coupling which arise due to the presence of the infinite set of massive string modes. The reference scale M is renormalization scheme dependent, although to our level of approximation it is basically the same in the various schemes of interest and is given by [22]

$$M = 1.03 \times g \times 10^{18} \text{ GeV}. \quad (2)$$

In Ref. [15] Δ_a was calculated explicitly and is given by

$$\Delta_a = \int_{\Gamma} \frac{d^2\tau}{\tau_2} (\mathcal{B}_a(\tau, \bar{\tau}) - b_a), \quad (3)$$

where the integral is over the modulus $\tau = \tau_1 + i\tau_2$ of the worldsheet torus, and it is restricted to the fundamental domain $\Gamma = \{\tau_2 > 0; |\tau_1| < 1/2; |\tau| > 1\}$; $\mathcal{B}_a$ is related to the string partition function by insertion of helicity and gauge group generators.

Unfortunately, evaluation of Δ_a using Eq. (3) requires the knowledge of the complete string spectrum, an information which is very often lacking or can only be obtained with great difficulty. In fact, a few examples have been worked out only in two classes of models (orbifolds [15] and free fermionic constructions [22]). A more fruitful approach has been to examine

the dependence of Δ_a on different parameters of the model, especially on the moduli fields T [16,18,23]. Indeed, general arguments based on the geometry of the compactified space lead to the following generic expression in orbifold models [16]

$$\Delta_a = \tilde{\Delta}_a + c_a + Y. \quad (4)$$

It has been shown that nontrivial dependence of Δ_a on the untwisted moduli fields T_I arises only if the theory contains sectors which preserve N=2 supersymmetry [16,24,23]. This dependence is partially encoded in $\tilde{\Delta}_a$ which is given by [16,24]

$$\tilde{\Delta}_a = - \sum_I \frac{1}{2} b_a'^I \ln [|\eta(iT_I)|^4 \text{Re } T_I] , \quad (5)$$

where T_I are the untwisted moduli fields for each of the N=2 sectors of the orbifold model, and $b_a'^I$ are the corresponding N=2 β -function coefficients. (This expression is a one-loop effect and remains uncorrected at higher orders in the loop expansion [23].) The universal (gauge-independent) piece Y can also depend on untwisted (and twisted) moduli but its effect on observable physics can be masked out by absorbing it into the also unknown string coupling g . The gauge-dependent piece c_a does not depend on the untwisted moduli (although it may depend on twisted moduli) and generally receives contributions from N=2 and N=1 sectors of the orbifold model. For free fermionic models (which by construction are at a particular point in the moduli space) a stronger statement can be made regarding the constant piece c_a : the contribution from N=1 sectors vanishes identically and thus only the N=2 sectors contribute [22].

3. Threshold corrections: anomaly method

In a large class of models the information which is at our disposal are the low-energy effective Lagrangian and the symmetries of the theory. It would be of great value if Δ_a could be extracted from this limited information. It has been pointed out that using the modular symmetry and the low-energy effective action a substantial portion of Δ_a can be inferred. An

effective (bosonic) low-energy effective action is given by

$$S = \int d^4x \left\{ -\frac{R}{2\kappa^2} + \frac{1}{4g_a^2(T)} F_{\mu\nu}^a F_a^{\mu\nu} + \frac{i\theta_a}{32\pi^2} F_{\mu\nu}^a \tilde{F}_a^{\mu\nu} + \frac{1}{2} G_{IJ} \partial^\mu T_I \partial_\mu T_J^* + \frac{1}{2} g_{kl}(T) \partial^\mu \Phi_k \partial_\mu \Phi_l^* + V(T, \Phi) \right\}, \quad (6)$$

where $1/g_a^2$ is the inverse gauge coupling, θ_a is the vacuum angle, T_I are scalar (moduli) fields, Φ_k are the matter fields, G_{IJ} is the metric of the space of moduli T_I , and $g_{kl}(T)$ is the matter field metric. In terms of a manifestly supersymmetric action for the matter and moduli fields, the effective action can be rewritten as follows

$$S = \int d^4x \left[\int d^4\theta K_1(\Phi^\dagger, e^V \Phi, T, \bar{T}) + \int d^4\theta K_2(T, \bar{T}) + \int d^2\theta f^{ab} W_a^\alpha W_{b\alpha} + \int d^2\theta W(T, \Phi) + \text{h.c.} \right]. \quad (7)$$

String theory saddles the above action with the modular symmetry [5,6]

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad ad - bc = 1, \quad a, b, c, d \in \mathbf{Z}. \quad (8)$$

However, when one tries to reconstruct the geometry of the moduli space from the S-matrix approach, one is forced to choose a coordinate system in which [25]

$$K = \sum_i \Phi_i' \Phi_i'^\dagger + \sum_I t_I \bar{t}_I + \dots. \quad (9)$$

The above form of K is a reflection of the fact that the vertex operators used to obtain the S-matrix elements are chosen to generate properly normalized fields. This is the so-called “string basis” as opposed to the traditional “supergravity basis” (primed fields refer to the string basis, while unprimed ones to the supergravity basis). Interpretation of the t_I as the moduli fields in the string basis allows us to infer their Kähler function to be $K(t) = -\sum_I \ln(1 - t_I \bar{t}_I)$. In the string basis typically one gets [25,6,3]

$$S = \int d^4x \left[\int d^4\theta \Phi_i'^\dagger e^V \Phi_i' + \int d^4\theta t \bar{t} + (\text{higher orders}) \right]$$

$$+ \int d^2\theta W_{ijk} \Phi'_i \Phi'_j \Phi'_k + \text{h.c.} + (\text{higher orders}) \Big]. \quad (10)$$

Passage from Eq. (10) to Eq. (7) clearly involves (possibly) nonlinear transformations. Furthermore, if one wishes to avail oneself of the machinery developed for non-compact supergravity models, where K (correct to first order) is chosen to be [26,6]

$$K = -\ln(T + \bar{T} + (T + \bar{T})^p \Phi \Phi^\dagger + \dots), \quad (11)$$

the connection between t and T fields is definitely nonlinear and perhaps even non-holomorphic.

Since the modular invariant properties of the theory are evident only in the supergravity basis, it is necessary to determine the precise relation between matter and moduli fields in these bases. By carefully keeping track of the modular transformations of t, T, Φ' , and Φ , this connection has been unraveled in (2,2) orbifold models [6,27]. We now do this for free fermionic models. From the connection between the fermionic formulation and orbifolds [3], one infers the existence of three untwisted moduli for the class of models of interest [3,22], which will be denoted in the supergravity basis by T_I , $I = 1, 2, 3$ (related to the string basis fields t_I). Defining $F_I = \ln(ic_I T_I + d_I)$, the connection between the two bases becomes

$$t_I = \frac{1 - T_I}{1 + \bar{T}_I} \rightarrow t_I e^{\bar{F}_I - F_I}, \quad (12)$$

$$\Phi_i = \Phi'_i \prod_J (T_J + \bar{T}_J)^{\frac{1}{2} w_J^i} e^{i w_J^i \phi_J} \rightarrow \Phi_i \prod_J e^{-w_J^i F_J}, \quad (13)$$

where the arrows indicate the result of a modular transformation of T_I , and the phases ϕ_I transform as $e^{i\phi_I} \rightarrow e^{i\phi_I} e^{(\bar{F}_I - F_I)/2}$. In this same notation, we also have $(T_I + \bar{T}_I) \rightarrow (T_I + \bar{T}_I) e^{-(F_I + \bar{F}_I)}$. Also, the Φ'_i are inert under modular transformations [6]. The charges w_I^i are those of the vertex operator for the scalar component of Φ'_i under the three $U(1)$'s which make up the N=2 worldsheet supersymmetry $U_J(1)$ current [3]. These charges are permutations of $\{1, 0, 0\}$ for untwisted fields, and permutations of $\{\frac{1}{2}, \frac{1}{2}, 0\}$ for twisted ones [3].

Using the above connection, the Kähler function in the string basis (Eq. (9)) can be rewritten in the supergravity basis as follows (modulo a Kähler transformation)

$$K = \sum_I -\ln(T_I + \bar{T}_I) + \sum_i \prod_I (T_I + \bar{T}_I)^{-w_I^i} \Phi_i e^V \Phi_i^\dagger + \dots \quad (14)$$

Using Eqs. (12) and (13), this Kähler function is easily seen to be covariant under the modular transformations^{*} of T_I (see Eq. (8)) with

$$K \rightarrow K + \sum_I (F_I + \bar{F}_I). \quad (15)$$

At first glance, the modular transformation appears to be a subset of the Kähler transformations, *i.e.*, if the theory is of the Kähler class, then it would appear to be invariant under modular transformations. However, the Kähler transformation is one-loop anomalous [17,18,19]. A one-loop triangle diagram among one T field and two vector fields generates the following non-local term [17,18,19,20]

$$\mathcal{L}_{nl} = \frac{1}{8} \frac{1}{16\pi^2} \sum_a \sum_I \alpha_a^I \int d^4\theta (W^\alpha W_\alpha)_a \frac{1}{\square} D^2 \ln(T_I + \bar{T}_I) + \text{h.c.}, \quad (16)$$

with

$$\alpha_a^I = -c_V^a - \sum_i c_i^{a,I} (2w_I^i - 1)_a, \quad (17)$$

where c_V^a is the quadratic Casimir of the gauge group G_a and the sum over i runs over all matter fields in the N=2 sector I which belong to representations of the gauge group with Casimir $c_i^{a,I}$. Under the modular transformations $\mathcal{L}_{nl}$ yields an anomaly which can be canceled by adding a counterterm [18]

$$\mathcal{L}_{ct} = \frac{1}{4} \sum_a \int d^2\theta f_a (W^\alpha W_\alpha)_a + \text{h.c.}, \quad (18)$$

* The effect of modular transformations on the superpotential is rather nontrivial due to the presence of nonrenormalizable terms, but it can be shown to be such that $G = K + \ln |W|^2$ is modular invariant [28]. For the cubic terms this follows straightforwardly from Eq. (13), *i.e.*, $\Phi_i \Phi_j \Phi_k \rightarrow \Phi_i \Phi_j \Phi_k \prod_I e^{-F_I(w_I^i + w_I^j + w_I^k)} = \Phi_i \Phi_j \Phi_k \prod_I e^{-F_I}$. Thus $\ln |W|^2 \rightarrow \ln |W|^2 - \sum_I (F_I + \bar{F}_I)$.

$$f_a = \frac{-2}{16\pi^2} \sum_I \alpha_a^I \ln \eta^2(iT_I). \quad (19)$$

The two pieces $\mathcal{L}_{nl}$ and $\mathcal{L}_{ct}$ then contribute to the one-loop threshold corrections as follows [18]

$$\begin{aligned} \tilde{\Delta}'_a &= \Delta_{a(nl)} + \Delta_{a(ct)} = \tilde{\Delta}_a + Y' \\ &= - \sum_I \alpha_a^I [\ln(|\eta(iT_I)|^4) + \ln(T_I + \bar{T}_I)]. \end{aligned} \quad (20)$$

Here $\Delta_{a(nl)}$ includes a portion (Y') of the universal piece Y which may depend on T . The effect of massive string states is encoded in $\mathcal{L}_{(nl)}$ (and consequently in $\Delta_{a(nl)}$) since its structure and the weights w^i are determined by integrating out the massive fields, whereas $\Delta_{a(ct)}$ contains the effect of mixing between massive and massless states induced by modular transformations. Hence, both $\Delta_{a(nl)}$ and $\Delta_{a(ct)}$ in tandem incorporate the effects of the tower of massive states. Alternatively put, even though an effective action loses most information about the heavier modes, some of it can be recaptured by demanding modular invariance.

Another place where the universal piece Y derives its origin from is the Chern-Simons term in the effective action [17]. Due to the non-holomorphicity of the gauge kinetic function f_{ab} , supersymmetrization of the theory necessitates the existence of a Chern-Simons term, which implements an anomaly cancellation mechanism similar to the ten-dimensional Green-Schwarz mechanism [17,19].* This procedure is gauge-independent (but it may be N=2 sector and thus T dependent [23]) and hence contributes to the universal piece Y by an amount Y_{GS} .

The total contribution to the threshold corrections from all sources is then given by

$$\begin{aligned} \Delta_a &= \Delta_{a(nl)} + \Delta_{a(ct)} + c_a + (Y - Y') + Y_{GS} \\ &= \tilde{\Delta}_a + c_a + Y + Y_{GS}. \end{aligned} \quad (21)$$

For $\mathbf{Z}_3$ orbifolds $\tilde{\Delta}_a = 0$ [16]. From the general arguments of Ref. [23], the T dependence of Y_{GS} and Y (if any) must cancel each other out, leaving Δ_a completely independent of any moduli. As we shall see below, in the flipped $SU(5)$ case the presence of Y_{GS} ensures that the universal piece Y remains independent of the untwisted moduli.

* This anomaly should not be confused with the modular transformation anomaly discussed above.

To summarize, after including the Green-Schwarz type of term in the effective action, the anomaly method gives the correct T dependence of the threshold corrections. Its failure to give c_a (which in all known cases is a rather small contribution) is compensated by the sheer simplicity of the method.

4. Application to Flipped $SU(5)$

In this section we compute the threshold corrections in the flipped $SU(5)$ string model [2] using the formulas for Δ_a obtained in the $N=2$ method (Eq. (5)) and the anomaly method (Eq. (20)). The model is specified by a set of eight basis vectors of boundary conditions for the worldsheet fermions: $B = \{S, \zeta, b_1, b_2, b_3, b_4, b_5, \alpha\}$. It is convenient to consider the subset $B_4 = \{S, \zeta, 1 = b_1 + b_2 + b_3 + \zeta, b'_4 = b_1 + b_4, b'_5 = b_2 + b_5, \alpha\}$ since it defines an $N=4$ space-time supersymmetric model [22]. The three $N=2$ sectors (of the underlying $\mathbf{Z}_2 \times \mathbf{Z}_2$ orbifold) are obtained by adding to B_4 the vectors b_1, b_2 or $b_1 + b_2$ respectively[22]. All the massless twisted sectors in the model then fall into three groups as follows:

$$\begin{aligned} I = 1 : & \quad b_{1,4}, b_{1,4} + \alpha, b_{1,4} + 2\alpha, \\ I = 2 : & \quad b_{2,5}, b_{2,5} + 2\alpha, b_{2,5} + b_1 + b_4 + \alpha, \\ I = 3 : & \quad S + b_4 + b_5, b_3, b_3 + 2\alpha, S + b_2 + b_4 + \alpha, b_2 + b_3 + b_5 + \alpha. \end{aligned}$$

The untwisted states (NS sector) also fall into the above groups according to their w_I^i charge, which is a permutation of $\{1, 0, 0\}$ for these states.

The $b_a'^I$ coefficients for the $SU(5) \times U(1) \times SO(10)_h \times SU(4)_h$ gauge groups of the model can be easily obtained from the massless spectrum of the model [2] using $\frac{1}{2}b_a'^I = -c_V^a + \sum_i c_i^{a,I}$. The result is: $\frac{1}{2}b_5'^I = \{0, 0, -1\}$, $\frac{1}{2}b_1'^I = \{7.5, 7.5, 6.5\}$, $\frac{1}{2}b_{10}'^I = \{-6, -6, -7\}$, and $\frac{1}{2}b_4'^I = \{0, 0, -1\}$. The α_a^I coefficients (Eq. (17)) that go into the anomaly determination of Δ_a (Eq. (20)) can be calculated in a similar way. The twisted states have charges $(0, \frac{1}{2}, \frac{1}{2}), (\frac{1}{2}, 0, \frac{1}{2}), (\frac{1}{2}, \frac{1}{2}, 0)$ with respect to the $U_I(1) (I = 1, 2, 3)$ respectively, hence they effectively split into three groups contributing to $\alpha_a^{I=1,2,3}$. These groups correspond precisely to the $N=2$ sectors above. The untwisted states, on the other hand, contribute to all α_a^I and in this they differ from the $N=2$ case. However, despite the difference in the role of the untwisted states and the different

formulas for $\frac{1}{2}b_a'^I$ and α_a^I , one can verify that the result is exactly the same, that is $\frac{1}{2}b_a'^I = \alpha_a^I$, for $I = 1, 2, 3$, as expected from consistency arguments [18].

To make numerical estimates transparent, we will consider the case where the three moduli take the same value T , as is the case in free fermionic models. In this case the sum over I involves only the $b_a'^I$ and we have $\sum_I \frac{1}{2}b_a'^I = b_a$ [24,10], where b_a is the N=1 β -function coefficient. The threshold corrections then reduce to*

$$\Delta_a = -b_a \ln[|\eta(iT)|^4 \text{Re } T] + c_a + Y \equiv -b_a \Delta(T) + c_a + Y, \quad (22)$$

with $b_5 = -1$, $b_1 = \frac{43}{2}$, $b_{10} = -19$, and $b_4 = -1$. The constant piece c_a is not determined by these methods but in free fermionic models it only receives contributions from the N=2 sectors [22]. Furthermore, in symmetric orbifold models the N=2 contributions to c_a are just constants multiplying the logarithm in Eq. (5) [16], *i.e.*, $c_a = \sum_I \frac{1}{2}b_a'^I \cdot c = cb_a$. In what follows we will assume this relation holds, even though fermionic constructions actually correspond to asymmetric orbifolds. The T -dependent function in Δ_a can be evaluated from the known series expansion of the η -function, as follows

$$-\Delta(T) = \frac{\pi}{3} \text{Re } T - \ln(\text{Re } T) - 2 \sum_{n=1}^{\infty} \ln\{1 + e^{-4\pi n \text{Re } T} - 2e^{-2\pi n \text{Re } T} \cos(2\pi n \text{Im } T)\}. \quad (23)$$

Since T takes values in the fundamental domain $\mathcal{D} = \{\text{Re } T > 0, |\text{Im } T| < 1/2, |T| > 1\}$, $\text{Re } T > \sqrt{3}/2$ and the series of logarithms gives a $\lesssim 1\%$ correction to the first two terms in $\Delta(T)$ for $\text{Re } T = \sqrt{3}/2$ and dies out very quickly for increasing values of $\text{Re } T$. In fact, $-\Delta(T) \approx \frac{\pi}{3} \text{Re } T - \ln(\text{Re } T)$ is a monotonically increasing function of $\text{Re } T$ with minimum value $-\Delta(T) = \frac{\pi}{3}$ at $\text{Re } T = 1$, which is where the free fermionic models are constructed.[†] At this point in the moduli space we obtain $\delta\Delta_{1-5} = \Delta_1 - \Delta_5 \approx \frac{\pi}{3}(b_1 - b_5) + c(b_1 - b_5) = 22.5(\frac{\pi}{3} + c)$, which compared with the exact calculation $\delta\Delta_{1-5} = 24.13$ [22] gives $c = 0.025$, a rather small number. Let us recall that the only other existing calculation of threshold corrections [15]

* We have implicitly assumed that the contribution of Y_{GS} to Y renders it independent of T .

† This is true for $\text{Re } T > 1$. For $\sqrt{3}/2 < \text{Re } T < 1$, $\Delta(T)/\Delta(T=1)$ can be as small as 0.98, a negligible effect.

is for a $\mathbf{Z}_3$ orbifold model with gauge group $SU(3) \times E_6 \times E_8$. In this case $\delta\Delta_{3-8}$ receives contributions only from the c_a piece of Δ_a since the $\mathbf{Z}_3$ orbifold does not have any N=2 sectors. The numerical result is $\delta\Delta_{3-8}/(b_3 - b_8) = (c_3 - c_8)/(b_3 - b_8) = 0.068$, another small number. These two examples lure us into conjecturing that $|(c_a - c_{a'})/(b_a - b_{a'})| \ll 1$ in these classes of models.

At the free fermionic point we then get $\Delta_a = b_a \frac{\pi}{3}$ (dropping Y which can be absorbed into $1/g^2$, and neglecting c) and thus Eq. (1) can be rewritten as follows

$$\frac{16\pi^2}{g_a^2(\mu)} = k_a \frac{16\pi^2}{g^2} + b_a \ln \frac{M_{SU}^2}{\mu^2}, \quad (24)$$

where

$$M_{SU} = M e^{\pi/6} = 1.69M = 1.74 \times g \times 10^{18} \text{ GeV}, \quad (25)$$

is the common unification scale for *all* gauge couplings in the flipped $SU(5)$ string model. This result should be compared with an exact calculation at the particular point in the moduli space where the fermionic construction is valid, which gives $M_{SU} = 1.76 \times g \times 10^{18} \text{ GeV}$ [22]. Thus, if our above conjecture is indeed true in general, this would open the way to simplified evaluations of threshold corrections in this class of models.

From the expression for Δ_a (Eq. (22)) and our above remarks regarding $\Delta(T)$, it is clear that any point in the moduli space away from the fermionic formulation (*i.e.*, $T \neq 1$), leads to a larger value of Δ_a and correspondingly $M_{SU}(T \neq 1) > M_{SU}(T = 1) \approx 1.7 \times g \times 10^{18} \text{ GeV}$. This implies among other things that for a model away from the free fermionic point, the disparity between the $SU(2)$ and $U(1)$ gauge couplings at the scale M_{GUT} (defined by the partial unification of $SU(3)$ and $SU(2)$), which is of great relevance to the prediction of $\sin^2 \theta_W$, will be larger than that for the corresponding model at the free fermionic point. The latter is already larger than the one obtained in the approximation of neglecting all threshold effects [22].

5. Conclusions

One of the diacritical properties of the flipped $SU(5)$ string model is the presence of an anomalous $U_A(1)$ factor. At the one-loop level, this anomaly may lead to a non-zero vacuum

energy and the destabilization of the vacuum. This anomaly is canceled by ‘shifting’ the vacuum by giving vacuum expectation values to some of the singlet scalar fields. One may be concerned about the effect of this shifting on the evaluation of Δ_a . However, the correction induced by the vacuum shift arises only at a scale $\langle\Phi\rangle$ which is typically an order of magnitude smaller than M_{SU} [2].

In a related note, it is interesting to observe that the shifting of the vacuum, necessary to cancel the $U_A(1)$ anomaly and which breaks many of the $U(1)$ symmetries [2], also breaks the modular symmetry. A complete analysis of modular symmetry breaking is beyond the scope of this paper and will be presented elsewhere [28]. However, it is clear that since shifted fields carry non-zero charges under the modular symmetry $PSL(2, \mathbf{Z})^3$, part or all of this symmetry is broken. The breaking of modular symmetry at a high scale may have great influence in our understanding of the low-energy effective action [11].

In conclusion, the scale M_{SU} obtained in the previous section is where gauge couplings for all the gauge groups converge to g . It should be noted that this scale is considerably larger than M_{GUT} (where the extrapolated low-energy values of the $SU(3)$ and $SU(2)$ gauge couplings meet). Thus, to obtain a precise prediction for $\sin^2 \theta_W$ further extrapolation to much higher energies will be necessary. Another place where a higher value of the unification scale has nontrivial consequences is in the determination of the scales Λ_h of non-abelian hidden sector condensation (*e.g.*, $SO(10)$ and $SU(4)$ in the flipped $SU(5)$ string model). A reliable determination of Λ_h would impact on our understanding of the cosmological role played by cryptons (hadrons of the non-abelian hidden sector groups [29]), and may explain the breaking of supersymmetry [9]. Hence the study of the interplay of modular symmetry continues to be remunerative.

Acknowledgments: We would like to thank I. Antoniadis, L. Dixon, J. Ellis, and J. Louis for useful discussions. This work has been supported in part by DOE grant DE-FG05-91-ER-40633.

REFERENCES

1. M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory*, (Cambridge Univ. Press, 1987).
2. I. Antoniadis, J. Ellis, J. Hagelin, and D. V. Nanopoulos, Phys. Lett. B **231** (1989) 65; J. L. Lopez and D. V. Nanopoulos, Phys. Lett. B **251** (1990) 73.
3. S. Kalara, J. Lopez, and D. V. Nanopoulos, Phys. Lett. B **245** (1990) 421, Nucl. Phys. B **353** (1991) 650.
4. D. Gross and J. Sloan, Nucl. Phys. B **291** (1987) 41; J. Lauer, D. Lüst, and S. Theisen, Nucl. Phys. B **304** (1988) 236; A. Givernon and M. Porrati, Phys. Lett. B **246** (1990) 54 and Nucl. Phys. B **355** (1991) 422.
5. See *e.g.*, J. Schwarz, Caltech preprint CALT-68-1581 (1989).
6. S. Ferrara, D. Lüst, A. Shapere, and S. Theisen, Phys. Lett. B **225** (1989) 363; S. Ferrara, D. Lüst, and S. Theisen, Phys. Lett. B **233** (1989) 147 and Phys. Lett. B **242** (1990) 39.
7. J. Lauer, J. Mas, and H. P. Nilles, Phys. Lett. B **226** (1989) 251; W. Lerche, D. Lüst, and N. P. Warner, Phys. Lett. B **231** (1989) 417.
8. P. Candelas, X. De La Ossa, P. S. Green, and L. Parks, University of Texas preprint UTTG-25-1990 and Phys. Lett. B **258** (1991) 118.
9. I. Antoniadis, J. Ellis, A. B. Lahanas, and D. V. Nanopoulos, Phys. Lett. B **241** (1990) 24; L. Dixon, in Proceedings of The Rice Meeting, ed. by B. Bonner and H. Miettinen (World Scientific, 1990), p. 811; C. P. Burgess and F. Quevedo, Phys. Rev. Lett. **64** (1990) 2611; S. Ferrara, N. Magnoli, T. R. Taylor, and G. Veneziano, Phys. Lett. B **245** (1990) 409; H. P. Nilles and M. Olechowsky, Phys. Lett. B **248** (1990) 268; P. Binetrui and M. K. Gaillard, Phys. Lett. B **253** (1991) 119; D. Lüst and T. R. Taylor, Phys. Lett. B **253** (1991) 335; B. de Carlos, J. A. Casas, and C. Muñoz, Phys. Lett. B **263** (1991) 248.
10. M. Cvetič, *et. al.*, CERN preprint CERN-TH.5999/91.
11. L. Ibáñez and D. Lüst, CERN preprint CERN-TH.6120/91.

12. P. Ginsparg, Phys. Lett. B **197** (1987) 139.
13. See *e.g.*, J. Ellis, J. Hagelin, S. Kelley, and D. V. Nanopoulos, Nucl. Phys. B **311** (1988/89) 1.
14. J. Minahan, Nucl. Phys. B **298** (1988) 36.
15. V. Kaplunovsky, Nucl. Phys. B **307** (1988) 145.
16. L. Dixon, V. Kaplunovsky, and J. Louis, Nucl. Phys. B **355** (1991) 649.
17. J. Derendinger, S. Ferrara, C. Kounnas, and F. Zwirner, CERN preprint CERN-TH.6004/91.
18. J. Louis, SLAC preprint SLAC-PUB-5527 (1991).
19. G. Lopes Cardoso and B. Ovrut, University of Pennsylvania preprint UPR-0464T (1991).
20. S. J. Gates, M. Grisaru, M. Rocek, and W. Siegel, *Superspace* (Benjamin-Cummings, 1983).
21. I. Antoniadis, C. Bachas, and C. Kounnas, Nucl. Phys. B **289** (1987) 87; I. Antoniadis and C. Bachas, Nucl. Phys. B **298** (1988) 586; H. Kawai, D.C. Lewellen, and S.H.-H. Tye, Phys. Rev. Lett. **57** (1986) 1832; Phys. Rev. D **34** (1986) 3794; Nucl. Phys. B **288** (1987) 1; R. Bluhm, L. Dolan, and P. Goddard, Nucl. Phys. B **309** (1988) 330; H. Dreiner, J. L. Lopez, D. V. Nanopoulos, and D. Reiss, Nucl. Phys. B **320** (1989) 401.
22. I. Antoniadis, J. Ellis, R. Lacaze, and D. V. Nanopoulos, CERN preprint CERN-TH.6136/91.
23. I. Antoniadis, K. S. Narain, T. Taylor, Northeastern University preprint NUB-3025 (1991).
24. I. Antoniadis, Phys. Lett. B **246** (1990) 377.
25. L. Dixon, V. Kaplunovsky, and J. Louis, Nucl. Phys. B **329** (1990) 27.
26. For a review see A. Lahanas and D. V. Nanopoulos, Phys. Rep. **145** (1987) 1.
27. L. Ibáñez, W. Lerche, D. Lüst, S. Theisen, Nucl. Phys. B **352** (1991) 435.
28. S. Kalara, J. L. Lopez, and D. V. Nanopoulos, in preparation.

29. J. Ellis, J. L. Lopez, and D. V. Nanopoulos, *Phys. Lett. B* **247** (1990) 257.