ANOMALOUS COLOUR TRANSPARENCY

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ABSTRACT

Colour transparency is not expected to occur in a conventional treatment of exclusive nuclear π^0 photoproduction $\gamma A \rightarrow \pi^0 A$. I argue that a measurement may nonetheless show a signal of colour transparency due to the anomalous $\gamma\gamma\pi^0$ vertex involved in the production. Estimates of the dependence on energy and nuclear number A are presented. The signals of anomalous transparency are a rise in the cross-section with energy, and a weaker than expected fall with A from attenuation, in a specific kinematic region.

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The axial anomaly has a long and interesting history. When first discovered by Steinberger [1] it may have appeared to be a peculiarity of a model in which pions were bound states of nucleons. In the quark basis the anomaly reappears as a consequence of the ultraviolet regularization. However, in a massless theory the anomaly sometimes seems to be coming from the infra-red region [2]. The 't Hooft conditions [3] offer another example of how the anomaly's interpretation is subtle in a hadronic basis. Thus, while everyone agrees the anomaly is inevitable, there is a history of the physical picture depending on the basis of quantum mechanical states used in the calculation.

This paper proposes a direct experimental probe of the spatial characteristics of an anomalous interaction, the $\pi^0\gamma\gamma$ vertex. This interaction will be studied in the neighbourhood of a large nucleus with charge Z and nuclear number A . I claim there is a possibility of observing anomalously small attenuation of π^0 's photoproduced on the nucleus. The argument goes as follows.

For many years, π^0 photoproduction at small momentum transfer t has been recognized to proceed in the nucleus' Coulomb field by the $\pi^0\gamma\gamma$ vertex [4]. From a 1950's point of view, we could predict π^0 photoproduction by measuring the π^0 decay rate (and vice-versa). From such a phenomenological viewpoint, the on-shell π^0 produced in photoproduction should have an utterly normal, 20 mb or so, inelastic cross-section to interact with the protons and neutrons in the target. For a wide range of reasonably high π^0 energies, the production and attenuation cross-section is fairly flat, and the elastic π^0 photoproduction should be similarly independent of energy.

Going beyond this picture, I consider the same experiment using the quark triangle graph to calculate the vertex. One must keep in mind that *this is a model* for the interaction, and a very peculiar model indeed. One replaces the full π^0 wave function by insertion of the local operator $\partial^\mu J_\mu^5$ in the loop. The numerical value of the vertex is dominated by large quark momenta in the loop. That is, most of the amplitude in the calculation comes from a configuration of the quarks separated by a small space-time region, approaching a single point. While this mathematical fact may be physically misleading, I will adopt it provisionally, qualifying it at the proper moment.

Any quarks produced at a point are not really a π^0 but have an amplitude (parametrized by f_π) of evolving into a π^0 . Other states to which the $q\bar{q}$ system could evolve are excluded by the experiment asking for elastic π^0 production. With this qualification understood, in the triangle graph model I say we begin with a "small- π^0 ". This emphasizes that the triangle graph model for the vertex has more information than the phenomenology of using the vertex alone.

The small π^0 state is associated with a time scale τ_0 to evolve the photoproduced $q\bar{q}$ pair into a normal π^0 . Such a time-scale is non-perturbative, but should be of order Λ_{QCD}^{-1} , say. Moreover, once the small π^0 is photoproduced it is moving quickly in the lab and the time scale should appear extended by a boost factor of $\gamma = E_\pi/m_\pi$. Now if the energy E_π is large enough, one should see little expansion of the small π_0 until it has moved considerably, and even beyond the vicinity of the nucleus. For example, if $E_\pi = 5$ GeV and $\tau_0 \simeq \Lambda_{QCD}^{-1} \simeq 0.2$ GeV, then the small π_0 would stay small for up to 30 fm. The range of the small π_0 is about 14.6 fm ($E_\pi/10$ GeV) (τ_0/GeV^{-1}), while the size of a nucleus is of order 1.1 fm $A^{1/3}$. Thus the concept of the small π_0 may be

relevant provided we have

$$E_\pi/\text{GeV} \gg 0.75A^{1/3}/(\tau_0/\text{GeV}^{-1}) \quad (1)$$

which is very feasible, technically.

Under these conditions we expect the small π^0 to exhibit colour transparency [5]. That is, destructive interference of gluon radiation from the closely separated $q\bar{q}$ pair should be nearly complete, extinguishing the system's "old-fashioned" strong interactions. The large nucleus still acts as a source of photons to produce the π^0 , but disappears as an attenuator of small π^0 's, for high enough energy satisfying (1). The possibility of measuring "anomalous" transparency as a test of the production of small π^0 's is the point of this paper.

The term "anomalous" for this kind of interaction certainly applies, because one would not expect colour transparency here on the basis of conventional arguments. Previous discussions of colour transparency have focused on using large momentum transfer (Q^2) to enhance the exclusive production of spatially small, colour singlet states. In photoproduction there is no large Q^2 : we are interested in $|t| \gtrsim m_\pi^2$ to get the photon pole. With no large Q^2 there is no apparent "short time" coming from the off-shellness of quarks being produced under the kinematic conditions of the experiment. That is, without invoking at all the triangle graph picture, a light-cone calculation of π^0 photoproduction would not necessarily lead to colour transparency. Instead, the vertex *might* come from details of an especially tuned "omniscient" soft wave function capable of giving the correct anomalous value from an ordinary small transverse momentum region.

Physically, the small and large π^0 distinction has probably been idealized so far. It staggers the imagination to think that the $\pi^0\gamma\gamma$ vertex could *literally* be in the ultraviolet, and the triangle graph picture could *literally* be true. If it were true, we would not need the smoothness properties of PCAC at all; other experiments, such as inserting a far off-shell photon, or even [6] an on-shell Z^0 , into the graph would show experimentally that the vertex acted pointlike. Assuming this is not the case, how do we interpret the success of the triangle graph phenomenology? It must have some truth, because it mysteriously produces the right answer, absolutely normalized, including the colour factors and phase.

A reasonable interpretation is that the process is indeed small, but only *small on the scale of chiral symmetry breaking*. The characteristic scale being $\langle\psi\psi\rangle$, this means that the small π^0 , at the moment of production, needs only to be small compared to a normal hadron size. Then the mathematics of the triangle graph could be reasonably true: the system looks a little bit ultraviolet, but it is not so ultraviolet as to cause other problems.

This concept can be made more definite by a thought calculation. Take one's model for the complete light-cone $q\bar{q}$ pair wave function in the π^0 , constructed so that attachment of photons will give a correct $\pi^0\gamma\gamma$ vertex. To separate the regions that contribute, add and subtract a real number λ times the usual PCAC pointlike π^0 wave

²I thank Al Mueller for insisting that a soft light cone wave function could be sufficiently omniscient.

function (Fig. 1). We have a "two-component" π^0 model if we crudely identify the subtracted amplitude with the large π^0 and the PCAC one as the small π^0 . Varying parameter λ , I consider the sum over components as a sort of quantum mechanical superposition. (The key assumption is that the small π^0 , once small, can safely be replaced by the local production since the calculation is smooth, as I will show.)

Fig. 1: Decomposition of a realistic $\gamma\gamma\pi^0$ vertex using the PCAC triangle model (weight λ) and everything else (weight $1 - \lambda$). The symbol "x" indicates that the short-distance contribution has been subtracted.

That is, for the production vertex without accounting for interaction with the nucleus I write

$$V^{\mu\nu} = \lambda V^{\mu\nu}_{\text{small}} + (1 - \lambda) V^{\mu\nu}_{\text{large}}, \quad (2)$$

where μ and ν are photon indices.

This decomposition becomes meaningful when we implement the different time evolution of the two components due to interaction with the nucleus. I will estimate the survival amplitude of the $q\bar{q}$ to emerge as a π^0 in the two cases.

Some minimal Feynman diagrams, useful for setting up the momentum flow, are shown in Fig. 2. Note there are three loops, minimum. Various techniques exist to sum rescatterings if production is via the small π^0 vertex, resulting in exponentiation. This is not an important issue. The interesting question is whether the small π^0 concept is relevant at all, and whether it can be measured in an experiment.

First I must specify the kinematics. Let q^μ be the momentum difference between the outgoing π^0 and incoming photon, so $|q^2| \cong |t|$, where $\perp$ denotes the transverse coordinate. To make sure the π^0 penetrates the nucleus we need $q_1 \gtrsim A^{-1/3}/\text{fm}$. To see the one-photon exchange is an experimental question, resolved by fitting the region $q_1 \lesssim m_\pi$. Multiple scatterings in the nucleus should not dominate (the predominant effect should be forward scattering and attenuation) but no amount of multiple scattering will mimic the photon t -channel pole, which I assume is observed. Thus, the kinematic region of interest is a t -interval in $d\sigma/dt$ satisfying $A^{-2/3}/fm^2 \lesssim |t| \lesssim m_\pi^2$.

I am interested in production outside the nucleus, before the photon reaches the front surface. Production inside the nucleus is not anomalous and not small in any model (Fig. 2b). Production outside implies $|q_2| \lesssim A^{-1/3}/fm$, where $\hat{z}$ is the direction along the photon beam. This region already predominates, since there is a strong

tendency for q^2 to be small from the photon pole. Once $q_1^2 \gtrsim A^{-1/3}$ is demanded, the region of q_2^2 allowed can only range up to q_1^2 . Thus, the production is predominantly outside, and I will not consider production inside; one can easily see that this just makes a small correction to the measured value of λ .

Fig. 2: Two processes producing π^0 's near a nucleus.

- (a) Production outside the nucleus via the usual anomalous vertex and scattering in the target;
- (b) A graph involving production inside the nucleus. The nucleus is shown as the heavy spectator lines, interacting with a $q\bar{q}$ pair in the loop.

The interesting physics is in the growth with time of the small π^0 . The transverse coordinate b of the quark separation is the most important. Suppose the typical b^2 grows linearly with time in the $q\bar{q}$ rest frame, as the system evolves in free space. As observed in the lab, we must correct for the time evolution with a boost factor γ .

To model this, I use an ansatz for the amplitude $\psi(b, x - x')$ for the quarks which have travelled a distance $x - x'$ after being created at x' :

$$\psi(b, x - x') \sim \frac{\mu \gamma e^{-\frac{1}{2} \gamma^2 b^2 \mu (x - x')}}{(x - x')} \quad (3)$$

for $x - x' < \gamma\tau_0$. The normalization is such that ψ begins as $\delta(b^2)$ at $x - x'$. The choice of a delta function is a simplification, since the same result will be obtained no matter what small size for the initial system is chosen, so long as the size is much smaller than a fermi. The mass scale μ is a parameter. The ansatz (3), reminiscent of "quantum diffusion" [7], is a model of the free space time evolution. It would not apply inside the nucleus.

Inside the nucleus, filtering effects of attenuation become important. Let the coordinates be chosen so the nucleus sits between $x = 0$ and $x = A^{1/3}$. The effects of

transmission through the nucleus are a factor f_A of

$$f_A \sim e^{-b^2 A^{1/3} \pi \sigma' / 2}$$

where $\sigma' b^2$ is the cross-section, and n the nuclear number density. After transmission through the nucleus one must take the overlap with the soft π^0 wave function $\phi_\pi(b^2)$, which can be represented by

$$\phi_\pi(b^2) \sim e^{-m^2 b^2 / 2}$$

The product $\psi f_A \phi_\pi$ needs to be integrated over transverse separation b , and then integrated over the production point x' . Let the x -space amplitude to find the Coulomb photon of the nucleus be $\mathcal{A}(x')$. We need, now,

$$\mathcal{M}_{\text{small}} = \int_{A^{1/3} \text{fm}}^{\tau_0} dx' \mathcal{A}(x') \int_0^\infty db^2 \psi f_A \phi_\pi \quad (4)$$

The x' integral is essentially one-dimensional, since production is taking place over a long tube with transverse size $x'_\perp \sim A^{1/3}$ fm, the target radius. The x' integral is the q' integral of Fig. 2 and the Coulomb potential $\mathcal{A}(x') \sim 1/\sqrt{x'^2 + x_\perp^2} \sim 1/x'$. Now all the dependence on large scales, namely the boost ν and the nuclear size A , can be extracted. The asymptotic limit of $\mathcal{M}_{\text{small}}$ for $\gamma \gg 1$, $A \gg 1$ is

$$\mathcal{M}_{\text{small}} \sim \frac{1}{1 + A^{2/3}/\gamma} \ln\left(\frac{\tau_0 \gamma}{A^{1/3}}\right) \quad (5)$$

The logarithm is an infrared singularity, set by the closest and farthest points of production, as can be anticipated from the q' loop in Fig. 2. Such singularities are general, slowly varying, and will cancel out when I take the ratio of cross-sections to study the transparency. The only dependence of the transparency then is a "scaling" dependence on the variable $\gamma/A^{2/3}$. This can be compared to another scaling result predicted [8] in ordinary, large Q^2 and large A processes such as electroproduction. In that case, deliberately averaging over any hadron expansion in the high energy limit, the dependence is $\gamma/A^{1/3}$ (since $\gamma \sim Q^2$).

The two results differ, because the free space expansion of the system has been put in for π^0 photoproduction. The new scaling result can be understood in a simple way³. Integrating over the production point, the typical attenuation cross-section including expansion is set by the duration of the expansion in units of the effective lifetime, $A^{1/3}/\gamma$. The attenuation depends on the effective cross-section times the target length, which is $A^{2/3}/\gamma$.

The approximation (5) is complemented by evaluating (4) and numerically studying it as a function of the several parameters. I then estimate the cross-section by summing the amplitudes and squaring,

$$|M_{\text{tot}}|^2 = |\lambda \mathcal{M}_{\text{small}} + e^{i\delta} (1 - \lambda) \mathcal{M}_{\text{large}}|^2, \quad (6)$$

where $\mathcal{M}_{\text{large}}$ is the case of the large π^0 . This should be exponentially attenuated with an energy independent cross-section of about 20 mb. The relative phase δ of the two

³The same argument can be applied to the intermediate energy region in ordinary electroproduction or other transparency measurements such as $pp \rightarrow pp$. This will be considered elsewhere.

amplitudes is too delicate for a crude estimate so I will treat it as a parameter. The transparency ratio $T(E, A)$ will be defined as

$$T(E, A) = \frac{1}{Z_{\text{eff}}^2} \frac{d\sigma(\gamma A \rightarrow \pi^0 A)/dt}{d\sigma(\gamma A \rightarrow \pi^0 p)/dt} \quad (7)$$

where Z_{eff}^2 is the square of the photon-nucleus form factor at the t -value of the experiment, which can be obtained from electron scattering experiments. I set my normalization by dividing (4) by $2 + \ln(\gamma/A^{1/3})$ and requiring that $T(E, A) \rightarrow 1$ for $\lambda = 1$, $A = 1$ and $E \rightarrow \infty$, in which limit the model gives zero attenuation. For parameters $1/\tau_0$, μ , and m I use 300 MeV.

The most important behaviour is the energy dependence, which should reveal a rise in production with increasing energy. This is shown in Fig. 3. The curves show the result of setting the small π^0 amplitude λ to different values. As λ is varied from one to zero the energy dependence becomes flatter; above $\lambda = 0.1$, it would become possible to detect a small π^0 component. In these plots, $\delta = 0$.

Fig. 3: Energy dependence of the π^0 production at fixed $A = 200$. T is the transparency ratio [Eq. (7)]. Curves are shown for different values of the parameter λ , which is the amplitude for the initial $q\bar{q}$ pair to be small. For $\lambda \gtrsim 0.5$ there is a substantial rise of production with energy, and even for $\lambda \gtrsim 0.1$ there is a signal.

In Figs. 4a-b, I explore the effects of the relative phase δ . An objective procedure is to let δ vary over its full range, $0 < \delta < 2\pi$, and present the extreme range of the predictions. I also show a curve for the "typical" value $\delta = \pi/2$. For λ below 0.5, say, the range includes the possibility of transparency decreasing with energy, i.e., destructive interference. However, one does not really expect a phase shift conspiracy to occur, and for $\lambda \gtrsim 0.5$ the characteristic rise of transparency with energy should not be concealed by interference effects.

because the large π^0 amplitude is attenuated away. Thus, the A dependence can also be used as a signal of anomalous transparency.

Fig. 5: (a) Nuclear number A dependence of the transparency ratio [Eq. (7)] at fixed energy $E_{\pi^0} = 10$ GeV; λ values as shown. For $\lambda \neq 0$ the attenuation with A is much less rapid than the exponential attenuation conventionally expected (shown for $\lambda = 0$).

(b) Effects of the phase δ . Varying over all δ , $0 < \delta < 2\pi$, the production is less sensitive to the effects of phases at large A .

The estimates presented here are not the last word on this subject. One anticipates that some experimental work may be required to more precisely locate the region of energy and momentum transfer over which the signal could be observed. Certainly much work can be done to improve the calculations, but they are probably adequate representations of the physics. Other applications of the same "anomalous" physics - such as η, η' photoproduction - would also be interesting.

Fig. 4: Effects of a relative phase δ [Eq. (6)]. At each energy the extreme range of variations of the predicted production for any phase $0 < \delta < 2\pi$ is shown, as well as the choice $\delta = \pi/2$. For $\lambda \gtrsim 0.5$ there is a signal whatever the phase. $A = 200$.

The A dependence is very interesting. Plots include the region of small A (where the approximations of neglecting few-body effects are too crude) to show that in this region nothing depends on λ . For $\lambda = 0$ and large A the A dependence falls exponentially (Fig. 5). However, for $\lambda \neq 0$ this behaviour changes to a power law for large A , at fixed energy. The asymptotic power $A^{-2/3}$ is not reached for $A < 200$ because one needs $A^{2/3}/\gamma \gg 1$ to see it; still, there is a clear departure from the exponential fall. Sensitivity of the production to the relative phase δ begins to disappear at large A ,

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References

- [1] J. Steinberger, *Phys.Rev.* **76** (1949) 480;
H. Fukuda and Y. Miyamoto, *Progr.Theor.Phys. (Kyoto)* **4** (1949) 347;
J.S. Bell and R. Jackiw, *Nuovo Cimento* **60a** (1969) 47;
S. Adler, *Phys.Rev.* **177** (1969) 2426.
- [2] A.D. Dolgov and V.I. Zakharov, *Nucl.Phys.* **B27** (1971) 525.
- [3] G. 't Hooft, in *Recent Developments in Gauge Theories* (Plenum, New York, 1980), edited by G. 't Hooft et al.;
Y. Frishman, A. Schwimmer, T. Banks and S. Yankielowicz, *Nucl.Phys.* **B177** (1981) 157;
S. Coleman and B. Grossman, *Nucl.Phys.* **B203** (1982) 205.
- [4] H. Primakoff, *Phys.Rev.* **81** (1951) 899.
For a review, see:
A. Donnachie and M. Shaw, *Electromagnetic Interactions of Hadrons*, Vol. 1, Sec. 3.2.3. (Plenum, New York, 1978).
- [5] S.J. Brodsky and A.H. Mueller, *Phys.Lett.* **B206** (1985) 685 and references therein;
S.J. Brodsky and G. de Terra, *Phys.Rev.Lett.* **60** (1988) 1924;
J.P. Ralston and B. Pire, *Phys.Rev.Lett.* **65** (1990) 2343; **61** (1988) 1823;
B. Jennings and G. Miller, *Phys.Lett.* **B236** (1990) 209 and Triumf Preprint (1990). See also Ref. [7].
For data on colour transparency in $pA \rightarrow p'p''(A-1)$ see:
A.S. Carroll et al., *Phys.Rev.Lett.* **61** (1988) 1698.
- [6] M. Jacob and T.T. Wu, *Phys.Lett.* **B232** (1989) 529.
- [7] G.R. Farrar, H. Liu, L. Frankfurt and M. Strikman, *Phys.Rev.Lett.* **61** (1988) 61.
- [8] B. Pire and J.P. Ralston, *Phys.Lett.* **B256** (1991) 523.