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NATURAL RANGE OF HIGGS MASSES IN SUPERSYMMETRY

Ulrich Ellwanger*
CERN, Geneva

Michel Rausch de Traubenberg**
Institut für Theoretische Physik
Universität Heidelberg
Philosophenweg 16, D-6900 Heidelberg

Abstract: We calculate the masses of the Higgs scalars and the lightest chargino for a large number of parameters of the supersymmetric standard model extended by a singlet. This allows us to estimate in which range we have to expect these masses naturally, and which region has to be covered experimentally in order to rule out a large fraction of the parameter space of low energy supersymmetry.

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I. The supersymmetric extension of the standard model [1] is motivated by the absence of quadratically divergent quantum contributions to the Higgs masses squared. This allows to contemplate about Lagrangians, valid at some GUT or even Planck scale, without having to fine-tune the Higgs masses at these scales to an incredible accuracy.

The search for the corresponding new particles, essentially the susy partners of the fields of the standard model, is one of the most interesting tasks of future accelerators. For the masses of these particles one expects an upper limit of the order of 1 TeV, because otherwise the Higgs masses would no longer be protected from large quantum corrections. Of course, numerous models exist which predict susy particles with masses far below 1 TeV, but these are of little help if one just wants to test the general assumption of supersymmetry around the electroweak scale.

It is interesting to ask which portion of the generally allowed parameter space has been ruled out, if no non-standard particle has been observed up to a certain energy. The most fruitful sector for such an investigation is the supersymmetrically extended Higgs sector: The same dimensionful parameters, which determine the masses of the new Higgs and Higgsino particles required by supersymmetry, also appear in the potential of the Higgs fields. Since the vacuum expectation values of the Higgs fields are fixed by the known W and Z masses, these parameters cannot be arbitrarily large; otherwise the dimensionless couplings have to enter the nonperturbative regime (upsetting one of the motivations of supersymmetry) or fine-tuning is required (in contrast to the philosophy behind the introduction of supersymmetry).

In addition the increased number of fields in the Higgs sector makes it likely for some of them to be comparatively light. The best example is the minimal supersymmetric extension, where the Higgs sector consists out of two Higgs doublets and their superpartners. In the scalar sector one is left, after the Goldstone mechanism has taken place, with two scalar, one pseudoscalar, and one charged Higgs fields. According to the tree level potential, the lightest of the scalar Higgs fields necessarily has to have a mass below M_Z . It has recently been observed, however, that radiative corrections to the potential increase this upper bound to $\sim 120 - 130 \text{ GeV}$ depending on the top quark and top squark masses [2-5].

Even this upper bound is invalidated within a nonminimal supersymmetric extension, where a gauge singlet field is added to the Higgs sector [6-10]. The bound now depends on the value of a dimensionless coupling λ and becomes, e.g., $\sim 160 \text{ GeV}$ for $\lambda \sim .9$. Thus, if no Higgs field is found below 130 GeV , just the minimal supersymmetric extension of the standard model is ruled out, but not the general assumption of supersymmetry.

Actually, the nonminimal extension involving a singlet has a very attractive feature [7]: Now it is possible to assume that all supersymmetric couplings are dimensionless (no $\mu H_1 H_2$ term in the superpotential) and the electroweak scale appears just in the form of soft supersymmetry breakings, i.e. scalar masses, trilinear scalar couplings and gaugino masses. (The absence of a $\mu H_1 H_2$ term in the superpotential of the minimal extension would imply a nearly massless axion.) Besides, models of this type may emerge as effective low energy theories in string models [11]. It also contains the minimal supersymmetric model for special choices of the parameters.

Of course, one can think about more general extensions of the Higgs sector as, e. g., more than two Higgs doublets [8]. This will make, however, the discovery of Higgs particles

even more likely, hence the extension involving just a singlet is the appropriate model for testing the general assumption of supersymmetry.

For the masses of the additional Higgs fields and supersymmetric particles we now face the following situation: There are no strict upper limits in general, but weak upper limits if one assumes no fine-tuning in the Higgs potential. It is difficult, however, to derive analytic results for these weak upper limits: The model under consideration with one extra singlet and dimensionless supersymmetric couplings has seven parameters in the Higgs sector (see below) and the absence of fine-tuning is not easy to define precisely.

The corresponding model has already been investigated in some detail by Ellis et al. [7]. There the particle spectrum was presented a) for particular values of the Higgs VEVs (the ratio $\langle H_1 \rangle / \langle H_2 \rangle \equiv \tan \beta$ and the singlet VEVs) and the dimensionless couplings, and b) for particular boundary conditions of the running parameters of the model at $\mu = M_{Planck}$. In case a) it is difficult to control whether one is moving in some particular unlikely corner in the space of parameters, whereas in case b) the authors were restricting themselves to a narrow (and, in fact, in the meantime disproved) subclass of such models.

In this paper we will pursue the following strategy: We will take a large number of different values of the parameters of the model (within boundaries and with a measure discussed below) and calculate the masses of the six Higgs scalars for each choice of the parameters. From the “probability distributions” we can obtain the above mentioned weak upper limits on the masses, which are violated only for special (and hence unlikely) choices of the parameters. We did not make any particular assumptions on the initial values of the parameters in order to be as model-independent as possible.

In the fermionic sector involving higgsinos and the electroweak gauginos, the situation is, at first sight, somewhat unpleasant: Two more parameters enter the game (the masses of the $U(1)$ and $SU(2)$ gauginos), and the mass eigenvalues depend crucially on them. In the chargino sector, which is the easiest one to observe experimentally, we are, however, able to derive an analytic upper bound on the mass of the lightest eigenstate. This bound is independent of the gaugino mass and just depends on two parameters of the Higgs sector. (Actually a similar bound also holds in the minimal supersymmetric extension, where it depends just on the μ -parameter.) Hence we are able to obtain a distribution of this upper bound and thus a corresponding weak upper limit on the mass of the lightest chargino.

These numbers are helpful for estimating the discovery potential of future accelerators, and to find the natural range of parameters of the model to be used for the evaluation of cross sections.

Of course one will never be able to rule out softly broken supersymmetry as such experimentally. If, however, nature employs supersymmetry in order to avoid fine-tuning of the electroweak scale, our results show the allowed region of masses of the corresponding particles.

In the next section we will present the model and our procedure in more detail, and our results and conclusions will be given in sect. III.

II. As discussed in the introduction, we consider the supersymmetric standard model with an additional gauge singlet field in the Higgs sector. The particle content of the Higgs sector is thus given by the two doublets H_1, H_2 (which couple to the up and down

quarks, respectively) and the singlet S . We intend to make use of an attractive feature of the model, the possibility to choose just dimensionless supersymmetric couplings. The relevant part of the superpotential W reads thus simply [7,10]

$$W = \lambda H_1 H_2 S - \frac{k}{3} S^3 \quad (2.1)$$

(Only here the letters H_1, H_2 and S denote chiral superfields.) In addition we have the following soft supersymmetry breaking trilinear couplings and masses:

$$V_{soft} = (\hat{\lambda} H_1 H_2 S - \frac{\hat{k}}{3} S^3) + h.c. \quad (2.2)$$

$$+ m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_s^2 |S|^2$$

In (2.2) we have omitted a possible term of the form $m_3^2 H_1 H_2 + h.c.$, which is technically natural (stable under radiative corrections) [7]. This hardly influences our results, since a corresponding term is effectively regenerated through a VEV of S with $m_3^2 \rightarrow \hat{\lambda} \langle S \rangle$.

The complete scalar potential reads accordingly:

$$V = |\lambda H_1 H_2 - k S^2|^2 + |\lambda|^2 |S|^2 (|H_1|^2 + |H_2|^2) \quad (2.3)$$

$$+ \frac{g_1^2}{2} D_1^2 + \frac{g_2^2}{2} \sum_{k=1}^3 D_k^2 + V_{soft} + V_{rad}(|H_1|^2)$$

with

$$D_1 = \frac{1}{2}(|H_1|^2 - |H_2|^2), \quad D_k = \frac{1}{2}(H_1^+ \tau^k H_1 - H_2^+ \tau^k H_2)$$

where τ^k are the Pauli matrices.

V_{soft} is given by (2.2), and V_{rad} denotes the leading radiative corrections induced by a large top quark Yukawa coupling h_t [2,4]:

$$V_{rad}(|H_1|^2) = \frac{3}{16\pi^2} \left[(h_t^2 |H_1|^2 + m_{sq}^2)^2 \ln \left(\frac{h_t^2 |H_1|^2 + m_{sq}^2}{Q^2} \right) - h_t^4 |H_1|^4 \ln \left(\frac{h_t^2 |H_1|^2}{Q^2} \right) \right] \quad (2.4)$$

Here m_{sq} denotes the left- and right-handed top squark masses (which we assume to be degenerate) and Q^2 is the arbitrary renormalization point. Actually all parameters of the scalar potential depend on Q^2 such that the complete scalar potential is Q^2 -independent; it is easy to see that in the approximation where V_{rad} is given by (2.4) m_1^2 in (2.2) varies with Q^2 according to

$$m_1^2(Q^2) = \bar{m}_1^2 + \frac{3}{8\pi^2} m_{sq}^2 h_t^2 \ln(Q^2/Q_0^2) \quad (2.5)$$

with $\bar{m}_1^2, Q_0^2$ independent of Q^2 . The freedom to choose Q^2 most conveniently will be used below.

Next we search for VEVs of the form

$$\langle H_1 \rangle = \begin{pmatrix} h_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} h_2 \\ 0 \end{pmatrix}, \quad \langle S \rangle = n. \quad (2.6)$$

For the purposes of this paper it will be sufficient to assume the parameters $\lambda, k, \hat{\lambda}, \hat{k}$ and the VEVs h_1, h_2 and n to be real. Out of the equations which correspond to the conditions of extrema of the potential (2.3) at h_1, h_2 and n , we can form the following linear combinations:

$$m_1^2 + V'_{rad} + m_2^2 + 2\lambda^2 n^2 + (h_1^2 + h_2^2)(\lambda^2 - \frac{k\lambda n^2 + \hat{\lambda}n}{h_1 h_2}) = 0 \quad (2.7a)$$

$$m_1^2 + V'_{rad} - m_2^2 + (h_1^2 - h_2^2)(\hat{g} - \lambda^2 + \frac{k\lambda n^2 + \hat{\lambda}n}{h_1 h_2}) = 0 \quad (2.7b)$$

$$m_s^2 + \lambda^2(h_1^2 + h_2^2) + 2k^2 n^2 - \hat{k}n - h_1 h_2(\frac{\hat{\lambda}}{n} + 2k\lambda) = 0 \quad (2.7c)$$

with

$$\hat{g} = \frac{1}{2}(g_1^2 + g_2^2),$$

$$V'_{rad} \equiv \frac{\partial}{\partial h_1^2} V_{rad}(h_1^2).$$

Of course these equations are too complicated to be solved for h_1, h_2 , and n in terms of the other parameters. Hence one often uses the VEVs h_1, h_2 and n as inputs and computes the masses m_1^2, m_2^2 and m_s^2 from eqs. (2.7) [7] (after all we know the combination $h_1^2 + h_2^2$ from M_Z and/or M_W). For our purpose it would be desirable, however, to use as many of the original parameters as input as possible. In fact, with the help of eq. (2.7c), one can compute $h_1 \cdot h_2$ as a function of $m_s^2, h_1^2 + h_2^2$ and n . (Since $m_{top} \gg m_{bottom}$ we will assume $|h_1| > |h_2|$; this allows to obtain h_1 and h_2 individually.) This result can be inserted into eqs. (2.7a and b), thus we will use eqs. (2.7) to determine $h_1 \cdot h_2$ (or $\tan \beta$), m_1^2 and m_2^2 in terms of $h_1^2 + h_2^2, n$ and all the other parameters.

Next we will choose the renormalization scale $\hat{Q}^2$ implicitly such that $V'_{rad}(Q^2 = \hat{Q}^2)$ vanishes [2]. From eq. (2.5) this implies that subsequently m_1^2 is to be understood as $m_1^2(Q^2 = \hat{Q}^2)$. Since we will not fix $m_1^2(Q^2)$ at some large Q^2 , this will play no role in the following.

From the quadratic expansion of the scalar potential (2.3) around the VEVs h_1, h_2 and n it is straightforward to obtain the mass matrices M_s^2 of the three neutral scalar fields ReH_1^0, ReH_2^0 and ReS , M_p^2 of the two neutral pseudoscalars $Im\hat{H} = (\cos \beta \cdot ImH_1^0 + \sin \beta \cdot ImH_2^0)$ and ImS , and the mass M_c of the remaining charged scalar after the Goldstone bosons have been eaten by vector bosons Z, W^+ and W^- . Due to the above choice of Q^2 the radiative corrections (2.4) finally affect only M_s^2 . After m_1^2, m_2^2 and m_s^2 have been

eliminated using eqs. (2.7) M_s^2 becomes

$$M_s^2 = \begin{pmatrix} \hat{g}h_1^2 + n\frac{h_2}{h_1}(\hat{\lambda} + \lambda kn) & h_1h_2(2\lambda^2 - \hat{g}) & 2\lambda^2nh_1 - h_2(\hat{\lambda} + 2\lambda kn) \\ +2h_1^2V''_{rad} & -n(\hat{\lambda} + \lambda kn) & \\ \dots & \hat{g}h_2^2 + n\frac{h_1}{h_2}(\hat{\lambda} + \lambda kn) & 2\lambda^2nh_2 - h_1(\hat{\lambda} + 2\lambda kn) \\ \dots & \dots & \hat{\lambda}\frac{h_1h_2}{n} + 4k^2n^2 - \hat{k}n \end{pmatrix} \quad (2.8)$$

with

$$V''_{rad} \equiv \left(\frac{\partial}{\partial h_1^2} \right)^2 V_{rad}(h_1^2, \hat{Q}^2) = \frac{3m_t^4}{8\pi^2 h_1^4} \ln \left(\frac{m_t^2 + m_{sq}^2}{m_t^2} \right) \quad (2.9)$$

where $m_t^2 = h_t^2 h_1^2$ has been used. (M_s^2 is symmetric, and we employed the basis ReH_1^0, ReH_2^0 and ReS .) For M_p^2 one finds

$$M_p^2 = \begin{pmatrix} \frac{n(h_1^2 + h_2^2)}{h_1 h_2}(\hat{\lambda} + \lambda kn) & (h_1^2 + h_2^2)^{1/2}(\hat{\lambda} - 2\lambda kn) \\ (h_1^2 + h_2^2)^{1/2}(\hat{\lambda} - 2\lambda kn) & h_1 h_2(\frac{\hat{\lambda}}{n} + 4\lambda k) + 3\hat{k}n \end{pmatrix} \quad (2.10)$$

and the mass of the charged scalars is given by

$$M_c^2 = M_W^2 + (h_1^2 + h_2^2) \left(\frac{\hat{\lambda}n + \lambda kn^2}{h_1 h_2} - \lambda^2 \right). \quad (2.11)$$

Note that the charged scalars could be lighter than M_W in contrast to the minimal supersymmetric extension, and that an upper bound on the mass of the lightest scalar M_{sl} can be given [8]:

$$M_{sl}^2 \leq M_Z^2 [\cos^2(2\beta) + \frac{2\lambda^2}{g_2^2} \cos^2 \theta_W \sin^2(2\beta)] \quad (2.12)$$

In the fermionic sector, as discussed in the introduction, we will just consider the charged fermions. The corresponding Dirac mass matrix $\tilde{M}_c$ reads

$$\tilde{M}_c = (\tilde{H}_2^-, \lambda_-) \begin{pmatrix} \lambda n & g_2 h_2 \\ g_2 h_1 & \tilde{m}_2 \end{pmatrix} \begin{pmatrix} \tilde{H}_1^+ \\ \lambda_+ \end{pmatrix} \quad (2.13)$$

Here $\tilde{H}_1^+$ and $\tilde{H}_2^-$ are fermionic superpartners of the charged scalars H_1^+ and H_2^- , λ_+ and λ_- are SU(2) gauginos with soft supersymmetry breaking mass $\tilde{m}_2$. Instead of treating $\tilde{m}_2$ as another unknown parameter we have observed that it is possible to derive an upper bound on the lightest eigenvalue $\tilde{M}_{cl}^2$ of $\tilde{M}_c^+ \tilde{M}_c$, which is independent of $\tilde{m}_2$:

$$\tilde{M}_{cl}^2 < |\lambda n|^2 + g_2^2 \text{Min}(h_1^2, h_2^2) \quad (2.14)$$

(Due to our assumption $|h_1| > |h_2|$ we have $\text{Min}(h_1^2, h_2^2) = h_2^2$). In the minimal supersymmetric standard model the product $\lambda \cdot n$ gets replaced by μ , and $g_2^2 \text{Min}(h_1^2, h_2^2)$ is

bounded from above, in turn, by M_W^2 . This would allow to formulate a slightly weaker bound independent of $\tan \beta = h_1/h_2$. For generic values of $\lambda \cdot n$, $\tilde{m}_2$ and $\tan \beta$ we have found that the true value of $\tilde{M}_{cl}^2$ is far below the bound (2.14), if the signs of $\lambda \cdot n$ and $\tilde{m}_2$ agree, but that $\tilde{M}_{cl}^2$ approaches the bound (2.14) closely if the signs of $\lambda \cdot n$ and $\tilde{m}_2$ are opposite.

Now we have all the required formulas at hand in order to compute the masses of the six scalars and the upper bound on the lightest chargino in terms of the seven parameters $\lambda, k, \hat{\lambda}, \hat{k}, m_1^2, m_2^2, m_s^2$ and the VEVs h_1, h_2 and n , which, in turn, are determined by the above parameters through eqs. (2.7).

Ideally we would like to proceed as follows: let us denote by $M_i^2 = M_i^2(\lambda, k, \dots, m_s^2)$ the mass of the scalar i expressed through the original seven parameters, and by $w(\lambda, k, \dots, m_s^2) \equiv (h_1^2 + h_2^2)(\lambda, k, \dots, m_s^2)$ the combination $h_1^2 + h_2^2$ as a function of $\lambda, k, \dots, m_s^2$. This latter combination is known from M_W and M_Z to be $\sim 30100 \text{ GeV}^2$. Our aim is to compute the functions $f_i(\bar{M}_i)$ defined through

$$f_i(\bar{M}_i) = \int [d\lambda dk \dots dm_s^2] \delta(\bar{M}_i - M_i(\lambda, k, \dots, m_s^2)) \times \delta(30100 - w(\lambda, k, \dots, m_s^2)). \quad (2.15)$$

The functions $f_i(\bar{M}_i)$ will denote the probability for the scalar i to have a mass $\bar{M}_i$ for generic choices of the original parameters under the constraint that M_W and M_Z assume their known values.

Of course, any particular class of models will lead to some prejudice concerning the measure $[d\lambda \dots dm_s^2]$. We are only interested, however, in the way the functions f_i decrease for large values of $\bar{M}_i$ in order to extract the "weak upper limits" as discussed in the introduction: For masses above these weak upper limits unnatural fine-tuning between the original seven parameters has to occur, which violates the general philosophy behind the introduction of supersymmetry. These weak upper limits are not very sensitive to the choice of the measure.

Nevertheless we tried to make this choice as sensible as possible. We start with the natural assumption that all dimensionful parameters $\hat{\lambda}, \hat{k}, m_1^2, m_2^2$ and m_s^2 are proportional to some common scale $\sim M_W$: $\hat{\lambda} = c_{\hat{\lambda}} M_W$, etc.. For the unknown coefficients c_i we take a logarithmic measure dc_i/c_i , and this translates directly into a logarithmic (and scale-invariant) measure $d\hat{\lambda}/\hat{\lambda}$ etc. for the dimensionful parameters.

In view of the discussion below eqs. (2.7), it is convenient to make a change of variables in (2.15):

$$dm_1^2 dm_2^2 = \left| \frac{dm_1^2 dm_2^2}{dn dw} \right| dn dw \quad (2.16)$$

Now the dw integral can be performed trivially, and the VEV n appears as an input parameter. Eq.(2.15) thus becomes

$$f_i(\bar{M}_i) = \int \frac{d\lambda d\hat{\lambda} dk d\hat{k} dm_s^2 dn}{\hat{\lambda} \hat{k} m_s^2 m_1^2 m_2^2} \cdot \text{Det} \cdot \delta(\bar{M}_i - M_i(\lambda, \dots)). \quad (2.17)$$

The masses m_1^2, m_2^2 , and $Det = |dm_1^1 dm_2^2 / dndw|$ can be computed in terms of $\lambda, \hat{\lambda}, k, \hat{k}, m_s^2$ and n using eqs. (2.7) and the known value of w .

It remains to discuss the range of integration in (2.17). First we observe that we can make use of the three discrete symmetries of the model

- i) $\lambda \rightarrow -\lambda, \quad k \rightarrow -k$
- ii) $\hat{\lambda} \rightarrow -\hat{\lambda}, \quad \hat{k} \rightarrow -\hat{k}, \quad S \rightarrow -S$
- iii) $\lambda \rightarrow -\lambda, \quad \hat{\lambda} \rightarrow -\hat{\lambda}, \quad H_1 \cdot H_2 \rightarrow -H_1 \cdot H_2$

to choose the signs $\lambda > 0, k > 0$, and $\hat{\lambda} > 0$. Next, for the absolute values of the dimensionful parameters $\hat{\lambda}, \hat{k}, m_s^2$ and n we find natural upper limits, beyond which the integrand of (2.17) vanishes (at least after imposing the additional constraints below). In view of the logarithmic measure in (2.17), the lower limits are chosen such that the geometric mean of the upper and lower limits is given by M_W .

Since the original motivation for supersymmetry was the stability of the Higgs masses under radiative corrections, we will also require radiative stability for each set of soft supersymmetry-breaking parameters $\hat{\lambda}, \hat{k}, m_1^2, m_2^2$ and m_s^2 : Each of these parameters suffer from logarithmically divergent quantum corrections. Some of these quantum corrections depend on the unknown squark, slepton, or gaugino masses; these we will not consider. For the quantum corrections, which are due to the same set of susy breaking parameters, we will require, however, that these corrections are not bigger than the parameters themselves. This rules out strongly unbalanced combinations of the parameters (where some are very much larger or very much smaller than some others) which would vary rapidly with the renormalization scale Q^2 . We estimate these radiative corrections simply by multiplying the r.h.sides of the corresponding renormalization group equations [6] with $\ln(\Lambda/M_W)$, where Λ is a UV cutoff. (It does not make sense to do better in the context of this crude procedure.) Accordingly, the results will depend somewhat on the choice of this cutoff Λ . We will present the results for the two choices $\Lambda = 10$ TeV (weak restriction on the parameters) and $\Lambda = 10^{18}$ GeV (stronger restriction), there is no strong dependence on Λ , however.

For the dimensionless parameters λ and k one should require that they do not enter the nonperturbative domain > 1 . There is a motivation for particular upper limits of 0(1) from the infrared fix points of the renormalization group equations of the model, which read .87 and .63 for λ and k , respectively [7]. Again our results are not sensitive on modifications of these upper limits except for the mass of the lightest neutral scalar Higgs, whose upper bound depends on λ in the simple way given by eq. (2.12).

This concludes the discussion of the range of the parameters of the model; for convenience we display these ranges in table 1. There we have also included m_1^2 and m_2^2 (which appear as "output") and the range of the VEVs n and $\tan\beta = h_1/h_2$.

In order to compute the functions $f_i(\bar{M}_i)$ of eq. (2.17) numerically we proceed as follows: With the help of a computer program we generate $\sim 10^8$ different values of the input parameters $\lambda, k, \hat{\lambda}, \hat{k}, m_s^2$ and n within the ranges given above and with logarithmic distributions for $\hat{\lambda}, \hat{k}, m_s^2$ and n . For each parameter set we evaluate, using eqs. (2.7) and the known value of $h_1^2 + h_2^2, m_1^2, m_2^2, h_1$ and h_2 . (Note that V'_{rad} disappeared from eqs. (2.7a and b) after our choice of Q^2 .) Then we check whether the corresponding set of parameters satisfies our conditions of quantum stability.

Of course we then require positive definite masses squared for all scalars in order to ensure that we have found a local minimum of the scalar potential. In addition we compute the value of the energy density and compare it with the corresponding values at the extrema of the potential where either h_1, h_2 or n or any combination thereof vanishes. Since such minima are favoured during the high temperature phase of the early universe, we have to ensure that they lie higher than the desired one. (The situation where $n = 0, h_1$ and $h_2 \neq 0$ is, however, never an extremum of the potential.)

If the corresponding set of parameters satisfies all these constraints (typically just one pars per mille!), we compute the masses of the six scalar fields from eqs. (2.8), (2.10), and (2.11) as well as the upper limit on the lightest chargino from eq. (2.14). Each value then contributes to the final distribution $f_i(\bar{M}_i)$ with a weight proportional to the determinant of the r.h.s. of eq. (2.17).

III. The distributions of masses obtained according to the procedure of sect. II still depend somewhat on the cutoff Λ appearing in the conditions of quantum stability. In figs. 1-7 we present our results for two extreme values of $\Lambda, \Lambda = 10 \text{ TeV}$ and $\Lambda = 10^{18} \text{ GeV}$. The areas under these curves have been normalized to a common value.

The mass of the lightest Higgs scalar is affected by the leading top quark radiative corrections as included in the mass matrix (2.8); for our computation we have used the values $m_t = 150 \text{ GeV}$ and $m_{s,q} = 500 \text{ GeV}$. In order to make the effect of these corrections visible, we show in fig. 1 the mass distribution of the lightest Higgs for $\Lambda = 10^{18} \text{ GeV}$, but without top quark correction. One finds that, in the average, this correction increases the lightest Higgs mass by $\sim 10 \text{ GeV}$.

We have stressed in the introduction that the aim of this investigation is just to obtain weak upper limits of the particle masses, which are violated only for special and hence unlikely combinations of the initial parameters. Clearly our distributions, at least theoretically, are still finite for arbitrarily large particle masses (except for the lightest Higgs scalar), but it is easy to see from the figures which values become very unlikely.

A rough measure for the weak upper limit is provided by the mass $\hat{M}$, which is defined such that 90 % of the area under the curves is obtained by integrating up to $\hat{M}$. Hence, if we have not found the corresponding particle up to $\hat{M}$, we have ruled out 90 % of the parameter space. The values of $\hat{M}$ corresponding to the distributions of figs. 1-7 are shown in table 2.

Of course, any particular class of models corresponding to a particular subset of initial parameters will lead, in general, to a modification of the distributions below $\hat{M}$, but it is unlikely that a model increases the area under the distributions beyond $\hat{M}$.

We have observed that if for a given set of parameters the mass of some field exceeds $\hat{M}$, this tends to happen simultaneously for all fields except for the lightest scalar. In this unfortunate situation all particles predicted by supersymmetry are quite heavy; in addition, as we have checked, the electroweak couplings of the lightest Higgs scalar get automatically very close to the ones of the non-supersymmetric standard model Higgs field.

In the region of very light masses we have not imposed constraints which follow from the non-observation of scalar neutral Higgses, charged Higgses, or charginos with masses below $\sim 40 \text{ GeV}$. This allows us to estimate which fraction of the parameter space has

been ruled out to date, and in fact this fraction is obviously quite small (in the ten percent range).

Concerning the discovery potential of future colliders we can extract the following informations from our results: Among the new particles we have considered the lighter chargino generally is the lightest one. (Note again that for this particle we have just computed a gaugino-mass independent upper bound.) Since it is generally lighter than the charged Higgs, the corresponding chargino decay channel is probably not available. Nevertheless it is fortunately one of the easiest ones to detect. We see that with a e^+e^- collider of 200 GeV center of mass energy (LEP II) we have, in contrast to LEP I, a reasonable chance to produce these particles or rule out $\sim$ half of the parameter space. In order to cover more than 90 percent of the parameter space, however, colliders with center-of-mass energies above 700 GeV will be required.

The heavier neutral scalars and pseudoscalars tend to have typical masses in 150-250 GeV range, but the distributions are still finite in the 1 TEV region. As mentioned before, the experimental confirmation of the complete particle spectrum may thus become very difficult or even impossible.

The upper bound on the mass of the lightest Higgs which we have obtained is just a reflection of our upper bound on the coupling λ , cf. eq. (2.12). Here the situation is thus not different from the one in the non-supersymmetric standard model. A violation of this bound in nature, however, would rule out most of the perturbative supersymmetric scenarios involving physics at very large scales as a GUT or even the Planck scale. It is thus the most obvious hurdle which this large class of models has to surmount.

Let us note again that we choose this slightly nonminimal supersymmetric extension of the standard model, because it is less restrictive than the minimal one and sufficiently rich that it can be used for a testing ground for the general assumption of supersymmetry. We have tried to give an answer, as honest as possible, to the question which masses for the supersymmetric Higgs particles we have to expect, and how far we have to go until we have tested the bulk of the parameter space of low energy supersymmetry.

References

1. H.-P. Nilles, Phys. Rep. **110** (1984) 1;
H. Haber and G. Kane, Phys. Rep. **117** (1985) 75;
A. Lahanas and D. V. Nanopoulos, Phys. Rep. **145** (1987) 1;
N. Dragon, U. Ellwanger, and M. G. Schmidt, Part. Nucl. Phys. **18** (1987) 1
2. J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. **B257** (1991) 83 and CERN-TH.6002/91
3. R. Barbieri, M. Frigeni, and F. Caravaglios, Phys. Lett. **B258** (1991) 167
4. Y. Okada, M. Yamaguchi, and T. Yanagido, Progr. Theor. Phys. **85** (1991) 1, and Phys. Lett. **B262** (1991) 54
5. H. Haber and R. Hempfling, Phys. Lett. **66** (1991) 1815;
R. Barbieri and M. Frigeni, Phys. Lett. **B262** (1991) 54
6. J.-P. Derendinger and C. A. Savoy, Nucl. Phys. **B237** (1984) 307
7. J. Ellis, J. Gunion, H. Haber, L. Roszkowski, and F. Zwirner, Phys. Rev. **D39** (1989) 844
8. M. Drees, Int. J. of Mod. Phys. **A4** (1989) 3635
9. L. Durand and J. Lopez, Phys. Lett. **B217** (1989) 463
10. J. Gunion, H. Haber, G. Kane, and S. Dawson, The Higgs Hunter's Guide, Addison-Wesley, 1990
11. J. Ellis, J. Hagelin, S. Kelley, and D. V. Nanopoulos, Nucl. Phys. **B311** (1988) 1

Table Captions

Table 1: Range of the parameters and VEVs, which contribute to our scan, after all the constraints (see text) have been taken into account.

Table 2: Values of $\hat{M}$ (in GeV) corresponding to the distributions of figs. 1-7; $\hat{M}$ is defined such that in 90 % of all cases $M < \hat{M}$. Λ is the cutoff which appears in the condition of quantum consistency of the parameters (see text), the particles are denoted as follows: S1, S2, S3: The three neutral Higgs scalars (ordered according to their masses); P1, P2: the two neutral Higgs pseudo-scalars; C: the charged Higgs scalars; LC: the upper bound on the mass of the lightest charged Higgsino/gaugino.

Figure Captions

Figs. 1-7 show the distributions of the masses after the scan over the parameter range given in table 1 (see sect. II). The solid curve corresponds to $\Lambda = 10 TeV$, the dashed curve to $\Lambda = 10^{18} GeV$. The particles are denoted as in table 2. In the case of the lightest neutral Higgs scalar S1 we also show the mass distribution (for $\Lambda = 10^{18} GeV$) without the top/stop correction as a dotted curve.

Fig. 1: S1; Fig. 2: S2; Fig. 3: S3; Fig. 4: P1; Fig. 5: P2; Fig. 6: C; Fig. 7: LC.

Tables

Table 1

Parameter	Range
λ	0 - 0.87
k	0 - 0.63
$\hat{\lambda}$	20 - 150 GeV
$\hat{k}$	-300 - 210 GeV
m_1^2	-(900 GeV) ² - +(350 GeV) ²
m_2^2	-(500 GeV) ² - +(800 GeV) ²
m_3^2	-(1250 GeV) ² - +(150 GeV) ²
n	-2000 GeV - +750 GeV
$\tan\beta$	1 - 30

Table 2

Λ	$\hat{M}_{S1}$	$\hat{M}_{S2}$	$\hat{M}_{S3}$	$\hat{M}_{P1}$	$\hat{M}_{P2}$	$\hat{M}_C$	$\hat{M}_{CL}$
10 TeV	130	500	640	390	640	520	370
10^{18} GeV	120	430	610	370	550	470	300

fig. 1

fig. 2

fig. 3

fig. 4

fig. 5

fig. 6

fig. 7