EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN-PPE/91-123
CERN/LAA/MSL/91-015
July 15, 1991

THE EFFECTIVE EXPERIMENTAL CONSTRAINTS ON M_{SUSY} AND M_{GUT}

F. Anselmo¹⁾, L. Cifarelli^{1,2,3)}, A. Petermann^{1,4,5)} and A. Zichichi¹⁾

1) CERN, Geneva, Switzerland

2) Università di Napoli, Italy

3) INFN-Bologna, Italy

4) World Laboratory, Lausanne, Switzerland

5) CPT, CNRS - Luminy, Marseilles, France

Abstract

A comprehensive analysis of the world-data on α_s is reported together with its average value at the Z^0 mass. The effective constraints on M_{SUSY} and M_{GUT} are given. The care needed to reach any conclusion on M_{SUSY} is discussed. For example, taking for the $\alpha_1, \alpha_2, \alpha_3$ coupled equations a numerical solution (it should be the most reliable one) and the two standard deviation limits in the uncertainty of the $\alpha_s(M_{Z^0})$ world average, the expected M_{SUSY} values range from $10^{0.5 \pm 0.5}$ GeV to $10^{5 \pm 1}$ GeV, i.e. from GeV to PeV.

(Submitted to "Il Nuovo Cimento")

1. INTRODUCTION

The purpose of this paper is twofold:

- i) to report on a comprehensive analysis of the world-data on α_s (the QCD coupling constant);
- ii) to work out which prediction, if any, can be made on M_{SUSY} (the ad hoc energy chosen for the introduction of SuperSYmmetry breaking, assuming that all sparticles have the same mass) and on M_{GUT} (the unification mass corresponding to the energy where the effective electromagnetic (α_1), weak (α_2) and strong (α_3) couplings converge).

Let us specify these couplings in the $SU(3)_C \otimes SU(2)_L \otimes U(1)$ Standard Model:

$$\begin{aligned}\alpha_1 &= (5/3) (g'^2/4\pi) = 5\alpha/3\cos^2\theta_{\overline{MS}} \\ \alpha_2 &= g^2/4\pi = \alpha/\sin^2\theta_{\overline{MS}} \\ \alpha_3 &= g_s^2/4\pi\end{aligned}\tag{1}$$

where g' , g and g_s are the $U(1)$, $SU(2)_L$ and $SU(3)_C$ couplings, respectively; α is the fine structure constant, and $\sin^2\theta_{\overline{MS}}(M_Z^0)$ is the mixing angle in the $\overline{MS}$ model, the so-called modified minimal subtraction scheme [1]. The normalization factor $5/3$ comes from the $SU(5)$ condition that, at the unification scale, $\sin^2\theta_W$ be equal to $3/8$ [2].

In the 2-loop approximation, the basic "coupled" equations which relate α_1 , α_2 , α_3 and their "running" versus $\mu^2 = q^2$, are:

$$\mu (d\alpha_i/d\mu) = (b_i/2\pi) \alpha_i^2 + \sum_j (b_{ij}/8\pi^2) \alpha_j \alpha_i^2\tag{2}$$

where the indexes i and j run from 1 to 3, and the $\{b_i\}$ and $\{b_{ij}\}$ matrices are given in Table 1, both for the non-SUSY [3] and the SUSY [4] cases.

As discussed in section 3, these coupled equations (2) can be solved [5], [6] either via numerical integration [7] or via approximate analytic expressions. These possible solutions are given in Table 2. Recently an analysis [8] using one of these solutions has been reported.

2. ANALYSIS OF α_s

The experimental determination of α_s ranges from 1.5 GeV up to 91 GeV, i.e. the Z^0 mass range [9].

As reported in the review by S. Bethke [10], the values of α_s obtained by LEP and SLC experiments have been grouped according to the various final state analyses. These data are reproduced in Tables 3-5 for the convenience of the reader: Jet Rates (Table 3), Energy-Energy Correlations (EEC, Table 4) and Asymmetry in Energy-Energy Correlations (AEEC, Table 5). The average value relative to the so-called Event Shapes analysis [11, 12], which is however affected by a larger error, is reported in Table 6, together with the other averages [10]. The world average that can be derived from LEP and SLC data is:

$$\alpha_s(M_{Z^0}) = 0.118 \pm 0.008. \quad (3)$$

In addition Table 7 shows a summary of all LEP results, for each experiment separately. From the results obtained with Jet Rates, EEC and AEEC, the weighted and unweighted averages for each experiment have been worked out for the sake of comparison.

As indicated in Tables 3-5, there are essentially three possible sources of errors on α_s :

- experimental uncertainties (both statistical and systematic),
- uncertainties due to the μ scale truncation,
- theoretical uncertainties (in particular due to the various possible hadronization models).

Let us attribute the index j to specify each of the above sources of errors. The index i specifies a given process and analysis technique used to extract the value of α_s . Then, for each measured value $(\alpha_s)_i$, the error will be:

$$\sigma_i = \left(\sum_{j=1}^3 \sigma_{ij}^2 \right)^{1/2}. \quad (4)$$

Notice that when asymmetric errors (σ_{ij}^+ , σ_{ij}^-) appear in Tables 3-5, an average value ($\sigma_{ij} = (\sigma_{ij}^+ + \sigma_{ij}^-)/2$) is used. To compute the weighted averages, the weights $\{w_i = 1/\sigma_i^2\}$ are used. We have assumed in Table 7 the errors to be the same for both kinds of averages, weighted and unweighted. In fact, since the various sources of errors are not purely statistical, moreover since possible correlations might exist among the theoretical errors involved in the different analyses, the resultant error of each experiment is taken to be:

$$\sigma = \min [\sigma_i]_{i=1, 2, \dots, n} \quad (5)$$

where n is the total number of analysis techniques used in the experiment. Notice that the unweighted and weighted α_s averages in Table 7 are essentially the same for all LEP experiments but DELPHI, whose α_s values are smaller and in particular the weighted average used in Ref. 8 to compute the $\alpha_1, \alpha_2, \alpha_3$ evolution.

As an additional piece of information, the number of events analysed by each collaboration [13, 14, 15, 16] is reported in Table 7. Since this number is not always the same for different α_s estimates by the same collaboration, moreover since possible overlaps of event samples, if any, are not specified, the maximum number of events analysed (N) is quoted in Table 7, i.e.:

$$N = \max [N_i]_{i=1, 2, \dots, n} \quad (6)$$

We have also analysed non-LEP data in order to get an extrapolated value of α_s at the Z^0 mass, $\alpha_s(M_{Z^0})$, from α_s measured at lower centre-of-mass energies. This has been done by fitting the 1.5 GeV to 59.5 GeV α_s values with the 2-loop QCD formula for α_s running [17]:

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2 n_f) \ln(\mu^2/\Lambda_{\text{QCD}}^2)} \times \left(1 - \frac{6(153 - 19 n_f)}{(33 - 2 n_f)^2} \frac{\ln[\ln(\mu^2/\Lambda_{\text{QCD}}^2)]}{\ln(\mu^2/\Lambda_{\text{QCD}}^2)} \right) \quad (7)$$

n_f being the number of flavours with mass smaller than μ . The world average, $\alpha_s(M_{Z^0})$, has obviously not been included in the fit. The only free parameter in the fit is Λ_{QCD} , which turns out to be:

$$\Lambda_{\text{QCD}} = (260 \pm 40) \text{ MeV}. \quad (8)$$

The results are shown in Figure 1a. We call this fit, where all data from 1.5 to 59.5 GeV are included, "fit 1".

Notice that the value of Λ_{QCD} remains practically unaltered if the world average, $\alpha_s(M_Z^0)$, is also included in the fit. The $\pm\sigma$ error on Λ_{QCD} is represented in Figure 1a via the two dashed-line curves.

The same data and fit are shown in Figure 1b but with a different error-band. This is obtained by randomly excluding one or more data points from the fit and by evaluating the spread of the fit results obtained in this way. As shown in Figure 1b, this spread turns out to be asymmetric with respect to the central fit curve. Therefore, the extrapolated α_s value at the Z^0 mass can be quoted, according to Figures 1a and 1b, respectively, as:

$$[\alpha_s(M_Z^0)]_{\text{fit 1}} = 0.115 \pm 0.002 \text{ (symmetric error)} \quad (9)$$

$$[\alpha_s(M_Z^0)]_{\text{fit 1}} = 0.115^{+0.011}_{-0.002} \text{ (asymmetric error)}. \quad (10)$$

The same exercise is repeated using the more reliable higher energy data: i.e. those between 29 and 59.5 GeV, only. This fit is called "fit 2". The results are shown in Figure 2. In this case Λ_{QCD} is (*):

$$\Lambda_{\text{QCD}} = (390 \pm 70) \text{ MeV}. \quad (11)$$

Here again, the value of Λ_{QCD} remains practically unchanged when, in the best fit, the world average, $\alpha_s(M_Z^0)$, is included. The $\pm\sigma$ error-band is shown via the dotted-line curves in Figure 2. Using the technique of random exclusion of data points (described above for Figure 1b), this error-band remains almost unchanged. The corresponding extrapolated $\alpha_s(M_Z^0)$ values are:

(*) Note that the values of Λ_{QCD} for "fit 1" [$\Lambda_{\text{QCD}} = (260 \pm 40) \text{ MeV}$] and for "fit 2" [$\Lambda_{\text{QCD}} = (390 \pm 70) \text{ MeV}$] are such as to indicate possible sources of unquoted "systematic" errors. In fact the inclusion of the low energy data lowers the value of Λ_{QCD} .

$$[\alpha_s(M_{Z^0})]_{\text{fit } 2} = 0.121 \pm 0.004 \text{ (symmetric error)} \quad (12)$$

$$[\alpha_s(M_{Z^0})]_{\text{fit } 2} = 0.121^{+0.005}_{-0.004} \text{ (asymmetric error)} \quad (13)$$

in fair agreement with the previous results.

Figure 3 shows a compilation of LEP data (see Table 7), where the asymmetric error-bands from Figures 1b and 2 are superimposed. Clearly there is no reason to select a particular value of α_s (as done in Ref. 8, with the DELPHI result) as the most significant one. The conclusion is that the world average for $\alpha_s(M_{Z^0})$ (which by the way is consistent with the extrapolated $[\alpha_s(M_{Z^0})]_{\text{fit } 1}$ or $[\alpha_s(M_{Z^0})]_{\text{fit } 2}$ values), should be the basis for all speculations on possible new physics. This world average (3) is indicated in Figure 3.

3. CONSEQUENCES ON M_{SUSY} AND M_{GUT}

The $\alpha_1, \alpha_2, \alpha_3$ evolution versus μ can be worked out in the Standard Model framework, using as input the values of $\alpha_s(M_{Z^0})$ (see section 2) with given hypotheses on N_H (number of Higgs doublets), N_F (number of flavour families) and using, as final input, the value of $\sin^2\theta_{\overline{\text{MS}}}(M_{Z^0})$, as indicated in (1).

Since 1978 SuperSYmmetry and its importance was emphasized [18]. The convergence of $\alpha_1, \alpha_2, \alpha_3$ using SuperSYmmetry was computed [18], and published for the first time in 1981 [19]. These papers showed that the couplings evolve better towards a common Grand Unification point (M_{GUT}) when SuperSYmmetry is introduced.

However, the values of M_{SUSY} and M_{GUT} will obviously depend on the many ingredients adopted. In fact, in addition to the experimental determination of $\alpha_s(M_{Z^0})$ and $\sin^2\theta_{\overline{\text{MS}}}(M_{Z^0})$, there are the number of Higgs doublets (N_H), the number of Families (N_F) and the choice of the particular analytical expression used as an approximate solution of the coupled 2-loop equations (Table 2).

Moreover, these differential equations can also be solved numerically, via a Runge-Kutta-Merson method of integration [7]. This method should provide the most reliable approximation.

In the present study, we have investigated many possibilities, using:

i) a set of fixed ingredients, i.e.:

$$\begin{aligned} N_F &= 3 \\ N_H &= 2 \\ \sin^2\theta_{\overline{MS}}(M_Z^0) &= 0.2334 \pm 0.0008^{(*)} [20]; \end{aligned} \quad (14)$$

ii) the different values of $\alpha_s(M_Z^0)$ and the various possibilities for the solution of the coupled $\alpha_1, \alpha_2, \alpha_3$ 2-loop equations. Four different values of $\alpha_s(M_Z^0)$ have been considered, in order to scan the region allowed by the present experimental and theoretical uncertainties. The values chosen range from the lowest ($\alpha_s = 0.108$, see Table 7 and footnote) to the highest ($\alpha_s = 0.120$) LEP result, with an intermediate value at $\alpha_s = 0.115 \pm 0.008$ and the world average: $\alpha_s = 0.118 \pm 0.008$. The masses of the lightest and the heaviest Higgs doublet have been assumed to be M_Z^0 and M_{SUSY} , respectively.

A simple fit procedure, applied to $\alpha_1, \alpha_2, \alpha_3$ evolution, with M_{GUT} and M_{SUSY} as free parameters, has been performed. The results are shown in Figures 4a-4d in terms of M_{SUSY} versus M_{GUT} , for the various possibilities discussed so far. As can be seen from Figure 5, M_{SUSY} can range within one standard deviation from very small values up to several tens of TeV and, accordingly, M_{GUT} from 10^{17} down to almost 10^{15} GeV.

In order to see clearly the route followed by the three couplings $\alpha_1, \alpha_2, \alpha_3$, towards the common point of convergence at M_{GUT} , we have plotted a three dimensional graph with $\alpha_1, \alpha_2, \alpha_3$ as orthogonal axes. Figure 6 shows an example worked out for $M_{SUSY} = 1$ TeV. The various points are at energies specified by increasing darkness: the higher the energy, the darker the point. The first point corresponds to 0.1 TeV, the

(*) This error includes the uncertainty on m_{top} and m_H .

second to 1 TeV, the third to 10 TeV and so on, up to the unification point (*).

We have also studied the convergence of the three running couplings $\alpha_1, \alpha_2, \alpha_3$ without SUSY and found that this is still possible, provided $N_H = 6$. The detailed analysis of these and other results is reported elsewhere [21].

4. DISCUSSION ON THE CARE NEEDED TO REACH CONCLUSIONS ON M_{SUSY}

The use of one standard deviation limit for the study of possible consequences of quantities experimentally measured with their standard errors, has become an accepted trend, as if these errors had more precise meanings than those specified by the well-known laws of statistics. The minimum range of safety is the 95% confidence level, i.e. $\pm 2\sigma$.

Therefore we have studied what would happen in terms of M_{SUSY} versus M_{GUT} , when applying our fit procedure not only with $\alpha_s(M_Z^0) = 0.118 \pm 0.008$ (the world average given in section 2), but also with the central value shifted by two standard deviations up and down: $\alpha_s(M_Z^0) = 0.134 \pm 0.008$ and $\alpha_s(M_Z^0) = 0.102 \pm 0.008$, respectively. The results are shown in Figures 7 a,b,c. The four possible solutions of the coupled 2-loop equations for $\alpha_1, \alpha_2, \alpha_3$ evolution (discussed in section 3) are all included. The values of M_{SUSY} and M_{GUT} scan a large range of possibilities. For instance, if $(\alpha_s + 2\sigma)$ is taken at M_Z^0 (Figure 7b), the values $M_{\text{SUSY}} = 10^{0.5 \pm 0.5}$ GeV and $M_{\text{GUT}} = 10^{16.8 \pm 0.1}$ GeV are obtained using the Runge-Kutta-Merson method of integration. The corresponding values for $(\alpha_s - 2\sigma)$ are $M_{\text{SUSY}} = 10^{5 \pm 1}$ GeV and $M_{\text{GUT}} = 10^{15.4 \pm 0.2}$ GeV, as shown in Figure 7c.

We would like to emphasize that if new knowledge on M_{SUSY} should seriously be claimed by present experimental data [8], this new knowledge should be based on the 95% confidence level. This is why in Figure 8 we

(*) We are aware that the requirement of a unique crossing of $\alpha_1, \alpha_2, \alpha_3$ trajectories is not a compulsory condition for GUT.

give a summary of the results derived in our analysis with $\alpha_s(M_Z^0) \pm 2\sigma$ as the basic scenario to study the evolution of $\alpha_1, \alpha_2, \alpha_3$ with SuperSYmmetry introduced at the various energy levels, as specified.

From these studies it comes out that care is needed before quoting [8] a value of M_{SUSY} where the expected SUSY-break would occur.

5. CONCLUSIONS

As reported in Figure 3, the values of $\alpha_s(M_Z^0)$ have a large range of uncertainty. No value of α_s can be considered as privileged with respect to any other in the quoted figure. The only correct choice which takes all data with their errors into account is the world average (3).

Moreover, the various approximate solutions of the coupled equations for $\alpha_1, \alpha_2, \alpha_3$ evolution must be considered as another source of uncertainty.

Consequently, the values of M_{SUSY} and M_{GUT} are in the range reported in Figure 8 for 95% confidence level. Claims for better knowledge on M_{SUSY} are unjustified.

6. ACKNOWLEDGEMENTS

We are grateful to S. Bethke, J. Ellis and W.J. Stirling for a series of interesting discussions. We would also like to acknowledge the collaboration of G. La Commare, M. Marino and G. Xexeo.

STANDARD MODEL

Without SUSY [3]		
$\{b_i\}$	$= \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -22/3 \\ -11 \end{pmatrix} + N_F \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} + N_H \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix}$	
$\{b_{ij}\}$	$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -136/3 & 0 \\ 0 & 0 & -102 \end{pmatrix} + N_F \begin{pmatrix} 19/15 & 3/5 & 44/15 \\ 1/5 & 49/3 & 4 \\ 11/30 & 3/2 & 76/3 \end{pmatrix} + N_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 13/6 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	

With SUSY [4]		
$\{b_i\}$	$= \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -6 \\ -9 \end{pmatrix} + N_F \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} + N_H \begin{pmatrix} 3/10 \\ 1/2 \\ 0 \end{pmatrix}$	
$\{b_{ij}\}$	$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -24 & 0 \\ 0 & 0 & -54 \end{pmatrix} + N_F \begin{pmatrix} 38/15 & 6/5 & 88/15 \\ 2/5 & 14 & 8 \\ 11/15 & 3 & 68/3 \end{pmatrix} + N_H \begin{pmatrix} 9/50 & 9/10 & 0 \\ 3/10 & 7/2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$	

Table 1

The $\{b_i\}$ and $\{b_{ij}\}$ matrices for the two cases: without SUSY and with SUSY

Formula 1:

$$\frac{1}{\alpha_i(\mu')} = \frac{1}{\alpha_i(\mu)} + \frac{b_i}{2\pi} \ln\left(\frac{\mu}{\mu'}\right) + \sum_j \frac{b_{ij}}{4\pi b_j} \ln\left(\frac{\alpha_j(\mu)}{\alpha_j(\mu')}\right)$$

[Reference 5]

Formula 2:

$$\frac{1}{\alpha_i(\mu')} = \frac{1}{\alpha_i(\mu)} + \beta_{0i} \ln\left(\frac{\mu}{\mu'}\right) + \frac{\beta_{1i}}{\beta_{0i}} \ln\left(\frac{1/\alpha_j(\mu') + \beta_{1i}/\beta_{0i}}{1/\alpha_j(\mu) + \beta_{1i}/\beta_{0i}}\right)$$

$$\beta_{0i} = \frac{b_i}{2\pi} + \sum_{j \neq i} \frac{b_{ij}}{8\pi^2} \alpha_j(\mu)$$

$$\beta_{1i} = \frac{b_{ii}}{8\pi^2}$$

[Reference 8]

Formula 3: same as Formula 2 but with $\alpha_j(\mu)$ in the β_{0i} expressions replaced by the 1-loop equation solution:

$$\alpha_j(\mu) \rightarrow \alpha_j(\mu') = \frac{1}{\frac{1}{\alpha_j(\mu)} + \frac{b_j}{2\pi} \ln\left(\frac{\mu}{\mu'}\right)}$$

Table 2

Approximate analytical solutions of eqs. (2) in the text.

Group	Value	Experimental	Scale	Hadronization
Mark II	0.123	± 0.009	$\pm \text{---}$	± 0.005
ALEPH	0.121	± 0.004	± 0.008	± 0.007
DELPHI	0.114	± 0.005	± 0.011	± 0.003
L3	0.115	± 0.005	± 0.010	± 0.003
OPAL	0.118	± 0.003	± 0.007	± 0.004
Average	0.119	± 0.002	± 0.007	± 0.003

Table 3

$\alpha_s(M_z)$ Determination from Jet Rates

Group	Value	Experimental	Scale	Theoretical
DELPHI	0.106	± 0.004	$+0.003$ -0.000	$\pm \text{---}$
L3	0.115	$+0.008$ -0.006	$+0.002$ -0.000	$+0.003$ -0.005
OPAL	0.117	$+0.007$ -0.009	± 0.000	$+0.006$ -0.002
Average	0.113	± 0.007	± 0.001	± 0.004

Table 5

$\alpha_s(M_z)$ Determination from AEEC

Group	Value	Experimental	Scale	Theoretical
ALEPH	0.117	± 0.005	$+0.006$ -0.009	$\pm \text{---}$
L3	0.121	± 0.004	$+0.009$ -0.006	± 0.006
OPAL	0.124	± 0.006	± 0.007	± 0.007
Average	0.121	± 0.004	± 0.007	± 0.006

Table 4

$\alpha_s(M_z)$ Determination from EEC

Method	Value
Jet Rates	0.119 ± 0.008
EEC	0.121 ± 0.010
AEEC	0.113 ± 0.008
Event shapes	0.126 ± 0.016

Table 6

Final Compilation on $\alpha_s(M_z)$

	DELPHI	ALEPH	OPAL	L3
α_s unweighted average	0.110 ± 0.004	0.119 ± 0.009	0.120 ± 0.009	0.117 ± 0.008
α_s weighted average	$0.107^{\#} \pm 0.004$	0.119 ± 0.009	0.119 ± 0.009	0.117 ± 0.008
Number $\cdot 10^5$ of events x 10	0.05	1.06	0.70	0.83

* Maximum number of events used either in Jet Rates or EEC or AEEC analysis by the four LEP experiments.

The weighted α_s value quoted in Ref. 8 is 0.108 ± 0.005 . This slight difference is likely due to rounding off when using the values of Tables 3-5.

Table 7

Summary of α_s results at LEP

REFERENCES

1. W.A. Bardeen, A. Buras, D. Duke and T. Muta, Phys. Rev. D18 (1978) 3998.
2. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32 (1974) 438.
3. D.G. Unger and Y.P. Yao, University of Michigan, Report No. UMHE 81-30 (unpublished). See also D.R.T. Jones, Phys. Rev. D25 (1982) 581.
4. M.B. Einhorn and D.R.T. Jones, Nucl. Phys. B196 (1982) 475.
J. Ellis, D.V. Nanopoulos and S. Rudaz, Nucl. Phys. B 202 (1982) 43;
For lectures and references on all these topics see: Superworld I, Ed. A. Zichichi (Plenum Press, New York-London, 1986); Superworld II, Ed. A. Zichichi (Plenum Press, New York-London, 1987); Superworld III, Ed. A. Zichichi (Plenum Press, New York-London, 1988).
5. W.J. Marciano and A. Sirlin, Invited talk, Second Workshop on Grand Unification, Ann Arbor, April 1981.
6. J. Ellis, S. Kelley and D.V. Nanopoulos, Phys. Lett. B249 (1990) 441 and B260 (1991) 131 and P. Langacker, University of Pennsylvania Preprint UPR-0435T (1990). In their analyses of LEP data these authors avoid overoptimistic estimates in the context of Supersymmetry.
7. See G.N. Lance, Numerical Methods for High Speed Computers, Iliffe & Sons, London 1960.
8. U. Amaldi, W. de Boer and H. Furstenau, Phys. Lett. B260 (1991) 447.
9. For a review, see for instance: S. Bethke, Preprint LBL-28112, Nov. 1989 and W.J. Stirling and M.R. Whalley, Preprint RAL-87-107 and DPDG/87/01.

10. S. Bethke, Preprint CERN-PPE/91-36.
11. ALEPH Collaboration, Phys. Lett. B255 (1991) 623.
12. N. Magnoli, P. Nason and R. Rattazzi, Phys. Lett. B252 (1990) 271.
13. DELPHI Collaboration, Phys. Lett. B247 (1990) 167 and Phys. Lett. B252 (1990) 149.
14. ALEPH Collaboration, Phys. Lett. B255 (1991) 623 and Phys. Lett. B257 (1991) 479.
15. OPAL Collaboration, Phys. Lett. B235 (1990) 389 and Phys. Lett. B252 (1990) 159.
16. L3 Collaboration, Phys. Lett. B248 (1990) 464 and L3 Preprint #023, December 1990.
17. W.J. Marciano, Phys. Rev. D29 (1984) 580.
18. A. Zichichi, Closing Lecture at the EPS Conference (York, UK, September 25-29, 1978); also Opening Lecture at the EPS Conference (Geneva, CH, 27 June - 4 July, 1979) - CERN Proceedings; and "New Developments in Elementary Particle Physics", Rivista Nuovo Cimento, Vol 2, No. 14 (1979) 1. The statement on page 2 of this paper, "*Unification of all forces needs first a Supersymmetry. This can be broken later, thus generating the sequence of the various forces of nature as we observe them*", was based on a work by A. Petermann and A. Zichichi where the renormalization group running of the couplings using Supersymmetry was studied with the result that the convergence of the three couplings improved. This work was not published.

19. S. Dimopoulos and H. Georgi, Nucl. Phys. B193 (1981) 150-162. L. Ibañez and G.G. Ross, Phys. Lett. B105 (1981) 439. See also J. Ellis, D.V. Nanopoulos, S. Rudaz, Nucl. Phys. B202 (1982) 43, and the three volumes of the Proceedings of the "Ettore Majorana" International School of Subnuclear Physics (reference 4). In all these volumes the fact that Supersymmetry allowed a better convergence of the basic running couplings (α_1 , α_2 , α_3) was fully discussed by many authors, using different models.
20. P. Langacker and M. Luo, Preprint UPR 0466-T.
21. F. Anselmo, L. Cifarelli, A. Petermann and A. Zichichi, Preprint in preparation.

FIGURE CAPTIONS

- Fig.1a: The α_s (fit 1) with 1.5-59.5 GeV data (symmetric error).
- Fig. 1b: The α_s (fit 1) with 1.5-59.5 GeV data (asymmetric error).
- Fig. 2: The α_s (fit 2) with 29-59.5 GeV data (symmetric and asymmetric errors coincide).
- Fig. 3: The allowed $\alpha_s(M_Z)$ ranges from low-energy data (fit 1 and fit 2), compared with the $\alpha_s(M_Z)$ measurements from LEP experiments. The world average for $\alpha_s(M_Z)$ is also shown.
- Fig. 4a: Results for M_{SUSY} and M_{GUT} using Formula 1 (Table 2) as solution of equations (2) in the text, for different values of $\alpha_s(M_Z)$, as specified.
- Fig. 4b: Results for M_{SUSY} and M_{GUT} using Formula 2 (Table 2) as solution of equations (2) in the text, for different values of $\alpha_s(M_Z)$, as specified.
- Fig. 4c: Results for M_{SUSY} and M_{GUT} using Formula 3 (Table 2) as solution of equations (2) in the text, for different values of $\alpha_s(M_Z)$, as specified.
- Fig. 4d: Results for M_{SUSY} and M_{GUT} solving equations (2) in the text by the Runge-Kutta-Merson method, for different values of $\alpha_s(M_Z)$, as specified.
- Fig. 5: Figures 4 a,b,c,d, superimposed.
- Fig. 6: The $\alpha_1, \alpha_2, \alpha_3$ evolution curve represented as a series of points whose darkness increases with energy. Also shown as a series of smaller points the $\alpha_1 = \alpha_2 = \alpha_3$ straight line. The two curves meet at the unification point (M_{GUT}).
- Figs. 7: Results for M_{SUSY} and M_{GUT} using for $\alpha_s(M_Z)$ the central value of the world average (Figure 7a) and two other values shifted upwards (Figure 7b) and downwards (Figure 7c) by two standard deviations. The four possibilities for each value of α_s

correspond to the four possible solutions of the coupled equations (see Table 2). Notice that two solutions (Formula 1 and Numerical Solution) produce exactly the same results for the case of $\alpha_s = 0.102$ and $\alpha_s = 0.134$.

Fig. 8: Figures 7 a,b,c superimposed.

Figure 1a

Figure 1b

Figure 2

Figure 3

Figure 4a

Figure 4b

Figure 4c

Figure 4d

Figure 5

Figure 6

Figure 7a

Figure 7b

$\alpha_s(M_{Z^0})$
0.102 ± 0.008

Figure 7c

Figure 8