QUANTUM MECHANICAL TUNNELING AT HIGH ENERGY

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ABSTRACT

We study tunneling processes for a quantum-mechanical particle in a double-well potential. We show that the instanton estimate of the transition rate between excited states in the two wells is reliable at energies much below the barrier height and weak coupling, in that it is asymptotic to the WKB result. We illustrate how the growth of the amplitude with energy can in a generic basis be perceived as partly due to the growth of available phase space, and we comment on the infrared divergences that appear in the calculation of radiative corrections. We then calculate the rate of tunneling from the ground state in one well to an excited state in the other, induced by an external periodic force, and show that it grows with energy like the geometric mean of the vacuum-to-vacuum and excited-to-excited-state rates. This slower growth is due to the difficulty of converting the energy, initially stored in a single quantum, to large occupation number. We argue by extrapolation that even at energies near the barrier height, induced tunneling should be suppressed by a fraction of the instanton action.

Recently there has been a surge of interest in baryon- and lepton-number violating processes due to the instantons of the $SU(2) \times U(1)$ electroweak model. Although 't Hooft [1] showed a number of years ago that such processes carry a huge suppression factor, it now appears [2, 3] that this could have been overcome in the early universe, at temperatures just below the electroweak phase transition. Much more controversial is the possibility raised by several authors [4, 5, 3] that the exponential suppression factor can also be overcome in high-energy scattering, so that baryon- and lepton-number violation could be observed in future colliders. The key technical remark [5] is that (spherically symmetric) instantons induce to leading order effective point-like interactions with an arbitrary number of bosonic legs. As a result inclusive cross-sections appear to grow exponentially fast with energy, and could conceivably become unsuppressed if this behaviour were to persist to sufficiently high energies of order the sphaleron [6] mass. For such high energies and correspondingly large multiplicities one is, however, faced with the well-known breakdown of perturbation theory due to the large number of graphs*. Thus, despite the recent effort to determine radiative corrections to the leading-order result [7, 9, 3], the hope of resolving the issue by an accurate calculation of the effect appears at present to be rather remote.

Nevertheless, there is by now growing consensus on at least two points: (a) That the instanton calculation is indeed valid in some energy range, as is the exponential growth of cross-sections and partial cancellation of the suppression factor, and (b) that the rate of growth may depend crucially on the nature of the initial state. The first statement is not a priori obvious, because the instanton calculation involves an analytic continuation and LSZ reduction of Euclidean Green's functions, a procedure whose validity has only been established in ordinary perturbation theory. As for the second statement, several authors [6, 10, 11, 12, 13] share the intuition that the available energy, initially stored in a few high-frequency quanta, is "misdirected" and cannot be readily converted to the large amplitude or occupation number of the sphaleron field. Put differently, although tunneling from some optimal "jumping" state grows rapidly with energy and becomes eventually unsuppressed, the growth can be partly compensated by the overlap of this optimal state with a few-particle initial state. This relative suppression factor is a priori expected to survive all the way up to the sphaleron mass making the effect unobservably small, a point most clearly advocated by Banks et al [11].

* The situation is even worse in the standard model, where the size of the instanton is not fixed, so that important corrections set in already at low multiplicities of order one [7]. It has, however, been claimed that in some intermediate-energy range a well defined perturbative expansion may exist [8].

The purpose of this letter is to demonstrate points (a) and (b) in the context of the double-well potential of quantum mechanics. Although our results may not be surprising, they clarify some technical and conceptual issues, and further elucidate the arguments of ref. [11]. We will, specifically, show how to compare the one-instanton with a more standard WKB result for the transition amplitude between excited states in the two wells, and prove that these results are asymptotic to each other in the region where both approximations can be trusted. We will comment on the origin of infrared divergences in the calculation of corrections to the one-instanton result, and explain how the reduction of the effective barrier with energy can appear as partly due to the growth of available phase-space. We will then compute the amplitude for stimulated tunneling from the ground state in one well to an excited state in the other, in the presence of an external rapidly oscillating force. We find that for times of the order of the harmonic frequency, this amplitude grows with energy only like the square root of the excited-to-excited-state amplitude. A naive extrapolation shows that when the latter amplitude becomes of order one, the former is still suppressed by half the instanton action. A similar conclusion has been reached recently in refs. [14, 15].

Let us consider the double-well potential problem in quantum mechanics, with Lagrangian density

$$\mathcal{L} = \frac{1}{2} \left(\frac{d\phi}{dt} \right)^2 - \frac{1}{2} \lambda (\phi^2 - v^2)^2 \equiv \frac{1}{2} \left(\frac{d\phi}{dt} \right)^2 - V(\phi), \quad (1)$$

where $\phi(t)$ is the position of the particle in a 1-dimensional space. Instead of the parameters λ and v , we will use the frequency of small oscillations in each well $\omega = 2v\sqrt{\lambda}$, and the dimensionless coupling constant $g^2 = \frac{1}{v\sqrt{\lambda}}$. The weak-coupling approximation corresponds to $g \rightarrow 0$ for fixed frequency ω . In this limit the two minima of the potential $V(\phi)$ are infinitely separated, while the energy barrier between them becomes infinitely high. Thus the system reduces to a pair of independent harmonic oscillators.

We want to calculate, in the one-instanton approximation, the S-matrix element $S_{n \rightarrow n} \equiv + < n | S | n >_-$ between the n th-excited states in the two wells.* To talk about an S-matrix one should in principle assume that the coupling is adiabatically switched off at $t \rightarrow \pm\infty$, so that the asymptotic states are harmonic oscillator states in the two wells; we will come back to this point later. Following the analysis of refs. [5] we may obtain the above S-matrix element by applying the LSZ procedure to the analytic continuation of the Euclidean Green's function

* This calculation has been previously performed by Shaposhnikov [12], but we include it here for completeness.

$$G_E(\tau_1, \dots, \tau_n; \tau'_1, \dots, \tau'_n) = \mathcal{N} e^{-S_0} \int d\tau_0 \delta_+ \phi_0(\tau_1 - \tau_0) \dots \delta_+ \phi_0(\tau_n - \tau_0) \delta_- \phi_0(\tau'_1 - \tau_0) \dots \delta_- \phi_0(\tau'_n - \tau_0). \quad (2)$$

Here,

$$\phi_0(\tau - \tau_0) = \frac{1}{g} \sqrt{\frac{2}{\omega}} \tanh\left(\frac{\omega}{2}(\tau - \tau_0)\right) \quad (3)$$

is the instanton field interpolating between the vacua at $\phi = \pm v \equiv \pm \frac{1}{g} \sqrt{\frac{2}{\omega}}$ and centered at Euclidean time τ_0 , $S_0 = \frac{4}{3g^2}$ is its Euclidean action, $\delta_{\pm} \phi_0 = v \mp \phi_0$, and $\mathcal{N} = \frac{2\omega}{g} \sqrt{\frac{2}{\pi}}$ is the contribution of quadratic fluctuations around the instanton calculated for instance in ref. [16].

The LSZ procedure gives:

$$\begin{aligned} S_{n \rightarrow n} &= + < 0 | \frac{(\mathbf{a}_+)^n}{\sqrt{n!}} S \frac{(\mathbf{a}_+^\dagger)^n}{\sqrt{n!}} | 0 > - \\ &= \frac{1}{n!} \prod_{j=1}^n \left(\lim_{t_j \rightarrow +\infty} \frac{i}{\sqrt{2\omega}} e^{i\omega t_j} \frac{\vec{d}}{dt_j} \right) \left(\lim_{t'_j \rightarrow -\infty} \frac{-i}{\sqrt{2\omega}} e^{-i\omega t'_j} \frac{\vec{d}}{dt'_j} \right) G_E(it_1, \dots, it_n; it'_1, \dots, it'_n), \end{aligned} \quad (4)$$

where $\mathbf{a}_+(\mathbf{a}_+^\dagger)$ is the annihilation (creation) operator in the remote future (past). Using the asymptotic form of the instanton in eq. (3) one finds

$$S_{n \rightarrow n} = iT \frac{2\omega}{g} \sqrt{\frac{2}{\pi}} \exp\left(-\frac{4}{3g^2}\right) \frac{1}{n!} \left(\frac{4}{g}\right)^{2n} (1 + \mathcal{O}(g)), \quad (5)$$

where T is the interaction time, and $\frac{4}{g}$ is the contribution of each instanton field in (2). It is convenient to rewrite eq.(5) as a weak-coupling expansion in the following form:

$$- \log\left(\frac{S_{n \rightarrow n}}{i\omega T}\right) = \frac{4}{3g^2} + (2n+1) \log\left(\frac{g}{4}\right) + \log(\sqrt{2\pi n!}) + \mathcal{O}(g). \quad (6)$$

Note that this expansion is valid for $n < \frac{1}{g}$ so that the energy of the n th excited state is near its harmonic-oscillator value and much below the potential barrier $\omega/4g^2$.

In order to check the validity of the instanton approximation we will now present an alternative calculation of the scattering amplitude $S_{n \rightarrow n}$, based on the standard WKB approach [17]. Our starting point is the fact that tunneling mixes the n th-excited states in the two wells, $|n >_+$ and $|n >_-$, so that the true energy eigenstates

are the (normalized)* even and odd combinations: $|e, n > = \frac{1}{\sqrt{2}}(|n >_+ + |n >_-)$ and $|o, n > = \frac{1}{\sqrt{2}}(|n >_+ - |n >_-)$, with energies $E_n \pm \frac{\delta E_n}{2}$. Here $E_n = (n + \frac{1}{2})\omega + \epsilon_n < < \frac{\omega}{4g^2}$, where $\frac{\delta E_n}{\omega} = \mathcal{O}(g)$ are perturbatively small corrections due to unharmonic terms, while the energy splitting δE_n is non-perturbatively small. The amplitude for $|n >_-$ to evolve to $|n >_+$ in time T is

$$+ < n | e^{iHT} | n >_- = ie^{iE_n T} \sin\left(\frac{\delta E_n T}{2}\right), \quad (7)$$

so that the system oscillates with period $4\pi/\delta E_n$. To make contact with the previously computed S-matrix element we remove the free-field evolution and keep the leading order in the coupling to find

$$S_{n \rightarrow n} \simeq iT \frac{\delta E_n}{2}. \quad (8)$$

Strictly speaking this is a good approximation for sufficiently small times compared to both ϵ_n^{-1} and δE_n^{-1} , although the former requirement can be relaxed if we are only interested in the modulus but not in the phase of the amplitude. We will discuss corrections to this result in a minute.

The standard WKB approximation gives [17]

$$\delta E_n = \frac{\omega}{\pi} \exp\left(-\int_{-a_n}^{a_n} \sqrt{2[V(\phi) - E_n]} d\phi\right) \equiv \frac{\omega}{\pi} e^{-I_n}, \quad (9)$$

where $\phi = \pm a_n$ are the classical turning points defined by the equation $E_n = V(\pm a_n)$ (see fig.1). Performing explicitly the integral one finds:

$$I_n = \frac{4\sqrt{2}}{3g^2(1+k^2)^{\frac{3}{2}}} [(k^2+1)E(k) + (k^2-1)K(k)], \quad (10)$$

where

$$k^2 = \frac{a_n^2}{2v^2 - a_n^2} = \frac{1 - 2g\sqrt{\frac{E_n}{\omega}}}{1 + 2g\sqrt{\frac{E_n}{\omega}}}, \quad (11)$$

and E, K are the complete elliptic integrals. In the weak-coupling limit the arguments of the elliptic integrals approach 1. Using the appropriate asymptotic expansions of

* Here we assume that $+ < n | n >_- = 0$. It is easy to check that an exponentially suppressed overlap will not affect the leading-order, linear-in-time result eq.(8).

these functions one finds after some algebra

$$I_n = \frac{4}{3g^3} + (2n+1)\log\left(\frac{g}{4}\right) + \left(n + \frac{1}{2}\right)\log\left(\frac{1}{2}\right) - \left(n + \frac{1}{2}\right) + \mathcal{O}(g). \quad (12)$$

It is worth pointing out that the unharmonic corrections to the energy cancel out at this order of the approximation.

Putting together eqs. (8), (9) and (12) we can finally compare the WKB result for $S_{n \rightarrow n}$ with the one-instanton approximation, eq. (6). In the limit of large n , where the former can be trusted, one finds using Stirling's formula: $n! = \sqrt{2\pi}(n+1)^{n+\frac{1}{2}}e^{-n-1}$, perfect agreement between these results up to, and including, their constant prefactor. This is evidence that the instanton calculation does give an answer which is asymptotic to the true result.

Perturbative corrections to eq. (5) can be calculated using the propagator in the instanton background, and ϕ^4 -vertices with one leg possibly replaced by the instanton field. Due, however, to the oscillatory behaviour of the amplitude one encounters infrared divergences cut-off by the total time of the interaction T . Such divergences also occur in standard perturbation theory. Consider for example the transition amplitude for the second-excited state to go to itself, in the case of a single-well quartic unharmonic oscillator with $V(\phi) = \frac{1}{2}\phi^2 + \frac{1}{4}\lambda\phi^4$. Ignoring possible transitions during the adiabatic switching on and off of the coupling, this amplitude reads

$$<2|S|2> \simeq e^{i\epsilon_2 T} = 1 + i\epsilon_2 T - (\epsilon_2 T)^2/2 + \dots, \quad (13)$$

with ϵ_2 the perturbatively small corrections to the harmonic-oscillator energy. The three terms in the expansion (13) correspond to leading order to the amputated on-shell diagrams of fig. 2. For instance diagram (2b) is proportional to $i\lambda\omega^{-2}\delta(0) = i\lambda\omega^{-2}T$. Diagram (2c) on the other hand is proportional to

$$\lambda^2 \omega^{-2} \delta(0) \int_{-\infty}^{+\infty} \frac{d\tilde{E}}{(\tilde{E}^2 + \omega^2)([\tilde{E} - \tilde{E}]^2 + \omega^2)} = \lambda^2 2\pi \frac{1}{\omega^3 \tilde{E}^2 + 4\omega^2 T}, \quad (14)$$

with $\tilde{E}$ the (Euclidean) incoming energy, and diverges when all external legs are put on shell, so that $\tilde{E} = 2i\omega$. This divergence is due to the extra factor of T in the quadratic term of the expansion (13), as can be checked by calculating the diagram explicitly in the time, rather than energy axis. One expects similar infrared divergences from the expansion of the exponential factor in eq. (7), while higher-order terms in the

expansion of the sine come from configurations of one instanton, followed by a number of anti-instanton-instanton pairs, whose zero-mode integrations yield extra factors of T^2 .

In general, for finite interaction time T , all perturbative corrections to eq. (5) are finite. The result will, however, depend on the precise way of switching on and off the coupling $g(t)$, so that extracting time-independent quantities such as the energy-splitting δE_n can be trickier than the leading-order result, eq. (8). This difficulty reflects the fact that a conventional S-matrix cannot be defined for this problem.

Returning now to the transition amplitude $S_{n \rightarrow n}$ given by eq. (5), or eqs. (8) and (9), we note that it grows with energy as the two turning points $\pm a_n$ move closer to each other, and the effective barrier suppressing tunneling is gradually reduced. It is amusing that this growth can also be perceived as due in part to the growth of available phase space, in direct analogy with Ringwald's calculation [5]. To see how, let us add to the theory a new decoupled coordinate $\tilde{\phi}$ with harmonic potential $V(\tilde{\phi}) = \frac{1}{2}\omega^2 \tilde{\phi}^2$, whose frequency is chosen equal to the one in the direction ϕ near the bottom of the two wells. Denote by $\tilde{n}$ the corresponding stationary excitation level. Clearly, starting from the state $|n, \tilde{n} >_-$, the probability to tunnel to $|n, \tilde{n} >_+$ is still given, to leading order, by eq. (5), while the amplitude to tunnel to any other state in the right well is negligible in the large-time limit. Suppose now that we change coordinates near the right well to: $\phi^{(1)} = \frac{\tilde{\phi} + \phi}{\sqrt{2}}$ and $\phi^{(2)} = \frac{\tilde{\phi} - \phi}{\sqrt{2}}$, so that the asymptotic states in the far future are labelled by the corresponding harmonic-excitation levels $n^{(1)}$ and $n^{(2)}$. The amplitude for tunneling from $|n, \tilde{n} = 0 >_-$ to any such state $|n^{(1)}, n^{(2)} >_+$ with $n^{(1)} + n^{(2)} = n$ is

$$+ < n^{(1)}, n^{(2)} | S | n, \tilde{n} = 0 >_- = \sqrt{\frac{n!}{2^n n^{(1)}! n^{(2)}!}} S_{n \rightarrow n}, \quad (15)$$

where $S_{n \rightarrow n}$ is given by eq. (5), while the extra factor comes from the normalization of the outgoing state and the modified residue of the instanton fields, and is precisely equal to the overlap $+ < n, \tilde{n} = 0 | n^{(1)}, n^{(2)} >_+$. For large n the amplitudes (15) are much smaller than $S_{n \rightarrow n}$, but one recovers the total tunneling rate $|S_{n \rightarrow n}|^2$ after summing over the large number of allowed final states: $n^{(1)} + n^{(2)} = n$. Thus the growth of the amplitude with energy, which in the original coordinates was solely due to the reduction of the energy barrier, appears in the $\phi^{(1)}, \phi^{(2)}$ -basis to be due in part to the growth of the available phase-space. This of course is a technical illusion, since the outcome of the tunneling process is a normalized coherent state, whose "decay" into various channels simply occurs with probability one.

Up to now we considered tunneling starting from the n th-excited state in the left well, and we found a rate which, if reasonably extrapolated, should become unsuppressed when the energy reaches the top of the barrier, and the two classical turning points $\pm a_n$ collapse to zero. Let us now ask a somewhat different question: what is the probability that a weak but rapidly oscillating force,

$$\delta \mathcal{L} = -\zeta \int_{-\frac{T}{2}}^{\frac{T}{2}} \phi e^{iEt} dt + h.c. , \quad (16)$$

triggers tunneling from the *ground state* in one well to some excited state in the other. This question is closer in spirit to the field-theory problem, because the large energy is initially stored in only a few quanta, and must be converted to large occupation numbers in the process of scattering. To linearized order in the weak external force, the quantity of interest is

$$S_{0 \rightarrow n}(\zeta) = -2i\zeta \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \cos(Et) + \langle n | U(\frac{T}{2}, t) \phi(t) U(t, -\frac{T}{2}) | 0 \rangle , \quad (17)$$

with U the evolution operator in the interaction representation, and $E = E_n - E_0$ the energy-splitting between the initial and final states. In the large-time limit the calculation of (17) corresponds to diagrams with one incoming off-shell particle at energy E , and n outgoing on-shell amputated legs, as shown in fig.3. For sufficiently low energy, the transition of eq. (17) is mediated by an instanton, and the amplitude to leading order reads

$$S_{0 \rightarrow n}(\zeta) = \frac{1}{\sqrt{n!}} \prod_{j=1}^n (i m_j + \infty) \frac{1}{\sqrt{2\omega}} e^{i\omega t_j} \frac{d}{dt_j} \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \, 2\zeta \cos(\omega t) G_E(it_1, \dots, it_n; it) , \quad (18)$$

with G_E the Euclidean Green's function of eq.(2). Now the Fourier transform of the instanton field:

$$\delta_- \phi_0(\mathcal{E}, \tau_0) = \frac{2\pi i}{\omega g} \sqrt{\frac{2}{\omega}} \frac{e^{i\mathcal{E}\tau_0}}{\sinh(\frac{\pi \mathcal{E}}{\omega})} \quad (19)$$

formally defined in the upper-half complex plane, has poles at all integer multiples of the harmonic frequency, $\mathcal{E} = n\omega$, so that expression (18) diverges in the infinite-time

limit. To regulate this divergence we must, as in the case of higher-loop corrections, keep the total interaction time T finite. Combining eqs. (2), (18) and (19) we thus obtain

$$\begin{aligned} S_{0 \rightarrow n}(\zeta) &= e^{-\frac{1}{32}} \left(\frac{2\omega}{g} \sqrt{\frac{2}{\pi}} \right) \frac{\zeta}{\sqrt{n!}} \left(\frac{4}{g} \right)^n \int_{-\frac{T}{2}}^{\frac{T}{2}} d\mathcal{E} \delta_- \phi_0(\mathcal{E}, 0) \int_{-\frac{T}{2}}^{\frac{T}{2}} dt e^{(\mathcal{E} - i n \omega)t} \int_{-\frac{T}{2}}^{\frac{T}{2}} d\tau_0 e^{(i\mathcal{E} + n\omega)\tau_0} \\ &= \frac{16}{g^2} \sqrt{\pi \omega} \zeta T^2 e^{-\frac{1}{32}} \frac{1}{\sqrt{n!}} \left(\frac{4}{g} \right)^n , \end{aligned} \quad (20)$$

which is valid as usual for small g and $n < \frac{1}{g^2}$.

The crucial point about the amplitude $S_{0 \rightarrow n}(\zeta)$ is that, even though it grows exponentially with energy, it does so at only half the rate of $S_{n \rightarrow n}$, demonstrating the importance of the initial state. Using eq.(5) we may indeed rewrite (20) in the suggestive form

$$S_{0 \rightarrow n}(\zeta) = \frac{4\pi}{g} \sqrt{\frac{2}{\omega}} \zeta T \sqrt{S_{0 \rightarrow 0} S_{n \rightarrow n}} . \quad (21)$$

Thus, even though the naive extrapolation of expression (20) outside its range of validity would indicate that $S_{0 \rightarrow n}(\zeta)$ may eventually become strong, the more plausible extrapolation of eq. (21) would imply that at energies comparable to the barrier height, where $S_{n \rightarrow n} \sim \mathcal{O}(1)$, and for times $\sim \omega^{-1}$, $S_{0 \rightarrow n}(\zeta)$ is still suppressed by half the instanton action. A similar conclusion has been reached recently in different contexts in refs.[14] and [15].

Another way of making this result-physically transparent is to rewrite the amplitude $S_{0 \rightarrow n}(\zeta)$ as follows:

$$S_{0 \rightarrow n}(\zeta) = \frac{4\pi}{g} \sqrt{\frac{2}{\omega}} \zeta T \sqrt{n!} \left(\frac{g}{4} \right)^n S_{n \rightarrow n} , \quad (22)$$

which shows that it is of the same order as the amplitude of tunneling from $|n\rangle \rightarrow |n\rangle$ to $|n\rangle$, times the amplitude of exciting $|n\rangle \rightarrow |n\rangle$ from the vacuum $|0\rangle \rightarrow |n\rangle$ inside the *same well*. Indeed this latter receives contributions from perturbative diagrams which, at tree level and for large n , are of order $\sim g^n \sqrt{n!} \omega T$, where the factorial comes from the normalization of the final state and the number of permutations of the outgoing legs [18]. This single-well transition amplitude vanishes in the absence of unharmonic terms except when $n = 1$, and decreases rapidly with increasing energy at least until

$n \sim \frac{1}{g}$, where the tree-level approximation breaks down. Furthermore it has been recently shown [19] by semiclassical methods that it continues to decrease with energy, even for $n > \frac{1}{g}$, so that the transition to highly-excited levels within a single well is always very much suppressed. It would be interesting to extend the analysis to the double-well potential. Nevertheless, it would be truly paradoxical if exciting a highly-energetic state in a single well is very much suppressed, but becomes unsuppressed for a state in a nearby well.

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Figure Captions

Fig.1: The double-well potential with minima at $\pm v$, separated by an energy barrier $\delta V = \frac{\omega^2}{4g}$. The classical turning points $\pm a_n$ corresponding to the energy E_n are the boundaries of the integration region in the WKB formula eq.(9).

Fig.2: Leading-order contributions to the three terms of eq. (13).

Fig.3: Diagrams contributing to the transition from the ground- to the nth-excited state, in the presence of an external oscillating force. The blob includes an instanton, when the initial and final states lie in different wells.

(a)

(b)

(c)

Fig. 2

Fig. 1

Fig. 3