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ON THE EFFECTIVE GAUGE COUPLING CONSTANT IN STRING THEORIES

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Abstract

We extend to all orders previous one-loop modular invariance arguments for determining the form of the string threshold corrections to the gauge coupling constant. In this way we obtain vanishing higher loop corrections as the most plausible possibility. We also discuss the phenomenological implications of the all order expressions for gauge couplings at the GUT scale and re-examine the form of the corresponding all order modular invariant gaugino condensation with and without matter.

CERN-TH.6187/91

IMT-FT-45/91

July 1991

In the last year the understanding of the four dimensional gauge couplings from strings has improved dramatically [1-7]. Radiative threshold corrections have been calculated [1], exhibiting a quite attractive connection with modular invariance arguments [2,3]. Furthermore, the unexpected non-holomorphicity of the corrections has led to the re-discovery of previous and very deep work on the structure of supersymmetric field theories [8]. All this progress has proved to be extremely relevant for the comprehension of the supersymmetry breaking mechanism by gaugino condensation [2,3,9,10], a crucial ingredient for the connection of superstring theories with the low energy physics. In this letter we extend previous one-loop modular invariance arguments [2] to all orders, obtaining quite satisfactory results, consistent with very recent string calculations [5]. According to these, higher loop threshold corrections to the gauge coupling should vanish. We also discuss the form of the (all order) effective gauge couplings at the GUT scale and the consequent phenomenological implications. Finally we revise the form of the modular invariant gaugino condensation at all orders with and without matter.

One loop corrections of the effective gauge coupling constant have the general expression

$$\frac{1}{g_a^2(\mu)} = \frac{1}{g_o^2(M)} + \beta^a \log\left(\frac{\mu}{M}\right)^2 + \Delta_{1-loop}^a \quad (1)$$

where a labels the different factors of the gauge group $G = \prod_a G_a$, M is the renormalization point, $\beta^a = \frac{1}{16\pi^2}(3T(G_a) - \sum_R T(R_a))$ are the one-loop coefficients of the corresponding beta functions¹ and Δ_{1-loop}^a are the one-loop threshold corrections which may come from both massive and massless modes. The Δ_{1-loop}^a piece has recently been calculated [1,7] for N=1 supersymmetric orbifold vacua. Restricting ourselves, for simplicity, to the untwisted matter contributions (incidentally, this is the only kind of matter in smooth compactifications, e.g. Calabi-Yau manifolds), and working with $M = M_S$, i.e the string mass, the dependence on the overall modulus T is

$$\Delta_{1-loop}^a = \beta^a \log[(T + \bar{T})|\eta(T)|^4] \quad (2)$$

This one-loop result was essentially reproduced in [2] by using the following renormalization group (RG) argument. Setting the renormalization point M at the scale below which the effective four dimensional theory is valid, i.e. $M_{GUT} = [(S + \bar{S})(T + \bar{T})]^{-1/2} M_P$, which seems to be the sensible choice, and substituting $g_a^{-2}(M_{GUT}) = g_o^{-2}(M_{GUT}) + \delta_{1-loop}^a$ with $g_o^{-2}(M_{GUT}) = S_R \equiv \text{Re}S$, i.e. its tree level value [11], it is clear from (1) that in order to be g modular invariant at any modular invariant scale μ (as it is physically desirable), δ_{1-loop}^a

¹ $T(G)$ is the Casimir operator in the adjoint representation of the gauge group G , R labels the chiral supermultiplets transforming like representations of G and $T(R)$ is defined by $\text{Tr}(T^i T^j) = T(R)\delta^{ij}$.

should be different from zero. More precisely, choosing $\mu = M_S$, the gauge coupling $g^{-2}(\mu)$ receives a non-modular-invariant contribution $\beta^a \log(M_S/M_{GUT})^2 = \beta^a \log(T + \bar{T})$, which should be compensated by δ_{1-loop}^a . If we demand holomorphicity of the gauge coupling the value of δ_{1-loop}^a is basically fixed

$$\delta_{1-loop}^a = \beta^a \log |\eta(T)|^4 \quad (3)$$

Absorbing the term $\beta^a \log(T + \bar{T})$ in the value of δ^a we obtain exactly expression (2).

One of the purposes of this paper is to extend the previous RG argument of Font et al. [2] to all order corrections. Fortunately enough the *exact* (all order) RG equations for $D = 4$ $N = 1$ non-Abelian supersymmetric Yang-Mills theories (SSYM) are completely known from the work of [8]. They simply read

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M)} + \frac{3T(G)}{16\pi^2} \log \left(\frac{\mu}{M} \right)^2 + \frac{2T(G)}{16\pi^2} \log \left(\frac{g^2(M)}{g^2(\mu)} \right) \quad (4)$$

Naively, one might proceed as before by substituting $M = M_{GUT}$, $g^{-2}(M) = S_R + \frac{3T(G)}{16\pi^2} \log |\eta(T)|^4 + \delta_{higher\ loops}$ and imposing modular invariance of $g^{-2}(\mu = M_S)$. This procedure leads to non-trivial $\delta_{higher\ loops}$ but, in fact, it is not correct. As we will see now the correct ansatz at all orders is

$$\frac{1}{g^2(M_{GUT})} = S_R + \frac{3T(G)}{16\pi^2} \log |\eta(T)|^4 + \frac{2T(G)}{16\pi^2} \log g^{-2}(M_{GUT}) + \delta_{higher\ loops} \quad (5)$$

This comes from the following. As it has recently been stressed in [12] we have to distinguish between the usual effective action S_Γ , i.e. the generator of the 1PI irreducible Feynman diagrams, and the Wilson action $S_W(\mu)$ in which the infrared divergences are excluded by means of the infrared cutoff μ [8]. The quantities appearing in S_W play the role of holomorphic variables in the supersymmetric theory (e.g. $g_W^{-2} = \text{Ref}$) and do not get radiative corrections beyond the one loop level². In particular the Wilson gauge coupling, $g_W(\mu)$, satisfies

$$\frac{1}{g_W^2(\mu)} = \frac{1}{g_W^2(M)} + \beta \log \left(\frac{\mu}{M} \right)^2 \quad (6)$$

By definition g_W does not get infrared threshold corrections from massless modes. g_W and g (the effective gauge coupling) are related, in general, in the following way

$$\frac{1}{g^2(\mu)} = \frac{1}{g_W^2(\mu)} + \sum_i c_i \log Z_i(\mu) \quad (7)$$

²It is worth stressing the fact that a similar result was obtained in ref.[13] using completely different arguments, in particular the Peccei-Quinn symmetry associated with the dilaton field.

where $Z_i(\mu)$ is the renormalization factor for the i th-field and c_i is an appropriate coefficient. The Z factors contain non-holomorphic infrared contributions at all orders. In a SSYM theory eq.(7) becomes

$$\frac{1}{g^2(\mu)} = \frac{1}{g_W^2(\mu)} + \frac{2T(G)}{16\pi^2} \log g^{-2}(\mu) + \Delta^{massless} \quad (8)$$

where we have added possible threshold corrections coming from massless modes (thus, not contributing to g_W^{-2}) through graphs like those considered in ref.[7]³. Note that $\log g^{-2}$ induces modifications from the 2-loop level. Now, at the 1-loop level the ansatz was

$$g^{-2}(M_{GUT})\big|_{1-loop} = S_R + \delta_{1-loop} \quad (9)$$

Taking into account (8) this means

$$S_R + \delta_{1-loop} = \frac{1}{g_W^2(M_{GUT})} + \Delta_{1-loop}^{massless} \quad (10)$$

Therefore, the ansatz at all orders should be (see eq.(8))

$$g^{-2}(M_{GUT})\big|_{all\ order} = S_R + \delta_{1-loop} + \frac{2T(G)}{16\pi^2} \log g^{-2}(M_{GUT}) + \delta_{higher\ loops} \quad (11)$$

where $\delta_{1-loop} = \frac{3T(G)}{16\pi^2} \log |\eta|^4$ (see eq.(3)) and $\delta_{higher\ loops}$ coincides with $\Delta_{n>1}^{massless}$. This gives eq.(5). Now we can substitute (11) in (4) with $\mu = M_S$ and look for possible $\delta_{higher\ loops}$ contributions cancelling eventual deviations from modular invariance. However it is clear that with the ansatz (11) they are not needed. The simplest choice is, of course,

$$\delta_{higher\ loops} = 0 \quad (12)$$

Another possibility is $\delta_{higher\ loops} = f(T)$ with $f(T)$ a modular invariant function. This is much less plausible since it seems reasonable that a modular invariant contribution comes from the whole spectrum. In any case (12) shows that the simple RG argument produces sensible results at all orders, compatible with a very recent calculation of Antoniadis et al. [5].

It is also interesting to write explicitly the relation between $g_W^{-2} = \text{Re} f_W$ and g^{-2} at different scales. For this it is necessary to take into account that $\Delta^{massless}$ receives two different contributions [1,7]

$$\Delta^{massless} = \frac{3T(G)}{16\pi^2} [\log(T + \bar{T}) + \log(S + \bar{S})] \quad (13)$$

³More precisely, the supergraphs of ref.[7] have three external legs: two with vector superfields and one with a modulus field (T) acquiring a vacuum expectation value.

Then, from eq.(8)

$$g^{-2}(\mu) = g_W^{-2}(\mu) + \frac{3T(G)}{16\pi^2} \log[(S + \bar{S})(T + \bar{T})] + \frac{2T(G)}{16\pi^2} \log g^{-2}(\mu) \quad (14)$$

Now, using the RG equation (6) for $g_W^{-2}(\mu)$, which is exhausted at one loop, we obtain from (11-14)

$$g_W^{-2}(M_P) = S_R + \frac{3T(G)}{16\pi^2} \log |\eta(T)|^4 \quad (15)$$

while

$$g^{-2}(M_{GUT}) = S_R + \frac{3T(G)}{16\pi^2} \log |\eta(T)|^4 + \frac{2T(G)}{16\pi^2} \log g^{-2}(M_{GUT}) \quad (16)$$

Clearly the whole massless threshold contribution can be understood as a redefinition of M_P (the natural scale for the Wilson action) to $M_{GUT} = [(S + \bar{S})(T + \bar{T})]^{-1/2} M_P$ for the effective action. In the absence of massive string corrections ($\sim T(G) \log |\eta(T)|^4$) and higher loop corrections ($\sim \log g^{-2}$) the effective field theory has all the Wilson couplings unified at M_P with a value S_R , whereas all the effective (physical) couplings unify at M_{GUT} with the same value S_R .

It is, however, worthwhile to stress the fact that, when all the corrections are included, the effective coupling constants, $g_a(\mu)$, do *not* unify at M_{GUT} or at any other mass. As it is clear from (16) this is due to two different effects. First, the "stringy" correction $\frac{3T(G_a)}{16\pi^2} \log |\eta(T)|^4$ which is different for each group factor and whose magnitude depends on the vacuum expectation values of the moduli. Second, the purely supersymmetric effect $\frac{2T(G_a)}{16\pi^2} \log g^{-2}(M_{GUT})$, whose magnitude is perfectly calculable e.g. for the Standard Model. Besides these, a third contribution $\sim \sum T(R) \log Z_R(M_{GUT})$ should be included when light matter is present [8]. It is not unconceivable that these signals might be found out from experimental data. Notice also that all these deviations should be taken into account when one investigates supersymmetry breaking mechanisms able to lead to a phenomenologically viable value of S_R (see e.g. references [14,15,10])

The result (12) for higher loop contributions was obtained for a SSYM effective theory (which is not the usual case). It is straightforward to extend the argument when untwisted light matter is present. In this case the RG equation is modified from (4) to

$$\begin{aligned} \frac{1}{g^2(\mu)} &= \frac{1}{g^2(M)} + \frac{1}{16\pi^2} (3T(G) - \sum_R T(R)) \log \left(\frac{\mu}{M} \right)^2 + \frac{2T(G)}{16\pi^2} \log \left(\frac{g^2(M)}{g^2(\mu)} \right) \\ &+ \frac{-2}{16\pi^2} \sum_R T(R) \log \left(\frac{Z_R(\mu)}{Z_R(M)} \right) \end{aligned} \quad (17)$$

with

$$g^{-2}(M_{GUT}) = S_R + \frac{1}{16\pi^2} (3T(G) - \sum_R T(R)) \log |\eta(T)|^4 + \frac{2T(G)}{16\pi^2} \log g^{-2}(M_{GUT})$$

$$+ \frac{-2}{16\pi^2} \sum_R T(R) \log Z_R(M_{GUT}) + \delta_{higher\ loops} \quad (18)$$

Again modular invariance of $g^{-2}(\mu = M_S)$ demands the result (12) as the simplest and most plausible choice at all orders. Note that $\log Z_R(\mu)$ admits a perturbative expansion

$$\gamma_R(g(\mu)) = \frac{-d \log Z_R(\mu)}{d \log \mu} = -C_2(R) \frac{g^2(\mu)}{4\pi^2} + \dots \quad (19)$$

where $C_2(R)$ is the quadratic Casimir operator and $g^2(\mu = M_S)$ is modular invariant by hypothesis.

Finally, let us write the all order expressions of the gaugino condensate and the corresponding non-perturbative superpotential. First, we consider the case of a pure SSYM low energy theory [2,3]. As usual, the physical condensate can be estimated by the scale at which the effective gauge coupling becomes strong. This is proportional to the RG invariant scale Λ_c , see eq.(4)

$$\langle \lambda\lambda \rangle_\Gamma = \Lambda_c^3 = M^3 g^{-2}(M) \exp \left\{ \frac{-8\pi^2}{T(G)} g^{-2}(M) \right\} \quad (20)$$

where $g^{-2}(M)$ satisfies the boundary condition (16). On the other hand in the effective Lagrangian approach [3] the gaugino composite bilinear field $(\lambda\lambda)$ corresponds to the scalar component of a chiral composite superfield U , such that $U = (\delta_{ab} W_\alpha^a \epsilon^{\alpha\beta} W_\beta^b)$. Then the superpotential which describes gaugino condensation is

$$W_{eff} = \frac{1}{4} Y^3 \left(f_W + \frac{T(G)}{8\pi^2} \log \frac{Y^3}{M_P^3} \right) \quad (21)$$

where $Y^3 = U/S_o^3$, S_o is the chiral compensator (in the superconformal gauge $|S_o|^2 = e^{K/3}$), and

$$f_W = S + \frac{3T(G)}{16\pi^2} \log \eta(T)^4 \quad (22)$$

is an exact expression at all order. Therefore, everything works here as at the one-loop order. As it has been demonstrated in refs.[15,9,10] it is equivalent to work with (21) than with

$$W \propto M_P^3 \exp \left\{ -(8\pi^2/T(G)) f_W \right\} = M_P^3 \exp \left\{ -(8\pi^2/T(G)) S \right\} \eta(T)^{-6} \quad (23)$$

this comes from substituting $\partial W_{eff}/\partial Y = 0$, i.e.

$$Y^3 = \frac{1}{e} M_P^3 \exp \left\{ -(8\pi^2/T(G)) f_W \right\} \quad (24)$$

This can be understood as a minimization condition [10]. It is easy to write now the relationship between the condensates at all order

$$(S + \bar{S}) < \lambda \lambda >_{\Gamma} = |Y^3| e^{K/2} \quad (25)$$

which coincides with the one-loop expression given in ref.[7]

Let us consider now the case when massive matter is present [9,10]⁴. Notice that the expression within parentheses in eq.(21) is simply the RG equation for g_W between M_P and the condensation scale. For the case at hand we have to substitute that expression by the corresponding RG equation.

For the sake of simplicity, we work here with a single non-Abelian gauge group G , whose matter representations acquire all the same mass M_I through a Higgs mechanism. For example, if ξ_1, ξ_2 are two matter representations, they acquire mass through the term $A\xi_1\xi_2$ in the superpotential, when the G -singlet field A gets dynamically a non-vanishing expectation value. (Again, we are assuming here that A, ξ_1, ξ_2 are untwisted fields; the modification for the twisted case is straightforward.) We will call $\tilde{\beta}$ and β the one-loop coefficients of the beta functions above and below M_I respectively. Let us focus on the case of "quark" masses higher than the condensation scale (Λ), but smaller than the Planck scale [10], i.e. $\Lambda < M_I < M_P$. Then, the decoupling theorem ensures that at any scale $\mu < M_I$ the theory should behave as a pure SSYM theory, thus satisfying eq.(8), i.e.

$$\frac{1}{g^2(\mu)} = \frac{1}{g_W^2(\mu)} + \frac{2}{3}\beta \log g^{-2}(\mu) + \Delta^{massless} \quad (26)$$

where $\beta = \frac{3T(G)}{16\pi^2}$ and $\Delta^{massless}$ is given by (13). On the other hand g satisfies the following RG equations, valid at all order (see eqs.(17,4))

$$\begin{aligned} \frac{1}{g^2(M_I)} &= \frac{1}{g^2(M_P)} + \tilde{\beta} \log \left(\frac{M_I}{M_P} \right)^2 + \frac{2}{3}\beta \log \left(\frac{g^2(M_P)}{g^2(M_I)} \right) \\ &+ 2(\tilde{\beta} - \beta) \log \left(\frac{Z_R(M_I)}{Z_R(M_P)} \right) \end{aligned} \quad (27)$$

$$\frac{1}{g^2(\mu)} = \frac{1}{g^2(M_I)} + \beta \log \left(\frac{\mu}{M_I} \right)^2 + \frac{2}{3}\beta \log \left(\frac{g^2(M_I)}{g^2(\mu)} \right) \quad (28)$$

where we have used $\tilde{\beta} - \beta = \frac{-1}{16\pi^2} \sum_R T(R)$. Hence

$$\begin{aligned} \frac{1}{g^2(\mu)} &= \frac{1}{g^2(M_P)} - \frac{-2}{16\pi^2} \sum_R T(R) \log Z_R(M_P) - \frac{2}{3}\beta \log g^{-2}(M_P) \\ &+ \frac{2}{3}\beta \log g^{-2}(\mu) + \beta \log \left(\frac{\mu}{M_P} \right)^2 + \frac{-2}{16\pi^2} \sum_R T(R) \log \left(\frac{M_I}{M_P} \right) \end{aligned} \quad (29)$$

⁴The case of massless matter is not yet well understood. See e.g. ref.[16].

where M_I^O is the complex bare mass of the matter fields (in our case $M_I^O/M_P = A$), satisfying

$$\frac{M_I^O}{M_I} = Z_R(M_I) \quad (30)$$

The first three terms of (29) are simply $g_W^{-2}(M_P) + \Delta^{massless}$. Therefore, using eq.(26), we obtain the RG equation for g_W

$$\frac{1}{g_W^2(\mu)} = \frac{1}{g_W^2(M_P)} + \beta \log \left(\frac{\mu}{M_P} \right)^2 + \frac{-2}{16\pi^2} \sum_R T(R) \log \left(\frac{M_I^O}{M_P} \right) \quad (31)$$

Thus, the effect of the massive matter shows up as a threshold effect, which is holomorphic and meaningful in its real and imaginary parts. This effect is consistent with the one given in ref.[12] for the analogue case in SQED, where it was stressed that it is exact at all orders. Consequently, we have to substitute (21) by

$$W_{eff} = \frac{1}{4} Y^3 \left(f_W + \frac{2}{3} \beta \log \frac{Y^3}{M_P^3} + 2(\tilde{\beta} - \beta) \log A \right) \quad (32)$$

and eq.(22) by

$$f_W = S + \tilde{\beta} \log \eta(T)^4 \quad (33)$$

These equations give the complete description of dynamical gaugino condensation when massive matter is present. Again, using the minimization condition $\partial W_{eff}/\partial Y = 0$, which in this case leads to

$$Y^3 = \frac{1}{e} A^{3(1-\tilde{\beta}/\beta)} \frac{e^{-\frac{3S}{2\tilde{\beta}}}}{\eta^{6\tilde{\beta}/\beta}} \quad (34)$$

eq.(32) is equivalent to

$$W \propto A^{3(1-\tilde{\beta}/\beta)} \frac{e^{-\frac{3S}{2\tilde{\beta}}}}{\eta^{6\tilde{\beta}/\beta}} \quad (35)$$

The physical gaugino condensate $\langle \lambda\lambda \rangle_\Gamma$ is again proportional to the RG invariant scale, which in this case is to be derived from eqs.(17) (for the range $M_I < M < M_P$) and (4) (for the range $\Lambda < M_I$).

$$\langle \lambda\lambda \rangle_\Gamma = \Lambda_c^3 = M^3 \left(\frac{M_I}{M} \right)^{3(1-\tilde{\beta}/\beta)} g^{-2}(M) e^{-\frac{3}{2\beta g^2(M)}} \prod_R \left(\frac{Z_R(M_I)}{Z_R(M)} \right)^{T(R)/T(G)} \quad (36)$$

where, for example, at $M = M_{GUT}$, $g^{-2}(M)$ is given by eq.(18) (with $\delta^{higher\ loops} = 0$). It is now straightforward to see that $\langle \lambda\lambda \rangle_\Gamma$ still satisfies the relation of eq.(25), with Y^3 given by eq.(34).

ACKNOWLEDGEMENTS

We thank L.E. Ibañez and D. Lüst for useful comments.

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