CERN-TH-6185/91

Higgs boson effects in $e^+e^- \rightarrow t\bar{t}$ close to the threshold

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Abstract

The influence of the standard Higgs boson on the cross-section for $e^+e^- \rightarrow t\bar{t}$ is discussed at the threshold and above. The large enhancement of the radiative corrections at the threshold from light Higgs bosons decreases rapidly for $s > 4m_t^2$.

CERN-TH-6185/91

July 1991

The process of top pair production will be of particular relevance for future high energy e^+e^- colliders. Recent experimental results from the CDF collaboration [1] have constrained the mass of the top quark in the Standard Model to the range $m_t > 89$ GeV. Due to the weak decay $t \rightarrow W^+b$ the width of the top quark exceeds Λ_{QCD} for $m_t > 120$ GeV. Hence, the top will predominantly decay before hadronization if it is heavy. This allows a clean analysis of the $t\bar{t}$ production process [2] due to the characteristic features of the weak top quark decay.

Sufficiently far above the threshold, the production cross-section for $e^+e^- \rightarrow t\bar{t}$ as predicted by the Standard Model is obtained by adding the QCD corrections [3] to the electroweak cross-section involving the electroweak loop corrections and the real photon bremsstrahlung contributions. Near the threshold of a quark-antiquark system, as known from charm and bottom quarks, the structure of the cross-section is controlled by non-relativistic quark dynamics with a spectrum of bound states and resonances. For a very heavy top quark, however, its weak decay width causes the individual bound states to disappear and to merge into a broad and smooth threshold enhancement as a remnant. Earlier studies of the threshold region for a “light” top quark [4, 5] have also been extended recently to the more realistic higher top masses [6–11]. Special attention was given in particular to the strong interaction effects. In addition, the top-Higgs Yukawa coupling becomes sizeable for large m_t . The effect of the neutral Higgs exchange has been estimated in [8–11] by adding a top-Higgs Yukawa potential to the static QCD potential. These studies have shown a strong sensitivity not only to α_s but also to the Higgs mass, in particular if $m_t > 150$ GeV.

A complementary approach was performed in [12, 13] where the electroweak 1-loop diagrams were evaluated at $s = 4m_t^2$ in order to obtain the 1-loop corrected electroweak cross-section at the top threshold. As discussed in [13], the non-relativistic potential approach can reproduce qualitatively the result of the relativistic treatment after correcting the bound-state wave function for short distance effects which become relevant for Higgs masses $M_H > 48$ GeV, as required by the experimental search at LEP [14]. The most striking Higgs effect for $m_t \gg M_H$ is a cubic dependence of the correction to the Born cross-section $\sim m_t^3/M_W^2 M_H$ on the top mass, yielding threshold enhancements of more than 20% for $m_t > 200$ GeV and $M_H < 100$ GeV.

In the present paper, we study in more detail the structure of the short distance effects from the Higgs boson in the cross-section for open top production at the threshold and above, taking into account the full s -dependence of the electroweak 1-loop corrections. The expressions for the complete electroweak 1-loop corrections to $e^+e^- \rightarrow t\bar{t}$ at $s > 4m_t^2$ have been given in [15] where the general case of heavy fermion production was considered. Directly at the top threshold we recover the results of [13]. Above the threshold, we find a strong s -dependence in the Higgs part of the corrections which diminish the large threshold enhancement considerably just a few GeV above the kinematical threshold.

In order to outline the discussion of the Higgs effects we write the integrated cross-section for $e^+e^- \rightarrow t\bar{t}$ in the following way:¹

$$\sigma = \frac{4\pi\alpha(s)^2}{3s} N_C \beta \left\{ \frac{1}{2}(3 - \beta^2) \sigma_V(s) \rho_V^{QCD} + \beta^2 \sigma_A(s) \rho_A^{QCD} + \Delta\sigma \right\} \quad (1)$$

where

$$s = (p_t + p_{\bar{t}})^2, \quad \beta = \sqrt{1 - \frac{4m_t^2}{s}}, \quad \alpha(s) = \frac{\alpha}{1 + \tilde{\Pi}_{ferm}^{\gamma}(s)} \quad (2)$$

with the renormalized (subtracted) fermionic photon vacuum polarization $\tilde{\Pi}_{ferm}^{\gamma}$. The QCD correction factor, ρ_V^{QCD} , cancels the factor β for $s \rightarrow 4m_t^2$ in front of σ_V [4], thus keeping the cross-section at the threshold finite. We do not consider the QCD effects here but refer to the literature [4–8].

The electroweak lowest-order contributions are contained in the quantities $\sigma_{V,A}$ together with those loop corrections which can be incorporated into effective couplings for the photon and the Z boson maintaining a Born-like structure. The remaining part of the corrections, consisting of the WW , ZZ box diagrams and the part of the vertex corrections not of V,A structure, is collected in $\Delta\sigma$. At $s = 4m_t^2$ the Higgs boson enters (1) only via σ_V . Using a compact notation we get the following expressions:

$$\begin{aligned} \sigma_V &= (Q_e^{V^2} + Q_e^{A^2}) Q_t^{V^2} \\ &\quad + 2(Q_e^V V_e + Q_e^A A_e) Q_t^V V_t \frac{s}{s - M_Z^2} \\ &\quad + (V_e^2 + A_e^2) V_t^2 \left(\frac{s}{s - M_Z^2} \right)^2, \\ \sigma_A &= (Q_e^{V^2} + Q_e^{A^2}) Q_t^{A^2} \\ &\quad + 2(Q_e^V V_e + Q_e^A A_e) Q_t^A A_t \frac{s}{s - M_Z^2} \\ &\quad + (V_e^2 + A_e^2) A_t^2 \left(\frac{s}{s - M_Z^2} \right)^2. \end{aligned} \quad (3)$$

The effective coupling constants in these formulae

$$\begin{aligned} Q_f^V &= Q_f \left[1 - \frac{1}{2} \tilde{\Pi}_{bos}^{\gamma}(s) \right] - F_V^{\gamma}(s) \\ Q_f^A &= -F_A^{\gamma}(s) \\ V_f &= \bar{v}_f + F_V^{Zf}(s) \\ A_f &= \bar{a}_f + F_A^{Zf}(s) \end{aligned} \quad (4)$$

¹We drop the QED corrections since they are not relevant to our discussion

with

$$\begin{aligned}\bar{a}_f &= \left(\frac{\sqrt{2}G_\mu M_Z^2}{4\pi\alpha(s)} \right)^{1/2} \bar{p}^{1/2} I_f^f \\ \bar{v}_f &= \left(\frac{\sqrt{2}G_\mu M_Z^2}{4\pi\alpha(s)} \right)^{1/2} \bar{p}^{1/2} (I_3^f - 2Q_f s^2)\end{aligned}\quad (5)$$

contain the propagator corrections

$$\begin{aligned}\bar{p} &= 1 - \Delta r - \hat{\Pi}^Z(s), \\ \bar{s}^2 &= s_W^2 - s_W c_W \hat{\Pi}^{\gamma Z}(s)\end{aligned}\quad (6)$$

from the self-energies renormalized according to the on-shell scheme

$$\begin{aligned}\hat{\Pi}^Z &= \text{Re} \frac{\hat{\Sigma}^Z(s)}{s - M_Z^2}, \\ \hat{\Pi}^{\gamma Z} &= \text{Re} \frac{\hat{\Sigma}^{\gamma Z}(s)}{s},\end{aligned}$$

together with the bosonic photon vacuum polarization $\hat{\Pi}_{\text{bos}}^\gamma$, and the V, A form factors from the vertex corrections (real parts). For the explicit expressions we refer to [15-18]. Δr is the correction to the relation between $s_W^2 = 1 - M_W^2/M_Z^2$ and M_Z, G_μ [19].

A few comments are in order:

1. The incorporation of weak loop effects into effective coupling constants allows a compact and transparent formulation. For a 1-loop discussion, however, the expressions (3) have to be understood as expanded to first order in the loop contributions and to be combined with the remaining terms of $\Delta\sigma$ in order to keep the electroweak cross-section gauge-invariant. The only exception where higher-order terms appear is the running electromagnetic coupling constant $\alpha(s)$ in the photon exchange amplitude, in accordance with the leading log summation from the light fermions. The corrections to the Z exchange amplitude are free of these large corrections. 2-loop terms in the Z coupling constants (5) from a heavy top in the propagator corrections [17,18] are neglected since they are not of practical importance in view of the experimental accuracy. Moreover, for a complete discussion an explicit calculation of the leading 2-loop vertex corrections would also be required.
2. The Higgs boson enters the cross-section via propagator corrections to the Z couplings (5) and vertex corrections to the final state. Initial-state vertex corrections involving Higgs bosons can be neglected because of the small Yukawa coupling to the electron. For the same reason box diagrams with Higgs exchange are also negligible.

3. The Higgs dependence of the universal propagator corrections (6) is essentially that of the effective coupling constants for the light fermions at the Z resonance. The Higgs contributions to $\bar{p}$ and $\bar{s}^2$ in (6) are small, of the typical size of $O(1\%)$, and smooth with respect to s and M_H . Large corrections with rapid variation with s and strong sensitivity to M_H are obtained from the vertex corrections to the $\gamma(Z)\ell\ell$ vertices with virtual Higgs bosons (Figure 1).

For $s \rightarrow 4m_t^2$ only σ_V and part of $\Delta\sigma$ remain in (1). The Higgs dependence is only associated with σ_V since the magnetic form factor contributions contained in $\Delta\sigma$ are suppressed by a relative factor β^2 . At $s = 4m_t^2$, therefore, only the vector form factors enter the integrated cross-section. From the diagrams of Figure 1 we obtain the following UV-finite and gauge-invariant expressions for the Higgs contributions to the vector current of photon and Z :

$$\begin{aligned}F_V^{\gamma\ell}(H) &= -Q_\ell \cdot \frac{\alpha}{4\pi} \frac{m_t^2}{4s_W^2 M_W^2} f_1(h) \\ F_V^{Z\ell}(H) &= v_\ell \cdot \frac{\alpha}{4\pi} \frac{m_t^2}{4s_W^2 M_W^2} (f_1(h) + f_2(h, z))\end{aligned}\quad (7)$$

where

$$h = \frac{M_H}{m_t}, \quad z = \frac{M_Z}{m_t}$$

and with v_ℓ from (5) for $\bar{p} = 1$, $\bar{s}^2 = s_W^2$. The functions f_1 and f_2 are given by

$$\begin{aligned}f_1 &= 4 - \frac{4h^2}{3} + \left(4 - 3h^2 + \frac{2h^4}{3} \right) \log(h^2) \\ &\quad + \left(2 - \frac{5h^2}{3} + \frac{2h^4}{3} \right) \beta_h \log(-x_h),\end{aligned}\quad (8)$$

$$\begin{aligned}f_2 &= \frac{1}{4 - z^2 - h^2} \cdot \\ &\quad \left\{ z^6 \left[\log(z^2) + \beta_z \log(-x_z) \right] - z^2 h^2 (h^2 - 4) \left[\log(h^2) + \beta_h \log(-x_h) \right] \right. \\ &\quad \left. + z^2 (4 + z^2 - h^2) \cdot \right. \\ &\quad \left. \left[-\frac{4 + z^2 - h^2}{4} \log(z^2) - \frac{4 + h^2 - z^2}{4} \log(h^2) + \frac{4 - (h - z)^2}{2} \beta_z \log(x_z) \right] \right\}\end{aligned}\quad (9)$$

with

$$\begin{aligned}\beta_h &= \sqrt{1 - \frac{4}{h^2}}, \quad x_h = \frac{\beta_h - 1}{\beta_h + 1} \\ \beta_s &= \sqrt{1 - \frac{4hz}{4 - (h - z)^2}}, \quad x_s = \frac{\beta_s - 1}{\beta_s + 1} + i\epsilon\end{aligned}$$

and β_z, x_z obtained from β_h, x_h by the substitution $h \rightarrow z$. These results are in agreement with the corresponding results of Ref. [13].

Due to the dominance of the photon exchange amplitude, the main features of the final-state Higgs corrections to (1) are attributed to F_V^t . Moreover, the term resulting from f_2 in the Z vertex is much smaller than from f_1 .

Large corrections arise for small M_H from the diagrams leading to f_1 . These are the only ones which have a non-relativistic limit for $M_H \ll m_t$. In this limit the leading terms are given by

$$F_V^t(H) \simeq -Q_t \cdot \frac{\alpha}{4\pi} \cdot \frac{m_t^2}{s_W^2 M_W^2} \left(\pi \frac{m_t}{M_H} + 2 \log \frac{M_H}{m_t} \right) \quad (10)$$

and the corresponding expression for F_V^{Zt} is obtained by the substitution $Q_t \rightarrow -v_t$. The other limit $M_H \gg m_t$ yields:

$$F_V^t(H) \simeq -Q_t \cdot \frac{\alpha}{4\pi} \cdot \frac{m_t^2}{s_W^2 M_W^2} \cdot \frac{5m_t^2}{6M_H^2} \left(2 \log \frac{M_H}{m_t} + \frac{31}{30} \right). \quad (11)$$

The first term of (10) $\sim m_t^2$ is also obtained by the non-relativistic potential approach [13]. The quality of this approximation is illustrated in Figure 2 where the ratio of the exact expression (7) and the m_t^2 term of (10)

$$r = \frac{h}{4\pi} \cdot f_1(h) \quad (12)$$

is displayed. For $M_H = m_t/2$ the result deviates already by -40% from the non-relativistic approximation.

We now consider the Higgs contributions for $s > 4m_t^2$. For this purpose, we calculate the electroweak cross-section in (1) as a function of $\sqrt{s}$ based on the complete set of electroweak 1-loop corrections, as discussed after Eq. (1) for various Higgs masses. For illustration we plot in Figure 3 the relative difference, Δ , of the cross-section for a given Higgs mass with respect to a reference cross-section which is calculated for a fixed Higgs mass of $M_H = 1000$ GeV:

$$\Delta = \frac{\sigma(s, M_H) - \sigma(s, 1000 \text{ GeV})}{\sigma(s, 1000 \text{ GeV})}. \quad (13)$$

The large mass for the reference point was chosen because there the vertex corrections from Higgs exchange are smallest and have very little variation with $\sqrt{s}$. The points at $s = 4m_t^2$ correspond to the threshold form factors in (7). The strong variation with $\sqrt{s}$ is almost completely due to the s -dependence of the vertex form factors in (4) with virtual Higgs bosons. This demonstrates that the extrapolation of the Higgs form factors, evaluated at the threshold $s = 4m_t^2$, to the continuum region becomes questionable for light Higgs masses and advocates the use of the exact s -dependent expressions. Note that Δ also contains the small variation with M_H resulting from the propagator corrections in (6).

In order to point out the behaviour far away from the threshold, we show in Figure 4 the relative Higgs dependence, Δ , for a wide range of $\sqrt{s}$. After a fast decrease within a relatively narrow range, the s -dependence of the electroweak corrections becomes very smooth for light Higgs masses as well, and the variation with M_H reaches the typical size of a few per cent at large s .

After finishing this paper we received a preprint by Fadin and Yakovlev [20] where the electromagnetic $\gamma\gamma t\bar{t}$ vector form factor from Higgs boson exchange was calculated at $s = 4m_t^2$. The result agrees with our Eqs. (7) and (8), and with the corresponding result obtained by Guth and Kühn [13].

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Figure captions:

Figure 1:

Vertex corrections with virtual Higgs bosons to the γtt and Ztt vertex

Figure 2:

The vector form factor at $s = 4m_t^2$ from Higgs diagrams normalized to the m_t^3 term, Eq. (12)

Figures 3 a - d:

s dependence Δ , Eq. (13), of the Higgs effect in the cross-section for $e^+e^- \rightarrow t\bar{t}$ for various Higgs and top masses

Figure 4:

s dependence Δ , Eq. (13), of the Higgs effect in the cross-section for $e^+e^- \rightarrow t\bar{t}$ for $m_t = 200$ GeV and various Higgs masses close to and far from the threshold

Figure 1

Figure 2

Figure 3a

Figure 3b

Figure 3c

Figure 3d

Figure 4