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The supersymmetric charged Higgs boson mass and LEP phenomenology

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Abstract

We calculate the leading radiative corrections, originating from top and bottom quark and squark loops, to the mass of the charged Higgs boson $H^\pm$ in the Minimal Supersymmetric Standard Model, which is given at the tree-level by $m_{H^\pm}^2 = m_W^2 + m_A^2$, where A is the physical CP-odd Higgs boson. We find that these corrections are numerically small, at most a few GeV, for generic values of the parameters, although they can be large in some special regions of parameter space. Our study indicates that generically, if the $H^\pm$ are to be accessible to LEP II ($m_{H^\pm} \lesssim 90$ GeV), then $m_A \lesssim 40$ GeV. In this case it is likely that $m_A < m_h$, where h is the lighter CP-even Higgs boson, and quite possible that $m_A < m_h/2$. After including radiative corrections to the hAA vertex, we find that the $h \rightarrow AA$ decay could dominate over $h \rightarrow b\bar{b}$ or $\tau^+\tau^-$, thereby changing LEP Higgs search strategies.

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It has been recently pointed out [1–3] that the masses of the Higgs bosons in the Minimal Supersymmetric extension of the Standard Model (MSSM) are subject to large radiative corrections associated with the top quark and its $SU(2)$ and supersymmetric partners¹. Several papers [8–13] have subsequently investigated various aspects of these corrections and their implications for experimental searches.

These radiative corrections are particularly important for the two CP-even neutral Higgs bosons (h, H), for which the leading corrections to the squared-mass terms, in the limit $m_W^2 \ll m_t^2$ and m_t/m_A fixed, are proportional to $(\alpha/\pi)(m_t^4/m_W^2)$. They have dramatic implications for the lighter CP-even neutral Higgs boson h , whose mass can be increased by tens of GeV, so that it can well weigh more than m_Z . This also means that the LEP searches published so far² [14] cannot be used to infer a lower bound on the ratio $\tan \beta = v_2/v_1$ of Higgs vacuum expectation values (VEVs) in the model. The radiative corrections to the mass of the CP-odd neutral Higgs boson only have a quadratic dependence on the top quark mass, and are linked to the mixing between superpartners of the left-handed and right-handed t and b quark components. Moreover, they can be reabsorbed in the definition of the physical A mass as an input parameter, together with the renormalized value of the ratio $\tan \beta = v_2/v_1$. Explicit formulae for all these corrections are given in ref.[10], which extends the previous results of refs.[1,2,3,9]. Among the other things, one can see that it is quite possible that $m_h > m_A$, in contrast to the tree-level mass inequality $m_h < m_A$.

Once the CP-even and CP-odd Higgs boson mass matrices are calculated to the one-loop level, the cross-sections for the LEP production processes $e^+e^- \rightarrow hZ$ and $e^+e^- \rightarrow hA$ can be recalculated [9,10] using the tree-level ZZh and ZhA vertices modified by the one-loop radiagonalization of the (h, H) mass matrix. This approximation gives the correct leading-order cross-section throughout the allowed phase space, and enables LEP's prospects of discovering neutral supersymmetric Higgs bosons to be reassessed.

In this paper we extend the previous calculations to include the radiative corrections to the mass of the charged supersymmetric Higgs boson. Calculating the $(t, b, \tilde{t}, \tilde{b})$ loop contributions to $m_{H^\pm}$ in an effective potential approach, we find that the corrections to the tree-level sum rule $m_{H^\pm}^2 = m_W^2 + m_A^2$ are numerically small, at most a few GeV, for generic values of the parameters, although they can be large in some special regions of parameter space. This implies that in general, if the $H^\pm$ are to be accessible to LEP II, which would require $m_{H^\pm} \lesssim 90$ GeV, then $m_A \lesssim 40$ GeV.

In the light of this observation, we conclude the paper with a few comments about the neutral Higgs phenomenology in the case of a light A . For $m_A \lesssim 40$ GeV, the calculations of refs.[9,10] indicate that the decays $Z \rightarrow hZ^*$ and $Z \rightarrow hA$ may well be observable at LEP I, and that probably $m_h > m_A$. The conventional h decay pattern may be significantly modified. In particular, if $m_h >$

¹Previous studies [4–7] either neglected the case of a heavy top quark [4–6], or concentrated on the violations of the neutral Higgs mass sum rule [7] without computing corrections to individual Higgs masses.

²After the completion of the work described in this paper, the first experimental analyses which include the effects of radiative corrections have appeared [15].

$2m_A$, $h \rightarrow AA$ decays can compete with the usually-expected $h \rightarrow b\bar{b}$ or $\tau^+\tau^-$ decays. We calculate the leading radiative corrections to the hAA vertex and find that indeed the $h \rightarrow AA$ decay mode can easily dominate when kinematically accessible. Therefore the h decay signatures may well be different from those normally considered in experimental searches [14], so that the hZ^* final state could be $f\bar{f} + [2(b\bar{b}) \text{ or } b\bar{b}\tau^+\tau^- \text{ or } 2(\tau^+\tau^-)]$, and the hA final state could be $n(b\bar{b}) + (3-n)(\tau^+\tau^-)$ ($n = 0, 1, 2, 3$).

The effective potential method for computing the one-loop corrected Higgs masses has been extensively discussed in refs.[2,10], whose notation we shall follow here. The entries of the charged Higgs mass matrix are well approximated by the second derivatives of the effective potential with respect to the fields H_i^+ and H_j^- ($i, j = 1, 2$), evaluated at the one-loop minimum

$$< H_1 > = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad < H_2 > = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad (1)$$

where v_1 and v_2 can be assumed real and positive. To account for loop diagrams with top, bottom, stop and sbottom exchange, we need to compute the top-bottom and stop-sbottom mass matrices, taking into account the full dependence on the neutral and charged Higgs fields.

In the case of the top and bottom quarks, we can write down the corresponding field-dependent eigenvalues directly

$$\bar{m}_a^2 = \frac{1}{2} [k_a^2(|H_2^0|^2 + |H_2^\pm|^2) + k_b^2(|H_1^0|^2 + |H_1^\pm|^2) + \delta_a] \quad (a = 1, 2, 3, 4), \quad (2)$$

where

$$\begin{aligned} \delta_1 &= \sqrt{k_t^2(|H_2^0|^2 - |H_2^\pm|^2) + k_b^2(|H_1^-|^2 - |H_1^0|^2)}^2 + 4k_t^2H_2^0H_2^- + k_b^2H_1^0H_1^-]^2 \\ \delta_2 &= \sqrt{k_t^2(|H_2^0|^2 + |H_2^\pm|^2) - k_b^2(|H_1^-|^2 + |H_1^0|^2)}^2 + 4k_t^2k_b^2H_2^0H_1^- + H_1^0H_2^-]^2 \end{aligned} \quad (3)$$

At the minimum (1), we obtain the familiar expressions of the top and bottom masses: $\bar{m}_1^2 = \bar{m}_2^2 = k_t^2v_2^2 \equiv m_t^2$, $\bar{m}_3^2 = \bar{m}_4^2 = k_b^2v_1^2 \equiv m_b^2$.

In the case of the stop and sbottom squarks, we have to deal with a single hermitian 4×4 field-dependent mass matrix $\bar{M}^2$, whose independent entries are

$$\begin{aligned} \tilde{t}_L^* \tilde{t}_L : & \quad k_t^2|H_2^0|^2 + k_b^2|H_1^-|^2 + m_Q^2 \\ & \quad + (-\frac{g^2}{4} + \frac{g'^2}{12})(|H_2^0|^2 - |H_1^0|^2) + (\frac{g^2}{4} + \frac{g'^2}{12})(|H_2^\pm|^2 - |H_1^\pm|^2), \\ \tilde{t}_R^* \tilde{t}_R : & \quad k_t^2(|H_2^0|^2 + |H_2^\pm|^2) + m_U^2 - \frac{g^2}{3}(|H_2^0|^2 - |H_2^\pm|^2 - |H_1^\pm|^2), \\ \tilde{b}_L^* \tilde{b}_L : & \quad k_t^2|H_2^\pm|^2 + k_b^2|H_1^0|^2 + m_Q^2 \\ & \quad + (\frac{g^2}{4} + \frac{g'^2}{12})(|H_2^0|^2 - |H_1^0|^2) + (-\frac{g^2}{4} + \frac{g'^2}{12})(|H_2^\pm|^2 - |H_1^\pm|^2), \\ \tilde{b}_R^* \tilde{b}_R : & \quad k_b^2(|H_1^0|^2 + |H_1^\pm|^2) + m_D^2 + \frac{g^2}{6}(|H_2^0|^2 - |H_2^\pm|^2 + |H_2^\pm|^2 - |H_1^\pm|^2), \end{aligned}$$

$$\begin{aligned}
\tilde{t}_L^+ \tilde{b}_R^- &: h_t(\mu H_1^0 + A_t H_1^{0*}), \\
\tilde{t}_L^+ \tilde{b}_L^- &: -h_t^2 H_2^{0*} H_2^+ - h_b^2 H_1^+ H_1^{0*} + \frac{g^2}{2}(H_1^0 H_1^+ + H_2^+ H_2^{0*}), \\
\tilde{t}_L^+ \tilde{b}_R^- &: h_b(\mu H_2^+ - A_b H_1^+), \\
\tilde{t}_R^+ \tilde{b}_L^- &: h_t(\mu H_1^+ - A_t H_2^+), \\
\tilde{t}_R^+ \tilde{b}_R^- &: -h_t h_b (H_2^+ H_1^{0*} + H_2^0 H_1^+), \\
\tilde{b}_L^+ \tilde{b}_R^- &: h_b(\mu H_2^0 + A_b H_1^{0*}),
\end{aligned} \tag{4}$$

We denote by $\tilde{m}_a$ ($a = 1, \dots, 4$) the four eigenvalues of the matrix (4). At the minimum (1), diagonalization is simple and the eigenvalues are the stop and sbottom masses ($\tilde{m}_1 \equiv m_{\tilde{t}_1}$, $\tilde{m}_2 \equiv m_{\tilde{t}_2}$, $\tilde{m}_3 \equiv m_{\tilde{b}_1}$, $\tilde{m}_4 \equiv m_{\tilde{b}_2}$), whose explicit expressions are

$$\begin{aligned}
m_{\tilde{t}_{1,2}}^2 &= m_t^2 + \frac{1}{2}(m_Q^2 + m_U^2) + \frac{1}{4}m_Z^2 \cos 2\beta \\
&\pm \sqrt{\left[\frac{1}{2}(m_Q^2 - m_U^2) + \frac{1}{12}(8m_W^2 - 5m_Z^2) \cos 2\beta\right]^2 + m_t^2(A_t + \mu \cot \beta)^2}, \\
m_{\tilde{b}_{1,2}}^2 &= m_b^2 + \frac{1}{2}(m_Q^2 + m_D^2) - \frac{1}{4}m_Z^2 \cos 2\beta \\
&\pm \sqrt{\left[\frac{1}{2}(m_Q^2 - m_D^2) - \frac{1}{12}(4m_W^2 - m_Z^2) \cos 2\beta\right]^2 + m_b^2(A_b + \mu \tan \beta)^2}.
\end{aligned} \tag{5}$$

Including only the contributions from top, bottom, stop and sbottom loops, the one-loop effective potential reads

$$V_1(Q) = V_0(Q) + \frac{3}{32\pi^2} \left[\sum_{a=1}^4 \tilde{m}_a^4 \left(\log \frac{\tilde{m}_a^2}{Q^2} - \frac{3}{2} \right) - \sum_{a=1}^4 \tilde{m}_a^4 \left(\log \frac{\tilde{m}_a^2}{Q^2} - \frac{3}{2} \right) \right], \tag{7}$$

where $V_0(Q)$ is the tree-level potential evaluated with parameters renormalized at some scale Q , and field-independent terms have been neglected. The one-loop corrected mass matrix in the charged Higgs sector is then given by

$$\begin{aligned}
\mathcal{M}_{\pm}^2 &\equiv \left[\frac{\partial^2 V_1}{\partial H_1^+ \partial H_1^-} \right]_{v_1, v_2} = \left(-m_3^2 + \frac{g^2 v_1 v_2}{2} \right) \frac{v_1 v_2}{v_1 v_2} \\
&+ \frac{3}{16\pi^2} \sum_{a=1}^4 \left[\left(\frac{\partial^2 \tilde{m}_a^2}{\partial H_1^+ \partial H_1^-} - \frac{\delta_{ij}}{v_i} \frac{\partial \tilde{m}_a^2}{\partial H_1^0} \right) \tilde{m}_a^2 \left(\log \frac{\tilde{m}_a^2}{Q^2} - 1 \right) - (\tilde{m}_a \rightarrow \bar{m}_a) \right]_{v_1, v_2}, \tag{8}
\end{aligned}$$

where we have exploited the one-loop minimization conditions and the fact that

$$\left[\frac{\partial \tilde{m}_a^2}{\partial H_1^+} \right]_{v_1, v_2} = \left[\frac{\partial \bar{m}_a^2}{\partial H_1^+} \right]_{v_1, v_2} = 0. \tag{9}$$

The only non-trivial part of the calculation is the evaluation of the second derivatives of the field-dependent squark masses with respect to H_1^+ and H_1^- . They can

be obtained by differentiating twice the eigenvalue equation for the 4×4 squark mass matrix of eq.(4):

$$\det(\tilde{M}^2 - \tilde{m}_a^2) = \tilde{m}_a^6 + A\tilde{m}_a^4 + B\tilde{m}_a^2 + C\tilde{m}_a^2 + D = 0, \tag{10}$$

where A, B, C, D are calculable functions of $H_1^\pm$. Using eqs.(10) and (9) we get, for example

$$\left[\frac{\partial^2 \tilde{m}_1^2}{\partial H_1^+ \partial H_1^-} \right]_{v_1, v_2} = - \left[\frac{A_{ij} \tilde{m}_1^6 + B_{ij} \tilde{m}_1^4 + C_{ij} \tilde{m}_1^2 + D_{ij}}{(\tilde{m}_1^2 - \tilde{m}_2^2)(\tilde{m}_1^2 - \tilde{m}_3^2)(\tilde{m}_1^2 - \tilde{m}_4^2)} \right]_{v_1, v_2}, \tag{11}$$

and analogous formulae for the other squark mass eigenvalues. The subscripts ij on A, B, C, D mean differentiation with respect to H_1^+ and H_1^- . After lengthy calculations, we obtain

$$\mathcal{M}_{\pm}^2 = \begin{bmatrix} \sin^2 \beta & \sin \beta \cos \beta \\ \sin \beta \cos \beta & \cos^2 \beta \end{bmatrix} m_{H^\pm}^2, \tag{12}$$

where

$$m_{H^\pm}^2 = m_W^2 + m_A^2 + \Delta^2, \tag{13}$$

$$\begin{aligned}
\Delta^2 &= \sum_{a=1}^4 \frac{3H_a \tilde{m}_a^2}{8\pi^2 \sin 2\beta (m_t^2 - m_b^2)} \left(m_t^2 \log \frac{\tilde{m}_a^2}{m_t^2} - m_b^2 \log \frac{\tilde{m}_a^2}{m_b^2} \right) \\
&- \frac{3g^2}{64\pi^2} \left(\frac{m_t^2}{\sin^2 \beta} + \frac{m_b^2}{\cos^2 \beta} - m_W^2 \right) \left[\frac{f(m_t^2) - f(m_b^2)}{m_t^2 - m_b^2} \right],
\end{aligned} \tag{14}$$

$$f(m^2) = 2m^2 \left(\log \frac{m^2}{Q^2} - 1 \right), \tag{15}$$

$$m_W^2 = \frac{g^2}{2}(v_1^2 + v_2^2), \tag{16}$$

and

$$H_{1,2} = \left[\frac{\partial^2 \tilde{m}_{1,2}^2}{\partial H_1^+ \partial H_1^-} \pm \frac{g^2}{2 \sin^2 \beta} \frac{m_t^2}{m_W^2} \frac{A_t \mu}{\tilde{m}_1^2 - \tilde{m}_2^2} \right]_{v_1, v_2}, \tag{17}$$

$$H_{3,4} = \left[\frac{\partial^2 \tilde{m}_{3,4}^2}{\partial H_1^+ \partial H_1^-} \pm \frac{g^2}{2 \cos^2 \beta} \frac{m_b^2}{m_W^2} \frac{A_b \mu}{\tilde{m}_3^2 - \tilde{m}_4^2} \right]_{v_1, v_2}. \tag{18}$$

The mass m_A is the one-loop corrected mass of the CP-odd Higgs boson A , as given in ref.[10]. Notice that the result of eq. (13) contains an explicit dependence on the renormalization scale Q through the function $f(m^2)$. However, this dependence is actually reabsorbed [up to terms $O(g^4)$], by the implicit Q -dependence of $\tilde{m}_W^2$ in eq. (16); besides, the implicit Q -dependence of m_A^2 is negligible by itself, so that the global Q -dependence of $m_{H^\pm}^2$ is numerically very small. A convenient choice of Q is $Q \sim m_W$, so that m_W in eq.(16) can be identified with the physical $W^\pm$ mass. Notice also that the matrix in eq. (12) has the same structure as the corresponding tree-level mass matrix. One of the two eigenvalues is zero, as it

should be for the Goldstone mode, and the corresponding eigenvector has formally the same expression, as a function of $\tan \beta$, as in the tree-level analysis. The other eigenvalue, $m_{H^\pm}^2$, is precisely the one-loop corrected charged Higgs mass squared. In eqs.(17), we have used the factorization property of eq.(12) to write down the coefficients H_a of eq. (14) in terms of the derivatives $(\partial^2 \tilde{m}_a^2 / \partial H_1^\pm \partial H_2^\mp)$ only. We then need the explicit expressions only for the coefficients $A_{12}, B_{12}, C_{12}, D_{12}$, which are given in the Appendix. This completes our calculation of the one-loop corrected charged Higgs mass, whose result is summarized by eqs.(14-26). As a final remark, we remind the reader that our results have been obtained in the effective potential approximation and considering only top and bottom quark and squark virtual effects. The complete diagrammatic computation of the $(t, b, \tilde{t}, \tilde{b})$ loop effects and an extensive comparison with the effective potential approach will be given in ref.[16]: we expect further corrections of at most a few GeV. Other loop contributions have been estimated in ref.[6], and found to be at most a few GeV.

To obtain more manageable formulae and an easier physical interpretation, we now consider some special limits.

- i) $A_b = A_t = \mu = 0$
In this case, we find

$$\Delta^2 = \frac{3g^2}{64\pi^2 \sin^2 \beta \cos^2 \beta m_W^2} \times \left\{ \frac{(m_b^2 - m_W^2 \cos^2 \beta)(m_t^2 - m_W^2 \sin^2 \beta)}{m_{\tilde{t}_1}^2 - m_{\tilde{b}_1}^2} [f(m_{\tilde{t}_1}^2) - f(m_{\tilde{b}_1}^2)] \right. \\ \left. + \frac{m_t^2 m_b^2}{m_{\tilde{t}_2}^2 - m_{\tilde{b}_2}^2} [f(m_{\tilde{t}_2}^2) - f(m_{\tilde{b}_2}^2)] - \frac{2m_t^2 m_b^2}{m_{\tilde{t}_1}^2 - m_{\tilde{b}_1}^2} [f(m_{\tilde{t}_1}^2) - f(m_{\tilde{b}_1}^2)] \right\}. \quad (19)$$

As an example, for $m_t = 180$ GeV, $\tan \beta = 20$ and $m_{\tilde{t}} = 1$ TeV, the one-loop correction amounts to $\Delta^2 = 1089$ GeV². For $m_A = 40$ GeV, this would imply $m_{H^\pm} = 95.3$ GeV (with a likely uncertainty of at most a few GeV) instead of the tree-level result $m_{H^\pm} = 89.4$ GeV.

- ii) $h_b = 0, m_Q^2 = m_{\tilde{t}}^2 = m_{\tilde{b}}^2 \equiv m_{\tilde{t}}^2$
Neglecting further the D-term contributions to the stop and sbottom mass matrices, our result takes the simple form

$$\Delta^2 = -\frac{3g^2 \mu^2 m_t^4}{32\pi^2 \sin^4 \beta m_W^2 (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)} \\ \times \left[\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} - \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2} \log \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right]. \quad (20)$$

This is a good approximation for large mixing in the stop mass matrix and for $g, h_b \ll h_t$, i.e. $\tan \beta \ll m_t/m_b$. Taking for example $m_{\tilde{t}} = 150$ GeV, $m_t =$

150 GeV, $\mu = 500$ GeV, $\tan \beta = 2$ and $A_t = 0$, we find $\Delta^2 = -2066$ GeV², which would correspond, for $m_A = 40$ GeV, to a one-loop corrected value $m_{H^\pm} = 77$ GeV, instead of the tree-level prediction $m_{H^\pm} = 89.4$ GeV. Note that in this case $m_{H^\pm} < m_W$.

- iii) $h_b = h_t = 0, \mu = 0$ but $h_b A_b \neq 0, h_t A_t \neq 0$

As in the previous case, we neglect the D-term contributions to the stop and sbottom mass matrices. This case is somewhat artificial, but it corresponds to the one previously considered in ref.[6], so we consider it here for the sake of comparison. Specializing our general formulae and putting for simplicity $m_Q^2 = m_{\tilde{t}}^2 = m_{\tilde{b}}^2 \equiv m_{\tilde{t}}^2$, we obtain

$$\Delta^2 = \frac{3g^2}{128\pi^2 \sin^2 \beta \cos^2 \beta m_W^2} \frac{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)(m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)}{(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2 - (m_{\tilde{b}_1}^2 - m_{\tilde{b}_2}^2)^2} \\ \times \left[m_{\tilde{t}_1}^2 \left(m_{\tilde{b}_1}^2 \log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} - m_{\tilde{t}_2}^2 \log \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right) - m_{\tilde{t}_2}^2 \left(m_{\tilde{b}_1}^2 \log \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} - m_{\tilde{b}_2}^2 \log \frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2} \right) \right]. \quad (21)$$

We have checked that, after some manipulations, this formula coincides with eq.(A3) of ref.[6]. The latter was obtained by an explicit Feynman diagram computation, but in the limit in which the squark masses are much larger than m_W and the Higgs boson masses themselves. This corresponds to taking the self-energy diagrams at zero external momentum, which is precisely our effective potential approximation. In ref.[6] two additional contributions to Δ^2 were computed, eqs.(A5) and (A6). However, considering these corrections is not consistent with the approximation of neglecting the D-terms in the squark mass matrices. More specifically, the terms of the Lagrangian contributing to eq.(A5) originate from precisely the same vertices that lead to the squark D-term masses. Also, the terms in eq.(A6) are of the same order, and should therefore be neglected in a consistent approximation.

These results indicate that $m_{H^\pm}^2 < m_W^2 + m_A^2$ is possible, albeit by an amount that is highly model-dependent. We find that it is even possible that $m_{H^\pm} < m_W$, although this can occur only for small values of m_A and $(m_W - m_{H^\pm})$ is typically not larger than a few GeV. Fig.1 shows a portion of the (m_A, m_h) plane for four different choices of parameters, some of which were also used as examples in ref.[10]. Contours of $\tan \beta = 1, \sqrt{2}, 2, 4, m_t/m_b$ are shown, together with the regions of the (m_A, m_h) plane where $m_{H^\pm} < m_Z$ and $m_{H^\pm} < m_W$ ³.

We now reconsider LEP Higgs phenomenology in the light of these results. In view of the β^3 threshold factor in $\sigma(e^+e^- \rightarrow H^+H^-)$, and the large background from $e^+e^- \rightarrow W^+W^-$, it is likely to be difficult for LEP to find the $H^\pm$ unless $m_{H^\pm} \lesssim m_W$. Our calculations offer at least some small hope that this could occur

³When two different values of $\tan \beta > 1$ correspond to the same point in the (m_A, m_h) plane, we choose the one which gives the smaller value of $m_{H^\pm}$.

even in the MSSM, whereas at the tree-level $m_{H^\pm} > m_W$ in general. It is certainly unlikely that LEP II would find the $H^\pm$ unless $m_{H^\pm} \lesssim m_Z$. In this case, the one-loop relation (14) between $m_{H^\pm}$, m_W and m_A indicates that $m_A \lesssim 40$ GeV.

The results of refs.[9,10] indicate that if m_A is indeed this low, (i) the decays $Z \rightarrow hZ^*$ and $Z \rightarrow hA$ are likely to be accessible to LEP I, and (ii) it is quite likely that $m_h > m_A$, in contrast to what was expected on the basis of the tree-level mass formulae. Point (i) focuses interest on this corner of parameter space, and point (ii) suggests that, in order to scoop LEP II, searches at LEP I may need to take into account novel Higgs decay signatures. If $m_A > m_h/2$, then $h \rightarrow b\bar{b}$ and $\tau^+\tau^-$ decays dominate, as expected previously. However, if $m_h/2 > m_A$, the decay $h \rightarrow AA$ is kinematically accessible, and its rate is given by

$$\Gamma(h \rightarrow AA) = \frac{|\lambda_{hAA}|^2}{32\pi m_h} \left(1 - 4 \frac{m_A^2}{m_h^2}\right)^{1/2}, \quad (22)$$

where, at the tree-level

$$\lambda_{hAA} = \lambda_{hAA}^0 \equiv -\frac{igm_Z}{2 \cos \theta_W} \cos 2\beta \sin(\beta + \alpha), \quad (23)$$

and α is the CP-even Higgs mixing angle which also enters into the ZZh and ZhA vertices [proportional to $\sin(\beta - \alpha)$ and $\cos(\beta - \alpha)$, respectively].

Before discussing the phenomenology of $h \rightarrow AA$ decays, a few comments are in order. In the approximation of neglecting radiative corrections to the irreducible ZZh , ZhA and $Zf\bar{f}$ vertices, these can all be characterized using the tree-level expressions in terms of the renormalized $\tan \beta = v_2/v_1$, consistent with the minimization of the one-loop effective potential, and another mixing angle α , determined by diagonalizing the radiatively corrected (h, H) mass matrix. The formal expressions for the mixing angles between the neutral Goldstone and the A , and between the charged Goldstone and the $H^\pm$, are not altered by radiative corrections, so they are both still given in terms of β only. The angle α depends in a complicated but calculable way on the parameters $(m_t, m_{\tilde{t}}, A_t, A_b, \mu)$ that enter into the one-loop calculations. In principle, one should also compute radiative corrections to the proper vertices $VV\varphi$, $V\varphi\varphi$, $Vf\bar{f}$ and $\varphi\varphi\varphi$, where $V = W^\pm, Z^0$ and $\varphi = h, H, A$. For the vertices of the $VV\varphi$, $V\varphi\varphi$ and $Vf\bar{f}$ type, radiative corrections are at most of order $\alpha m_t^2/m_W^2$, so we expect them to be numerically not very large. On the other hand, vertices of the type $\varphi\varphi\varphi$ can receive corrections of order $\alpha m_t^4/m_W^4$, which will be computed in ref.[16]. For example, at zero external momentum, putting $h_b = 0$, $A_t = \mu = 0$, $m_{\tilde{Q}}^2 = m_{\tilde{U}}^2 = m_{\tilde{D}}^2 \equiv m_{\tilde{f}}^2$ and neglecting D-term contributions to stop and sbottom masses, the one-loop corrected hAA vertex is given by

$$\lambda_{hAA} = \lambda_{hAA}^0 + \Delta\lambda_{hAA}, \quad (24)$$

where

$$\Delta\lambda_{hAA} = -\frac{igm_Z}{2 \cos \theta_W} \frac{3g^2 \cos^2 \theta_W \cos \alpha \cos^2 \beta}{8\pi^2} \frac{m_t^4}{\sin^3 \beta} \log \frac{m_t^2 + m_{\tilde{f}}^2}{m_{\tilde{f}}^2}. \quad (25)$$

We are now ready to come back to our phenomenological discussion. We have seen that, for $m_h \lesssim m_Z/2$, $h \rightarrow AA$ can compete with $h \rightarrow b\bar{b}$ or $h \rightarrow \tau^+\tau^-$.

Taking into account radiative corrections in the form of eq.(25), we show in Fig.2 that $h \rightarrow AA$ does indeed dominate over $h \rightarrow b\bar{b}$ or $h \rightarrow \tau^+\tau^-$ in much of the parameter space. In this case, the dominant A decay mode is $A \rightarrow b\bar{b}$ if $m_A > 2m_b$, in which case the final states from h decay are mainly $2(b\bar{b})$, or $A \rightarrow \tau^+\tau^-$ if $2m_b > m_A > 2m_\tau$, in which case the dominant h final state is $2(\tau^+\tau^-)$. It is not clear whether the previously published searches for $Z \rightarrow hZ^*$ or $Z \rightarrow hA$, which assumed $h \rightarrow b\bar{b}$ or $\tau^+\tau^-$ decay, would have been sensitive to this more complicated decay patterns, which may warrant a new search [15].

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Appendix

We give here the explicit expressions of the coefficients $A_{12}, B_{12}, C_{12}, D_{12}$ introduced in the text. The contributions coming from the D-terms in the stop and sbottom mass matrices are denoted by $B_{12}^{(D)}, C_{12}^{(D)}, D_{12}^{(D)}$.

$$A_{12} = 0$$

$$B_{12} = B_{12}^{(0)} + B_{12}^{(D)}$$

$$B_{12}^{(0)} = \frac{g^2}{2m_W^2 \cos^3 \beta \sin^3 \beta} [A_b m_b^2 \mu \sin^2 \beta - m_b^2 m_t^2 \sin \beta \cos \beta + (b \leftrightarrow t)]$$

$$B_{12}^{(D)} = \frac{g^2}{2 \cos \beta \sin \beta} G$$

$$C_{12} = C_{12}^{(0)} + C_{12}^{(D)}$$

$$C_{12}^{(0)} = \frac{g^2}{2m_W^2 \cos^3 \beta \sin^3 \beta} \left\{ \mu^2 m_t^4 \sin^4 \beta - (m_D^2 + m_U^2 + 2m_t^2) \mu A_b m_b^2 \sin^3 \beta \cos \beta \right. \\ \left. + [(m_D^2 + m_U^2 + m_t^2) + \mu^2 - A_t(A_t + A_b)] m_t^2 m_b^2 \sin^2 \beta \cos^2 \beta \right. \\ \left. + (b \leftrightarrow t) \right\}$$

$$C_{12}^{(D)} = -\frac{g^2}{2 \cos^3 \beta \sin^3 \beta} \left\{ -\frac{\mu \cos 2\beta}{2 \cos^2 \theta_W} (A_t m_t^2 \cos^2 \beta - A_b m_b^2 \sin^2 \beta) \right.$$

$$+ \sin \beta \cos \beta [m_b^2 \sin^2 \beta (\mu^2 - A_t^2) + m_t^2 \cos^2 \beta (\mu^2 - A_t^2)]$$

$$+ \sin \beta \cos \beta (m_U^2 + m_D^2 + m_b^2 + m_t^2 + \frac{1}{3} m_W^2 \tan^2 \theta_W \cos 2\beta) G \Big\}$$

$$D_{12} = D_{12}^{(0)} + D_{12}^{(D)}$$

$$D_{12}^{(0)} = -\frac{g^2}{2m_W^2 \cos^3 \beta \sin^3 \beta} \{ \mu^2 (m_U^2 + m_t^2) m_b^4 \sin^4 \beta \\ + [A_t m_t^2 (\mu^2 + 2m_b^2 + 2m_t^2 + A_b A_t) - A_b (m_U^2 + m_t^2) (m_D^2 + m_t^2)] \mu m_b^2 \cos \beta \sin^3 \beta \\ + [\frac{1}{2} m_D^2 m_U^2 + m_b^2 (m_D^2 + m_U^2 + m_t^2 + 2\mu^2) - 2m_D^2 A_t A_b - A_b^2 (m_U^2 + m_t^2) \\ + \frac{1}{2} (m_D^2 + \mu^2 + A_t A_b)] m_t^2 m_b^2 \sin^2 \beta \cos^2 \beta + (b \leftrightarrow t) \}$$

$$D_{12}^{(D)} = \frac{g^2}{72 \cos^3 \beta \sin^3 \beta} \{ m_W^2 \tan^4 \theta_W \sin^2 \beta \cos^2 \beta \cos^2 2\beta (7m_t^2 m_b^2 - 8m_W^2 G) \\ + 6 \tan^2 \theta_W \cos^2 \beta \sin^2 \beta \cos 2\beta [-2m_W^2 G (m_U^2 + m_t^2 - 2m_D^2 - 2m_b^2) \\ + 2m_t^2 m_b^2 (m_U^2 + m_D^2 - 2m_b^2) + 3m_t^2 m_b^2 (m_t^2 - m_b^2)] \\ + 9 \cos^2 \beta \sin^2 \beta [m_t^2 m_b^2 m_W^2 \cos^2 2\beta + 2 \cos 2\beta m_t^2 m_b^2 (m_t^2 - m_b^2) \\ + 4(m_U^2 + m_t^2)(m_D^2 + m_b^2) G] \}$$

$$+ \frac{g^2}{6 \cos^3 \beta \sin^3 \beta} \left\{ -\tan^2 \theta_W \cos^2 \beta \sin^2 \beta \cos 2\beta \right. \\ \left. [2A_t^2 m_b^2 (m_W^2 \sin^2 \beta - m_t^2) - A_t^2 m_t^2 (m_W^2 \cos^2 \beta - m_b^2) + A_t A_b m_t^2 m_b^2] \right\}$$

$$\begin{aligned} & - 3 \sin^2 \beta \cos^2 \beta [A_t^2 m_b^2 \sin^2 \beta (m_U^2 + m_t^2) + A_t^2 m_t^2 \cos^2 \beta (m_D^2 + m_b^2)] \\ & - \frac{g^2 \mu^2}{36 \cos^2 \beta \sin^2 \beta} \left\{ \tan^4 \theta_W \cos^2 2\beta (2A_b m_b^2 \sin^2 \beta - A_t m_t^2 \cos^2 \beta) m_W^2 \right. \\ & - \tan^2 \theta_W \cos 2\beta [A_b m_b^2 \sin^2 \beta (6m_W^2 \cos 2\beta + 9m_t^2 + 12m_D^2 - 3m_U^2) \\ & + A_t m_t^2 \cos^2 \beta (3m_W^2 \cos 2\beta - 9m_b^2 - 6m_D^2 - 3m_U^2)] \\ & - 9 \cos 2\beta A_b m_b^2 \sin^2 \beta (m_U^2 + m_t^2) + 9 \cos 2\beta A_t m_t^2 \cos^2 \beta (m_D^2 + m_b^2) \\ & - 36 \cos^2 \beta \sin^2 \beta m_t^2 m_b^2 (A_b + A_t) \Big\} \\ & - \frac{g^2 \mu^2}{12 \cos^3 \beta \sin^3 \beta} \left\{ \tan^2 \theta_W \cos 2\beta [m_W^2 \cos^2 \beta \sin^2 \beta (2m_t^2 \cos^2 \beta - 4m_b^2 \sin^2 \beta) \right. \\ & - 2m_t^4 \cos^4 \beta + 4m_b^4 \sin^4 \beta - 2 \cos^2 \beta \sin^2 \beta m_t^2 m_b^2] \\ & - 6m_b^2 \sin^4 \beta \cos^2 \beta (m_U^2 + 2m_t^2) - 6m_t^2 \cos^4 \beta \sin^2 \beta (m_D^2 + 2m_b^2) \Big\}, \end{aligned}$$

where

$$G = m_b^2 \sin^2 \beta + m_t^2 \cos^2 \beta - m_W^2 \cos^2 \beta \sin^2 \beta$$

and $b \leftrightarrow t$ stands for the following set of substitutions:

$$\begin{aligned} m_b & \leftrightarrow m_t \\ A_b & \leftrightarrow A_t \\ m_D & \leftrightarrow m_U \\ \sin \beta & \leftrightarrow \cos \beta. \end{aligned}$$

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Figure Captions

Figure 1

Lines of constant $\tan \beta = 1, \sqrt{2}, 2, 4, m_t/m_b$ in the region of the (m_A, m_h) plane accessible to LEP I searches. The dotted lines correspond to $m_h = m_A$, the dashed ones to $m_{H^\pm} = 2m_A$. The dark areas correspond to $m_{H^\pm} < m_W$, the shaded ones to $m_{H^\pm} < m_Z$. Four different choices of parameters have been made:

- (a) $m_t = 120$ GeV, $m_{\bar{t}} = 300$ GeV, $A_t = A_b = \mu = 0$;
- (b) $m_t = 160$ GeV, $m_{\bar{t}} = 1$ TeV, $A_t = A_b = \mu = 0$;
- (c) $m_t = 140$ GeV, $m_{\bar{t}} = 1$ TeV, $A_t = A_b = 1$ TeV, $\mu = 200$ GeV;
- (d) $m_t = 140$ GeV, $m_{\bar{t}} = 300$ GeV, $A_t = A_b = -250$ GeV, $\mu = 250$ GeV.

Figure 2

The ratios $R_b \equiv \Gamma(h \rightarrow AA)/\Gamma(h \rightarrow b\bar{b})$ and $R_\tau \equiv \Gamma(h \rightarrow AA)/\Gamma(h \rightarrow \tau^+\tau^-)$, as functions of m_A , for fixed values of $\tan \beta = 1, \sqrt{2}, 2, 4, m_t/m_b$. The radiative correction (25) to the hAA vertex has been included, and two different choices of parameters have been made:

- (a) $m_t = 120$ GeV, $m_{\bar{t}} = 300$ GeV, $A_t = A_b = \mu = 0$;
- (b) $m_t = 160$ GeV, $m_{\bar{t}} = 1$ TeV, $A_t = A_b = \mu = 0$.

We have restricted the parameter space to those values for which the decays involved in the ratios R_b and R_τ are kinematically allowed.

Figure 1 (cont.)

Figure 1 (cont.)

Figure 1 (cont.)

Figure 2

Figure 2 (cont.)

Figure 2 (cont.)