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The primary purpose of the School is still to familiarize young postgraduate students of experimental physics with the current theoretical and experimental situation in studies of elementary particles. However, this year more of the lectures were devoted to experimental and technical subjects.

The present Proceedings are, as before, a photo-offset reproduction of the manuscripts provided by the lecturers. I would like to thank the authors for giving me the carefully typed notes so rapidly and also the Scientific Information Service of CERN for their excellent technical work.

W.O. Lock
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SOME RECENT TOPICS ON HADRON COLLISIONS

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INTRODUCTION

Much of what I shall speak about in these lectures has happened during the last year. Moreover, the production rate of physicists these days is so great that it has been quite impossible for me even to follow the literature, let alone appreciate it or pass critical judgement on it. Instead of attempting to review these developments, I therefore propose to give a brief sketch of the basic ideas underlying them. I shall do this only at a general level, ignoring most of the details. My reason for so doing is simply that of necessity -- I cannot hope to do otherwise.

The topics I shall discuss include, in particular: (i) finite energy sum rules, (ii) duality, and (iii) the Veneziano model. These are closely connected subjects, and together they offer an exciting new approach to strong interactions. Although the first results obtained so far are extremely encouraging, there still remain many unanswered questions of great importance. It will probably be quite some time before all these ideas can be knitted together to form a consistent whole. In my lectures, the topics are therefore kept separate, although I shall try my best to point out the relationship between them.

1. FINITE ENERGY SUM RULES

Finite energy sum rules are just another method of exploiting the analytic properties of scattering amplitudes. As such, they are thus little different in principle from ordinary dispersion relations which have been in common use for over ten years. In practice, however, they seem to be particularly useful for relating the asymptotic behaviour of scattering amplitudes to their values at low energies. They are thus a handy tool for checking the consistency of asymptotic models, such as the Regge model.
Consider for concreteness the case of πN scattering. In the following we shall ignore the complications due to the nucleon spin, which are not essential for illustrative purposes. The amplitude, as represented by Fig. 1

![Fig. 1](image)

is a function of the momenta $p_\pi$ and isospin indices $i_\pi$ of the four external lines. Notice that for convenience we have taken all the lines as incoming. Bose statistics require that the amplitude be asymmetric under the interchange of the two pions, namely under the simultaneous interchange of the momenta $p_1 \leftrightarrow p_4$ and the isospin indices $i_1 \leftrightarrow i_4$.

Because of relativistic invariance and isospin symmetry, it is usual to express the amplitude in terms of invariant quantities: thus, instead of $p_\pi$, we have $s = (p_1 + p_2)^2$, $t = (p_2 + p_3)^2$, and $u = (p_2 + p_4)^2$, and instead of the isospin indices $i_\pi$ we shall use the total isospin in the $t$-channel $I_t = 0, 1$.

The variables $s$, $t$, and $u$ are not independent, being related by the linear condition

$$s + t + u = 2m^2 + 2\mu^2 .$$

We shall choose as our independent variables, $t$ and $\nu = s - u$, the advantage of which will be apparent later. The analytic properties of the amplitude in the variables $t$ and $\nu$ are easily seen. For example, for fixed $t$, the amplitude is analytic in $\nu$ apart from singularities due to bound states and scattering thresholds in the $s$- and $u$-channels.

Now both the $s$- and $u$-channels correspond to πN elastic scattering; we have thus poles at both $s$ and $u = m^2$ corresponding to the nucleon state, and branch points at $s$ and $u = (m + \mu)^2$ corresponding to the πN threshold. On the $\nu$-plane for fixed $t$, therefore, the situation is as shown in
Fig. 2, where \( \nu_1 = t - 2\mu^2 \) and \( \nu_0 = t + 4m\mu \). The points \( \nu = \nu_1 (\nu = -\nu_1) \) and \( \nu = \nu_0 (\nu = -\nu_0) \) correspond to \( s = m^2 \) (\( u = m^2 \)) and \( s = (m + \mu)^2 \) (\( u = (m + \mu)^2 \)), respectively.

![Diagram](image)

Fig. 2

The following symmetry properties of the amplitude are useful later for our derivation of the finite energy sum rules:

i) Under the interchange of the momenta \( p_1 \leftrightarrow p_4 \), we have

\[
s = (p_1 + p_2)^2 \leftrightarrow (p_4 + p_2)^2 = u;
\]

or in other words, \( \nu \leftrightarrow -\nu \). Now the whole amplitude has to be invariant under the simultaneous interchange \( p_1 \leftrightarrow p_4 \) and \( i_1 \leftrightarrow i_4 \). This means that for \( I_t = 0 \), which is even under \( i_1 \leftrightarrow i_4 \), we have (even under crossing)

\[
A_0(\nu) = A_0(-\nu),
\]

whereas for \( I_t = 1 \), which is odd under \( i_1 \leftrightarrow i_4 \), we have (odd under crossing)

\[
A_1(\nu) = -A_1(-\nu).
\]

In particular, this explains why the singularities on the \( \nu \)-plane as exhibited in Fig. 2 are symmetrically situated about the origin.

ii) For both \( I_t = 0 \) and 1, the 'reality' property of the amplitude implies

\[
A(\nu^*) = A^*(\nu).
\]

This is a consequence of hermiticity.
Consider now the amplitude \( A_1(v,t) \) for \( I_t = 1 \). In the Regge model, this is supposed to be dominated at high energy and small momentum transfer by the exchange of the \( \rho \) trajectory. Now for fixed \( t \), and \( s \to \infty \), one sees from Eq. (1) that \( v \sim 2s \). Hence we have for the asymptotic form of the amplitude

\[
A_1(v,t) \to \beta_{\rho}(t) \frac{+1 - e^{-i\pi\alpha_{\rho}}}{\sin \pi\alpha_{\rho}} \left( \frac{s}{s_0} \right)^{\alpha_{\rho}}.
\]

(5)

Let us assume that there exists a certain value \( N \) such that for all \( |v| \geq N \), the asymptotic form (5) is already a good approximation to the amplitude. We then apply the Cauchy theorem to the function \( A_1 \) integrating along the semicircle of radius \( N \) in the upper-half \( v \)-plane, as shown in Fig. 2. Since the function \( A_1 \) is analytic inside the semicircle, we have

\[
\int_{-N}^{N} A_1(v + i\varepsilon, t) \, dv = \int_{\gamma} A_1(v,t) \, dv,
\]

(6)

where the right-hand integral is carried along the arc of the semicircle, namely \( |v| = N \). Since, by assumption, the asymptotic form (5) is already a good approximation to \( A_1 \) for \( |v| = N \), we can replace the integrand on the right by the asymptotic form, obtaining

\[
\int_{-N}^{N} A_1(v + i\varepsilon, t) \, dv = 2i \frac{\beta(t)}{\alpha_{\rho}(t) + 1} \left( \frac{s}{s_0} \right)^{\alpha_{\rho}(t) + 1}.
\]

(7)

Using now the symmetry (3) of \( A_1 \) under crossing, we can rewrite the left-hand integral as

\[
\int_{-N}^{N} A_1(v + i\varepsilon, t) \, dv = \int_{0}^{N} \left[ A_1(v + i\varepsilon, t) - A_1(v - i\varepsilon, t) \right] \, dv.
\]

(8)

The reality condition (4) then gives for Eq. (7) the equation

\[
\int_{0}^{N} \text{Im} \, A_1(v,t) \, dv = \frac{\beta_{\rho}(t) \left( \frac{s}{s_0} \right)^{\alpha_{\rho}(t) + 1}}{\alpha_{\rho}(t) + 1},
\]

(9)

which is the simplest example of a finite energy sum rule\(^2\).
The physical content of Eq. (9) is nothing more than just the analytic properties of $A_1(\nu,t)$ and its assumed asymptotic behaviour. The only new point is that these properties have been expressed in a manner particularly convenient for phenomenological exploitation. The left-hand side is an integral over the 'low-energy region' from 0 up to the energy $N$, while the right-hand side is expressed in terms of those parameters that are supposed to characterize the amplitude for high energies, i.e. $\nu > N$. Equation (9) therefore can be regarded as a consistency relation connecting scattering at high and low energies imposed by analyticity.

The finite energy sum rule (9) can be generalized in various ways. We mention those generalizations that have been particularly useful:

i) The function $\nu^{2m}A_1(\nu,t)$ for integral $m > 0$ has the same analytic properties and symmetries (3) and (4) as the original $A_1(\nu,t)$. One can thus apply the Cauchy theorem to $\nu^{2m}A_1(\nu,t)$ instead of $A_1(\nu,t)$ in the same manner, obtaining

$$\int_0^N \nu^{2m} \text{Im} A_1(\nu,t) \, d\nu = \beta_\rho(t) \frac{\alpha_\rho(t)+2m+1}{N^{\alpha_\rho(t)+2m+1}} \frac{\alpha_p(t)+2m+1}{\alpha_p(t)+2m+1}. \tag{10}$$

This will be an additional consistency relation to be satisfied by the $A_1$ amplitude. In principle, one has an infinite sequence of such integral moment sum rules. In practice, however, only the first few of these are useful; with the higher moment sum rules degenerating rapidly into approximate identities.

ii) By using odd moments instead of even moments as in point (i), one can obtain sum rules also for amplitudes that are even under crossing. Thus for $A_0(\nu,t)$, for example, assuming that for $\nu > N$ the amplitude is dominated by the P and P' trajectories, one would have for integral $m > 0$

$$\int_0^N \nu^{2m-1} \text{Im} A_0(\nu,t) \, d\nu = \beta_P(t) \frac{\alpha_p(t)+2m}{\alpha_p(t)+2m} + \beta_{P'}(t) \frac{\alpha_{p'}(t)+2m}{\alpha_{p'}(t)+2m}. \tag{11}$$
iii) With slight modifications, sum rules can also be derived for moments $\gamma A(\nu, t)$, where $\gamma$ is not an integer. These so-called continuous moment sum rules involve, in general, both the real and imaginary parts of the amplitude$^3$.

iv) Obviously, the methods discussed above are not restricted to $\pi N$ scattering, or to the $\rho$, $P$, and $P'$ Regge poles alone. Indeed, such sum rules can be derived for any assumed asymptotic form of the amplitude, such as sums of Regge poles or even Regge cuts. The sum rules derived in each case will afford a means for checking phenomenologically whether the assumed high-energy behaviour is consistent with the low-energy scattering data.

The applications of finite energy sum rules to phenomenological analyses can be divided into three types differing only by emphasis:

a) using low-energy data to predict high-energy parameters, such as Regge intercepts and residue functions;

b) resolving ambiguities in the low-energy region by means of high-energy data;

c) making simultaneous fits of high- and low-energy data in a manner consistent with analyticity.

Obviously, since high-energy data are in general less accurate than those at lower energies, points (a) and (c) far outweigh point (b) in importance.

The usefulness and accuracy of finite energy sum rules depend mainly on the available data. A particularly favourable case is the sum rule at $t = 0$, where by means of the optical theorem, the imaginary part of the amplitude is simply related to the total cross-section. With the accurate measurements of total cross-sections already available, one can make very accurate predictions of Regge parameters at $t = 0$. Thus, for example, the Pomeron intercept $\alpha_p(0)$ has been determined by such means$^b$ in $\pi p$ scattering to be $\alpha_p(0) = 1 \pm 0.02$.

For $t \neq 0$, the imaginary part of the amplitude is no longer given directly in terms of measured cross-sections. One then has to rely on phase-shift analysis, which for $\pi N$ scattering is already quite reliable up to 2 GeV/c. Feeding this into the left-hand side of Eq. (9), one can evaluate the integral and obtain the $\rho$-exchange parameters for various
values of \( t \). In this way, Dolen, Horn and Schmid\(^2\) were able to predict the so-called 'wrong-signature nonsense' dip in \( d\sigma/dt \) for \( \pi N \) charge-exchange scattering: \( \pi^- p \rightarrow \pi^0 n \).

The art of exploiting the finite energy sum rules is now highly developed. The sensitivity of the method is often enhanced by judicious use of the various moments. As an example, I wish to quote the work on photoproduction of charged pions by the Rome-Trieste group\(^5\). Using the low-energy phase-shift analysis obtained previously, they were able to determine the Regge parameters for both the pion and its 'conspirator' to remarkable accuracy, and to predict successfully the high-energy data up to 16 GeV. Their result is shown in Figs. 3 and 4.

In those reactions where reliable phase-shift analyses have not been performed, such as \( \pi N \) scattering, it is often found possible to make qualitative predictions by assuming that the imaginary part of the amplitude at low energy is completely dominated by resonances. This will not be a direct check of analyticity and Regge behaviour since it involves the further assumption of resonance dominance. However, its qualitative success has led to the important new concept of 'duality', which will be the subject of our next lecture.

2. **Duality and Exchange Degeneracy**\(^6\)

Finite energy sum rules, as discussed in the previous section, are based only on two theoretical concepts, namely (i) analyticity, and (ii) asymptotic behaviour (not necessarily Regge) of the scattering amplitude. Clearly, being so general, they can have few predictions, and will remain only as a phenomenological tool for data analysis, unless supplemented by further assumptions.

We now propose the following:

(A) down to fairly low energies, \( \sim 2 \) GeV/c, scattering amplitudes are already well-approximated by the exchange of a few Regge poles;

(B) the imaginary part of a scattering amplitude is dominated entirely by direct channel resonances;

both statements being subject to an important exception which we shall later specify.
Neither of these assumptions are new; they have frequently been made both in phenomenological analysis and in theoretical studies. However, when coupled with the finite energy sum rules, they take on an additional significance. Consider again as an example the $I_t = 1$ amplitude in $\pi N$ scattering, namely $A_1(\nu, t)$ in Section 1, which satisfies the sum rule (9). On the right, we have the $\rho$-exchange amplitude, which we shall assume to be approximately valid down to energies of $\sim 2 \text{ GeV/c}$. On the left, we shall assume that $\text{Im} \, A_1$ is dominated by the direct channel resonances, which occurs in $\pi N$ scattering. Equation (9) then becomes a relation between the masses and widths of nucleonic resonances and the Regge parameters $\alpha$ and $\beta$ of $\rho$-exchange. Since the resonance parameters are well-known from phase-shift analysis and $\alpha_\rho$ and $\beta_\rho$ from high-energy Regge fits, the relation is subject to a direct check with existing data, and is found to be approximately valid. Thus, for example, by feeding in the resonance parameters on the left-hand side of Eq. (9), one can predict with some certainty the dip in $d\sigma/dt$ at $t \sim -0.6 \text{ GeV}^2$ for the reaction $\pi^- p \to \pi^0 n$.

This relation between resonances in the direct channel and the exchanged Regge poles is of great theoretical significance since the Regge poles themselves are supposed to be connected to resonances in the exchange channels. This significance is best appreciated in reactions such as $\pi \pi \to \pi \pi$, where the direct and exchange channels are identical. Equation (9) then becomes a consistency requirement involving the $\rho$-trajectory on both sides, which can be used to restrict the trajectory parameters. This is then the so-called FESR bootstrap.

The implications of finite energy sum rules plus the assumptions (A) and (B) do not, however, stop there. As we have seen, analyticity implies, in addition to Eq. (9), further sum rules for various moments of the amplitude. In each case, the contributions of resonances on the left must add in such a way as to build up the Regge exchange on the right. Now the higher moment sum rules will emphasize the higher mass resonances. The only way then for all the sum rules to be satisfied will be to have the integrand itself approximately equal to the Regge amplitude.

Clearly, this 'duality' or equivalence between the direct channel resonances and Regge-pole exchange should not be taken too literally, at least in the low-energy region where the resonance amplitude shows large
fluctuations as a function of the energy. It is supposed to hold only in the average sense when the resonance amplitude is integrated over a small ($\sim 1$ GeV) interval. It is this 'semi-local average' over the resonance contribution which is supposed to be approximately equal to the Regge amplitude.

A dramatic demonstration of this, at first sight, amazing fact has been given by Schmid\(^7\). He took the Regge parameters as determined from fits at high energy to extrapolate the $\rho$-exchange amplitude in $\pi N$ scattering down to energies $\sim 2$ GeV. Then, performing a partial wave analysis on this, he obtained for each partial wave a loop on the Argand diagram very similar to those obtained by phase-shift analysis as evidence for nucleon resonances. Moreover, these 'pseudo-resonances' were shown to lie approximately on a linearly rising trajectory! Indeed, on closer examination it was found that such a behaviour of partial wave phases is an almost automatic consequence of the Regge form of the amplitude, for any exchanged trajectory with finite slope\(^8\).

At this point, we should turn back to specify the important exception mentioned at the beginning of this section. This concerns what is known as the Pomeranchuk trajectory in the theory of Regge poles. Now, it has long been accepted by theoreticians that elastic scattering at high energy is dominated by the diffractive mechanism, or in other words, by the shadow effects of multiparticle channels via unitarity. If one insists on representing this by a Regge trajectory carrying the quantum numbers of the vacuum, namely, the Pomeron, then its parameters can be determined phenomenologically from scattering data. It was found that the trajectory thus obtained is much flatter than all other known trajectories. Indeed, apart from perhaps the recent results reported from Serpukhov, existing data are not inconsistent with $\alpha_p' = 0$. If this is true, then the $P$ trajectory cannot be dual to resonances in the sense of Schmid, since a flat trajectory will not give rise to loops in the Argand plot. Another reason for this belief is as follows. Since the Pomeron carries the quantum numbers of the vacuum, its exchange has the same contributions in all isospin states in the direct channel. If it is dual to resonances then one expects resonances in all isospin states which are more or less degenerate. This is certainly contradictory to everything we know experimentally.
It is not yet clear in what way the Pomeron is going to affect our previous discussion of duality. Indeed, no answer to this question is likely until one understands more about diffractive scattering. However, as a first approximation, one may assume that the Pomeron contribution is additive to the dual part of the amplitude; namely, we write for the full amplitude: \[ A = A_{\text{Dual}} + A_{\text{Pomeron}}, \]
where \( A_{\text{Dual}} \) has then all the properties prescribed by Schmid. This hypothesis has been popularized by Harari and is found to be qualitatively valid in the cases where it has been checked\(^9\). Nonetheless, it should be accepted only as a first approximation and not as the final word on the Pomeranchuk problem.

This 'principle of duality', as formulated by Schmid, must rank with 'bootstrap' and 'maximal analyticity' as one of the most loosely defined principles in the history of physics. Nonetheless, like the others, it has proved extremely fruitful as a basis for the understanding of strong interaction dynamics\(^{10}\). I shall try here to summarize a few of its main consequences.

The first implications of 'duality' are negative. It destroys two accepted concepts which have been in common use for several years.

2.1 Interference models

This concerns the intermediate energy region, say 2-6 GeV/c in \( \pi N \) scattering. Below 2 GeV/c incoming energy, phase-shift analysis tells us that the scattering amplitude is dominated by a large number of resonances. In fact, many aspects of \( \pi N \) reactions can be qualitatively understood by taking account of these resonances alone, and neglecting everything else. This is the basic premise of the so-called isobar models. Whereas at energies > 6 GeV/c, the large number of Regge fits performed seem to show that the amplitude is dominated by the exchange of a few Regge poles. The question then arises: What about the intermediate energy region? One obviously needs here some sort of an interpolation between the wild fluctuations of the resonances below 2 GeV/c to the smooth behaviour of Regge amplitudes at higher energy. A natural assumption, at first sight, would seem to be the following: one just adds the resonance contribution to the Regge amplitude, thus:

\[ A = A_{\text{Res}} + A_{\text{Regge}}. \] (12)
At low energies, $A_{\text{Res}}$ would dominate and $A_{\text{Regge}}$ would be small, while at higher energies, $A_{\text{Res}}$ will diminish and $A_{\text{Regge}}$ will take over. In the intermediate region, say 2-6 GeV/c, $A_{\text{Res}}$ and $A_{\text{Regge}}$ are comparable and interfere. This is thus the so-called Interference Model, which for some years has enjoyed a fair amount of success. On closer examination, however, this assumption cannot be strictly correct in view of the discussion given above. According to Schmid, $A_{\text{Res}}$ and $A_{\text{Regge}}$ both represent one and the same thing; they cannot therefore possibly interfere. And, true enough, a closer scrutiny of the early successes of the interference model reveals that they are indeed independent of the interference assumption.

There is one point here which should be clarified and which has led to some misunderstanding. In our discussion, $A_{\text{Regge}}$ represents only the contribution of a 'small number' of leading poles. Otherwise the discussion would be completely meaningless. Had we allowed many non-leading poles in $A_{\text{Regge}}$, then

$$A_{\text{Regge}} = \sum_i \beta_i(t) \zeta_i(t) \alpha_i(t)$$

(13)

would itself already be a complete expansion of the amplitude. We shall not need Schmid to tell us that Eq. (12) is unfeasible. It is only when a small number of poles are concerned (e.g. the $\rho$-contribution in $A_1$ for $\pi N$ scattering) that the distinction between 'duality' and the 'interference model' takes on a meaning.

2.2 Subtraction of background from resonances in data analysis

A resonance in hadron collision data usually appears as a peak or hump in the effective mass distribution of its decay products, with the peak standing above some sort of smooth background. The question then arises: How much of the peak should be regarded as the resonance and how much as the background? The problem is accentuated where the 'background' itself has a hump at around the resonance position because of some 'kinematic enhancement'. A classic example of this is the $A_1$ hump in the reaction $\pi p \rightarrow \pi pp$; it is still not clear whether the $A_1$ is a true resonance or just a kinematic enhancement called the Deck effect. In the Regge language, the Deck effect arises as follows. Consider the
double-Regge diagram of Fig. 5. The fact that both the momentum transfers $t_1$ and $t_2$ are restricted to small values implies that the diagram is appreciable only in one corner of the Dalitz plot\textsuperscript{11), as shown in Fig. 6. It is clear that on the effect mass plot, the distribution will be enhanced at low values of the $\pi\rho$ mass. If indeed there is a resonance called the $A_1$, it will show up as a little peak on top of the hump. Formerly, one may have been tempted to consider the Deck effect as the background, and as such to be subtracted from the resonance. However, with the new concept of duality, this is no longer clear. Since the Regge amplitude and the resonance contribution are essentially one and the same thing, the subtraction of the Deck effect will necessarily remove most of the resonance as well. Indeed one becomes a little confused as to what one can actually call a resonance.

The two consequences of 'duality' just discussed, though of great importance both theoretically and experimentally, are destructive rather than creative. Duality claims the interference model to be wrong and background subtraction from resonances to be dangerous, but it does not yield a better method in either case. To do this, one has to have a more specific framework than just the loose statement given earlier of 'duality'. A good example of such is the Veneziano model, which we shall discuss later.

Even in its loose form adopted in this section, however, 'duality' is able to give some definite predictions of great significance. One of the most interesting of these concerns the exchange degeneracy of Regge
trajectories. The concept of exchange degeneracy was first introduced by Arnold from other considerations. What 'duality' does is to give the concept a somewhat sounder basis.

Consider again a specific example, say, Kπ scattering$^{12}$ for K mesons with $S = +1$. Since elastic Kπ scattering admits the exchange of vacuum quantum numbers, there will be a Pomeron contribution, which we assume to be additive as discussed above. What concerns us here is only the dual part of the amplitude. Now, one outstanding feature of Kπ scattering is the empirical fact that no resonance with $B = +1$ and $S = +1$ are known to exist. This means that in the direct channel for Kσ scattering, the contributions of resonant states are negligible. Since 'duality' suggests that the resonance contributions and Regge exchanges are essentially the same, the absence of resonances would imply that the imaginary part of the Regge amplitude is also zero. This statement is true for both Kp and Kσ scattering, and for both forward and backward scattering; it must therefore hold separately for each of the amplitudes with $I_t = 0,1$ and $I_u = 0,1$. Now for these amplitudes, the following trajectories are expected to contribute:

\begin{align*}
I_t = 0 & : \quad \omega \quad f_0 \\
I_t = 1 & : \quad \rho \quad A_2 \\
I_u = 0 & : \quad \Lambda \quad \gamma^*_0(1520) \\
I_u = 1 & : \quad \gamma^*_1(1385) \quad \gamma^*_1(1765),
\end{align*}

where the two trajectories occurring in each case have opposite signatures. For the imaginary part of each Regge amplitude to cancel for all $t$ and $u$, the trajectories must therefore be degenerate in pairs in both the trajectory function $\alpha$ and the residue $\beta$. The fact that the meson trajectories are indeed approximately degenerate is well known. For the baryons, also, it appears that the prediction is equally valid, as can be seen in Figs. 7 and 8.

Similar arguments have been applied with fair success to other reactions. However, there also exist some cases, such as baryon-antibaryon scattering, for which the predictions of 'duality' appear not to be valid. On the whole, I would say that the results in this direction have been quite impressive though not yet entirely overwhelming.
Fig. 7  (a) exchange-degenerate sequence of $Y_0^*$.  
(b) the residues of the sequence also approximately degenerate (Ref. 12).
Fig. 8  (a) exchange-degenerate sequence of $Y_1^*.$  
(b) the residues of the sequence also approximately degenerate (Ref. 12).
3. THE VENEZIANO MODEL

As a result of both theoretical and phenomenological studies carried out in recent years, especially those based on the finite energy sum rules and 'duality' that I have described in the last two sections, we have now quite a good knowledge of the properties possessed by two-body collision amplitudes. I shall list below some of these that are relevant for our present discussion:

i) analyticity,
ii) crossing symmetry,
iii) Regge asymptotic behaviour,
iv) resonances on linear rising trajectories,
v) 'duality' in the sense of Schmid.

The Veneziano model\textsuperscript{13}) is a particular example that satisfies all these properties. It is thus a good theoretical laboratory in which to study hadron collision amplitudes. Moreover, since it already has so much in common with physical amplitudes, one may reasonably hope that it may serve also as a phenomenological model for the description of experimental data.

Again, instead of introducing the subject in full generality, I prefer to begin by giving a specific example. For this, I have chosen $\pi\pi$ scattering\textsuperscript{14}), which is particularly suitable for our illustrative purpose.

Let $i$ and $p_r$ ($r = 1, 2, 3, 4$) denote the isospin indices and the four-momenta of the external pions, respectively. The isospin indices $i$ can take the values 1, 2, or 3, where as usual $\pi_1 = \frac{1}{2}(\pi^+ + \pi^-)$, $\pi_2 = (1/2i) (\pi^+ - \pi^-)$, $\pi_3 = \pi^0$. To each value of $i$ we shall associate the $2 \times 2$ Pauli matrix $\tau_i$. We shall take all the momenta as ingoing, so that the Mandelstam variables $s$, $t$, and $u$ are given as

\begin{equation}
\begin{align*}
  s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\
  t &= (p_2 + p_3)^2 = (p_1 + p_4)^2 \\
  u &= (p_1 + p_3)^2 = (p_2 + p_4)^2.
\end{align*}
\end{equation}

The only known trajectories that are strongly coupled to the $\pi-\pi$ system are: (i) the $\rho$-trajectory with isospin $I = 1$ and negative signa-
ture; and (ii) the \( f_0 \)-trajectory with isospin \( I = 0 \) and positive signature. The two trajectories are empirically almost degenerate on the Chew-Frautschi plot. Moreover, 'duality' arguments similar to those given in the last lecture, plus the fact that no meson resonances with double charge are known, imply that the \( \rho \) and \( f_0 \) trajectories are exchange degenerate partners. We shall therefore introduce a common trajectory function \( \alpha \) for both the \( \rho \) and \( f_0 \), and assuming \( \alpha \) to be linear, we have \( \alpha(x) = \alpha_0 + \alpha'x \), where as usual \( \alpha_0 \) is the intercept and \( \alpha' \) the slope of the trajectory.

In this notation then, the Veneziano amplitude for \( \pi\pi \) scattering, as first given by Lovelace\(^{14}\), takes the following form:

\[
T = V(s,t) + V(t,u) + V(u,s) , \tag{15}
\]

where

\[
V(s,t) = \beta \text{ Tr}(\tau_1 \tau_2 \tau_3 \tau_4) \times \frac{\Gamma[1 - \alpha(s)] \Gamma[1 - \alpha(t)]}{\Gamma[1 - \alpha(s) - \alpha(t)]} \tag{16}
\]

\[
V(t,u) = \beta \text{ Tr}(\tau_1 \tau_3 \tau_2 \tau_4) \times \frac{\Gamma[1 - \alpha(t)] \Gamma[1 - \alpha(u)]}{\Gamma[1 - \alpha(t) - \alpha(u)]} \tag{17}
\]

\[
V(u,s) = \beta \text{ Tr}(\tau_1 \tau_3 \tau_4 \tau_2) \times \frac{\Gamma[1 - \alpha(u)] \Gamma[1 - \alpha(s)]}{\Gamma[1 - \alpha(u) - \alpha(s)]} \tag{18}
\]

In the formulae, \( \beta \) is a constant, \( \text{Tr} \) stands for trace, and \( \Gamma \) is the gamma-function as defined by Euler. We shall show that this function \( T \) does have all the desired properties of the \( \pi\pi \) amplitude listed at the beginning of the section.

Since all the lines are identical, crossing symmetry here is equivalent to the statement that the amplitude is symmetric under any of the
$4! = 24$ permutations of the external lines. It is convenient to distinguish between (i) cyclic permutations, e.g. $(1234) \rightarrow (2341)$, and reversals, e.g. $(1234) \rightarrow (4321)$; and (ii) other permutations, such as $(1234) \rightarrow (2134)$. Those orderings of the four external lines which are related by type (i) permutations we shall call equivalent orderings.

It can then readily be checked that the 24 different orderings are divided into three disjoint equivalent classes, which can be represented by the diagrams in Fig. 9. One notes that these three diagrams correspond exactly to the three terms in Eq. (15). Indeed, the two variables occurring in each term are just the c.m. energies for the two Mandelstam channels which can be formed in the diagram without changing the ordering of the external lines. Thus, for example, for the first diagram, the variables are just $s$ corresponding to $12-34$ and $t$ corresponding to $23 \rightarrow 41$, as required. From the properties of traces of matrices, it is then seen that each term in Eq. (15) is invariant under a permutation of type (i). Moreover, a permutation of type (ii) transforms a term in Eq. (15) into another, leaving the sum invariant. In other words, one has then fully established the crossing symmetry of the amplitude.

![Diagrams](image)

**Fig. 9**

Next, to study the analytic properties of the amplitude $T$ in Eq. (15), we first note that the gamma-function $\Gamma(z)$ is analytic in $z$ on the whole complex $z$-plane apart from simple poles at $z = 0, -1, -2, \ldots$, and that $\Gamma(z)$ has no zeros anywhere. Consider then $V(s,t)$ in Eq. (16); being a ratio of $\Gamma$'s, $V(s,t)$ must be analytic except where the $\Gamma$ functions in the numerator have poles, i.e. $\alpha(s) = 1, 2, 3, \ldots$ and $\alpha(t) = 1, 2, 3, \ldots$. It does not have double poles, however, since when $\alpha(s)$ and $\alpha(t)$ are both positive integers, the denominator $\Gamma[1 - \alpha(s) - \alpha(t)]$ will also have a pole to cancel it. Similar properties will also hold for $V(t,u)$ and $V(u,s)$, and eventually also for $T$. 
What then are these poles in $T$? They must represent the resonances lying on our degenerate $\rho$-$f_0$ trajectory. Since the amplitude is completely symmetric, we need consider only one Mandelstam channel; say, for example, the $s$-channel. At $\alpha(s) = 1$, which should give us the $\rho$-meson, we have no pole in $V(t,u)$, while the poles in $V(s,t)$ and $V(u,s)$ give us together for $T$ the residue

$$\text{Res } T = \delta \left[ \text{Tr}(\tau_{i_1 i_2} \tau_{i_3 i_4}) \alpha(t) \right. $$

$$\left. + \text{Tr}(\tau_{i_1 i_3} \tau_{i_4 i_2}) \alpha(u) \right],$$

(19)

where we have used the relation $\Gamma(1 + z) = z \Gamma(z)$. Remembering that $\alpha(x) = \alpha_0 + \alpha'x$, namely a linear function in $x$, and that $t = -2k_s^2(1 - \cos \theta_s)$, $u = -2k_s^2(1 + \cos \theta_s)$, one sees that the residue of $T$ at $\alpha(s) = 1$ is a linear function of $\cos \theta_s$. Hence the pole must represent some particle with maximum angular momentum 1. Collecting the leading terms in $\cos \theta$, one has

$$\text{Res } T \sim 2\beta k_s^2 \alpha' \cos \theta_s \times$$

$$\times \left[ \text{Tr}(\tau_{i_1 i_2} \tau_{i_3 i_4}) - \text{Tr}(\tau_{i_1 i_3} \tau_{i_4 i_2}) \right].$$

(20)

Projecting into the isospin states in the $s$-channel, one obtains for:

$I_s = 0$:

$$2\beta k_s^2 \alpha' \cos \theta_s \left[ \text{Tr}(\tau_{i_1 i_2}) \text{Tr}(\tau_{i_3 i_4}) - \text{Tr}(\tau_{i_1 i_3}) \text{Tr}(\tau_{i_4 i_2}) \right]$$

$$= 2\beta k_s^2 \alpha' \cos \theta_s \left[ \delta_{i_1 i_2} \delta_{i_3 i_4} - \delta_{i_1 i_3} \delta_{i_2 i_4} \right] = 0$$

$I_s = 1$:

$$2\beta k_s^2 \alpha' \cos \theta_s \left[ \sum_x \text{Tr}(\tau_{i_1 i_2} \tau_x \tau_{i_3 i_4}) \text{Tr}(\tau_{i_1 i_3} \tau_x \tau_{i_4 i_2}) - \sum_x \text{Tr}(\tau_{i_1 i_2} \tau_x \tau_{i_3 i_4}) \text{Tr}(\tau_{i_1 i_3} \tau_x \tau_{i_4 i_2}) \right]$$

$$= 2\beta k_s^2 \alpha' \cos \theta_s \left[ \sum_x (i \epsilon_{i_1 i_2 x})(i \epsilon_{i_3 i_4}) - \sum_x (i \epsilon_{i_1 i_3 x})(i \epsilon_{i_2 i_4}) \right]$$

$$= 2\beta k_s^2 \alpha' \cos \theta_s (-2) \sum_x \epsilon_{i_1 i_2 x} \epsilon_{i_3 i_4} \neq 0.$$
We have then shown that the pole at \( \alpha_s = 1 \) has maximum spin 1, and that the spin 1 part has pure isospin 1, namely the same quantum number of the \( \rho \) meson as required.

Repeating the arguments for \( \alpha(s) = 2 \), one easily sees that the leading term in \( \cos \theta_s \) in the residue now becomes

\[
\text{Res} \quad \alpha(s) = 2 \quad \sim -\beta(2k_s)^2 \alpha'' \cos^2 \theta_s \times
\]

\[
\times \left[ \text{Tr}(\tau_{i_1 \tau_{i_2} \tau_{i_3} \tau_{i_4}}) + \text{Tr}(\tau_{i_1 \tau_{i_1} \tau_{i_3} \tau_{i_4}}) \right]
\]

(21)

with a + sign between the two traces. The pole having now a residue of second order in \( \cos \theta_s \) must represent a particle of maximum spin 2, and the spin 2 part, because of the change in sign, has now \( I_s = 0 \), i.e. the same quantum numbers of the \( f_0 \) meson.

In general, at \( \alpha(s) = \ell \), the residue of the pole is a polynomial in \( \cos \theta_s \) of degree \( \ell \), thus representing a particle of maximum spin \( \ell \). The leading term in \( \cos \theta_s \) has isospin 1 if \( \ell \) is odd, but isospin 0 if \( \ell \) is even. In other words, our amplitude \( T \) contains two linear trajectories in the \( s \)-channel, an odd signature trajectory with \( I = 1 \), and an even signature trajectory with \( I = 0 \). The trajectories are degenerate with the same intercept \( \alpha_0 \) and the same slope \( \alpha' \). We shall identify them with the \( \rho \) and \( f_0 \) trajectories, respectively.

Next we wish to show that the amplitude (15) has the proper Regge behaviour. Again because of symmetry, we need only establish this for one channel, say for \( s \to \infty \) and fixed \( t \). One must, however, be careful in taking this limit, for, as we have just shown, the function \( T \) has an infinite sequence of poles on the real \( s \)-axis so that the limit \( s \to \infty \) on the real axis cannot strictly exist. We shall therefore define instead as the 'Regge limit' the following: \( \sigma \to \infty \) for \( s = \sigma + i\epsilon \sigma \) and some infinitesimal \( \epsilon > 0 \). Namely, instead of approaching \( \infty \) on the \( s \)-plane strictly along the real axis, we shall approach \( \infty \) along a ray at an infinitesimal angle to the real axis. That this can indeed be taken as the Regge limit is non-trivial and needs some justification. I shall return to this later when we discuss the problem of unitarity for the Veneziano model.
Accepting for the moment our new definition of the Regge limit, we return now to the amplitude $T$ in Eq. (15). We shall consider only the first two terms, for, as can be shown, the third term $V(u,s)$ vanishes exponentially as $s \to \infty$. Projecting out the isospin states, this time however in the $t$-channel, we obtain:

\begin{equation}
I_t = 0 : \quad T_0 = \frac{\Gamma(1 - \alpha(s))\Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha(s) - \alpha(t))} + \frac{\Gamma(1 - \alpha(t)) \Gamma(1 - \alpha(u))}{\Gamma(1 - \alpha(t) - \alpha(u))}
\end{equation}

\begin{equation}
I_t = 1 : \quad T_1 = \frac{\Gamma(1 - \alpha(s)) \Gamma(1 - \alpha(t))}{\Gamma(1 - \alpha(s) - \alpha(t))} - \frac{\Gamma(1 - \alpha(t)) \Gamma(1 - \alpha(u))}{\Gamma(1 - \alpha(t) - \alpha(u))}
\end{equation}

Notice the difference in sign for $I_t = 0$ and 1, which, as we shall see, will give us the different signature of the $\rho$ and $f_0$ trajectories.

To go further, we need the following properties of the $\Gamma$ function:

\begin{equation}
\Gamma(z) \Gamma(1 - z) = \pi (\sin \pi z)^{-1}
\end{equation}

\begin{equation}
\lim_{z \to \infty} \frac{\Gamma(z + a)}{\Gamma(z + b)} = z^{-a-b} \left[ 1 + \frac{2}{2z} (a - b)(a + b - 1) + O(z^{-2}) \right]
\end{equation}

which can be found in standard text-books.

Using Eq. (24), we can rewrite Eqs. (22) and (23) as

\begin{equation}
T_I = \frac{\pi}{\Gamma(\alpha(t)) \sin \pi \alpha(t)} \left\{ (-1)^I \frac{\Gamma(1 - \alpha(u))}{\Gamma(1 - \alpha(t) - \alpha(u))} + \frac{\sin \pi(\alpha(t) + \alpha(s))}{\sin \pi \alpha(s)} \cdot \frac{\Gamma(\alpha(t) + \alpha(s))}{\Gamma(\alpha(s))} \right\}
\end{equation}

Then using Eq. (25), we have

\begin{equation}
\lim \frac{\Gamma(\alpha(t) + \alpha(s))}{\Gamma(\alpha(s))} = \left[ \alpha(s) \right]^{\alpha(t)} = (\alpha'_s)^{\alpha(t)}
\end{equation}
Moreover, since \( s + t + u = 4\mu^2 \), \( s \to \infty \) above the real axis for fixed \( t \) implies \( u \to -\infty \) below the real axis. This gives again by Eq. (25)

\[
\lim_{t \to \infty} \frac{\Gamma[1 - \alpha(u)]}{\Gamma[1 - \alpha(t) - \alpha(u) - 1]} = \left[ -\alpha(u) \right]^{-\alpha(t)} = (\alpha')^{-\alpha(t)},
\]

(28)

Using then the identity \( \sin(A + B) = \sin A \cos B + \cos A \sin B \), we can rewrite:

\[
\frac{\sin \pi[\alpha(t) + \alpha(s)]}{\sin \pi \alpha(s)} = \cos \pi \alpha(t) + \sin \pi \alpha(t) \cot \pi \alpha(s),
\]

(29)

where on taking the limit as previously defined, \( \cot \pi \alpha(s) \to -i \).

Altogether then for \( T_I \), one has the limit

\[
\lim_{t \to \infty} T_I \cong \frac{\pi}{\Gamma[\alpha(t)]} \cdot \frac{(-1)^I + e^{-i\pi \alpha(t)}}{\sin \pi \alpha(t)} \cdot (\alpha')^{-\alpha(t)}
\]

(30)

which is identical to the Regge limit, e.g. Eq. (5), provided we identify \( \beta(t) \) to \( 1/\Gamma[\alpha(t)] \) and \( s_0 \) to \( \alpha^{-1} \). Notice we do have negative signature for \( I = 1 \) (the \( \rho \)-trajectory) and positive signature for \( I = 0 \) (the \( f_0 \)-trajectory) as anticipated.

Having now proved the Regge behaviour and also the existence of resonance poles in the amplitude, 'duality' becomes an automatic consequence, since the same function has been shown explicitly to contain both the \( s \)-channel resonance poles and \( t \)-channel Regge exchange amplitude. Indeed, the loosely formulated 'principle of duality' receives in the Veneziano model the first concrete realization.

Besides the properties just proved, the Veneziano-Lovelace formula (15) has, in addition, the following attractive features:

i) Absence of exotic states. The term \( V(s,t) \) in Eq. (16) has poles only in the \( s \)- and \( t \)-channels and not in the \( u \)-channel. In the \( s \)-channel, the complete isospin structure can be seen by the identity

\[
\frac{1}{2}\text{Tr}(\tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4}) = \frac{1}{2}\text{Tr}(\tau_{i_1} \tau_{i_2}) \cdot \frac{1}{2}\text{Tr}(\tau_{i_3} \tau_{i_4}) + \sum_x \left[ \frac{1}{2}\text{Tr}(\tau_{i_1} \tau_{i_2} \tau_{x}) \cdot \frac{1}{2}\text{Tr}(\tau_x \tau_{i_3} \tau_{i_4}) \right],
\]

(31)
which shows that the s-channel poles can have only isospin 0 or 1 and no higher. A similar statement will hold also for the other terms in Eq. (15) and for all these Mandelstam channels. As we have already seen in the last lecture on 'duality', the absence of exotic states is closely connected with exchange degeneracy. The facts that in Eq. (15) no resonance occurs with $I > 1$ and that the $\rho$ and $f_0$ trajectories are degenerate are connected, and represent just a particular realization of the arguments presented earlier.

ii) **The Adler consistency condition.** A condition derived by Adler from current algebra states that the $\pi\pi$ scattering amplitude has to vanish at the symmetry point $s = t = u = \mu_\pi^2$ when one pion has zero mass. One notes from Eq. (16) that $V(s,t)$ does indeed have a zero at

$$1 - \alpha(s) - \alpha(t) = 0,$$

which can be identified with the Adler zero if

$$\alpha(\mu^2) = \frac{1}{2}.$$  \hspace{1cm} (33)

Assuming the $\rho$ mass to be 764 MeV, this gives $\alpha(0) = 0.48$, which is not far from the intercept of the $\rho$-trajectory determined empirically from Regge fits. Once Eq. (33) is satisfied, one sees that the whole amplitude $T$ vanishes at the Adler point.

iii) Existence of daughter trajectories. As we have seen above, the residue at $\alpha_s = \ell$ is in general a polynomial of order $\ell$ in $\cos \theta_s$. The maximum spin content is thus $\ell$, as already stated. However, the polynomial being in general not identical to the Legendre polynomial $P_\ell(\cos \theta_s)$, there will also be components of lower spins. These are called daughter states which are degenerate with the parent states. Thus, for example, the $\rho$ meson in Eq. (15) will be degenerate with another spin zero state with $I = 0$ called the $\epsilon$, while the $f_0$ is degenerate with a spin 1 state with $I = 1$ called the $\rho'$. Now although there is no clear evidence for the existence of these states, it has long been conjectured that the $\epsilon$ can explain the asymmetry in $\rho^0$ decay, while the $\rho'$ has often been invoked to improve Regge fits and to 'explain' the $1/q^4$ dependence of the proton electromagnetic form factors. The occurrence of daughter trajectories seems to be a general feature of dual resonance models. Whether the daughter states can be regarded as real resonances, however, remains to be clarified.
In spite of its many beautiful features, the Veneziano model is not free from diseases. By far the most serious of these are (i) the problem of ambiguities, and (ii) the question of unitarity.

i) We have shown that the amplitude (15) does possess all the properties listed at the beginning of the lecture. The question naturally arises whether there are other functions satisfying the same condition, and if so how different they can be from (15). Unfortunately, the question has not been completely answered. A wide class of such solutions are indeed known, which however are not very different in structure from Eq. (15). Consider the modification

\[ V(s,t) \rightarrow V(s,t) + V'(s,t) \]

\[ V'(s,t) = \beta' \text{Tr}(\tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4}) \frac{\Gamma[m - \alpha(s)] \Gamma[n - \alpha(t)]}{\Gamma[\ell - \alpha(s) - \alpha(t)]}, \quad (34) \]

with \( m, n > 1, \ell > m + n \). It can readily be seen that the new \( V(s,t) \) when substituted in Eq. (15) will also possess all the required properties. It follows then that any convergent series of such terms as \( V' \) when added to \( V \) will give an equally good amplitude as far as the properties (i) to (v) are concerned. These are the so-called satellite terms. Such terms in general only modify the coupling of trajectories at the daughter levels. Thus, they may not be too important for a first approximation. Also, there are some weak theoretical arguments connected with factorization, which favour the original form (15) with no satellites. Nonetheless, the possibility of introducing satellites represents such a large degree of ambiguity, that the predictive power of the model is drastically weakened.

ii) The model amplitude (15) is not unitary. This can easily be seen since the function has only poles on the real axis, whereas a unitary amplitude should have cuts corresponding to thresholds of elastic and inelastic channels, while its poles corresponding to resonances should move off the real axis onto the unphysical Riemann sheet. For this reason, when taking the Regge limit of (15), we had to simulate this by artificially approaching infinity along a ray at an angle to the real axis. A connected question is that of the Pomeranchuk 'trajectory'.
On the one hand, we have good reason to believe that the Pomeron really represents the shadow effect of inelastic channels via unitarity. On the other, we know that the Pomeron is typically non-dual and therefore cannot be contained in such an amplitude as (15). Attempts have indeed been made to 'unitarize' the Veneziano model, but have as yet met with no success. For phenomenological purposes, one may simulate the effects of unitarity by artificially introducing an imaginary part to the trajectory function $\alpha$. The poles of (15) will then move off the real axis, but their residues will now, in general, not be polynomials in $\cos \theta$, and our arguments given above will be only approximately valid.

In addition to the two main problems discussed above, there are further problems in the Veneziano model such as the parity-doubling of certain trajectories, which are not realized in nature. Though serious in themselves, they may not be quite as overwhelming in comparison with the two preceding ones.

In view of these difficulties, it is perhaps not clear to many people why some theoreticians are so enthusiastic about the model. The reason for this enthusiasm is twofold:

i) At least for certain idealized cases, the Veneziano model can be generalized to processes with any number of external lines. This generalized model, in addition to all the properties possessed by the original Veneziano model for four-line processes discussed above, has the further attractive feature of being consistent with the bootstrap hypothesis in the following sense: If one takes a bound state between two particles in the amplitude for an $N$-line process, the residue at the pole reduces to just the amplitude for an $(N - 1)$-line process. Although the difficulties mentioned previously still remain, it is nonetheless highly non-trivial to have a model that consistently treats all hadronic processes on the same basis. For this reason, the theoretical interest in the model is understandable.

ii) In the few cases where the Veneziano model has been applied seriously in phenomenology, the success has been impressive. It is true that in all such applications, the difficulties mentioned above were only either ignored or artfully avoided. One may thus dispute the ultimate significance of the agreement found with experiment. However, at this
stage of utter confusion in our knowledge of hadronic processes, the value of a concise formula such as this, which can summarize so much information in terms of so few parameters, should not be underestimated. As an example, I quote the fit of Petersson and Törnqvist\(^{16}\) to the reaction \(K^- p \rightarrow \pi^+ \pi^- \Lambda\) in terms of only one parameter, the normalization. Two diagrams from their paper are quoted in Figs. 10 and 11.

\[
\begin{align*}
\text{Fig. 10} & \quad \text{The energy dependence of the total cross-section and of the partial cross-sections for } Y_1^{*\pm} \text{ production in the reaction } K^- p \rightarrow \pi^+ \pi^- \Lambda \text{ (Ref. 16).}
\end{align*}
\]
Fig. 11 The percentage effective mass distributions at 3 GeV/c (Ref. 16).
FOOTNOTES AND REFERENCES

1) On the Veneziano model alone, the list compiled by Lovelace contains more than 400 references between Aug. 1968 and July 1969.


3) e.g. A. Della Selva, L. Masperi and R. Odorico, Nuovo Cimento 54 A, 979 (1968).

4) A. Della Selva et al., contribution to the 14th Int. Conf. on High-Energy Physics, Vienna (1968), paper 621.


6) For a general review of duality, see M. Jacob, lecture notes at the Schladming Winter School; CERN preprint, TH 1010 (1969).


8) C.B. Chiu and A. Kotanski, Nuclear Phys. 7, 615 (1968);
   ibid. 8, 553 (1969).


10) This may be just a reflection of the special mentality of physicists working on strong interactions.

11) See, for example, Chan Hong-Mo, K. Kajantie and G. Ranft, Nuovo Cimento 49, 157 (1967).


15) See, for example, Chan Hong-Mo, CERN preprint, TH 1057 (1969).

CHIRAL DYNAMICS

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INTRODUCTION

Our present picture of the physics of elementary particles is based on the classification of interactions according to their strength:

- strong interactions (coupling of order 1);
- electromagnetic interactions (coupling $\alpha = e^2/(4\pi \hbar c) \approx 1/137$);
- weak interactions (coupling $G$, Fermi constant, where $Gm_N^2 \sim 10^{-5}$).

Perhaps the CP violating phenomena are due to a new, very weak, interaction. As a rule, whenever a new, weaker interaction is imagined switched on, some conservation law is violated.

The strong interactions conserve charge, baryon number, strangeness and isospin and are invariant under C, P and T separately. Because of their relative weakness, electromagnetic and weak interactions can be treated to lowest order and used as a probe to study strong interactions, or more precisely, the hadronic currents which act as sources for the electromagnetic and weak interactions. The aim of a theory of strong interactions is to calculate not only purely strong phenomena, such as scattering and creation processes of hadrons (S matrix of strong interactions), but also the matrix elements of currents (form factors). The absence of a completely dynamical theory of strong interactions, due mostly to the impossibility of using perturbation theory, forces us to use group theoretic arguments, which succeed in connecting S matrix parameters to form factor parameters.
CHIRAL TRANSFORMATIONS

The hadronic sources of the weak and electromagnetic interactions are the vector and the axial vector current. Restricting ourselves at first to processes with no change of strangeness, we have a vector isovector current \( V^i_\mu \) (\( i = 1, 2, 3 \)), and an axial vector current \( A^i_\mu \) which is also an isovector. The vector current is conserved and the axial vector current partially conserved (it would be conserved in the ideal limit of vanishing pion mass). One assumes further that \( V^i_\mu \) is actually the isospin current. The isospin group, having generators

\[
V^i = -\int V^i_o \, d^3x
\]

is also known to play an important role as a symmetry of the strong interactions. Its structure relations are

\[
[V^i, V^j] = i \varepsilon_{ijk} V^k
\]

where \( \varepsilon_{ijk} \) is the usual totally antisymmetric symbol (\( \varepsilon_{123} = 1 \), \( \varepsilon_{213} = -1 \), etc.). It is natural to assume that the axial vector charges

\[
A^i = -\int A^i_o \, d^3x
\]

also play a role as generators for a symmetry of the strong interactions. Clearly, since \( A^3 \) is an isovector,

\[
[A^i, A^j] = i \varepsilon_{ijk} A^k
\]

One usually completes the structure relations by assuming that

\[
[A^i, A^j] = i \varepsilon_{ijk} V^k
\]

a strong assumption, having far reaching consequences. With the above commutation relations \( V^i \) and \( A^i \) are the generators of the group \( SU_2 \times SU_2 \). The phenomenological fields associated with the physically known particles have well-known transformation properties under the isospin group. For instance, under an infinitesimal isospin transformation of parameters \( \beta = (\beta^1, \beta^2, \beta^3) \), the nucleon field

\[
N = \begin{pmatrix} p \\ n \end{pmatrix}
\]
and the pion field $\vec{\psi}$ transform as

$$\delta N = -i \left[ 2 \vec{V} \cdot \vec{\beta}, N \right] = i \vec{\beta} \cdot \vec{\tau} N,$$

and

$$\delta \vec{\varphi} = -i \left[ 2 \vec{V} \cdot \vec{\beta}, \vec{\varphi} \right] = -2 \vec{\beta} \times \vec{\varphi},$$

while an isoscalar (scalar) field $\sigma$, associated with a possible s-wave isoscalar pion-pion resonance, would remain unchanged

$$\delta \sigma = -i \left[ 2 \vec{V} \cdot \vec{\beta}, \sigma \right] = 0.$$

In general, a field $\psi_a$ will transform linearly with appropriate isospin matrices $(\vec{\Pi})_{ba}$

$$\delta \psi = -i \left[ 2 \vec{V} \cdot \vec{\beta}, \psi \right] = 2i \vec{\beta} \cdot \vec{\tau} \psi,$$

and the corresponding particles will form an isospin multiplet.

For the case of an infinitesimal transformation generated by the axial charges (a chiral transformation), one could similarly assume that the fields transform linearly. Thus, one could assign the pion field $\vec{\varphi}$, together with the field $\sigma$, to the four-dimensional representation of $SU_2 \times SU_2$. This is done, for instance, in the $\sigma$ model \(^1\), where the infinitesimal chiral transformations are taken to be

$$\delta \sigma = -i \left[ 2 \vec{A} \cdot \vec{\alpha}, \sigma \right] = 2 \vec{\alpha} \cdot \vec{\varphi},$$

$$\delta \vec{\varphi} = -2 \vec{\alpha} \cdot \vec{\sigma},$$

and

$$\delta N = i \vec{\alpha} \cdot \vec{\tau} \chi \delta N;$$

here $\vec{\alpha}$ is the parameter of the transformation. However, in recent times, it has been understood \(^2\) that the chiral transformations are more appropriately realized as non-linear transformations on the fields. Thus, the pion field can be taken to undergo the non-linear and inhomogeneous transformation

$$\delta \vec{\varphi} = -i \left[ 2 \vec{A} \cdot \vec{\alpha}, \vec{\varphi} \right] =$$

$$= \frac{1}{\alpha} \left[ \vec{\alpha} (1 - \alpha^2 \vec{\varphi}^2) + 2\alpha^2 \vec{\varphi} \cdot (\vec{\alpha} \cdot \vec{\varphi}) \right],$$
while the nucleon field transforms as

$$\delta N = -i \left[ 2 \vec{A} \cdot \vec{\sigma}, N \right] =$$

$$= i a \left( \vec{q} \times \vec{a} \right) \cdot \vec{t} N.$$

Here $a$ is a constant which, as we shall see later, is given by the reciprocal of the pion decay constant

$$a = \frac{1}{F_p}.$$

In general, a field $\psi$ having isospin $\vec{T}$ transforms as

$$\delta \psi = -i \left[ 2 \vec{A} \cdot \vec{\sigma}, \psi \right] =$$

$$= 2 i a \left( \vec{q} \times \vec{a} \right) \cdot \vec{t} \psi$$

The non-linear chiral transformation of any field is completely determined by its isospin properties and there is no enlargement of the isospin multiplets when one includes chiral transformations. For instance, since the $\sigma$ is an isoscalar, one has also

$$\delta \sigma = -i \left[ 2 \vec{A} \cdot \vec{\sigma}, \sigma \right] = 0$$

The physical consequences of chiral invariance are not of the same kind as those of a linearly realized symmetry. Instead of relations between different members of a linear multiplet, chiral invariance gives rise to connections between processes involving different numbers of (soft) pions. In order to derive these physical consequences, one proceeds to construct Lagrangians invariant under the above transformations.

Observe that, if one adopts the non-linear realizations of the group, it is not necessary to introduce the $\sigma'$ field, since the group can be realized on the pion degrees of freedom alone.

CHIRAL LAGRANGIAN

The invariant Lagrangian can be constructed very easily provided one introduces appropriate "covariant derivatives". Observe that the infinitesimal chiral transformation of the nucleon field can be written as
\[ S N = i \vec{R} \cdot \vec{\tau} N \]

where
\[ \vec{R} = a \vec{\varphi} \times \vec{a}. \]

Now, it is easy to verify that the vector
\[ \delta \vec{V}_\mu = a^2 \frac{\partial \vec{\varphi} \times \vec{\varphi}}{1 + a^2 \vec{\varphi}^2} \]
transforms quite simply as
\[ \delta \vec{V}_\mu = -2 \vec{R} \times \vec{V}_\mu + \partial_\mu \vec{R}. \]

The covariant derivative of the nucleon field is defined, in analogy
with electrodynamics or the Yang-Mills theory 3), as
\[ D_\mu N = \partial_\mu N - i \vec{V}_\mu \cdot \vec{\tau} N, \]
and transforms exactly like the nucleon field
\[ \delta (D_\mu N) = i \vec{R} \cdot \vec{\tau} (D_\mu N). \]
Similarly, for a field \( \psi \) of isospin matrices \( \vec{\tau} \), the covariant
derivative is
\[ D_\mu \psi = \partial_\mu \psi - i 2 \vec{V}_\mu \cdot \vec{\tau} \psi \]
The pion field has to be treated in a special way. Its covariant
derivative is defined as
\[ D_\mu \vec{\varphi} = \frac{\partial_\mu \vec{\varphi}}{1 + a^2 \vec{\varphi}^2} \]
and transforms as an isospin one field
\[ \delta (D_\mu \vec{\varphi}) = -2 \vec{R} \times (D_\mu \vec{\varphi}). \]

It is now immediate that the Lagrangian density
\[ L = -\frac{1}{2} (D_\mu \vec{\varphi})^2 + i \vec{N} (\gamma_\mu D_\mu + m_\nu) N - \frac{f}{m_\pi} \vec{N} \gamma_\mu \gamma_5 \vec{\tau} N. D_\mu \vec{\varphi} \]
is invariant under isospin and chiral transformations. The first two
terms are obtained simply by replacing the ordinary derivatives by co-
variant derivatives in the Lagrangian densities for a free massless pion
and for a free nucleon.
The last term is a covariant generalization of the pseudovector nucleon-pion interaction. Writing everything out, we have

\[
L = -\frac{1}{2} \left( 1 + \alpha^2 \phi^2 \right)^2 \left( \partial_\mu \phi \right)^2 + i \tilde{N} \left( \gamma_\mu \partial_\mu + m_N \right) N +
- \left( 1 + \alpha^2 \phi^2 \right)^{-1} \int \frac{f}{m_\pi} \tilde{N} \gamma_\mu \gamma_5 \tilde{\tau} N \cdot \partial_\mu \phi + \alpha^2 \tilde{N} \gamma_\mu \tilde{\tau} N \cdot \left( \tilde{\phi} \times \partial_\mu \phi \right)
\]

Expansion of the reciprocals generates multipion vertices which are related uniquely to the simplest vertices. These are the trilinear pseudovector pion-nucleon vertex

\[
- \frac{f}{m_\pi} \tilde{N} \gamma_\mu \gamma_5 \tilde{\tau} N \cdot \partial_\mu \phi,
\]

the quadrilinear current-current contact term

\[
- \alpha^2 \tilde{N} \gamma_\mu \tilde{\tau} N \cdot \left( \tilde{\phi} \times \partial_\mu \phi \right)
\]

and the pion-pion coupling

\[
a^2 \phi^2 \left( \partial_\mu \phi \right)^2.
\]

About the first two one knows, from low energy pion-nucleon scattering data, that the pseudovector constant is

\[
f \approx 1.0 \quad , \quad \frac{f^2}{4\pi} \approx 0.08
\]

and that \( \alpha \approx 0.3/m_\pi \); the current-current contact term describes correctly the isospin dependence of the low energy pion-nucleon scattering.

To \( L \) one must add a symmetry-breaking term which also gives the pion a mass

\[
L_{s.b.} = - \frac{1}{2} \frac{m_\pi^2}{1 + \alpha^2 \phi^2} \phi^2 + \cdots
\]

where the dots denote higher powers of \( \phi \). Different choices for \( L_{s.b.} \) give different pion-pion interactions. For instance, the function

\[
L_{s.b.} = - \frac{1}{2} \frac{m_\pi^2}{1 + \alpha^2 \phi^2} \phi^2
\]

is the sum of a chiral invariant and of the fourth component of a chiral four-vector and is appealing because of these simple transformation properties. It seems also to be favoured by experiment.
CURRENTS

Given a Lagrangian invariant under a group, one can construct by standard methods (see Appendix) the corresponding conserved currents. If we apply the general procedure to $L$, we obtain a vector current $\vec{V}_\mu$:

$$\vec{V}_\mu = \vec{N}_\mu - \frac{2}{1 + A^2 q^2} \left[ \frac{f}{m_\pi} \vec{q} \times \vec{N}_\mu - A q \vec{q} \times (\vec{q} \times \vec{N}_\mu) \right] - \frac{\vec{q} \times \partial \vec{\rho}}{(1 + A^2 q^2)^2}$$

and an axial vector current $\vec{A}_\mu$ given by

$$\frac{a m_\pi}{f} \vec{A}_\mu = \vec{N}_\mu - \frac{2 A^2 q}{1 + A^2 q^2} \left[ \frac{m_\pi}{f} \vec{q} \times \vec{N}_\mu - \vec{q} \times (\vec{q} \times \vec{N}_\mu) \right] +$$

$$+ \frac{m_\pi}{2f} \frac{\partial \vec{q}}{1 + A^2 q^2} + \frac{2 m_\pi}{f} \frac{A^2 q \vec{q} \times (\vec{q} \times \partial q)}{(1 + A^2 q^2)^2}.$$

Here we have used the abbreviation

$$\vec{N}_\mu = \vec{N} \gamma_\mu \frac{\bar{c}}{2} \bar{N}; \quad \vec{N}_\mu^S = \vec{N} \gamma_\mu \sigma \frac{\bar{c}}{2} \bar{N}.$$

The first terms in the expansion of the expressions for the currents are

$$\vec{V}_\mu = \vec{N} \gamma_\mu \frac{\bar{c}}{2} \bar{N} - \vec{q} \times \partial \vec{q}$$

and

$$\vec{A}_\mu = \frac{f}{a m_\pi} \vec{N} \gamma_\mu \sigma \frac{\bar{c}}{2} \bar{N} + \frac{1}{2} \partial \vec{q} \bar{q}.$$

Comparing the coefficients, we obtain

$$- \frac{G_A}{G_V} = \frac{f}{a m_\pi},$$

a relation connecting the strong $S$ matrix parameters $f$ and $a$ to the form factor parameter $-G_A/G_V \approx 1.2$; we see that it is well satisfied. This is a form of the Adler-Weisberger relation 4). We also must identify

$$F_\pi = \frac{i}{a}$$

with the pion decay constant, again in good agreement with experiment. Finally, combining the two above relations, we can write
\[
-\frac{G_{A}}{G_{V}} = \frac{f}{m_{\pi}} F_{\pi}
\]

which is the Goldberger-Treiman relation \(^5\) expressed in terms of the pseudovector pion-nucleon coupling constant.

**DECAY OF THE \(\pi^{0}\) INTO TWO \(\gamma\)'S**

For some time chiral invariance was thought to face a serious difficulty in the problem of the decay \(\pi^{0} \rightarrow 2\gamma\). This stems from the impossibility of finding a chiral invariant interaction which contains the desired term, proportional to

\[
\varphi^{3} \varepsilon_{\lambda\mu\nu\rho} F_{\lambda\rho} F_{\nu\rho},
\]

where \(\varphi^{3}\) is the field of the neutral pion, \(F_{\lambda\rho} = \partial_{\lambda} A_{\rho} - \partial_{\rho} A_{\lambda}\) is the electromagnetic field and \(\varepsilon_{\lambda\mu\nu\rho}\) is the totally antisymmetric tensor with \(\varepsilon_{0123} = 1\). Alternatively one could argue from the partial conservation equation

\[
\partial_{\mu} A_{\mu}^{3} = \frac{m_{\pi}^{2}}{2a} \varphi^{3}.
\]

Veltman and Sutherland \(^6\) showed that this equation, together with gauge invariance, implies that the amplitude for the decay \(\pi^{0} \rightarrow 2\gamma\), extrapolated to zero pion mass, vanishes. This difficulty is resolved by the realization that, with a minimal electromagnetic interaction, in a theory with charged spin one-half particles, the correct partial conservation equation is

\[
\partial_{\mu} A_{\mu}^{3} = \frac{m_{\pi}^{2}}{2a} \varphi^{3} + \frac{\alpha}{4\pi} \varepsilon_{\lambda\mu\nu\rho} F_{\lambda\rho} F_{\nu\rho}
\]

The extra term on the right-hand side is called the anomalous term. By using the same Veltman-Sutherland argument, it gives rise now to a finite amplitude for the \(\pi^{0}\) decay.

To see how the additional term arises, let us begin with a very simple problem. Consider the Dirac equation for a spinor under the influence of an external electromagnetic potential \(A_{\lambda}\):
\[
\left\{ \gamma^\lambda \partial_\lambda + m - i e \gamma^5 \partial_5 \right\} \psi = 0,
\]
and the adjoint equation
\[
\bar{\psi} \left\{ \gamma^\lambda \partial_\lambda - m + i e \gamma^5 \partial_5 \right\} \psi = 0.
\]
Multiplying the first equation by \( \bar{\psi} \) on the left, the second by \( \psi \) on the right and adding, one obtains the conservation equation for the vector current
\[
\partial_\lambda (\bar{\psi} \gamma^\lambda \psi) = 0.
\]
Similarly, if one multiplies the first by \(-\bar{\psi} \gamma_5\), the second by \( \gamma_5 \psi \) and adds, one obtains the partial conservation equation for the axial vector current
\[
\partial_\lambda (\bar{\psi} \gamma_5 \gamma^\lambda \psi) = 2m \bar{\psi} \gamma_5 \psi.
\]
One may wish to verify these equations in perturbation theory. No problem arises with the vector equation. However, it was observed by Bell and Jackiw \(^1\) and by Adler \(^2\) that the axial vector equation is not satisfied whenever, as in our case, the spinors are coupled to vector fields. One can already see this in the simplest matrix element, the vacuum expectation value.

The difficulty comes from the triangle diagram

\[\includegraphics{triangle_diagram.png}\]

which corresponds, up to numerical factors, to the integral
\[
-e^2 \int \frac{d^4q}{16\pi^2} \frac{1}{i(q + ik_1) + m} \frac{1}{i(q + ik_2) + m} \frac{1}{i(q - ik_3) + m}.
\]
By power counting this integral is linearly divergent, however, if one integrates symmetrically, it is actually convergent, but ambiguous.

Multiplication by $(k_1+k_2)\lambda$ should reproduce the corresponding matrix element of the right-hand side of the partial conservation equation, which is given, up to the factor $2m$, by the (convergent) diagram

\[ \begin{figure}[h] \centering \includegraphics[width=0.3\textwidth]{diagram.png} \end{figure} \]

This can indeed be verified formally, provided one performs some transformations which involve a shift of the integration variable. It is well known, however, that shifts of the integration variable may give incorrect results when performed in divergent integrals, and the result is actually ambiguous. The ambiguities occurring here had been observed essentially already long ago by Steinberger in his calculation of the $\pi^0 \rightarrow 2\gamma$ decay. To obtain an unambiguous answer, one must regularize the integral in a gauge invariant way. The idea is to replace the amplitude, which we call $F(m^2)$, by the more convergent expression

\[ F(m^2) - F(m'^2) = -\int_{m^2}^{M^2} \frac{d}{d(m'^2)} F(m'^2) \, dm'^2, \]

where $M$ is a large mass which ultimately will tend to infinity (in some cases more subtraction terms are necessary, but for our integral one is enough). An explicit expression for the regularized triangle integral was given by Rosenberg. In this expression, the limit $M\rightarrow\infty$ is already performed, and one can check directly that the partial conservation equation given above is not satisfied. Instead, one obtains the equation

\[ \partial_\lambda (\overline{\psi} f_5 \psi) = 2m \overline{\psi} f_5 \psi + \frac{\alpha}{4\pi} \varepsilon^{\mu\nu\rho} F_{\mu\nu} F_\rho, \]

where $\alpha = \frac{\alpha}{4\pi}$. 
In order to see how the extra term arises, we may proceed as follows. According to Gupta, the regularization procedure can be described as the introduction of a regularizing spinor field $\chi$ of mass $M$. To ensure that the closed $\chi$ loops contribute with a sign opposite to that of the $\gamma^5$ loops, $\chi$ is quantized according to Bose statistics (which requires an indefinite Hilbert space metric). The partial conservation equation is therefore replaced by

$$\partial_\lambda(\overline{\psi} \gamma^\lambda \gamma^5 \psi + \overline{\chi} \gamma^\lambda \gamma^5 \chi) = 2m \overline{\psi} \gamma^5 \psi + 2M \overline{\chi} \gamma^5 \chi.$$ 

As $M \to \infty$, the last term $2M \overline{\chi} \gamma^5 \chi$ does not vanish because the diagram

![Diagram](image)

behaves like $1/M$ for large $M$. Instead, one obtains in the limit the finite contribution $\alpha/4\pi \epsilon_{\lambda\mu\nu\rho} F_{\lambda\mu} F_{\nu\rho}$.

The fact that the gauge invariant regularization gives rise in perturbation theory to the additional term, whilst the formal use of the operator equations of motion gives no such term, is - at first - rather puzzling. However, it can be shown that a careful treatment of the operator equations also gives the correct additional term. This is achieved by giving a more precise definition of products of field operators at the same space time point. Typically, an expression such as $\overline{\psi}_\alpha(x) \gamma^\mu \psi_\beta(x)$ is to be understood as the symmetrized limit of the gauge invariant expression

$$\overline{\psi}_\alpha(x) \gamma^\mu \psi_\beta(x') e^{-ie\int_{x'}^{x} A_{\lambda} dx_{\lambda}}$$

as $x' \to x$, (the integral can be taken over a straight line). If we take the Dirac equation at the point $x'$ and its adjoint at the point $x$ and combine them, we obtain

$$[\partial_\lambda - ie A_\lambda(x) + \partial_\lambda' + ie A_\lambda(x')] \overline{\psi}(x) \gamma^\lambda \gamma^5 \psi(x') = 2m \overline{\psi}(x) \gamma^5 \psi(x').$$
As \( x' \to x \), the vector potential \( A_{\mu} \) does not drop out of the equation, because the product \( \bar{\psi}(x) \gamma_{\mu} \gamma_5 \psi(x') \) is singular for \( x' \to x \). More precisely

\[
\bar{\psi}(x) \gamma_{\mu} \gamma_5 \psi(x') \sim -\frac{ie}{2\pi^2} \frac{(x-x')_{\mu}}{|x-x'|^2} F^{\mu\lambda}_{\nu\rho} \]

where \( F_{\mu\nu} = \frac{i}{2} \varepsilon_{\mu\nu\lambda\rho} F_{\lambda\rho} \). Carrying out this argument one obtains the same additional term as with the regularization method.

The additional term in the partial conservation equation depends upon the kind of charged spinor fields which can contribute to the triangle diagrams. If the various spinor fields \( \psi_j \) have charges \( Q_j e \) and the axial vector current has the form

\[
\sum_j g_j \bar{\psi}_j \gamma_{\mu} \gamma_5 \psi_j + \ldots
\]

then the additional term in the partial conservation equation is:

\[
S \frac{\alpha}{4\pi} \varepsilon_{\lambda\mu\nu\rho} F_{\lambda\mu} F_{\nu\rho}
\]

where

\[
S = \sum_j g_j Q_j^2
\]

This permits one to distinguish among various models of strong interactions. If only the proton contributes one has \( S = 1 \); in the quark model \( S = \frac{1}{3} \). The experimental lifetime of the \( \pi^0 \), \( \tau^{-1} = (1.12 \pm 0.22) \times 10^{15} \text{sec}^{-1} \), corresponds to \( |S| = 0.88 \). The quark model comes out very badly. This argument assumes that higher order diagrams do not change the coefficient of the anomalous term. That this is so has been argued by Adler and Bardeen, who have studied the higher order diagrams and find no contributions from them.

Similar considerations can be applied to the decay \( \eta^0 \to 2\gamma \). However, here one has to take into account \( \text{SU}(3) \) symmetry breaking and \( \eta^0 - \chi^0 \) mixing. As a consequence one does not come up with a definite prediction but rather with relations among various amplitudes which include that for \( \chi^0 \to 2\gamma \).
APPENDIX

We show how to construct the currents from the Lagrangian. Let the Lagrangian density be a function of certain fields $\psi_i$ and of their first derivatives

$$
L = L(\psi_i, \partial_\mu \psi_i).
$$

Hamilton's principle states that

$$
\delta \int L \, d^4x = 0
$$

for arbitrary variations of the fields $\delta \psi_i$, which vanish outside of a finite space time volume. Now (sum over repeated indices)

$$
\delta \int L \, d^4x = \int \delta L \, d^4x = \int \left[ \frac{\partial L}{\partial \psi_i} \delta \psi_i + \frac{\partial L}{\partial (\partial_\mu \psi_i)} \partial_\mu (\delta \psi_i) \right] d^4x =
$$

$$
= \int \left[ \frac{\partial L}{\partial \psi_i} - \partial_\mu \frac{\partial L}{\partial (\partial_\mu \psi_i)} \right] \delta \psi_i \, d^4x,
$$

where the last equality involves an integration by parts. Since the $\delta \psi_i$ are arbitrary, subject to the mentioned boundary conditions, we obtain the Lagrange equations of motion

$$
\frac{\partial L}{\partial \psi_i} - \partial_\mu \frac{\partial L}{\partial (\partial_\mu \psi_i)} = 0.
$$

Now let us subject the fields to an infinitesimal transformation of some group, which may act on the fields linearly or non-linearly

$$
\delta \psi_i = F_i^s(\psi) y^s
$$

where $y^s$ are infinitesimal parameters of the group. The corresponding infinitesimal change of the Lagrangian density is
\[ \delta L = \frac{\partial L}{\partial \delta \psi_i} \delta \psi_i + \frac{\partial L}{\partial (\partial_\mu \psi_i)} \partial_\mu (\delta \psi_i) = \]

\[ = \partial_\mu \left( \frac{\partial L}{\partial (\partial_\mu \psi_i)} \delta \psi_i \right) + \frac{\partial L}{\partial (\partial_\mu \psi_i)} \partial_\mu (\delta \psi_i) = \]

\[ = \partial_\mu \left[ \frac{\partial L}{\partial (\partial_\mu \psi_i)} \right] \delta \psi_i = \partial_\mu \left[ \frac{\partial L}{\partial (\partial_\mu \psi_i)} F_{is} (\psi) \right] \phi_s . \]

Here we have made use of Lagrange's equations of motion. Now, if the Lagrangian density is invariant under the group transformations, then

\[ \delta L = 0 \]

for arbitrary values of the parameters \( \phi_s \). Therefore, if one defines the currents

\[ J_{\mu s} \equiv \kappa \frac{\partial L}{\partial (\partial_\mu \psi_i)} F_{is} (\psi) , \]

with \( \kappa \) a normalization constant, one obtains the conservation equations

\[ \partial_\mu J_{\mu s} = 0 \]

On the other hand, if the Lagrangian is not invariant, its change must be linear in the infinitesimal parameters, and so

\[ \delta L = h_s (\psi) \phi_s . \]

In this case one obtains the partial conservation equations

\[ \partial_\mu J_{\mu s} = \frac{1}{\kappa} h_s (\psi) . \]

There are as many currents as parameters in the group. The vector and axial vector currents given in the text can be obtained in this way, using the infinitesimal isospin and chiral transformations on the pion and nucleon fields.
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INTRODUCTION TO PHOTOPRODUCTION

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1. INTRODUCTION

Photoproduction has been covered in the last few years by a series of review articles 1). It is not the aim of this paper to produce out of this material another review article. It is rather intended to present the simple general framework of photoproduction and to supplement a few topics in the review papers in order to facilitate their understanding.

2. GENERAL MATRIX ELEMENT FOR PHOTOPRODUCTION

The general photoproduction reaction is of the form

\[ \gamma + \text{hadron} \rightarrow \text{hadrons}. \]

In most cases of practical interest the initial state hadron is a nucleon (state vector \( |N\rangle \)). The final state can be any hadron state allowed by the conservation laws, e.g., \( \pi N, \pi\pi \ldots N, N\bar{N}\pi\pi\ldots, K\bar{K}\pi\pi\ldots, K\gamma \ldots \). We shall denote this final hadron state by \( |N\pi\rangle \). In most cases it will be sufficient to treat photoproduction in lowest order of the electromagnetic coupling. In this approximation the \( S \) matrix element can be written in the form

\[ S_{fi} = \int_{\text{p}} \langle N\pi | J_{\mu}^{\gamma}(x) | N \rangle \langle 0 | A_{\mu}^\ast | f \rangle d^4 x. \]

Here \( J_{\mu}^{\gamma} \) is the electromagnetic current operator. The first term \( \langle N\pi | J_{\mu}^{\gamma}(x) | N \rangle \) contains only hadron states and is therefore determined by the strong interaction. It is this term
about which we learn something in photoproduction. The second term \( \langle 0 | A^\mu | g \rangle \) is purely electromagnetic, it is given by the wave function of the incident photon:

\[
\langle 0 | A^\mu | g \rangle = (2\pi)^{-3/2} \varepsilon^\mu \cdot e^{-i k \cdot x} \quad (2.2)
\]

- \( k \) = four momentum vector of the incident photon
- \( k \) = four vector of the incident photon
- \( \varepsilon \) = polarization four vector.

We have

\[
|\varepsilon|^2 = 1 \quad \text{(normalization)} \quad (2.3)
\]

\[
\varepsilon \cdot \hat{k} = 0 \quad \text{(transversality condition)} \quad (2.4)
\]

A possible base to describe the photon state are the two helicity states (component of the spin angular momentum of the photon on its direction of propagation = \( \hat{k}_{\pm} \) or = \( -\hat{k}_{\mp} \)). Such a state corresponds to circularly polarized radiation. For example, for a circularly polarized photon with energy \( E \), propagating in the \( z \)-direction, we would have:

\[
\hat{k} = \left[ E, 0, 0, 0 \right] \quad \varepsilon = \left[ 0, \sqrt{\pi}, \gamma \sqrt{\pi}, 0 \right]
\]

We now insert equ. (2.2) into equ. (2.1) and introduce the \( x \)-dependence:

\[
\langle N \pi | j_{\mu} | N \rangle = e^{i (\hat{P}_f - \hat{P}_i) \cdot x} \langle N \pi | j_{\mu}^{(0)} | N \rangle
\]

where \( \hat{P}_f \) and \( \hat{P}_i \) denotes the resultant four vector of the final and initial state hadrons, respectively.

Integrating over \( d^4 x \) we get

\[
S_{f,i} = (2\pi)^{3/2} \int d^4 (\hat{P}_i + k - \hat{P}_f) \cdot T_{f,i} \quad (2.5)
\]
The photoproduction amplitude $\overline{T_{f_i}}$ is given by

$$\overline{T_{f_i}} = \varepsilon^\mu \left< \mathcal{N} \pi \right| j_\mu (c) \left| N \right>$$

(2.6)

This amplitude contains the complete information. E.g., the differential CMS cross section for the process

$$J' + N = \text{nucleon + meson is}$$

$$\frac{d\sigma/dQ}{\text{CMSS}} = \frac{(2\pi)^2}{16} \cdot \frac{|q|}{|k|} \cdot |\overline{T_{f_i}}|^2$$

Here $|q|$ and $|k|$ are the magnitudes of the meson and photon three momenta in the CMS, $S = (\text{total CMS energy})^2 = (p_i^2 + k^2) = p_i^2$

3. GAUGE INVARIANCE

A transformation of the polarization four vector $\varepsilon$ of the form

$$\varepsilon \rightarrow \varepsilon' = \varepsilon + \lambda \hat{k}$$

($\lambda = \text{arbitrary scalar parameter}$)

does not change the physical situation.

It leaves the normalization and the transversality conditions equ. (2.3) and equ. (2.4) intact (because of $k^2 = 0$). The general matrix element equ. (2.6) must therefore be invariant under the transformation $\varepsilon \rightarrow \varepsilon'$. This leads to the condition

$$\overline{T_{f_i}}^\mu \left< \mathcal{N} \pi \right| j_\mu (c) \left| N \right> = 0$$

(3.1)

This is the condition which gauge invariance imposes on the photoproduction amplitude. This condition can also be stated as follows: If in the photoproduction matrix element $\varepsilon$ is substituted by $k$, the matrix element must become zero.

The gauge invariance condition equ. (3.1) can be derived directly from current conservation $\partial \bar{j}_\mu / \partial x_\mu = 0$. Applying this to the hadronic current in the photoproduction amplitude we get
\[ \frac{\partial}{\partial x^\mu} \left[ \langle N| J_\mu(x) | N \rangle \right] = \sum_{\nu} \left[ \langle N| J_\nu(x) | N \rangle \right] = 0 \]

Because of \( \frac{p_f}{c} - \frac{p_e}{c} = k \), this leads to equ. (3.1).

4. CURRENT-FIELD IDENTITY

The hadronic electromagnetic current can be connected with the phenomenological fields of the vector mesons \( \phi, \omega \) and \( \phi \), which have the same quantum numbers as the photon \( \phi_{\rho, \omega}^C = 1^- \).

This relation is

\[ J_\mu(x) = - \left[ \frac{m_\rho}{2 \omega} \bar{\psi}(x) \left( \gamma_\mu \gamma_5 \right) \psi(x) + \frac{m_\omega}{2} \bar{\omega}(x) \omega_\mu(x) + \frac{m_\phi}{2} \bar{\phi}(x) \phi_\mu(x) \right] \quad (4.1) \]

The masses \( m_\rho, \omega, \phi \) enter for dimensional reasons, \( \gamma_\mu, \omega, \phi \) are coupling constants. One inserts equ. (4.1) into equ. (2.6) and compares with the matrix element for the reaction (vector meson +N \rightarrow hadrons).

This leads to the relation between photoproduction and vector meson matrix elements

\[ T'(\gamma + A \rightarrow B) = \sum_{\nu = \rho, \omega, \phi} \sqrt{\frac{m_\nu}{2}} \bar{\nu} \bar{\nu} \left( \gamma_\mu \gamma_5 \right) \psi(x) \bar{\psi}(x) \bar{\omega}(x) \omega_\mu(x) \bar{\phi}(x) \phi_\mu(x) \quad (4.2) \]

The index \( \tau \) at \( \bar{\omega} \) means that transverse polarization states for the vector mesons have to be taken. The vector meson matrix element has to be taken for \( \left( \frac{p_A}{c} - \frac{p_B}{c} \right)^2 = 0 \) i.e. mass zero of the vector mesons. This far it is only formalism. In order to come to physical consequences, one must connect equ. (4.2) with the matrix element for real vector mesons with mass \( \neq 0 \). The standard assumption is that the matrix element is not changed much by this change of mass of the vector mesons.

This is the basic assumption of the vector dominance model.

Unfortunately the meaning of this assumption is not unambiguous.
5. **ISOSPIN**

Photoproduction does not conserve isospin. Nevertheless, isospin can be a useful concept. Consider the connection between charge $Q$ and the third component of isospin $I_3$:

$$Q = I_3 + \frac{3 + S}{2}$$

(5; 1)

If we operate on this expression with one of the operators in isospin space, which change $e.g.$ the third component of $I$, the r.h.s. transforms like a mixture of isovector and isoscalar, because changing the charge state of the member of an isospin multiplet does not change its strangeness $S$ or baryon number $B$. The charge operator $Q$ on the l.h.s. transforms therefore also like a mixture of isospin $0$ and $1$.

The charge operator is related to the fourth component of the electromagnetic current.

It is therefore very natural to assume, that the whole electromagnetic current transforms under rotations in isospin space like a mixture of isovector and isoscalar, i.e. like a mixture of isospin $1$ and isospin $0$. We have therefore the following selection rule for photoproduction:

$$\Delta I = 0 \quad \Rightarrow \quad \pm 1$$

This is not a trivial statement. It implies $e.g.$ that the electromagnetic current has no $I = 2$ components.

It would be of great interest to check this experimentally.

6. **SU$_3$ SYMMETRY**

One assumes today that the photon has certain invariance properties under unitary transformations, namely that it transforms like the member of an octet, and that it behaves like an U-spin scalar, i.e. that it has $U = 0$. The latter property is plausible, if one notes, that members of the same U-spin multiplet have the same charge.
The following is an (incomplete) list of tests for this assumed invariance property of the photon:

(i) Let us make the additional assumption that the mass differences in an isospin multiplet are due to electromagnetic effects only. The masses of the hadrons can then be written as a sum of two terms:

\[ m = m_S + m_e \]  \hspace{1cm} (6.1)

The term \( m_e \) is the contribution of electromagnetic effects to the mass, the term \( m_S \) must be the same for all members of an isospin multiplet. If the photon has U-spin = 0, the electromagnetic mass terms \( m_e \) must be the same for all members of an U-spin multiplet. Therefore

\[ m_e (\Sigma^+) = m_e (\Xi^0) \]
\[ m_e (\eta) = m_e (\Xi^-) \]
\[ m_e (\Xi^-) = m_e (\Xi^-) \]

Together with equ. (6.1) we get the relation

\[ m(\rho) - m(\eta) = m(\Sigma^+) - m(\Sigma^-) + m(\Xi^-) - m(\Xi^-) \]

Experimentally\(^3\):

1.h.s = -1.29 \pm 0.01 MeV  \hspace{1cm} r.h.s = 1.37 \pm 0.8 MeV

(The main contribution to the experimental error comes from the \( \Xi^- \) mass). There is a significant agreement, as one sees, if one compares its accuracy (0.8 MeV) with the magnitude of the mass differences of the isospin multiplets (1.3 MeV \( p - \eta \), 7.9 MeV \( \Sigma^+ \Xi^- \), 6.6 MeV \( \Xi^0 \Xi^- \)).

(ii) By similar arguments, one derives for the magnetic moments

\[ \mu_\eta = \mu_\Xi^- \]
\[ \mu_p = \mu_{\Xi^+} \]
\[ \mu_{\Sigma^-} = \mu_{\Xi^-} \]

A bit more involved is the derivation of
\[ \mu_\Lambda = \mu_n / 2 \]

\[ \mu_p + \mu_n = - \mu_\Sigma^- \]

The relations \( \mu_\Lambda = \mu_n / 2 \) and \( \mu_p = \mu_\Sigma^+ \) have been checked experimentally. They should not be exactly satisfied because of mass breaking \(^3\).

\[
\begin{align*}
\mu_\Lambda &= -0.73 \pm 0.16 \\
\mu_n / 2 &= -0.95 \pm 0.043 \\
\mu_p &= 2.79 \pm 0.043 \\
\mu_\Sigma^+ &= 2.5 \pm 0.5 \\
\text{(unit: } e^\pm / 2 m_p c \text{)}
\end{align*}
\]

(iii) \( \Delta^0 \) (1236) and \( \gamma^c \) (1385) are members of an \( U \) spin triplet inside the \( 3/2^+ \) baryon decuplet, similarly \( K^+ \) and \( \pi^+ \) are members of an \( U \) spin doublet inside the \( 0^- \) meson octet.

Consider the reactions

\[
\begin{align*}
\gamma p &\rightarrow \gamma^c K^+ \\
\gamma p &\rightarrow \Delta^0 \pi^+
\end{align*}
\]

If the photon has \( U = 0 \), the initial state is a pure \( U = 1/2 \) state.

Clebsch Gordan coefficients give then for the ratio of the two amplitudes

\[
A(\Delta^0 \pi^+) : A(\gamma^c K^+) = \sqrt{\frac{2}{3}} : -\sqrt{\frac{1}{3}} = -\sqrt{2}
\]

By the same method many other relations can be derived, e.g.

\[
\begin{align*}
A(\gamma p \rightarrow \Delta^c \pi^+) : A(\gamma p \rightarrow \gamma^c K^+ \pi^{\pm}) &= -\sqrt{2} \\
A(\gamma n \rightarrow \Delta^- \pi^+) : A(\gamma n \rightarrow \gamma^c K^+) &= -\sqrt{3} \\
A(\gamma p \rightarrow \Delta^- \pi^+ \pi^+) : A(\gamma p \rightarrow \gamma^c K^+ \pi^+) &= -\sqrt{3/2} \\
A(\gamma p \rightarrow \gamma^c K^+ \pi^+) : A(\gamma p \rightarrow \gamma^c \pi^{\pm} K^+) &= -\sqrt{2} \\
(\Sigma A(\gamma p \rightarrow \pi^+ n) = -\sqrt{3} \Sigma A(\gamma p \rightarrow K^+ \Lambda) - A(\gamma p \rightarrow K^+ \Sigma^\circ) \)
\]
The last equation leads only to a triangular inequality for the corresponding cross sections. This inequality has been checked experimentally at high energies\(^4\). The data are consistent with the inequality for large momentum transfers, but violate it for small ones. This effect is commonly attributed to mass breaking, whose influence should be greatest at small momentum transfers.

(iv) Photon vector meson coupling constants: Consider the coupling constants \(\chi', \gamma, \lambda\) appearing in the current field identity equ. (4.1). In order to establish a connection with \(SU_3\) symmetry, consider the octet of vector mesons, and their U-spin transformation properties. We shall use the quark model (with \(p, n, \lambda\) as basic quark states) to show this.

We want to exhibit first the isospin structure. In terms of quark states we have in the octet:

\[
\begin{align*}
|\rho^c\rangle &= (n\bar{n} - p\bar{p}) / \sqrt{2} & \text{Isospin } I = 1 \\
|\phi'\rangle &= (p\bar{n} + n\bar{p} - 2\lambda\bar{\lambda}) / \sqrt{6} & \text{Isospin } I = 0
\end{align*}
\]

(6.2)

By analogy, we have for the U-spin

\[
\begin{align*}
|A'\rangle &= (n\bar{n} - \lambda\bar{\lambda}) / \sqrt{2} & \mathcal{U} = 1 \\
|B'\rangle &= (n\bar{n} + \lambda\bar{\lambda}) - 2p\bar{p}) / \sqrt{6} & \mathcal{U} = 0
\end{align*}
\]

(6.3)

\[
= -\frac{i}{\sqrt{2}} (|\phi'\rangle + \sqrt{3} |\rho^c\rangle)
\]

The state \(|\phi'\rangle\) is a linear combination of \(|\omega\rangle\) and \(|\Phi\rangle\). Commonly one chooses a \(\omega - \phi\) mixing angle of \(\tan\Theta = 1 / \sqrt{2}\), which will make the physical \(\Phi\) a pure \(\lambda\bar{\lambda}\) state.

Then

\[
|\phi'\rangle = \sqrt{3} |\omega\rangle - \sqrt{\frac{3}{3}} |\Phi\rangle
\]
Inserting this into equ. (6.3) leads to
\[ |B'\rangle = -\frac{i}{\omega} \left( \sqrt{\frac{3}{2}} |\omega\rangle - \sqrt{\frac{3}{2}} |\phi\rangle + \sqrt{3} |\rho\rangle \right) \]  
(6.4)

The photon, being U-spin scalar, transforms the same as \(|B'\rangle\).

Therefore
\[ \sqrt{\frac{3}{2}} : \sqrt{\frac{3}{2}} : \sqrt{3} = 3 : 1 : -\sqrt{2} \]  
(6.5)

Exponentially, this ratio is
\[ \sqrt{3} : \sqrt{3} : \sqrt{3} = 9 : 1 : 2 \]

Experimentally, this ratio is
\[ \sqrt{3} : \sqrt{3} : \sqrt{3} = 9 : (1.1 \pm 0.3) : (1.5 \pm 0.4) \]

consistent with equ. (6.5). Mass breaking changes these ratios.

Independent of mass breaking and of the assumed \(\omega-\phi\) mixing angle the following relation holds, subject to the following three assumptions: 1) Weinberg's first sum rule 2) the photon is a member of a unitary octet 3) Vector meson dominance.

\[ \frac{m_\omega}{3} \overline{T}(\gamma \rightarrow e^+e^-) = m_\omega \overline{T}(\omega \rightarrow e^+e^-) + m_\phi \overline{T}(\phi \rightarrow e^+e^-) \]

Experimentally, it is satisfied within the errors.

Appendix: Selected Topics on the Photoproduction of Vector Mesons.

A1. Connection between \(\bar{\sigma}_{T}(\gamma p)\) and Vector Meson Photoproduction.

This relation offers a very clean test of the vector dominance model.

The amplitudes for forward Compton scattering on the proton and for vector meson photoproduction in the forward direction are connected as follows (vector dominance model, see equ. (4.2)):

\[ T(\gamma p \rightarrow \gamma p)\bigg|_c = \sum_{V=\rho,\omega,\phi} \frac{V_{\pi}\pi}{\mathcal{S}_V} \overline{T}(\gamma p \rightarrow V p)\bigg|_c \]  
(A1)

On the other hand, the total cross section for photoproduction of hadrons \(\sigma_T(\gamma p)\) is connected with the imaginary part of
\[ T(y^p \rightarrow y^p) \bigg|_o \sim \text{via the optical theorem. Calling } \beta = \frac{Re T}{Im T}, \]
the optical theorem reads in terms of cross sections:
\[ \left( \frac{16 \pi}{1+\beta^2} \right) \frac{d\sigma}{dt} (y^p \rightarrow y^p) \bigg|_o^{1/2} = \sigma_T (y^p) \]  
(A2)
Substituting the vector meson dominance relation equ.(A1) into equ. (A 2), we get
\[ \sigma_T (y^p) = 4 \pi \sqrt{\alpha} \sum_{V=\rho,\omega,\phi} \left[ \frac{1}{(1+\beta^2)} \frac{d\sigma}{dt} \bigg|_0 \right]^{1/2} \]  
(A3)

A 2. Photoproduction of Vector Mesons on Heavy Nuclei \(^7\))

The amplitude for photo-rho production in the forward direction on a heavy nucleus can be calculated in terms of a nuclear model, in which one sums over the contributions to rho production from the individual volume elements of the nucleus.
Since rho production proceeds mainly via diffraction, the amplitude builds up in a particularly simple way. The forward production amplitude in such a model is given by
\[ f_A = 2\pi \int_{H} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(z, \ell) \cdot e^{i A_m z} \rho(z', \ell') \cdot e^{i \varphi(z', \ell')} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(z', \ell') \, dz' \, \ell' \]  
(A4)

The term \( f_A \) is the forward production amplitude off a single nucleon, \( \rho(z, \ell) \) is the density distribution of nucleons in the nucleus. The integral sums up rho production over the whole nucleus. The production amplitudes are added with a phase factor
\[ e^{i \Delta m \bar{z}} \], which takes into account the difference of the wave vector of the photon and the rho wave 'inside nuclear matter'.

We have

\[ \Delta m \approx \frac{m_p}{2k} \]  \hspace{1cm} (A 5)

\[ k = (E_{\gamma B}) \text{ photon energy}. \]

The rho amplitude is attenuated, as it propagates through nuclear matter, by the exponential factor, which contains the total rho-nucleon cross section \( \Sigma \). Cross sections for rho production on a number of nuclei and at various energies can be fitted in terms of this model and the parameter \( \Sigma \) determined. This is a way to measure the total rho- (more generally \( \text{vector meson} - \) ) nucleon cross section.

A 3. Total Cross Section for Photoproduction on Heavy Nuclei.

The total cross section of a hadron on a heavy nucleus (mass number \( A \)) can be calculated by the optical model in terms of the total hadron-nucleon cross section. For example the rho-nucleus total cross section is

\[ \Sigma_{\gamma N}^{(pA)} = 4\pi \int_0^\infty \frac{k \, dk}{k} \, \left[ 1 - \exp \left( - \Sigma_{pN} \int f(k, \bar{z}') \, d\bar{z}' \right) \right] \] \hspace{1cm} (A 6)

The notation is the same as in equ. (A 4), with

\[ \Sigma_{pN} = \Sigma = \text{total rho nucleon cross section}. \]

Now we try to bring equ. (A 4) into a similar form (Ting, ref. 1):

We shall assume, that the energy is sufficiently high so that

\[ \Delta m \bar{R} \ll 1 \] (complete coherence). We have then
\[ f_A = \left( \frac{d \sigma}{dt} (yA \to \rho A) \right)^{1/2} = 2 \pi \int_H \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(\tau, k) \exp \left( -\frac{\tau}{2} \right) d\tau \]

(A7)

Under the integral, the term \[ \ldots \ldots = \frac{d}{dz} \left( \frac{e^{\rho}}{\sqrt{2}} \exp \left( -\frac{\rho}{2} \right) \right) \]

therefore the integration over \( z \) in equ. (A 7) can be carried out, the result is

\[ \left( \frac{d \sigma}{dt} (yA \to \rho A) \right)^{1/2} = \frac{\mu_n f_H}{\sqrt{2}} \int_0^\infty \lambda d\lambda \left[ 1 - \exp \left( -\frac{\rho}{\rho_N} \right) \right] \int_0^\infty \rho(\tau, k) d\tau \]

(A8)

There is the remarkable fact (Ting, Ref. 1), that the rho production amplitude (equ. A 8) and the total rho-nucleus cross section (equ. A 6) depend on the nuclear radius, and hence, on \( A \), in the same way. E.g., for very heavy nuclei, the total rho-nucleus cross section depends on \( A \) roughly like \( A^{2/3} \), since the rho is strongly absorbed in nuclear matter. According to equ. (A 8), the forward rho production amplitude has the same dependence \( A^{2/3} \). Now in the vector dominance model, we can write in first approximation (neglecting the \( \omega \) and \( \phi \) contributions):

\[ \left( \frac{d \sigma}{dt} (yA \to \rho A) \right)^{1/2} \approx \left( \frac{d \sigma}{dt} (yA \to yA) \right)^{1/2} = \frac{\sigma_T(yA)}{\sqrt{16\pi}} \]

(A9)

\[ = \rho_{prop} \sigma_T(\rho A) \]

The last part in equ. (A 9) follows from the optical theorem and the assumption of diffractive rho production. Comparing equ. (A 6), (A 8) and (A 9), we see, that the total hadronic
photoproduction cross section on hadrons \( \sigma_T(\gamma A) \) depends on \( A \) in the same way as \( \sigma_T(\rho A) \), e.g. it depends on \( A \) for heavy nuclei like \( A^{2/3} \). This is very strange, since intuitively one would expect \( \sigma_T(\gamma A) \) to be proportional to \( A \), since photons are very weakly absorbed in nuclear matter and the total cross section in this naive picture would be just proportional to the number of nucleons available (no shielding!). The strange property of \( \sigma_T(\gamma A) \approx A^{1/3} \) was first pointed out by Stodolsky\(^6\), for more detailed theories for \( \sigma_T(\gamma A) \) see ref. 8). It will be interesting to see, whether experiments indeed support the existence of these sophisticated effects.
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1. INTRODUCTION

1.1 By studying electron-positron collisions one can investigate a large domain of high energy physics which seems difficult to reach in other ways. On the one hand, reactions such as $e^+e^-$ elastic scattering, $e^+e^-$ annihilation into 2 $\gamma$ rays or into a $\mu^+\mu^-$ pair, provide various checks of quantum electrodynamics (QED) and test the muon-photon vertex. On the other hand, the annihilation of $e^+e^-$ pairs leads to the production of virtual time-like photons which can in turn materialize into particle-antiparticle $A\overline{A}$ states; such reactions allow the study of the electromagnetic structure of hadrons and a systematic search for new particles with the only restriction that they interact with the electro-magnetic field.

While electron accelerators are being used to probe the structure of nucleons with virtual space-like photons, $e^+e^-$ colliding beam rings make the extension of form factor measurements in the time-like region possible, and these measurements are not limited to the study of nucleons since pairs of unstable particles can be created as well as $N\overline{N}$ pairs.

Clearly, it is not possible to cover all of this field in a single lecture. We shall restrict ourselves to a description of some specific features of $e^+e^-$ colliding beam rings and of the experiments done with these machines, to a rapid survey of the results obtained up to now and we shall present a few particular goals of the next generation of $e^+e^-$ storage rings.

1.2 The progress made in the operation of $e^+e^-$ storage rings as high energy physics facilities is best illustrated by the following historical survey. Collisions between an electron beam and a positron beam were recorded for the first time in 1964 with the small Italian ring AdA.
operating at Orsay\textsuperscript{1}); single bremsstrahlung events, $e^+ e^- \rightarrow e^+ e^- \gamma$, of low momentum transfer were detected with a counting rate of a few counts per minute. While several hundred large angle $e^- e^-$ scattering events at a total $C$ of $M$ energy $E = 2 \times 300$ MeV were reported in 1965 by the Princeton-Stanford group\textsuperscript{2}, a few $e^- e^-$ scattering events and annihilations into $\pi^+ \pi^-$ pairs at $E = 2 \times 380$ MeV were observed the following year at Novosibirsk \textsuperscript{3} with VEPP2. Two years later, several thousand $e^- e^-$ large angle scattering events and annihilations into either pion pairs, kaon pairs or $\pi^+ \pi^- \pi^0$ states were measured at Orsay\textsuperscript{4-6} with ACO.

These steps could be made because of the following achievements: positron beams of higher intensity for injection, higher vacuum giving longer beam storing time, improved beam control and the change from weak focusing magnetic rings to strong focusing ones with separate functions. It must be pointed out that a large fraction of the time of operation of all existing rings has been devoted to machine studies which were being done alternately with high energy physics experiments. Important results have been obtained especially on beam instabilities and on the limitations of beam-beam interaction rates; one may thus be confident in the prediction that the "luminosities" which will be achieved in the rings presently in the process of being designed will be much higher than those of the first generation rings.

2. MAIN CHARACTERISTICS OF ELECTRON-POSITRON COLLIDING BEAM RINGS

2.1 MAGNETIC GUIDE FIELD AND BEAM STRUCTURE

High energy electrons and positrons produced in a linear or circular accelerator can be stored for many hours in a more or less circular vacuum chamber, the trajectories of the particles being closed by a series of bending magnets. The orbits of the stored particles are confined within a volume of small transverse dimensions by magnetic quadrupoles which focus the two beams alternately in the horizontal plane and the vertical plane. As an example, Figure 1 shows the lay out of the magnetic elements of ADONE\textsuperscript{7}), a 1.5 GeV $e^- e^-$ ring built at Frascati. The mean radius of this machine is 16 meters.
1. Linac
2. Injection optics
3. Pulsed inflectors
4. R. F. cavities
5. Vacuum pumps
6. Bending magnets
7. Quadrupoles
E. Experimental regions

Fig. 1 General lay-out of the Frascati $e^+ - e^-$ colliding beam ring ADONE.
The number of electrons and positrons stored is of the order of $10^{10} \div 10^{11}$ in the rings presently in operation; such values correspond to circulating currents $I_x$ of $10 \div 50$ mA. The area $S$ of a transverse cross section of the beams at an interaction point is typically a few mm$^2$.

Since the electrons have their trajectories deflected in the bending magnets, they emit photons; this radiation, called "synchrotron radiation", plays an essential role in the structure of the beams. The energy losses which it causes must be compensated by an acceleration provided by one or several radio-frequency cavities. It follows that the stored particles are longitudinally bunched by the periodic accelerating field. In the existing rings, the number of bunches is less than or equal to 3; the length of each bunch is of the order of $10 \div 50$ cm.

2.2 Injection, particle spill out and beam lifetime

Particles being injected in the ring at a certain distance from the ideal closed orbit, the injection process generates oscillations of a large amplitude; nevertheless these oscillations are progressively damped by the synchrotron radiation. Such damping does not conserve phase space and so permits multiple injection into the same orbit. One can thus build up high circulating currents from relatively weak sources of positrons. Typical values of injection repetition rates and injection times are $0.5 \div 50$ Hz and $1 \div 30$ minutes respectively.

Losses of stored particles are due to their collisions either with other stored particles which belong to the same bunch ("Touschek effect"), or with stored particles of the opposite beam (single bremsstrahlung: $e^+e^-\rightarrow e^+e^-\gamma$), or with the residual gas of the vacuum chamber (elastic or inelastic scattering). In order to achieve beam lifetimes of the order of 10 hours, ultra-high vacuum must be maintained in the chamber despite the high power of synchrotron radiation which hits the walls.

It is important to note that when the total intensity of stored particles is 50 mA with a lifetime of 10 hours, the average number of lost particles per second and per meter along the orbit is $3 \times 10^4$. This spill out of high energy particles turns out to be in general an important source of background since it can generate large showers which trigger the

*) Except in the BY-PASS where ~150 bunches are filled.
experimental set up used to detect beam-beam collisions.

2.3 Beam energy

The mean energy $E$ of the beams is entirely determined by the magnetic field $B$ in the bending magnets of the ring. By varying $B$, one changes $E$ at will between two limits which usually correspond to a drop of the interaction rate or, at high energies, to the saturation of the magnetic elements.

The RMS $\sigma_E$ of the energies of the stored particles does not critically depend upon the magnetic structure of the ring and is approximately equal to $E \gamma (x_0/2p)^{1/2}$ where $\gamma = E/m_0 c^2$, $x_0 = R/m_0 c$ and $p$ is the magnets' radius of curvature. This energy spread is therefore very small; for example, in the Orsay storage ring ACO, one has $\sigma_E/E < 4 \times 10^{-4}$. This is a valuable characteristic common to all $e^+e^-$ storage rings.

Absolute energy calibration of a ring with a precision better than 0.5% is difficult to achieve with magnetic measurements. Nevertheless, by such measurements one can tune a ring so that its energy is reproduced to a few parts in $10^4$ over periods of several weeks, so that one can take full advantage of the very good energy definition of the stored beams.

2.4 Luminosity and its limitations

The rate $\lambda$ of collisions $e^+e^- \rightarrow f$ per beam crossing region is given by:

$$\lambda = \sigma f_0 B \int \rho_+ \rho_- dS = \mathcal{L} \sigma \quad \text{(events/s), (1)}$$

where $\sigma$ is the cross section of the events considered, $f_0$ the frequency of revolution of the particles in the ring, $B$ the number of bunches per beam, $\rho_+$ and $\rho_-$ the transverse bunch densities; the quantity $\mathcal{L}$ measured in cm$^{-2}$ s$^{-1}$ is called the "Luminosity" of the ring: it includes all the other parameters just mentioned and is the prime parameter of a ring since it characterizes the collision rate between the 2 beams.

The variation of $\mathcal{L}$ as a function of the energy $E$ may be sketched very roughly in the following way.* In storage rings whose energies are limited to about 1.5 GeV (such as VEPP2, ACO and ADONE) the peak

*) For more details, the reader may consult the lectures given by Sands* on "Storage Ring Luminosity".
luminosity occurs at the maximum energy at which the ring can be operated. In higher energy rings, the peak luminosity is reached at an energy which lies somewhere between 1.5 GeV and 2.5 GeV. Below the energy $E_T$ corresponding to $\mathcal{L}_{\text{peak}}$, $\mathcal{L}$ is limited by the space charge effect of one beam acting on the other one, the so called "Amman-Ritson" effect\textsuperscript{10).} This effect sets a limit on the transverse density $\rho$ of each bunch which is proportional to the energy $E$ of the beams. Since the natural transverse area $S$ of the beams varies like $E^2$, it follows directly from Eq. (1) that

$$ \mathcal{L} = \left( \frac{E}{E_T} \right)^4 \mathcal{L}_{\text{peak}} \quad (E \leq E_T) \quad (2) $$

when the beams collide head on. This $E^4$ dependence is the one observed in ACO. If one makes the beam cross at a variable angle and if one operates the ring at constant currents, then one expects:

$$ \mathcal{L} = \left( \frac{E}{E_T} \right) \mathcal{L}_{\text{peak}} \quad (E \leq E_T). \quad (3) $$

The latter energy dependance is of course more favorable than Eq. (2) but it has not been achieved yet in $e^+ - e^-$ rings.

At energies larger than $E_T$, the power $P$ of the synchrotron radiation becomes so high that the limitation on $\mathcal{L}$ comes from the maximum current which can be accomodated by the RF cavity (or cavities). Since $P \propto E^4$, one has $I_s \propto E^{-4}$. Just above $E_T$, there is a domain of energy which extends up to some energy $E_c$ in which one may slow down the drop in luminosity due to the decrease in stored intensities by reducing $B$ and/or the coupling between vertical and horizontal oscillations of the particles. In this energy range, $\mathcal{L}$ varies in the following way:

$$ \mathcal{L} = \left( \frac{E}{E_T} \right)^3 \text{ to } 5 \mathcal{L}_{\text{peak}} \quad (E_T \leq E \leq E_c). \quad (4) $$

Above $E_c$, $\mathcal{L} \propto E^{-10}$ and therefore drops very rapidly.

2.5 Data on existing rings and rings being designed or proposed

Table 1 gives some of the main characteristics of the rings which are operated presently and of those being designed or proposed for construction.
<table>
<thead>
<tr>
<th>Name</th>
<th>Location</th>
<th>Energy (GeV)</th>
<th>Mean radius (m)</th>
<th>Focusing type</th>
<th>Number of interactions free for experiments</th>
<th>Luminosity (cm⁻²·s⁻¹)</th>
<th>Status</th>
<th>First high energy physics experiments started in 1969</th>
<th>First high energy physics experiments started in 1969</th>
<th>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</th>
<th>Multicycle injection achieved; design value at 3 GeV still to be achieved.</th>
<th>Design values: 3.10²⁻¹ in the first phase; 3.10⁻¹ in later phases.</th>
<th>Design values: Proposal to † 10⁻¹ in the first phase.</th>
<th>Proposal</th>
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<td>First high energy physics experiments started in 1967</td>
<td>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</td>
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<td>Shut down in 1965</td>
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<td>First high energy physics experiments started in 1967</td>
<td>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</td>
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<td>First high energy physics experiments started in 1967</td>
<td>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</td>
<td>Multicycle injection achieved; design value at 3 GeV still to be achieved.</td>
<td>Design values: 3.10²⁻¹ in the first phase; 3.10⁻¹ in later phases.</td>
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<td>1·10⁻⁹</td>
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<td>First high energy physics experiments started in 1967</td>
<td>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</td>
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<td>Design values: 3.10²⁻¹ in the first phase; 3.10⁻¹ in later phases.</td>
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<td>1·10⁻⁹</td>
<td>Shut down in 1965</td>
<td>First high energy physics experiments started in 1967</td>
<td>First high energy physics experiments started in 1967</td>
<td>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</td>
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<td>Design values: 3.10²⁻¹ in the first phase; 3.10⁻¹ in later phases.</td>
<td>Proposal</td>
<td></td>
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<tr>
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<td>3 to 4.5</td>
<td>46</td>
<td>2, low B</td>
<td>1</td>
<td>1·10⁻⁹</td>
<td>Shut down in 1965</td>
<td>First high energy physics experiments started in 1967</td>
<td>First high energy physics experiments started in 1967</td>
<td>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</td>
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<td>Shut down in 1965</td>
<td>First high energy physics experiments started in 1967</td>
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<td>Final tests on two beam operation, at an angle crossing, at an energy 1.1 GeV with beams crossing at an angle.</td>
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<td>Design values: 3.10²⁻¹ in the first phase; 3.10⁻¹ in later phases.</td>
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<td>2, low B</td>
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<td>1·10⁻⁹</td>
<td>Shut down in 1965</td>
<td>First high energy physics experiments started in 1967</td>
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<td>Proposal</td>
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</tbody>
</table>
3. DETECTION

3.1 Generalities

A detector to be used with an e⁺e⁻-colliding beam ring must satisfy various requirements which put severe restrictions on its structure.

i) Electron-positron reaction products have in general a broad angular distribution over the total 4π steradians of solid angle (see Fig. 2). Since the counting rates of many of the interesting events are usually low (see Table 2), the detector must sustain as large a solid angle as possible. The same condition follows from the fact that many of the particles produced are unstable; a good efficiency of detection of the decay products also requires a large solid angle detector.

ii) Collisions occur at random times (except for the effect of the bunching of the stored particles); this implies that the detector must be triggerable.

iii) Spill out of stored particles is in general an intense source of background, especially since condition i) above implies that the detector be close to the vacuum chamber and surround most or all of it. The most efficient way of rejecting this background is based on the fact that true e⁺ - e⁻ collisions must originate from the very well defined beam-beam interaction region whose transverse section has an area of only a few mm²; furthermore this interaction region is also limited in length. The probability that a lost particle generate tracks which cross within this small volume is very small. To take full advantage of this selection criterion, one has to be able to make a precise geometric reconstruction of the origin of the events detected.

iv) Several channels are open at the same time at moderate energies (E < 1 GeV), plenty of them are open at higher energies. Since the counting rates of many reactions are expected to be low, it is desirable that the same detector record several types of events and provide means of discriminating among them. At total C of M energies below 2 M, the particles to be detected are: γ, e⁺⁻, μ⁺⁻, π⁺⁻⁰, K⁺⁻⁰, η or their decay products. The identification of these particles can be based on electric charge detection, shower detection, range measurement and on the evidence of nuclear interactions. Kinematic constraints are also useful in
Fig. 2 Angular distributions of some electron-positron reactions.
(This figure is taken from the SLAC proposal for a high
energy \(e^+e^-\) colliding-beam storage ring, Stanford, 1965).
TABLE 2
Predicted rates of some $e^+e^-$ reactions with an assumed luminosity of $10^{33}$ cm$^{-2}$ s$^{-1}$.

<table>
<thead>
<tr>
<th>Beam energy $E$ (GeV)</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2$ (annihilation) (GeV/c)$^2$</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>final state</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$e^+e^-$</td>
<td>1.0(+6)</td>
<td>4.7(+5)</td>
<td>2.7(+5)</td>
<td>1.1(+4)</td>
</tr>
<tr>
<td>$\gamma\gamma$</td>
<td>1.3(+5)</td>
<td>5.8(+4)</td>
<td>3.2(+4)</td>
<td>1.4(+4)</td>
</tr>
<tr>
<td>$\mu^+\mu^-$</td>
<td>4.9(+4)</td>
<td>2.1(+4)</td>
<td>1.2(+4)</td>
<td>5.4(+3)</td>
</tr>
<tr>
<td>$\pi^+\pi^-$</td>
<td>6.1(+1)</td>
<td>1.1</td>
<td>6.1(-2)</td>
<td>1.1(-3)</td>
</tr>
<tr>
<td>$\pi^0\gamma$</td>
<td>9.3</td>
<td>3.6(-1)</td>
<td>3.6(-2)</td>
<td>1.4(-3)</td>
</tr>
<tr>
<td>$\rho^+\rho^-$</td>
<td>2.7(+3)</td>
<td>4.7(+2)</td>
<td>1.0(+2)</td>
<td>9.7</td>
</tr>
<tr>
<td>$p\bar{p}$</td>
<td>2.9(+2)</td>
<td>6.5</td>
<td>3.6(-1)</td>
<td>6.2(-3)</td>
</tr>
<tr>
<td>$\Lambda\bar{\Lambda}$</td>
<td>6.5(-1)</td>
<td>3.8(-2)</td>
<td>6.5(-4)</td>
<td></td>
</tr>
<tr>
<td>$\Sigma^+\bar{\Sigma}^+$</td>
<td>4.8</td>
<td>3.1(-1)</td>
<td>5.4(-3)</td>
<td></td>
</tr>
</tbody>
</table>
discriminating among the various possible final products. In the multigev energy range the identification of a particular reaction among the scores of those which are possible requires a kinematic analysis based on accurate momentum and angle measurements. However, such an analysis might sometimes be difficult to carry out because of the following point.

v) Many of the reactions lead, either directly or after the decay of unstable particles, to the production of $\gamma$ rays (for example: $e^+e^- \rightarrow \pi^0\gamma$, $e^+\pi^- \rightarrow \pi^+\pi^-\pi^0\pi^0$, $e^+\pi^- \rightarrow \pi^+\pi^-\pi^0\pi^0$, $e^+\pi^- \rightarrow \pi^+\pi^-\pi^0\pi^0$, $e^+\pi^- \rightarrow \pi^+\pi^-\pi^0\pi^0$, etc...). It is quite important that these $\gamma$ rays be detected with high efficiency $^\ast$). It follows that shower detectors must be so located as to be separated from the interaction region by little absorbing material. This condition implies that these detectors be inside the solenoid which produces the field necessary for momentum analysis, or at least inside the iron used for flux return.

3.2 First generation of detectors

A set of spark chambers triggered by multifold coincidences between scintillation counters provides the easiest way to build a detector which meets all the above requirements in a range of $C$ of $M$ energy limited to about 2 GeV. Figure 3 gives an example of such an experimental set up, namely the first one used at Novosibirsk to detect $e^+e^-$, $\mu^+\mu^-$ and $\pi^+\pi^-$ pairs.$^{11}$ It consists of a symmetrical arrangement of optical spark chambers triggered by four-fold coincidences. The spark chambers located closest to the ring vacuum chamber have thin plates in order to make accurate geometrical measurements. Further away, thick plate spark-chambers develop electromagnetic cascade showers and stop pions or muons; the latter spark chambers allow range measurements and can show interactions of pions with nuclei (stars, for instance). The whole set up is covered by a layer of scintillation counters which veto cosmic rays.

A similar arrangement was built at Orsay, except for the fact that the spark chambers being somewhat wider and closer to the beams sustained a larger solid angle. The corresponding increase of the solid angle of detection allowed the observation of three-body final states ($e^+e^- \rightarrow \omega + \pi^+\pi^-\pi^0$). Figure 4 shows some typical spark chamber pictures $^\ast$) Note that these $\gamma$ rays do not necessarily have a high energy.
Fig. 3 First experimental set-up used with VEPP 2 at Novosibirsk.

1) Anticoincidence scintillation counter; 2) Lead absorber 20 cm thick; 3) "Range" spark chamber; 4) "Shower" spark chamber; 5) Duraluminium absorber 2 cm thick; 6) Thin-plate spark chambers; 7) Window of outer vacuum chamber; 8) Interaction region; 9) Inner vacuum chamber; 10) Scintillation counters; 11) Storage ring magnet.
Fig. 4  Some typical spark-chamber pictures of events registered at \( E^* = 2 \times 382 \text{ MeV} \) with ACO (Orsay).
events registered at Orsay at a total C of M energy $E^* = 2 \times 382$ MeV, that is, on the lower tail of the $\omega$ resonance.

Other detectors of similar structure have been built or proposed\textsuperscript{12) for ACO, ADONE and higher energy rings; most of them can detect $\gamma$ rays as well as charged particles and sustain a very large solid angle since they surround almost completely the vacuum chamber. The present trend is to replace part or all of the optical spark chambers with wire spark chambers operated with a computer. The main advantages of this change are well known: direct digitization of the information, possibility of higher triggering rates, and absence of light collection problems.

3.3 Future detectors

Non-magnetic detectors of the type described in the preceding paragraph may also be used at higher energies ($E > 1$ GeV) to study QED reactions ($e^+ e^- \rightarrow e^+ e^-$, $\gamma\gamma$, $\mu^+ \mu^-$), to search new intermediate vector mesons, and to get a first picture of strong interaction reactions. Nevertheless, the untangling of all the possible channels which are open at high energy requires precise momentum measurements with a magnetic field.

The kinematic analysis should take into account the possibility that some energy and momentum may be taken away by undetected $\gamma$ rays, since the probability that the incoming electrons radiate in the direction of the beam line becomes high at high energy. In particular, there is a non negligible probability that the reactions which have the highest rates (viz. $e^+ e^- \rightarrow$ scattering and $\mu^+ \mu^- \rightarrow$ pair production) simulate\textsuperscript{*} rarer ones (for example: $e^+ e^- \rightarrow pp$) when accompanied with the emission of two photons of approximately opposite momenta.

It seems necessary to achieve momentum measurements within 1 % or so. Large solenoids must therefore be used; typical parameter values are: diameter and length $\%$ 3 m, field $\%$ 1.5 Tesla. Superconducting coils are desirable in order to avoid power dissipations of several megawatts.

Since it is difficult to collect the light from many optical spark

\textsuperscript{*})As far as the kinematics are concerned.
chambers and from scintillation counters and/or Čerenkov counters located inside a solenoid, efforts are being made to design wire proportional chambers of various shapes and to operate them in strong magnetic fields. It is hoped that it will be possible to use such chambers not only to locate tracks but also as triggering devices. A complete set up will require an order of magnitude of $10^5$ wires, and therefore will need an on-line computer to handle the storing and the checking of the information.

One then realizes that a detector to be used with a multigev $e^+ - e^-$ ring will be a very heavy and complicated apparatus whose size and complexity of operation will be somewhat similar to those of a bubble chamber. As is the case for a bubble chamber, the same detector will be used for a long period of time, without modification, to record a large amount of data to be analyzed later. This analysis might be done by groups of physicists working in different laboratories than the one where the ring is operated.

4. PURELY ELECTROMAGNETIC CHANNELS

4.1 Generalities

Among the various electromagnetic channels, the most interesting reactions are the three following one:

\[
e^+ + e^- \rightarrow e^+ + e^- \quad \text{(Bhabha scattering),} \quad (a)
\]
\[
e^+ + e^- \rightarrow \gamma + \gamma \quad \text{(Two-quantum annihilation),} \quad (b)
\]
\[
e^+ + e^- \rightarrow \mu^+ + \mu^- \quad \text{(Muon production).} \quad (c)
\]

The cross sections of these reactions can be exactly calculated by QED and the radiative corrections have also been evaluated $^{13-16}$. The latter are quite large for high resolution experiments.

The lowest order Feynman diagrams for these three processes are shown in Figure 5.
Fig. 5 Feynman diagrams for processes (a), (b) and (c).

The corresponding cross sections in the C of M are found to be:

(a) Bhabha scattering

\[
\frac{d\sigma}{d\Omega} = \frac{\mathcal{K} \alpha^2}{8\gamma^2} \left[ \frac{q^4 + s^4}{q^4} \left| F(q^2) \right|^2 + 2 \frac{q^4}{q^2 s^2} \text{Re} F(q^2) \mathcal{F}^*(s^2) \right] + \frac{q^4 + s^4}{s^4} \left| F(s^2) \right|^2 \right], \quad (5)
\]
(b) **Two-quantum annihilation** ($\gamma >> 1$ and $\theta >> \frac{1}{\gamma}$)

\[
\frac{d\sigma}{d\Omega} = \frac{r_0^2}{8\gamma^2} \left[ \frac{q^2}{q^2} |F(q^2)|^2 + \frac{q^2}{q'^2} |F(q'^2)| \right]
\]

\hspace{1cm} , \hspace{1cm} (6)

(c) **Muon production**

\[
\frac{d\sigma}{d\Omega} = \frac{r_0^2}{16\gamma^2} \beta \mu \left[ (1 + \cos^2 \theta) + (1 - \beta^2) \sin^2 \theta \right] |F(s^2)|^2
\]

\hspace{1cm} , \hspace{1cm} (7)

where $q^2 = -4E^2 \sin^2 \theta/2$, $q'^2 = -4E^2 \cos^2 \theta/2$, $s^2 = 4E^2$ and $\theta$ is the angle between the final particle momenta and the beam line.

Form factors, $F(q^2) = 1 / (1 + q^2/\Lambda^2)$, have been introduced in these formulas according to the usual procedure in order to parameterize possible deviations from pure QED predictions.

The Feynman diagrams of Fig. 5 show that reaction (a) can be used to test the e - $\gamma$ vertex function. It also tests the photon propagator both for space-like and time-like transfers. However, in an experiment which does not distinguish the charges of the final particles, the main contribution to the cross section is by far the one associated with time-like transfers. Reaction (b) tests the electron vertex and the electron propagator, while reaction (c) probes the muon structure with extremely time-like photons and provides also a very clean way to look for vacuum polarization effects in the photon propagator.

To illustrate the importance of the steps which can be made in this field by $e^+ - e^-$ colliding beam experiments, let us consider the measurement of $\mu$ pair production with ADONE as proposed by a Roma-Frascati team\(^{17}\). The goal of this proposal is to measure the cross section of reaction (c) within a few per-cent in the GeV region. To make it more precise, let us assume that $E_+ = E_- = 1$ GeV and that the overall accuracy of the measurement is 5 %. If this experiment leads to $\sigma_{\text{measured}} = \sigma_{\text{theory}}$, the lower limit of the cut-off parameter $\Lambda$ of Eq. (7) would be set at 13 GeV/c (68 % confidence level). The distance which corresponds to such
a cut-off is 0.015 fermi.

A similar experiment carried out with the BY-PASS at $E = 3$ GeV would bring this limit down to $5 \times 10^{-3}$ fermi.

4.2 Experimental results

4.2.1 Bhabha scattering

The absolute cross section of $e^+e^-$ elastic scattering has been measured \textsuperscript{18} with ACO at a total C of M energies close to the mass of the $\phi$ resonance. The identification of these two-body events was based on the collinearity of the tracks in the transverse view* and on the recognition of showers in thick plate spark chambers (see Fig. 4). The detector could not distinguish the electric charges carried by the final particles.

The $e^+e^-$ pairs detected were those emitted in a cone whose axis was perpendicular to the beam line, and which had a total angular aperture of $74^\circ$. The total number of accepted events was 443.

Simultaneously with the observation of these large angle scattering events, the luminosity of the ring was measured by recording the double bremsstrahlung reaction

$$e^+ + e^- + e^+ + e^- + \gamma + \gamma'$$

in which the two $\gamma$ rays are emitted back-to-back along the beam line.

The time integral of the luminosity was then used to determine the absolute Bhabha cross section according to Eq. (1). The comparison of the number thus obtained with the one deduced from the theoretical differential cross section integrated over the solid angle of detection leads to the following value \textsuperscript{+} of the cut-off $1/A^2$:

$$A^{-2} = (\pm 0.002 \pm 0.061 \text{ (statistical)} \pm 0.030 \text{ (systematic)}) \text{ (GeV/c)}^{-2}$$

For comparison we give the results of Barber et al.\textsuperscript{19} based on the

* In the "transverse view", the tracks are seen as projected on a plane perpendicular to the beam line.

+ I am very grateful to Dr. Buon for letting me quote this result prior to its publication.
study of $e^-e^-$ scattering at 556 MeV/particle (C of M). This experiment also tested the space-like photon propagator and the electron vertex function. Barber et al. found:

$$\Lambda^{-2} = (-0.06 \pm 0.06 \text{ (statistical)}) \text{ (GeV/c)}^{-2},$$

with a systematic error smaller than the statistical one.

4.2.2 Mu Pair Production

There has not been, up to now, any $e^+e^-$ colliding beam experiment which has had as the unique goal the detection of $\mu^+\mu^-$ pairs: the latter were obtained as by-products of an experiment carried out with ACO on $\pi^+\pi^-$ production. Muon tracks are easily differentiated from those of the other two-body events ($e^+e^-$ pairs and $\pi^+\pi^-$ pairs) by their regular aspect and by their longer range (see Fig. 4).

A total of 62 $\mu^+\mu^-$ events were recorded at an average energy (total C of M) of about 650 MeV. This number of events differs by less than 3% from the one computed by using Eq. (7) and the integral of the luminosity during the data taking time. One can then set the following limit to the cut-off parameter $\Lambda$ of Eq. (7):

$$|\Lambda^{-2}| \leq 0.16 \text{ (GeV/c)}^{-2} \text{ (68% confidence level).}$$

5. STRONG INTERACTIONS

5.1 Generalities

To the lowest order in the fine structure constant $\alpha$, hadron production in $e^+e^-$ collisions proceeds according to the graph of Fig. 6.

![Fig. 6 Graph of the one photon channel for hadron production. a, b, c... represent strong interacting particles.](image)
The electron and the positron annihilate into a virtual time-like photon which is then coupled to the strongly interacting particles. By such processes, one can probe the structure of stable and unstable hadrons with the electromagnetic current, and thus learn about strong interactions although the beam particles are not strongly-interacting particles themselves.

Of particular note is the multiplicity of open hadronic channels when the beam energy reaches $1.5 \pm 1.7$ GeV. Here is a non-exhaustive series of them:

$$e^+e^- \rightarrow \pi^+\pi^-, K^+K^-, K^{\ast}\bar{K}^0, \rho^+\rho^-, K^{\ast\ast}\bar{K}^0, \text{etc...}$$

$$e^+e^- \rightarrow p\bar{p}, n\bar{n}, \Lambda\bar{\Lambda}, \Sigma^+\Sigma^-, \Sigma^0\bar{\Lambda}, \text{etc...}$$

$$e^+e^- \rightarrow N^{\ast\ast}\bar{N}^{\ast\ast}, Y^{\ast\ast}\bar{Y}^{\ast\ast}, \Omega^{-}\bar{\Omega}^-, p\bar{N}^{\ast\ast}, \text{etc...}$$

In contrast with the experimental difficulties that one might encounter when dealing with such a variety of final states, the interpretation of the experiments concerning each channel is particularly simple because of the structure of the relevant Feynman diagram (see Fig. 6). Since hadron production passes mostly through one intermediate virtual photon\(^*\), the final particles must be in a state with $J^{PC} = 1^{--}$. One particular consequence of this constraint is the following. When the final state is made of $n$ pions, one has necessarily $T$ (isospin) = 1 for $n$ even and $T = 0$ for $n$ odd. Therefore the isovector resonances can be investigated by examining the $2\pi$ state, while isoscalar resonances are studied by examining the $3\pi$ state.

Theoretical predictions on hadron production cross sections are quite uncertain. While several models\(^{20-22}\) can accommodate the experimental results on the nucleon form factors as measured in the space-like region and described by the well known "dipole fit"

$$G_{Ep}(q^2) = \frac{G_{np}(q^2)}{\mu_p} = (1 - \frac{q^2}{0.71 (\text{GeV}/c)^2})^{-2}, \quad (8)$$

these models sometimes differ by orders of magnitude in the time-like

\(^*\) Putzolu\(^{15}\) has shown that the contribution to the cross section of the interference term between the one-photon diagrams and the two-photon diagrams cancels when the measurement does not distinguish the electric charges of the final particles.
region, which will be explored with $e^+ e^-$ colliding beam experiments. Such experiments appear as essential in clarifying the situation in this field.

Although absolute cross sections are difficult to estimate, symmetry schemes can relate in a precise way the form factors of particles which belong to the same multiplets. To illustrate this point, we write the $e^+ e^-$ annihilation cross sections in the form

$$
\sigma (e^+ e^- \to AB) = \sigma_{MB} (1 + \cos^2 \theta) + \sigma_{EAB} \sin^2 \theta
$$

which is valid in the lowest order of the fine structure constant $\alpha$. The quantities $\sigma_{MB}$ and $\sigma_{EAB}$ thus introduced are simply related to the magnetic and electric form factors of the $\gamma AB$ vertex. SU(3), for example, predicts relations among these quantities $\sigma_M$ and $\sigma_E$ such as:

$$
\sigma_{EpP} = \sigma_{E^+ \overline{E}^+} = \sigma_{E^- \overline{E}^-} = \text{etc...},
$$

$$
\sigma_{MnP} = \frac{9}{4} \sigma_{Mn\overline{n}} = 9 \sigma_{M\overline{A} \overline{A}} = 3 \sigma_{M\overline{O} \overline{O}} = \text{etc...},
$$

$$
\sigma_{EN\overline{N}} = \sigma_{EY\overline{Y}} = \sigma_{E\overline{O}} = \text{etc...},
$$

$$
\sigma_{E\overline{p}} = \sigma_{E^+ \overline{K}^-} = \text{etc...}
$$

The measurement of the relative yields of various hadron pairs at a given beam energy will thus allow very direct tests of these symmetries.

5.2 Vector meson production

5.2.1. The vector meson dominance model (VDM)

The vector meson dominance model\(^{22-26}\) is one of the first which has been proposed in view of understanding the hadron electromagnetic form factors, and since then it has played an essential role in this field.

This model is based on the existence of three neutral vector mesons ($\rho^0, \omega$ and $\phi$) which have the same quantum numbers ($J, P$ and $C$) as those of the photon. The vector dominance hypothesis assumes that, to a very
good approximation, the entire electromagnetic current of the hadrons is a linear combination of the fields which describe the three mesons. The basic equation of the VDM is therefore the following identity:

$$I_{\mu}^{\text{em}}(x) = \left[ \frac{m_{\rho}^2}{g_{\rho}} \rho_{\mu}^0(x) + \frac{m_{\omega}^2}{g_{\omega}} \omega_{\mu}(x) + \frac{m_{\phi}^2}{g_{\phi}} \phi_{\mu}(x) \right] .$$  \hspace{1cm} (9)

The coupling constants have been written in the form $m_{\mathcal{V}}^2/g_{\mathcal{V}}$, ($\mathcal{V} = \rho^0, \omega, \phi$) for convenience.

Kroll, Lee and Zumino\(^\dagger\) have constructed a Lagrangian field theory in which the identification implied by Eq. (9) can be made exactly\(^\dagger\). They have further shown that this identity is completely consistent with gauge invariance, provided that the vector mesons are coupled only to conserved currents. The $\rho$ is then assumed to be coupled to the isospin current with the coupling constant $g_{\rho}$, while the currents which are the sources of the $\omega$ and $\phi$ fields are taken as linear combinations of the baryon number current $N_{\mu}$ and the hypercharge current $Y_{\mu}$. Two angles, $\theta_{\omega}$ and $\theta_{\phi}$, and two coupling constants, $g_{\omega}$ and $g_{\phi}$, are in general necessary to describe the $\omega$-$\phi$ mixing in this formalism and it can be shown that

$$g_{\phi} = \frac{2g_{\omega}}{\cos\theta_{\omega}} ,$$  \hspace{1cm} (10)

and

$$g_{\omega} = \frac{2g_{\phi}}{\sin\theta_{\phi}} ,$$  \hspace{1cm} (11)

where $g_{\mathcal{V}}$ is the coupling constant of the octet central member $\mathcal{V}$ to the hypercharge current.

All these coupling constants but $g_{N}$ and the two mixing angles can be deduced from the measurement of the cross sections of the reactions which follow:

\begin{align*}
e^+ + e^- &\rightarrow \rho \rightarrow \pi^+ + \pi^- , \hspace{1cm} (d) \\
e^+ + e^- &\rightarrow \omega \rightarrow \pi^+ + \pi^- + \pi^0 , \hspace{1cm} (e) \\
e^+ + e^- &\rightarrow \phi \rightarrow \pi^+ + \pi^- + \pi^0 , \hspace{1cm} (f) \\
e^+ + e^- &\rightarrow \phi \rightarrow \pi^0 + K^0 , \hspace{1cm} (g) \\
e^+ + e^- &\rightarrow \phi \rightarrow K^- + K^+ . \hspace{1cm} (h)
\end{align*}

\(^\dagger\)Kroll, Lee and Zumino have identified the hadronic electromagnetic current operator with a linear combination of the renormalized vector meson fields.
The simplest analysis\(^*)\) of these reactions is carried out in the following way. Assuming vector dominance, and treating the vector mesons as essentially stable particles, the production of any hadronic final state \(f\) proceeds (in the lowest order in \(a\)) according to the graph of Fig. 7. This graph shows that if \(f\) represents a possible decay mode of a vector meson \(V\), the \(e^+e^-\rightarrow f\) reaction cross section will have a resonant

![Graph of hadron production](image)

Fig. 7 Graph of hadron production according to the VDM. \(V\) represents \(\rho^0\), \(\omega\) or \(\phi\). The quantity \(e^2/\sigma_V\) (resp. \(e^2/\sigma_{Vf}\)) is the coupling constant of the vector meson \(V\) to the photon (resp. final state \(f\)).

behaviour with a peak occurring at a total \(C\) of \(\rho\) energy \(E^* = 2E = m_V\). In the vicinity of this maximum, one can assume a Breit-Wigner description for the cross section,

\[
\sigma(E) = \frac{3\pi}{4} \frac{1}{E^2} \frac{\Gamma(V+f) \Gamma(V+e^+e^-)}{[(2E-m_V)^2 + \Gamma_V^2/4]} \quad ,
\]

(12)

where \(\Gamma(V+f)\) (resp. \(\Gamma(V+e^+e^-)\)) is the partial width of the \(V\) meson decay into the \(f\) state (resp. the \(e^+e^-\) state) and \(\Gamma_V\) is the total width of the \(V\) resonance. It follows that the peak cross section is given by

\[
\sigma_{\text{peak}} = \frac{12\pi}{m_V^2} \frac{\Gamma(V+e^+e^-) \Gamma(V+f)}{\Gamma_V^2} \quad .
\]

(13)

The excitation curve \(\sigma(E)\) which can be obtained by studying the reaction \(e^+e^-\rightarrow f\) leads to the measurement of the three quantities\(^**)\)

\(^*)\) A thorough study may be found in Gourdin's\(^28\) lectures on "Electron-positron annihilation into hadrons".

\(^**)\) The masses of the \(\omega\) and of the \(\phi\) mesons were accurately known prior to colliding beam experiments.
m_v, \Gamma_v and \sigma_{peak}. One can therefore compute the product \( B_f \times \Gamma(\nu \to e^+e^-) \) where \( B_f \) represents the branching ratio \( \Gamma(\nu \to f)/\Gamma_v \). This branching ratio is accurately known for the \( \rho \to \pi^+\pi^- \) decay and the \( \omega \to \pi^+\pi^-\pi^0 \) decay. So far, the information on the \( \phi \) meson partial decay modes is not very precise but the storage ring experiments have themselves provided a measurement of the branching ratios \( \Gamma(\phi \to \pi^+\pi^-\pi^0)/\Gamma(K_S^0 K_L^0) \) and \( \Gamma(\phi \to K^+K^-)/\Gamma(K_S^0 K_L^0) \), since the three processes (f), (g) and (h) have been studied with \( e^+e^- \) colliding beams.

Finally, from the knowledge of \( \Gamma(\nu \to e^+e^-) \) one can deduce the value of the coupling constant \( g_v \) which appears in Eq. (9). This is achieved by using a pole model for the \( \nu \to e^+e^- \) decay as illustrated by Fig. 8.

![Graph of the \( \nu \to e^+e^- \) decay.](image)

A straightforward calculation gives the following partial width:

\[
\Gamma(\nu \to e^+e^-) = \frac{4\pi \alpha^2}{3} \frac{m_v}{\sigma_v^2}.
\]  

Eqs. (10) and (11) are then used to compute \( g_y \) and \( \theta_y \). The other mixing angle, \( \theta_N \), can be computed\(^27\) from the partial decay width of the \( \phi \) going into \( K^+K^- \) or \( K_S^0 K_L^0 \) without having to make any assumption on the \( SU(3) \) symmetry-breaking interactions:

\[
\Gamma(\phi \to K^+K^- \text{ or } K_S^0 K_L^0) = \frac{1}{12} \frac{g_y^2}{4\pi} \frac{\cos^2 \theta_y}{\cos^2(\theta_y-\theta_N)} \frac{\beta_k^3 m_\phi}{\sigma_k^2}.
\]  

In the preceding analysis, the vector mesons were treated as infinitely narrow resonances. Finite width corrections must be made which
bring in some additional factors in the formulas derived above. Details on these corrections will be found in References 30 to 34.

5.2.2 Experimental results

The yield of $\pi^+\pi^-$ pairs at the $\rho$ resonance has been measured both at Novosibirsk\textsuperscript{11)} and at Orsay\textsuperscript{4).} Roughly speaking, these measurements cover the following range of energies (total C of M): 580 MeV to 1030 MeV by 30 MeV steps. In the Orsay experiment, special attention has been paid to the $\omega$ resonance region in order to look for a possible $\rho-\omega$ interference effect\textsuperscript{35).} A total of about 2500 $\pi^+\pi^-$ pairs have been detected in this set of experiments.

The yield of $\pi^+\pi^-\pi^0$ states has been measured at Orsay in a 60 MeV energy range covering the $\omega$ resonance\textsuperscript{5)} ($\simeq$ 200 events) and in a 15 MeV range covering the $\phi$ resonance\textsuperscript{5)} ($\simeq$ 50 events).

Finally, the yields of $K_S^0K_L^0$ and $K^+K^-$ pairs have also been measured\textsuperscript{6,29)} at the $\phi$ resonance energy ($\simeq$ 150 $K_S^0K_L^0$ pairs and $\simeq$ 2000 $K^+K^-$ pairs).

All these measurements lead to the excitation curves of reactions (d) to (g) as shown in Figs. 9 to 11. In Fig. 9, it is the square of the modulus of the pion electromagnetic form factor $|F_\pi|^2$ which has been plotted versus the C of M energy, rather than the cross section of reaction (d) itself. $F_\pi$ is a scalar function defined by

\begin{equation}
<\pi^+\pi^-|J_{\mu}^{\text{em}}(0)|0> = i(q_+ - q_-)_{\mu} F_\pi(s),
\end{equation}

where $q_+$ (resp. $q_-$) is the four-momentum of the ingoing positron (resp. electron) and $s = -(q_+ + q_-)^2 = 4E^2$. The cross section of reaction (d) is then easily found to be

\begin{equation}
\sigma_{\text{total}}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{\pi a^2}{3} \frac{1}{s} \left(1 - \frac{4m_p^2}{s}\right)^{3/2} |F_\pi(s)|^2.
\end{equation}

While the curves of Figs. 10 and 11 were obtained by using the standard Breit-Wigner expression multiplied by a phase space factor, the curve shown in Fig. 9 is a four parameter fit obtained by Parkinson\textsuperscript{33)}. \textcopyright
Fig. 9 Pion electromagnetic form factor $|F^2_\pi|$ versus energy. The curve shows a four parameter fit to the experimental points which has been obtained by Parkinson$^{33}$. 
Fig. 10 The $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section at the energy of the $\omega$ resonance. The experimental results have been fitted with a Breit-Wigner expression in which the width of the resonance was taken as a free parameter.
Fig. 11 Excitation curves of the $\phi$ resonance.
using a formula derived from a relativistic generalization of the Breit-Wigner formula. Other expressions of the pion form factor have been proposed by Gounaris and Sakurai \textsuperscript{30)}, Vaughn and Wali \textsuperscript{31}), and Roos and Piščú \textsuperscript{32}).

The extraction of the coupling constants and of the mixing angles from these data can be found in Refs. 36 and 37, as well as a comparison of these experimental results to theoretical predictions based on the VDM. We shall merely quote here the values obtained of the coupling constants and the mixing angles *) :

<table>
<thead>
<tr>
<th>$\frac{g_\omega^2}{4\pi}$</th>
<th>$\frac{g_\phi^2}{4\pi}$</th>
<th>$\frac{g_Y^2}{4\pi}$</th>
<th>$\frac{g_N^2}{4\pi}$</th>
<th>$\theta_Y$</th>
<th>$\theta_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.17±0.12</td>
<td>14.8±2.3</td>
<td>11.0±1.6</td>
<td>1.58±0.18</td>
<td>40.8°±3.5°</td>
<td>21.2°+9° -10°</td>
</tr>
</tbody>
</table>

Finally, let us mention some evidence of a $\rho$-$\omega$ interference which comes out of the pion form factor measurements carried out at Orsay \textsuperscript{35}). The data on pion pairs production (reaction (d)) obtained with ACO at the energy of the $\omega$ resonance have been left out of the analysis given above. The overall set of measurements including those made at the $\omega$ resonance are shown in Fig. 12. This figure also shows two curves which were fitted to these experimental results. The confidence level corresponding to fit (1) which does not assume any $\rho$-$\omega$ interference is 8 %, while the confidence level corresponding to fit (2) which assumes an interference effect is 47 %. When taken at their face values, the best fit parameters of fit (2) lead to a relatively high figure of the $\omega \to \pi^+\pi^-$ transition rate: $\Gamma^{1/2}(\omega \to \pi^+\pi^-) = (0.63 \pm 0.23) \text{ MeV}^{1/2}$.

*) The value of the coupling constant $g_\rho$ was deduced from Gounaris\textsuperscript{30b}) analysis of the pion form factor.
Fig. 12 Pion electromagnetic form factor. The complete experimental Orsay results have been fitted (1) without assuming any $\rho-\omega$ interference effect, (2) assuming a $\rho-\omega$ interference, the amplitude and phase of the $\omega\rightarrow\rho$ transition being taken as free parameters.
6. **ASYMPTOTIC BEHAVIOUR**

Last but not least, let us briefly mention the very important problem of the asymptotic behaviour of hadron production cross sections in $e^+ e^-$ collisions. Particular emphasis was recently given to it by Gatto\textsuperscript{38}).

Conjectures have been made on (i) the asymptotic behaviour of the cross section of each hadronic channel, (ii) the asymptotic behaviour of the total cross section of a set of reactions such as $e^+ e^- \rightarrow p + \text{"anything"}$, where "anything" indicates the sum over all possible hadron states, (iii) the asymptotic behaviour of the cross section summed over all hadronic channels\textsuperscript{*}.

Many of these predictions are based upon crossing from the $e^+ e^-$ annihilation channels to the elastic or deep inelastic electron-positron scattering. Of particular note are the results of Drell, Levy and Yan\textsuperscript{39}) which are based on Bjorken's\textsuperscript{40}) work on the structure functions of the nucleon. Drell et al. have found with their "parton" model that the cross section of the $e^+ e^- \rightarrow p + \text{"anything"}$ reaction should asymptotically be comparable in magnitude to that of lepton pair creation and very much larger than the "elastic" annihilation process which leads to a $\bar{p}p$ pair.

A similar conclusion on the total cross section of the $e^+ e^- \rightarrow$ hadrons process had been previously found by Gribov, Ioffe and Pomeranchuk\textsuperscript{41}) by studying the commutation relations between the components $j_0$ and $j_1$ of the hadron electromagnetic current and the properties of spectral functions of the Lehmann-Källén representation.

Gatto\textsuperscript{38}) has given a general classification of the field algebra in terms of asymptotic behaviours of the $e^+ e^-$ annihilation cross-section, and has pointed out that the algebra which predicts the slower decrease of the annihilation cross section as a function of energy ($\sigma(E) \sim \frac{1}{E^2}$) is the only one which has the virtue of leading to finite E.M. mass differences.

7. **CONCLUSION**

Luminosities intense enough to allow high energy $e^+ e^-$ experiments were achieved three years ago, first with VEPP 2 at Novosibirsk, and soon after with ACO at Orsay.

Important results have been obtained since then, which bear mostly

\textsuperscript{*}) The analysis can be carried out separately for the $T = 0$ channels and the $T = 1$ channels.
on the properties of the three neutral vector mesons. These experiments represent the first step of a systematic investigation of the hadron structure carried out with time-like photons. They have confirmed that \( e^+ e^- \) rings are by far the best suited machines in this field. The recent successful two-beam operation of ADCONE makes one confident that this investigation will be extended to higher energies in the near future, while the BY-PASS, VEPP 3 and DORIS promise further progress in the pursuit of still higher energies and luminosities.

* * *
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PLANS FOR PHYSICS WITH THE INTERSECTING STORAGE RINGS

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1. Introduction

In 1971 the Intersecting Storage Rings (ISR) at CERN will start operating. The ISR will allow us to study proton-proton collisions in an energy region, which is until now only accessible to cosmic ray experiments.

The total energy in the centre-of-momentum system of two particles with energies $E_1$ and $E_2$, and momenta $\vec{p}_1$ and $\vec{p}_2$, is

$$E_{CM}^2 = (E_1 + E_2)^2 - (\vec{p}_1 + \vec{p}_2)^2$$  \hspace{1cm} (1a)

For individual energies $E_1 = E_2 = E$, and for $E \approx |\vec{p}| = |\vec{p}|$, and for a collision angle $\approx \pi$ we obtain ($\theta = \pi$ - collision angle):

$$E_{CM} \approx 2E(1 - \frac{1}{8}E^2)$$  \hspace{1cm} (1b)

With $E = 25$ GeV we can consequently reach a CM energy of 50 GeV. This has to be compared to the CM energy of a 25 GeV proton incident on a proton at rest: $E_2 = M$

$$E_{CM} \approx \sqrt{2ME_1}$$  \hspace{1cm} (1c)

which is about $E_{CM} \approx 7$ GeV for $E_1 = 25$ GeV. In order to obtain $E_{CM} = 50$ GeV under these conditions one has to apply an energy of the incident proton of $E_1 = 1340$ GeV.

As we will see later, the ISR are not equivalent.
to a 1 TeV accelerator. Whereas the ISR can reach an interaction rate of roughly $10^6$ sec, at all intersects together, an accelerator can dump its full accelerated current, which may be as much as $10^{12}$/sec.

2. The Intersecting Storage Rings

2.1 ISR Parameters

The intersecting storage rings are essentially two alternating gradient synchrotrons, each being roughly a rounded square, one turned by 45° with respect to the other, and thereby intersecting at 8 points. Both rings are contained in the same tunnel. The beam circulates in a stainless steel vacuum tube of elliptic shape pumped by sublimation pumps to a vacuum of $10^{-9}$ Torr all around and $10^{-11}$ Torr at the intersects. Some parameters are listed in the table below:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>total circumference</td>
<td>942.6 meters</td>
</tr>
<tr>
<td>intersection angle</td>
<td>14.8 degrees</td>
</tr>
<tr>
<td>vertical betatron amplitude</td>
<td>± .5 cm</td>
</tr>
<tr>
<td>horizontal betatron amplitude</td>
<td>± 1.0 cm</td>
</tr>
<tr>
<td>vertical betatron angle</td>
<td>± .4 mrad</td>
</tr>
<tr>
<td>horizontal betatron angle</td>
<td>± .5 mrad</td>
</tr>
<tr>
<td>momentum dispersion</td>
<td>2.3 cm/°/o</td>
</tr>
<tr>
<td>total momentum bite</td>
<td>2 °/o</td>
</tr>
<tr>
<td>maximum momentum</td>
<td>28 GeV/c</td>
</tr>
<tr>
<td>max. number of stacked particles</td>
<td>$4.10^{14}$/ring</td>
</tr>
<tr>
<td>max. current</td>
<td>20 Amp</td>
</tr>
<tr>
<td>luminosity</td>
<td>$4.10^{30}$ cm$^{-2}$ sec$^{-1}$</td>
</tr>
<tr>
<td>half life of luminosity</td>
<td>20 hours</td>
</tr>
</tbody>
</table>
2.2 The Stacking Process

Protons are injected from the inside of the rings on a stable orbit by means of a kicker magnet. Before injecting the next PS burst these protons have to be removed from this injection orbit, since the next pulse of the kicker would dump them on the wall. The shifting of the injected beam away from the injection orbit is achieved by a programmed R.F. acceleration. It is known, that the equilibrium orbit in a synchrotron is given by the frequency of the acceleration voltage. To shift the beam the R.F. is switched on with a frequency corresponding to the injection orbit. Then a slow decrease in frequency will move the beam to a larger circumference and, correspondingly, to a higher energy. Having reached the final orbit, the R.F. is switched off. In order to keep the momentum spread of the beam as small as possible, two conditions have to be fulfilled:

(i) not to induce synchrotron oscillations with big amplitude, the R.F. amplitude has to be small;

(ii) to capture all the beam by the R.F., the stable phase has to include almost $2\pi$, i.e. the equilibrium phase is almost $\pi$; the acceleration has to be slow.
This process is trivial if there is only one PS pulse to be stacked. The aim is, however, to stack as many pulses as possible side by side. To do that a new pulse must be brought close to the previous one without affecting its position. To accelerate a particle with an energy and an orbit which is slightly different, it has not only to be in a stable phase region, but also the R.F. voltage has to be sufficiently high. Otherwise the particle slips in phase and obtains on the average no acceleration. From that we conclude the condition for not affecting the previously stacked pulse:

(iii) the R.F. amplitude has to be sufficiently low just to accelerate the new pulse, but not the previous one.

By this operation, described here in a simplified way, one obtains a number of stacked pulses side by side with slightly different energies.
The number of stacked particles is:

\[ N_{ST} = N_{PS} n_{stacks} = N_{PS} \frac{(\Delta p \Delta \varphi)}{(\Delta p \Delta \varphi)_{PS}} \approx 10^{12} \cdot 0.400 \]  

(2a)

It is proportional to the phase space occupied:

\[ N_{ST} = 2.10^{16} \frac{\Delta p}{p} \]  

(2b)

The circulating electric current is \((e = 1.6 \cdot 10^{-19} \text{Coulomb})\)

\[ I = \frac{N_{ST} e c}{2\pi R} \approx 20 \text{ Ampere} \]

2.3 Interaction Rate and Luminosity

The number of beam beam interactions can be easily calculated from the known relation

\[ N_{\text{int}} \ [\text{sec}^{-1}] = \sigma [\text{cm}^2] n_1 \ [\text{sec}^{-1}] \rho_2 [\text{cm}^{-3}] \ell_2 [\text{cm}] \]  

(3a)

\[ = \sigma \cdot \frac{N_1 c}{2\pi R} \frac{N_2}{2\pi R w} \frac{w}{\tan \frac{\theta}{2}} \]  

(3b)
where \( N_{1,2} \) are the number of particles in each beam, \( 2\pi R \) the total circumference of one ring, \( w \) the beam width, \( \Theta \) the angle of intersect, \( \ell \) the effective length

\[
\ell = \frac{w}{\tan \frac{\Theta}{2}} ,
\]

and \( h \) the effective height, defined by

\[
\rho = \frac{N}{2\pi R w h_{\text{eff}}} = \frac{N}{2\pi R w} \cdot \frac{\int \rho_2(y) \rho_1(y) \, dy}{\int \rho_1(y) \, dy \int \rho_2(y) \, dy} \quad (5)
\]

Here \( \rho_{1,2}(y) \) is a one-dimensional vertical particle density on one beam. For equal numbers in either beam

\[
N_{\text{int}} = \frac{\sigma c}{h \tan \frac{\Theta}{2}} \left( \frac{N}{2\pi R} \right)^2 \quad (3c)
\]

\[
= \sigma L \quad (6)
\]

where \( L [\text{cm}^{-2} \text{ sec}^{-1}] \) is the luminosity, i.e. the rate per second for a cross section of 1 cm\(^2\). Inserting numbers, and using \( \sigma = \sigma_{\text{tot}} = 40 \text{ mb} \), we obtain

\[
L = 10^{34} \left( \frac{\Delta p}{p} \right)^2, \quad N_{\text{int}} = 4.10^8 \left( \frac{\Delta p}{p} \right)^2
\]

\[
L_{\text{max}} = 4.10^{30}, \quad N_{\text{int}} = 1.6 \cdot 10^5
\]

In order to see this result in the right proportion we calculate a "luminosity" \( L \) for a hydrogen target of 1 foot \( \approx 30 \text{ cm} \) at a PS beam of \( 10^{12} \) protons/second: (one foot of liquid \( \text{H}_2 \) \( \rightarrow 1 \) event/sec. barn)

\[
L_T = 10^{36}
\]

This is more than five orders of magnitude higher than the ISR luminosity.
2.4 Background

Besides the beam-beam interactions there are of course collisions of the circulating protons with the rest gas in the vacuum chamber, predominately hydrogen desorbed from the stainless steel walls. The vacuum is supposed to be $10^{-9}$ Torr all around the ring and $10^{-11}$ Torr at the intersects. The resulting effects are the following:

(i) beam loss by nuclear interactions;

(ii) beam blow-up and subsequent beam loss by multiple Coulomb scattering. Although the beam loss by this effect is not considerable, the increase of beam height and the corresponding decrease in luminosity is expected to determine the beam (luminosity) life time to 20 hours;

(iii) beam gas background reactions. The rate of these reactions is at $10^{-11}$ Torr ($N_{\text{Avogadro}} = 2.7 \cdot 10^{19} \text{ cm}^{-3}$)

\[
N_{\text{BG}} = \sigma n_B \rho_g \tau_g
\]

\[
= \sigma \frac{N \sigma}{2\pi R} 2 \cdot N_{\text{Avogadro}} P_g \left( \frac{\text{kg}}{\text{cm}^2} \right) \tau_g
\]

\[
\approx 300 \text{ [sec}^{-1} \text{ m}^{-1}]
\]

This tells us that the serious background is not produced at the intersect, but comes from particles produced with small angles upstream in $10^{-9}$ vacuum region.

3. Present Knowledge about Physics at Ultra High Energies$^3$

Cosmic ray experiments have been performed for many years and yielded information about high energy reactions. What is known at 1000 GeV proton energy?
(i) the total pp cross section is unequal zero and probably between 30 and 50 mbarn;

(ii) the multiplicity of charged particles, mostly pions, produced in an interaction is \( \langle n^\pm \rangle \approx 14 \). Its dependence from energy may be

\[
\langle n^\pm \rangle = 2 \sqrt{E_{CM}}
\]

One expects a corresponding number of neutral \( \langle n^0 \rangle \approx \sqrt{E_{CM}} \)

The distribution of multiplicity is rather uniform.

(iii) The angular distribution is usually parametrized by that of transverse and total momentum. One finds an exponential distribution function for \( p_T \):

\[
\frac{dN}{dp_T} \propto p_T \exp(-p_T/p_{T0})
\]

with

\[
p_{T0} = 0.15 \text{ GeV/c}
\]

and

\[
\langle p_T \rangle = 2 p_{T0} = 0.3 \text{ GeV/c}
\]

(iv) the distribution of total momentum is different for nucleons and pions. For nucleons it is found to be flat, which is equivalent to a uniform distribution of inelasticity. With an average inelasticity of 0.5 we expect a mean pion momentum of

\[
\langle p_\pi \rangle \approx \frac{0.5 \sqrt{E_{CM}}}{\langle n^\pm \rangle + \langle n^0 \rangle} = \frac{1}{6} \sqrt{E_{CM}}
\]

which yields about 1 GeV at the ISR. The distribution function of pion momenta seems to be gaussian or exponential.

The correlation between \( p_T \) and \( p \) is represented in the "Peyrou plot":

The present knowledge of high energy collisions is expressed by the "fire ball" model. This model claims that

nucleons continue as "leading particles" with little change in energy and direction, possibly exited to an \( N^x \);

pions evaporate from the fire ball which is an energy cloud at rest in the C.M., deposited by the peripheral collision of the initial nuclei.
4. Plans for Experiments at the ISR

4.1 The Total Cross Section

The experiment to measure the total pp cross section is probably the conceptually easiest. Since the total path length in the gas is about $10^2$ times more than that in the crossing beam, a transmission experiment is not possible. Therefore the total number of interactions is counted in an almost $4\pi$-counter-geometry. The cross section is then obtained from (6).

$$\sigma_{\text{tot}} = \frac{N_{\text{tot}}}{L}$$

A 1% statistical error can, in principle, be obtained in 0.1 sec.

In praxis there are however several difficult problems:

(i) the counters see always the sum of beam-beam and beam gas interactions. To exclude the beam-gas background the coincidence of both sides is required, since a target at rest will emit particles preferentially into one cone only. The validity of this hypothesis can be checked by one-beam operation.

(ii) some fraction of events all charged particles will remain inside the beam tube. This is for example the case with small angle elastic scattering. One estimates this fraction to be of the order of 10%o. This is corrected for by an extrapolation of detected particles to 0°.
the quality of the measurement of the total cross section is dependent on the normalization, i.e. the determination of luminosity (eqs. 5 and 6)

\[ L = \frac{c}{h_{\text{eff}} \tan \theta} \left( \frac{N}{2\pi R} \right)^2 \]

where \[ \frac{1}{h_{\text{eff}}} = \frac{\int \rho_1(y) \rho_2(y) \, dy}{\int \rho_1(y) \, dy \int \rho_1(y) \, dy} \]

The experimenters intend to determine \( h_{\text{eff}} \) numerically after having measured \( \rho_1(y) \) and \( \rho_2(y) \) independently. For this purpose they measure the \( y \)-distribution of beam gas interaction vertices along the upstream beam pipe by a spark chamber arrangement. They estimate that a 1\(^\circ\)/o accuracy can be obtained by this method.
4.2 Elastic Scattering

Four definitions to start with:

(i) **diffraction scattering**:

\[
\left( \frac{d\sigma}{dt} \right)_N = A e^{bt} \tag{11}
\]

\[
b = \frac{R^2}{4} \quad \text{with } R \text{ the "radius" of the scattering body}
\]

\[
-t = 4 k^2 \sin^2 \left( \frac{\theta}{2} \right) \approx k^2 \frac{\theta^2}{2} \quad \tag{12}
\]

with \( k = p_{CM} \) and \( \theta \) the scattering angle

| Note : \( \frac{d\sigma}{d\Omega} = \frac{k^2}{\pi} \frac{d\sigma}{dt} \) \tag{13} |

(ii) **optical theorem**:

\[
\text{Im } F(0) = \frac{k}{4\pi} \sigma_{tot} \tag{14a}
\]

with \( F(\theta) \) the scattering amplitude

For a scattering on a black disk \( F(\theta) \) is imaginary and

\[
\frac{d\sigma}{d\Omega}(0^\circ) = \frac{k^2}{16\pi^2} \sigma_{tot}^2 \tag{14b}
\]

(iii) **real part of forward scattering**:

\[
F_R = \alpha F_I \quad (|\alpha| \approx 0.1 \text{ at } 25 \text{ GeV})
\]

\[
F = F_R + i F_I
\]

\[
\frac{d\sigma}{d\Omega}(0^\circ) = \frac{k^2}{16\pi^2} \sigma_{tot}^2 (1 + \alpha^2) \tag{14c}
\]

where the real part adds of the order \( \alpha^2 \approx 0.01 \) to equ.(14b).
iv) Coulomb scattering:

\[ \frac{d \sigma}{dt} = \frac{4 \pi \hbar^4}{\beta^2 t^2} G_E^4(t) \]  

(15)

where \( G_E(t) \) is the proton electric form factor, which is equal unity at the values of \( t \) under consideration.

From here we can elaborate the following experimental objectives:

(a) determination of the slope of the diffraction peak. At PS energies \( b \) is found to be of the order of 10. At 50 GeV (Serpukhov) a recent experiment established \( b = 11 \). A logarithmic extrapolation to ISR energies yields \( b = 15^6 \). This shrinkage of the diffraction peak may indicate an increase in the proton size7. The value of \( b = 15 \) limits experiments to \( t < 1.5 \) i.e. to \( \Theta < 50 \) mrad.

<table>
<thead>
<tr>
<th>( \Theta ) mrad</th>
<th>( -t \left[ \text{GeV}^2/c^2 \right] )</th>
<th>( \frac{d\sigma}{d\Omega} \left[ \text{mb/sterad} \right] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1.6 \cdot 10^4</td>
</tr>
<tr>
<td>1</td>
<td>0.6 \cdot 10^{-3}</td>
<td>1.58 \cdot 10^4</td>
</tr>
<tr>
<td>2</td>
<td>2.3 \cdot 10^{-3}</td>
<td>1.54 \cdot 10^4</td>
</tr>
<tr>
<td>5</td>
<td>15 \cdot 10^{-3}</td>
<td>1.26 \cdot 10^4</td>
</tr>
<tr>
<td>10</td>
<td>0.06</td>
<td>6.3 \cdot 10^3</td>
</tr>
<tr>
<td>20</td>
<td>0.25</td>
<td>3.75 \cdot 10^2</td>
</tr>
<tr>
<td>50</td>
<td>1.56</td>
<td>1.07 \cdot 10^{-6}</td>
</tr>
</tbody>
</table>

(b) measurement of the Coulomb scattering, which dominates for angles of less than 2 mrad at 25 GeV. This experiment yields an absolute and calculable cross section and is therefore a measurement of the luminosity.
(c) extrapolation of the nuclear cross section to $0^\circ$ which yields independent determination of the total cross section by the optical theorem.

**pp scatt. $E^* = 50 \text{ GeV}$**

- Coulomb scattering
- Nuclear scattering
- Sum ($\alpha = 0$)

$\sigma_{\text{tot}} = 40 \text{ mb}$

$b = 15 \text{ (GeV/c)}^{-2}$

![Graph showing pp scattering with various cross sections at $E^* = 50 \text{ GeV}$]

Fig. 3
The pp scattering experiment, is in principle, straightforward by measuring the coincidence of two collinear (in the CM!) particles from the intersect. Practical difficulties arise for small angles from the fact that in order to measure at 2 mrad the counters have to be at a distance of 2 cm from the central axis of the beam if 10 meters downstream. So the beam shape has to be carefully determined, and the absolute measurement of the scattering angle requires a calibration.

At larger angles the cross section drops so steeply that above 20 mrad events giving accidentally a collinear configuration such as

$$pp \rightarrow N^X N^X$$

is \(N^X\) decay produces protons with larger transverse momentum, will dominate in the detection. Two ways are foreseen to improve the discrimination:

- anticounters against other particles emerging from the vertex;
- magnetic analysis of the protons.

![Diagram of magnet for pp scattering](Darvinlat et al.)
4.3 Inelastic Scattering, Diffraction and Exchange Reactions

One starting remark:

Two-body and quasi two-body processes can be split into the classes of diffraction and exchange reactions:

(i) Diffraction reactions are interpreted to occur at the boundary of the scattering body (black or opaque disk) and the cross section should be independent from energy, if the size of the scattering body does not change with energy. The differential cross section follows the exponential law

$$\frac{d\sigma}{d\Omega} \propto e^{bt}$$

Besides momentum there is only orbital angular momentum and the corresponding parity $P = (-1)^{\Delta L}$ which can be transferred in a diffraction reaction (diffraction dissociation, or Pomeranchon exchange).

(ii) Exchange processes are invoked if there are other quantum numbers such as charge, isospin and intrinsic parity are to be transferred. These processes are known to have a cross section dropping with energy.

Turning now to pp interactions we see that the prominent quasi two body process is the production of isobars:

$$pp \rightarrow p N^{x+}$$

$$pp \rightarrow N^x N^x$$

Considering the conservation of quantum numbers we can separate the two mechanisms:
The diffraction process should be detectable at the ISR with a cross section of $\sim 1$ mbarn; the exchange process should be suppressed.

To discuss the detection of the isobar production we consider the reaction

$$pp \rightarrow p N^x(1480)$$

$$\rightarrow n \pi^+$$

Momentum and direction of the incident protons are known. If one measures the directions of the final proton, pion and neutron, one constraint is left after the application of 4-momentum conservation laws and the event can be reconstructed. The
distribution of the effective \((n \pi^+)\) mass indicates if a \(N^x\) was produced. We now discuss the problems of resolution, rate and background:

(i) the resolution is mainly determined by the momentum spread of the incident protons \(p/p = \pm 1\%\). This error is reflected after the fit in the fitted values of the particle momenta. The effective \((n \pi^+)\) mass \(M\) is:

\[
M^2 = m_n^2 + m_{\pi^+}^2 + 2E_{n}E_{\pi^+} - 2p_{n}p_{\pi^+}\cos \phi
\]

\[
\approx m_n^2 + p_{n}p_{\pi^+}\phi^2
\]

\[
M \approx \frac{\phi}{2m} \left( p_{n}^2 \phi^2 p_{\pi^+}^2 + p_{n}^2 \phi^2 p_{\pi^+}^2 + 4p_{n}^2 p_{\pi^+}^2 \delta \phi^2 \right)^{1/2} + \text{correlations}
\]

with assumed values we obtain for example

\[
\phi \approx 0.05 \quad \delta \phi \approx 0.002
\]

\[
p_n \approx 7 \quad \delta p_n \approx 0.07
\]

\[
p_n \approx 18 \quad \delta p_n \approx 0.18
\]

\[
\delta M \approx 0.03 \text{ GeV}
\]

(ii) the cross section of \(N^x\) production at PS energies is about 1 mb, and the diffraction mechanism invoked predicts the same cross section, which is about one order of magnitude down from the elastic, at ISR energies. The differential cross section should follow the same exponential law (see table 2)

\[
\frac{d\sigma}{dt} = Ae^{bt} \quad \text{with} \quad b \approx 15 \text{ GeV}^{-2}
\]

(iii) similar to the elastic scattering the limit in the four momentum transfer \(t\) is given by events which accidentally fulfil the kinematical conditions for this reaction. Similar are also the methods to suppress these events: anti-coincidence counters against additional particles, and,
especially, momentum analysis of the charged secondaries. This makes the fit to have 3 constraints with only the neutron energy unmeasured. A large magnetic analysis system is being constructed to serve for this and other experiments.

4.3.1 The Split-Field-Magnet (SPM)

A big magnetic analysis system is foreseen at one of the intersects. It consists of one 5 m magnet on each of the outgoing beams. The magnets are magnetically coupled such that the same flux passes the two gaps, but in opposite direction (fig. 4). Two compensator magnets in each line keep the outgoing proton beam in its original position and direction. The magnet will be equipped with proportional wire chambers, which are self triggering and do not require scintillation counters. Additional particle identification detectors, such as Čerenkov and neutron counters, can be easily added. The detector is connected to a computer and should serve as a general facility.
Fig. 4
4.4 Particle Production

Particle production has been parametrized by empirical formulae and is theoretically described by the thermodynamical model. The experimental aim is to measure absolute yields of stable particles, such as $\pi^\pm$, $K^\pm$, $p$, $\bar{p}$ and $\gamma$ rays, and to determine their production cross section as function to their energy and angle. Experiments to provide these data are planned, and they will cover a range from 15 mrad up to $90^\circ$. Magnetic spectrometers equipped with wire spark chambers will measure the particle production angle and momentum, Cerenkov counters will determine its nature.
Another aspect of particle production is concerned with the production of massive new particles with \( M > M_{\text{proton}} \), such as:

(i) quarks

\[
pp \rightarrow NN \; q\bar{q} \\
pp \rightarrow N \; q_{1/3} \bar{q}_{2/3}
\]

(ii) heavy bosons:

\[
pp \rightarrow NN \; W \rightarrow \mu^+ \mu^- \\
pp \rightarrow NN \; V^0 \rightarrow e^+ e^-
\]

(iii) vector mesons:

It is obvious that the high obtainable CM energy of the ISR shifts the threshold of produced masses to higher energies. It does not, however, in the frame of a statistical model, change the mass dependence of the production cross section

\[
\sigma \propto \exp \left( - \frac{M}{0.15} \right)
\]

where \( 0.15 \rightarrow kT \) is the "temperature" of the system. The formula predicts a \( \pi/\bar{p} \) ratio of 250, but a \( \pi/M \) ratio for \( M = 5 \text{ GeV} \) of \( 10^{-13} \)!

There is no quark experiment proposed at the ISR at the moment (see ref. 11). There are proposals for the search of heavy bosons and vector mesons\(^1,2\). These experiments do not necessarily try to reconstruct a particle from its decay products but try to find evidence for the decay of a heavy mass. The particle emitted in a two body decay energetic leptons, and these leptons do not have the limitation in transverse momentum, as have particles produced in strong interaction; secondary pions are bound to have small \( p_T \), and so are their decay muons because of the small decay energy. The heavy boson W may be produced with small \( p_T \), but not its decay muon because of the high decay energy.
So the experiments search for leptons with high $p_T$ there, where the tail of the exponential production distribution from strong interaction has become very small. The experiments aim at a limit $\sigma \cdot BR \approx 10^{-34} \text{ cm}^2$. 
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The CERN/ISRC documents are unpublished and available from the authors only.
NEW DEVELOPMENTS IN ELECTRONIC POSITION DETECTORS

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University College of Swansea
Wales

1. INTRODUCTION

In the fields of nuclear and elementary particle physics a knowledge of the spatial distribution, identity and intensity of particles, both relative to each other and to some arbitrary fixed position is clearly of vital importance. This is especially the case in high energy accelerators of electrons and protons where the ever increasing energy scale demanded by physicists requires larger machines and larger beam transport paths of possibly several kilometers in length.

For example in the CERN Intersecting Storage Ring project the proton beam, after ejection from the Proton Synchrotron has to be lifted over the linear accelerator injector and thence, through a height of twelve metres to allow for the difference in level between the PS and ISR sites. The beam transfer system has a total length of almost two kilometres and consists of four complicated transport channels. It is necessary to have reliable information about the beams at very many points in this complex arrangement for stacking 28 GeV protons. The handling and steering of such beams usually requires a continuous visual display and automatic or computer control. This control system necessitates initial detection of the beam and its fluctuations in space and time.

At the other end of the energy scale (\$ keV) we now find that considerable use is made of concentrated ion and electron beams for ion implantation and welding where a knowledge of the beam position is equally important.

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In these two lectures I will be concerned principally with high energy beams and I wish to discuss some methods and systems of electronic position detectors which are currently being developed or coming into use soon in high energy physics laboratories such as the Stanford Linear Accelerator Centre and CERN.

I will not confine the discussion to an account of the measurement of the position of electron and proton beams, but I will also comment on such devices as spark and streamer chambers and on electronic methods of examining bubble chamber film records. In doing so I am interpreting the title electronic position detectors in a very wide sense to include anything which involves the movement of electrons. This motion can be an atomic transition from one excited state to another as in a laser or the bulk movement of electrons in a linear accelerator.

The particular type of detector selected for an accelerated beam depends upon several factors, including:

a) The type of beam being accelerated, charged or uncharged, stable or unstable particles,
b) The machine itself, i.e. whether it is pulsed or continuous,
c) The energy of the particles constituting the beam - because of the threshold energy of possible detecting reactions - e.g. Synchrotron and Cerenkov radiation,
d) The intensity of the beam.

Once decisions on these have been taken a selection of one or more of the following methods may be made:

1) Electrostatic and Electromagnetic Induction
   Induced charges - potential measurement
   Induced magnetic fields - current measurement
   Microwave cavities

2) Ionisation Phenomena
   Inelastic collisions with residual gas molecules
   Secondary electron ejection from probes
3) Radiation Phenomena
   Cerenkov radiation
   Synchrotron radiation

4) Direct optical Observation
   Photographic plates
   Phosphorescent screens

2. SECONDARY ELECTRON EMISSION MONITORS

The physical principle underlying these is very simple.

Ions incident upon a conductor can, provided they have sufficient potential or kinetic energy, eject secondary electrons. If the conductor is insulated it acquires a positive charge. The number of electrons ejected and the mechanism of ejection depends upon the identity and energy of the ions and upon the nature of the conductor. For example, low energy atomic He\(^+\) ions (\(
\lesssim\)
100 eV) can eject on the average \(\gamma \sim 0.2\) electrons per ion from gold and silver by an Auger interaction in which the ion potential energy is exchanged with an electron in the metal. The ion is neutralized and may be reflected or absorbed at the surface and the electron ejected, usually with a few electron-volts of kinetic energy. This is secondary emission due to potential ejection.

At much higher energies kinetic ejection occurs and a proton incident at, say 20 GeV, on an aluminium foil, will eject, on average, \(\gamma\), about 0.04 electrons and the proton, after multiple scattering will continue in the forward direction. The emitted secondary electrons can be swept away by a small electric field leaving the aluminium foil positively charged.

Thus a proton beam of cross-sectional area \(A\) comprising \(m\) bunches each containing \(N\) protons incident on a foil of electrostatic capacity \(C\) (including parallel connections) which intercepts a fraction \(a/A\) of the beam will produce a voltage \((\epsilon Y a mN/CA)\) on the foil. This is simply proportional to the number of protons which have traversed the foil and may typically lie between a few millivolts and a volt or so depending upon beam conditions. It can be measured by means of a sampling amplifier
(if there are several insulated foils) with high input impedance and displayed on an oscillograph. In this way the charge left on the foil can give a measure of the intensity of the beam at a particular place.

A geometrical arrangement of several insulated conductors in the form of a grid can thus yield valuable information about the full spatial, and temporal structure of the beam, i.e. its width, height and intensity as a function of time.

Particular requirements for monitors based on this principle have arisen in the CERN Intersecting Storage Rings and two types of Secondary Electron Emission (SEM) beam position detectors have been developed to work under very stringent conditions for the proton beam transfer system from the PS to the rings.

The devices are required to operate with fast extracted beam pulses of 2.1 μsec duration or with 10 nanosecond single bunches as well as with slow extracted beam pulses of a few hundred milliseconds duration. In order to facilitate computer control for certain operations the output signals can be digitized and displayed oscillographically. The monitors have to operate in regions where the residual gas pressure is very low (∼10⁻¹⁰ Torr) in order to avoid multiple scattering and beam blow-up and consequently must be bakeable at temperatures up to 250°C.

1) SEM GRID PROFILE MONITOR

This device, which is shown in Figure 1, consists of a grid of 15 horizontal and 15 vertical spring loaded aluminium foils mounted individually on two ceramic (magnesium silicate) frames. The foils are 2 mm wide, 0.02 mm thick and are set 5 mm apart. They intercept, but do not stop, about one-tenth of the beam cross-section. Secondary electrons liberated by the passage of the protons are swept to three collector plates by a small sweeping field. The two outer collectors are in the form of rings which surround the beam. The third collector is a 5 micron thick aluminium foil placed between the two sets of signal foils and is traversed by the beam. It serves not only as an electron collector
but also as an electrostatic screen between the vertical and horizontal signal foils. It is also used to apply test pulses which are capacitatively coupled to the signal foils.

The introduction of the foils naturally causes some multiple scattering of the proton beam. This amounts to between 4 and 12 microradians depending on whether the beam traverses only one or all three intercepting zones.

**Electronics**

Each of the measuring foils is connected to a high input impedance amplifier via a cable the length of which is adjusted to provide a capacitance of 100 pF on which the positive charge is stored pending measurement.

The output signals of the sampling amplifier are switched sequentially into a cable at a rate of 20 kHz and can be displayed and recorded and stored in an analogue-digital recorder generator.

Between each measurement the capacitors are discharges by means of reed relays. Arrangements are made so that the timing of the relays and of the signal switching at the output of the amplifiers can be adjusted so that a beam profile of a slow-ejected beam can be measured as a function of time during the ejection.

The spatial resolution achieved is naturally governed by the foil separation of 5 mm for this particular device but this can be improved by interpolation measurements which are effected by mounting the assembly on a pendulum which permits simultaneous movement in the x and y planes. The movement takes place between two beam pulses and effects a resolution of 2.5 mm.

The same pendulum arrangement allows the grid to be moved completely out of the beam.

Typical results are shown in Figure 2.
4. SECONDARY ELECTRON EMISSION PROBE BEAM SCANNER

The probe beam scanner is essentially a secondary emitter detector which is arranged to traverse the proton beam in steps. The probe - an aluminium cylinder 3 mm dia and 0.02 mm wall thickness is moved along the arc of a circle to preselected positions in the beam and fixed relative to the walls of the vacuum chamber. Movement is effected by a stepping motor the digital drive circuits of which are controlled by the beam pulses (one per second to three per second) in such a way that the probe is always moved into its new position between two successive pulses.

This monitor when used with one probe for each beam axis can provide the same information as the grid monitor but is much slower insofar as it requires 9 consecutive pulses to locate the beam edge and a further 10 to 20 pulses to measure the intensity profile - during which time the beam should remain stable. It is much simpler to make than the grid monitor.

The scanning action is illustrated in Figure 3 and is as follows: On receipt of the command "Search Beam Edge" the logic and control box applies 256 pulses to the stepping motor which moves the probe through an angle $\alpha/2$ from its lateral rest position. If no signal is detected in this position on the next beam pulse it is moved inwards by 128 steps through an angle $\alpha/4$. If it now lies in the beam and detects a signal it moves through an angle $\alpha/8$ in the reverse direction and so on until the edge of the beam is located. This location is recorded on a helipot system and the "SCAN" command given on receipt of which the probe moves across the beam in steps taking one or two steps per pulse. The signals from the amplifier and helipot can be used to draw the intensity profile on an x-y recorder or memoscope or fed to the recorder-generator for storage.

This particular monitor can also be used as an adjustable limit switch to detect drift of the beam edge. For this application the probe first locates the edge and is then set say one centimetre away. If the beam drifts over this distance a warning signal is produced. This is of considerable use for slow-ejected beams.
The electronic circuitry associated with this probe detector and with the grid monitor is miniaturized and exceedingly compact. The whole unit together with a system of luminescent screens, mirror telescope and closed circuit television monitor is built as a single unit.

5. SYNCHROTRON RADIATION 5)

For initial alignment and monitoring of relativistic electron beams it is necessary to have direct visual observation of the beam shape and position at a number of places along its path.

Advantage may be taken of the fact that the accelerated electrons become self-luminous due to synchrotron radiation; the spectrum of radiation is continuous down to a critical wavelength

$$\lambda_{\text{crit}} \approx \frac{4\pi R}{3} \left(\frac{m_c^2}{E}\right)^{3/2} \approx 6 \text{ metres} \cdot \frac{E_{\text{GeV}}^3}{(10^5 \text{ gauss} E_{\text{GeV}}^2)}$$

Angstroms,

for electrons of energy $E$ moving in a path of radius $R$ determined by a magnetic field $H$. A fraction of the total energy radiated, proportional to $(H/E)^{2/3}$ lies in the visible region ranging from about 4000 to 6000 Å. For example 30 MeV electrons in the 30 inch accelerator form a concentrated narrow cone of dark red radiation changing to blue at about 80 MeV.

Synchrotron radiation, which is entirely non-intercepting thus provides the most direct form of electronic position detector. It is not related to the properties of the surrounding environment through which the electrons accelerate as is the case for Cerenkov radiation. It is emitted immediately by the electron which becomes "luminous" in the most literal sense. This property makes it highly desirable although in a linear accelerator it can only provide information about beam dimensions and position inside magnetized regions where the beam is deflected.

The energy radiated per second by an electron

$$\frac{2}{3} \frac{C e^2}{R} \left(\frac{E}{M_0 c^2}\right)^4$$

so that the amount of synchrotron radiation emitted in travelling unit distance is
\[ W = \frac{2}{3} \frac{e^2}{R} \left( \frac{E}{M_e c^2} \right)^4 = \frac{2}{3} e^4 \left( \frac{E}{M_e c^2} \right)^2 \left( \frac{H}{M_e c^2} \right)^2 \]

\[ \approx 0.015 \frac{E_{\text{GeV}}}{R_{\text{m}}^2} \approx 1.4 \times 10^{-11} \frac{H_{\text{gauge}}}{\text{MeV/metre/electron}} \]

The number of photons within the visible part of the spectrum emitted by one electron in travelling one metre is

\[ n \approx 0.4/R^{2/3} \]

so that the number emitted per second per unit path length per microampere of beam current is

\[ N \approx 2.5 \times 10^{12}/R^{2/3} \text{ photons/sec/metre/\muA}. \]

These photons are emitted tangentially to the electron orbit i.e. in the forward direction and the zeros of the radiation pattern are located in the orbital plane at angles

\[ \theta = (1 - \beta^2)^{1/2} = (M_e c^2/E) \]

from the direction of motion which approximately defines a narrow cone of light which is roughly symmetrical about \( \theta = 0^\circ \). The time during which the radiation reaching an observer is emitted is given by

\[ 2 R (1 - \beta^2)^{1/2}/c \]

and the pulse duration is thus approximately

\[ R (1 - \beta^2)^{3/2}/c = (R/c)(M_e c^2/E)^3. \]

In the case of the Beam Switchyard of the Stanford Linear Accelerator, with a bending radius of 57 metres the half angle \( \theta \) amounts to about 1.5 millirads in cases of interest and the light output amounts to about \( 5 \times 10^{10} \) photons/sec/\muA from a path length of 40 cm.

This light is transmitted via a series of front surface aluminized mirrors of reflectivity 90% and 95% transmission quartz window to a vidicon which is sufficiently sensitive to observe linac beam currents as low as 0.01 \muA.
At peak power operation some 21 watts of synchrotron radiation, covering the full spectrum is incident on the first mirror but this is not sufficient to cause serious damage.

The resolution of the system is governed principally by the vidicon and TV monitor used for display and is about 0.5 mm at the position of the beam.

Substitution of a photomultiplier for the TV camera converts the system into a beam intensity monitor as well as position detector. It can then be used at extremely low current levels where other conventional methods are ineffective. The necessary calibration can be effected at high intensities by comparison with a current transformer.

6. POSITION DETECTORS BASED ON RESIDUAL GAS IONISATION

There have been a number of interesting advances in the last two years in the development of beam position detectors based upon ionisation caused by the passage of proton beams through residual gas in accelerator chambers.

Some of the gas in the region traversed by the beam is excited and ionised so that fast collection of the ions or electrons created can give information about the intensity and position of the beam. The number of electron-ion pairs created per unit volume in a given region of the proton beam is proportional to the proton density in that region.

This is the essential principle of three types of so called "non-destructive" detectors developed for use on the zero gradient Synchrotron at Argonne and on the CERN PS.

N protons traversing L metres of a gas at a pressure p the specific energy loss of which is S will create

$$3.56 \times 10^6 \frac{pLNS}{\delta}$$

electron-ion pairs requiring an energy $\delta$ per pair for liberation. Under the conditions of the Argonne Synchrotron this corresponds to about $2 \times 10^{13}$ ion pairs per second or an electron current of
about 3 μA/metre. The corresponding current obtainable under CERN PS conditions is about half a microampere per metre. These constitute a convenient and measurable signal.

7. PHOSPHOR SCREEN DETECTION SYSTEM

One system consists of an aluminized phosphorized (type 31) glass screen lying above a mesh electrode which, with another plate electrode is located above and below the proton beam and parallel to the orbital plane.

A small (∼1 kV) potential difference is applied between the mesh and plate so that the electrons liberated by the proton beam are accelerated through the mesh into a region of strong electric field. They acquire energy up to about 15 keV before impinging upon the screen to cause the emission of light directly over the region occupied by the proton beam. Thus one obtains a visual measure of the width of the proton beam, and it is observed by means of a closed circuit television camera together with fiducial marks on the screen.

The sensitivity is such that at a pressure of $2 \times 10^{-6}$ torr beam images for intensities as low as $5 \times 10^{10}$ protons per pulse are observable.

8. STRIP ELECTRODE DETECTION SYSTEM

Separate radial and vertical detection schemes are used to monitor beam position and profile. In each case an earthed and an HT plate are used to collect ions or electrons, liberated by the proton beam onto a series of 10 vertical and ten radial strip electrodes which are mounted separately on, and individually insulated from, the earthed plate. The strip electrodes are each 10 inches long and separated by 0.1 inch. On the radial system they are 1.5 inches wide and 0.5 inches wide on the vertical system and lie parallel to the beam direction.

The currents to individual strips are converted to voltages and connected to the Main Control Room and multiplexed to a single multipurpose output which operates in three modes.
In the first mode the multiplexer sums the input and gives an output proportional to the beam intensity.

In a second mode of operation the multiplexer sequentially samples the ten strip signals continuously and connects the outputs to the vertical signal of a synchronized oscilloscope to provide a dynamic display of the proton beam. It can also be used to obtain a time display of the beam position and profile for the entire machine cycle. For this purpose a 20 kHz sawtooth voltage generated in synchronism with multiplexer is connected to the vertical input of the oscilloscope resulting in a series of vertical lines. Each vertical line can be visualized as being divided into equal segments each representing a strip on the electrode. The vertical lines are intensity modulated by connecting the multiplexer output to the oscilloscope Z axis so that the intensity of a given vertical line segment corresponds to the associated strip electrode current.

In the third mode of operation the system is triggered and the multiplexer sequentially samples to provide measurements at chosen values of the magnetic field with the aid of high speed photography of the oscillograph display, this method is providing non-destructive measurements of the beam profile and position in two planes. Previously unobservable behaviour of the beam can now be seen quite readily.

9. THE CERN PS GAS IONISATION BEAM SCANNER 3).

A very ingenious development of the gas ionisation scanner has been installed in the CERN PS at Easter 1969. The requirements are rather more stringent than at the AZGS and the CERN method achieves high resolution and at the same time avoids the complication of multiplexer scanning by using an elegant and delightfully simple system incorporating only one collecting electrode and orthogonal electric and magnetic fields to move the electrons created in the residual gas to the collector. It is shown in Figure 4. The principle of the system is as follows: a magnetic field $\mathbf{B}$ of about 100 to 150 Gauss parallel to the proton beam
direction is produced between two main electrodes, some 50 cm long set 16 cm apart, by external Helmholtz coils, and an electric field \( \vec{E} \) of 200 V cm\(^{-1}\) is set up orthogonally.

Under these fields the electron paths are a combination of a drift velocity \( \frac{\vec{E}}{B} \left( -\frac{\omega e}{1 + \omega^2 c^2} \right) = V_- \) along the equipotentials and a circular motion in the plane perpendicular to the \( \vec{E} \) vector with radius \( R \) given by

\[
R = \left[ x^2 + \left( z - \frac{E}{B} \left( \frac{\omega^2 c^2}{1 + \omega^2 c^2} \right) \right)^2 \right]^{1/2} \left( \frac{eB}{m} \right).
\]

With the values of \( \vec{E} \) and \( \vec{B} \) quoted \( V_- \) is approximately 2 \( \times \) \( 10^8 \) cm sec\(^{-1}\) and \( R \approx 0.3 \) cm and the electron collection time i.e. transit time from point of liberation to detector is \( \sim 100 \) nano-seconds. \( \omega \) and \( c \) are the electron cyclotron frequency and mean free time between collisions.

The collector electrode is maintained at earth potential at the top and in the centre of a series of guard electrodes which serve to reduce electric field distortion so that, initially, the equipotential surfaces lie vertically in the diagram.

Scanning is achieved by applying a triangular waveform of 3 kV at up to 20 kHz to the main electrodes superimposed up an equal amplitude static potential difference maintained between them. In this way the equipotential surfaces are swept laterally through the proton beam at a rate which is sufficiently low to allow collection, i.e. the collection time is much less than the period of the triangular waveform.

Since the detector is at earth potential it is only those electrons which moved along or near the earth equipotential surface which arrive at the collector. Thus as this particular equipotential surface moves through the beam electrons from each section of the beam move up to the collector. The number arriving at a particular time is proportional to the amount of ionisation and hence to the time averaged proton density.
Two detectors have been used: a simple strip electrode 10 cm long which gives a sufficiently large signal at scanning times down to 100 µs, and the first dynode of an electron multiplier which, with its associated dynodes, permits a 1 µsec scanning time even though the collection length is reduced to one centimetre in the proton beam direction.

With this improved sensitivity obtained with the electron multiplier it seems feasible to operate this device at pressures as low as $10^{-8}$ torr with $10^{12}$ circulating protons. The resolution achieved in beam radial position is 1 mm. The signals are amplified and fed to an oscillograph in the PS Control Room.

The whole system clearly forms an elegant and very convenient method of providing, in an immediately available visual form, the beam position and size during the whole of the PS machine cycle, and can range from looking at modulations in the beam intensity due to bunching and debunching to the overall gross movement of the beam. Typical displays for certain beam conditions are illustrated in Figures 5 and 6.

10. INDUCIVE BEAM POSITION MONITOR 4)

Inductive beam position monitors depend for their action the displacement of the beam, or rather the centroid of the beam from its equilibrium position, and modulation of the beam by pulsed operation or bunching. Electrostatic or magnetic potentials are produced which, within limits, are proportional to the product of the beam intensity and displacement.

An exceedingly compact device has been developed at the Stanford Linear Accelerator Centre to examine the behaviour of electron and positron beams. It measures the x and y displacements of the centroid of the beam within a clear aperture of 3.5 inches diameter and uses the beam modulation which is in the form of a rectangular current pulse of 0.5 to 2 µsec duration, repeated up to 360 times per second. It has a sensitivity of $2 \times 10^{-4}$ ampere millimetre and a spatial resolution of 0.1 mm.
The principle of operation is as follows:

A pick up coil is orientated so as to embrace the magnetic field set up by the beam current $I_B$. When this pulsed current passes along the axis of the coil (the self-inductance of which is $L$) the magnetic field couples symmetrically and no net emf is induced in the coil. However if the pulsed current axis is displaced laterally by an amount $x$ this symmetry is destroyed and an emf, proportional to $x$ and to the rate of change of beam current is generated across the coil terminals.

Thus if $M$ is the mutual inductance coupling the beam current and the coil the emf generated is $= \frac{MdI_B}{dt}$. This causes a current $I_L = \frac{\epsilon_L}{\omega L}$ to flow in the loop and so an output power $P = \sum P_w = (\frac{M^2}{L}) \sum \omega I_{Bw}^2 \cos \phi$ is available for detection purposes.

Here $I_{Bw}$ is the beam current component of frequency $\omega$ and $\cos \phi$ is the power factor, governed by the circuit in parallel with the coil.

To obtain the maximum output power $M^2/L$ should be made as large as possible. This is governed by the geometry. $M$ is maximized by bringing the loop wire as close to the beam as possible and $L$ is minimized by making the loop wire into a broad conducting strap of width $S$.

Now $M = 8 \times 10^{-9} x 1/W$ and $L = 4 \times 10^{-3} 1 \log_e (2 W/S)$ in centimetre so that the power available at a frequency $\omega$ is

$$P_\omega = \frac{1.6 \times 10^{-14} l^2 \omega I_{Bw}^2 \cos \phi}{W^2 \log_e (2 W/S)} \text{ watts.} \quad (I_B \text{ in amps}).$$

A combined unit incorporates both $x$ and $y$ (horizontal and vertical) pick up loops into a single unit which is milled from a piece of copper tubing. The electrical connection of the two loops in this way eliminates cross coupling because it is at the balanced centre of each loop.
With this arrangement coupled to two 10 : 1 pulse transformers and amplifier and integrator the minimum beam pulse size is \(2 \times 10^{-4}\) ampere millimetre.

11. **TRACK DETECTION EMPLOYING ELECTRICAL DISCHARGES IN GASES**

Several devices which involve the amplification of the number of ion pairs created by the passage of an incident charged particle through a gas to locate the position or track of the particle have been the subject of intensive development since they offer several advantages on bubble chambers.

The track is located, in some chambers by the light radiated from a succession of discharges created along its path by the application of a strong electric field. This results in the spatio-temporal growth of ionisation and in visible radiation.

The track following properties of the wide gap spark chamber are of particular interest. This phenomenon occurs even when the track of an incident particle is inclined at large angles to the applied electric field, though the luminosity of the discharges becomes fainter as this angle is increased. At present the limiting angle for relatively easy detection is about 40°.

A series of measurements of the temporal behaviour of track following sparks has recently been completed at CERN 6) and an explanation in terms of the space charge of the applied electric field by the discharge current has been proposed. A quantitative examination 7) of the spatio-temporal growth of ionisation has shown the track following can be explained on the basis of known values of electron-atomic collisional and transport coefficients.

In any electrical discharge in a gas the rate of change of concentration of active particles, e.g. electrons, ions, metastable atoms etc. can be described in terms of integro-differential equations describing the balance between the rate of generation by any process or processes and the rate of loss. For example 8), if we consider an elementary volume \(\text{d}V\) in a gas, the concentration \(n\) of, for example, electrons, may change continually due to their creation, at a rate \(q\) by ionizing
interactions and due to their arrival or departure by drift \((W_-)\) and diffusion \((D)\) and losses by such processes as recombination \((R)\) and attachment \((a)\).

Thus the net rate of change of electron concentration in a volume \(V\) bounded by a closed surface \(S\) can be written

\[
\frac{dn}{dt} \, dV = \int_V (q - R \, n^2 - an) \, dV - \int_S (-D \text{grad} \, n + nW_-) \, dS
\]

which can be transformed using Gauss' theorem into

\[
\frac{dn}{dt} = q - R \, n^2 - an + D \nabla^2 n - \text{div} \, nW_-
\]

In order to solve this we must specify the appropriate boundary conditions. Its solution, together with that of a similar equation for ions, has led to expressions for the spatio-temporal growth of ionization in a wide variety of discharge conditions ranging from breakdown under static electric fields \(^8\)), unit function impulse fields, microwave frequency fields and discharges created by focussed laser beams.

Under the conditions which obtain in spark chambers the situation is very complicated because the concentration of electrons and positive ions and their relative motion can result in highly distorted electric fields due to the formation of space charges. The relevant excitation, ionization, collisional and transport coefficients then become functions of position and time. Straightforward analytical solutions of the above continuity equation cannot be obtained and numerical computation must be used.

In order to simplify the very complex situation we will assume that recombination losses are negligible and that the only active ionisation process is single impact collisions between electrons and non-attaching gas molecules.

The continuity equations are thus:

\[
\frac{dn}{dt} = n \, \alpha \, W_- - \text{div} \, n \, W_- + D \nabla^2 n
\]

\[
\frac{dN}{dt} = n \, \alpha \, W_-
\]
where $\alpha$ is the probability that an electron will undergo an ionising collision in travelling unit distance along the direction of the electric field, i.e. it is the Townsend primary ionisation coefficient. If $V$ is the electrostatic potential at any point it is governed by Poisson's equation

$$\nabla^2 V = \frac{1}{e} \times 3 \times 10^{13} (N - n)$$

where $n$ and $N$ are written in Coulomb's $\text{cm}^{-3}$ and $\nabla^2 V$ is in volts $\text{cm}^{-2}$.

These three simultaneous equations have been solved using numerical procedures and supplementary equations relating the primary ionization coefficient $\alpha$ and the electron drift velocity $W_-$ to the electric field, subject to the boundary condition that at a time $t_1$ the centre of each avalanche has moved a distance $W_- t_1$ and the negative charge density at a distance $r$ from the centre is given by

$$n(r,t_1) = q_0 \left( \frac{4 \pi D t_1}{\alpha} \right)^{3/2} \exp \left\{ \alpha \frac{W_- t_1}{4Dt_1} - \frac{r^2}{4Dt_1} \right\}$$

where $q_0$ is the electronic charge and the value of $t_1$ is chosen so that at time $t_1$ space charge distortion of the field is negligible. As a further boundary condition we require the potential distribution to represent a uniform distribution at a large distances.

Typical results are shown in the following figures for neon at a pressure of 690 torr under an applied electric field of $6.73 \text{ kV/cm}$. The separation between ion pairs created along the track is $2 \text{ mm}$ for a track induced to the applied field at an angle of $30^\circ$.

Figure 7 shows the progressive change with time in the electric field along the axis of a single avalanche. The field is enhanced at the avalanche front and rear and decreased at the centre. The distribution of electron velocity is thus such as to lengthen the avalanche. At the centre the field appears to attain a constant value at late times. The rate of ionization in this region is almost exactly balanced by losses due to drift and diffusion leading to a region of almost uniform density moving forward.
The spatio-temporal distribution of light intensity along the axis is shown in Figure 8 which reveals the fact that most of the luminosity is restricted to a small region near the front of the avalanche but there is a second maximum at the back which is due to the rapid increase of field in this region. Any electrons liberated for example, by photo absorption, behind the advancing avalanche thus see a field which, in the particular example considered is some 20% in excess of that at the advancing head. Consequently avalanche growth from the photo electron would be faster. A succession of such photo initiated avalanches could then give rise to an apparently faster moving wave of luminosity in the reverse direction, since the appearance of luminosity at a point signifies the attainment of a certain electron concentration, and hence collision frequency at that point which yields detectable radiation.

The precise behaviour will of course be critically dependent upon the details of the experiment.

The track following characteristics are illustrated in Figures 9 and 10.

In the first, corresponding to a time of $82.8 \times 10^{-9}$ sec after application of the field the field is only slightly distorted but the avalanches are asymmetrical and have started to bend towards the track. In the second ($t = 97.9 \times 10^{-9}$ sec) the field lines are highly distorted and now follow the track very closely: the avalanches have merged to form a continuous tube of current.

Further information, of a more general character concerning the track following action may be obtained from graphs of the direction of the resultant current as a function of time for various track angles. Thus if $\theta$ is the angle between the applied field and the current and if $\bar{\theta}$ is the track angle we draw graphs of $\theta/\bar{\theta}$ for various times. Data for avalanches 1 mm apart are shown in Figure 11 which shows that the relative rotation away from the applied field and towards the track increases with time. Initially the rotation is very small, since the field distortion is minimal. The relative rotation is most pronounced when the track angle is small - not much greater than $37.5^\circ$ say.
On general grounds, we would expect an increased amount of field distortion at earlier times if the pressure were doubled and E/p kept constant. This is because the current would be the same, but the current density, which governs the space charge will be much larger. This effect is shown in Figure 12 where the track following is both earlier and more pronounced.

12. ELECTRONIC BUBBLE CHAMBER RECORD ANALYSIS

Sweepnik is a device which has been developed during the last few years at the Cavendish Laboratory in Cambridge. It is a semi-automatic device combining the advantages of a track following device with that of a spiral reader. It derives its name from its ability to sweep up information from bubble chamber film records with the speed of a sputnik.

A small on-line computer controls its operation and gives an output like that of a manual machine and it is claimed to be accurate to 1 μ in the film plane.

Once the operator has set the sweeping mechanism on a track it will follow it at 5 cm/sec measuring co-ordinates in steps of 1 mm. Each track is automatically measured in both directions to reduce systematic errors, and fiducial marks are automatically measured.

The principle of operation is as follows:

The circular beam of light from a low power single spatial mode helium - neon laser is first converted into a line image by passing it through an astigmatic lens. The bright line image so formed is about 1 mm long by 0.03 mm wide. The now long thin beam passes through a rotating Dove prism so that the line image rotates at about 48 revolutions/sec at a point 0.5 mm outside its own length. So what one would see of this image falls on a screen is a circle of red light formed by the line image rotating. The position i.e. orientation of the line at any time is measured by a coded disc attached to the prism.
An image of the rotating line is focused onto the bubble chamber film using an arrangement of mirrors the position of which is controlled by the on-line computer.

When the rotating line, i.e. the light probe is near a track and intercepts it, there is a reduction in the intensity of light transmitted through the film every time the spinning line crosses or touches the track. This produces a signal in a photomultiplier located behind the film. The angle of the line image, at the time of the pulse is recorded and read by the computer which then calculates the sine and cosine of the angle and sends an instruction to the steering mirrors which then move the rotating line to the position where the centre of rotation was, which is a good approximation to where the track lies. The steering mirrors take less than 15 ms to make this move and the photomultiplier is ready for the next signal.

In this way the probe follows the track in 1 mm steps 48 times a second, i.e. it tracks at about 5 cm sec\(^{-1}\).

In order to achieve an accuracy of 1 micron in the film plane the steering mirrors must be controlled to within 0.1 second of arc. An interferometric method is used using the beam from the other end of the laser. The interferometer is a Michelson mounted on the back of each mirror and the fringe pattern, which is governed by the mirror position is counted and used, via the computer to control the position of the steering mirrors.

The performance of Sweepnik may be summarized as follows:

1) Measurements using Sweepnik or other devices show that it is at least as accurate as the best of other automatic machines.

2) Tracks can be followed at about 4 to 5 cm sec\(^{-1}\) on the film through nearly all regions of confusion. For example, with 25 cm of track for a 4 prong event the actual time spent track following is about 12 sec per view. This together with 10 sec for 5 fiducial marks totals about 22 sec, so that roughly 1 minute is
occupied for all three views - about 4 times as fast as present methods.

1) Because of the laser source it has an excellent signal to noise ratio (20 : 1). This means that poor bubble definition is no longer a serious limitation.

2) All the optical components are relatively cheap.

3) Operator interaction is minimized and is only needed with distorted track shapes.

4) The track following with the integrated slit obviates problems of reconstruction of track segments since the line always points along the line.

5) An optimized system is expected to operate at up to 60 events/hour.

6) Data on ionization is immediately available in the form of pulse height and width averaged along the track.

13. THE POSSIBILITY OF USING LASER RADIATION TO DETECT PARTICLES

It is now well known that sufficiently intense laser beam radiation can excite and ionize atoms and cause the electrical breakdown of gases accompanied by the emission of visible radiation. Hot plasmas can be created in exceedingly short times (~nanoseconds). Experiments at CERN \textsuperscript{10,11} and elsewhere have shown that power densities approaching $10^{11}$ to $10^{12}$ watts cm\textsuperscript{-2} are required with ruby laser radiation to cause breakdown in air at atmospheric pressure. However, considerable amplification in the concentration of initiatory electrons resulting in intense visible radiation may be achieved with much lower power densities. It has been suggested \textsuperscript{12} that this phenomenon might be used as a particle track detector which would enable the track of the ionizing particle to be photographed readily - The laser produced plasmas grow symmetrically about the points at which the ion pairs are liberated.

In order to examine the feasibility further we consider some of the factors which govern the generation and growth of plasmas by laser radiation.
In treating the problem we can take advantage of the fact that although five parameters are involved, namely the ionisation potential of the gas atoms, the electric field created by the laser beam, the wavelength of the laser radiation, the mean free path of the electrons in the gas and the size of the volume irradiated by the laser light, there are only two dimensional variables — voltage and centimetres.

We can thus use the so called \(^{13}\) "proper variables" which are of considerable help in direct current and high frequency discharges, namely \(E/p\) or \(E/N\), the ratio of the electric field \(E\) to gas pressure \(p\), or atomic concentration \(N\) particles/CC \((N = 3.56 \times 10^6 \ p\), when \(p\) is measured in torr\), and \(p\lambda\) or \(N/\omega\) where \(\lambda\) and \(\omega\) are respectively the wavelength and angular frequency of the laser radiation.

The rate of gain of energy \(\frac{dE}{dt}\) by the electrons from the laser radiation field in making elastic and inelastic collisions with gas atoms is

\[
\frac{dE}{dt} = \frac{e^2 E^2}{m^2} \cdot \gamma_c \frac{\omega^2}{\omega^2 + \gamma_c^2}
\]

where \(\gamma_c\) is the electron — atom momentum transfer collision frequency. Thus we can introduce the concept of an effective electric field \(E_{\text{eff}}\) which produces the same energy transfer as a steady electric field. This is given by

\[
\frac{E_{\text{eff}}}{N} = \left[ \text{ERMS laser/N} \right] \left[ \frac{\gamma_c^2}{\gamma_c^2 + \omega^2} \right]^{1/2} \approx \left[ \text{ERMS laser/N} \right] \left[ \frac{\gamma_c}{\omega} \right]
\]

Since, for most case of interest \(\omega \gg \gamma_c\).

Consider now the dynamic characteristics of the electrons in a discharge as defined by the continuity equation

\[
\frac{dn}{dt} = q - R n^2 - an - D \nabla^2 n - \text{div} n W,
\]

and let us limit the discussion to the earliest stages of growth and thus neglect recombination and attachment.
Drift losses are eliminated because of the oscillatory nature of the electromagnetic field and, if the irradiated volume is large (several cubic centimetres of uniform radiation) we can neglect diffusion losses. The continuity equation is thus much simplified and is now merely
\[
\frac{dn}{dt} = n \mathcal{Y}_i(t) \text{ where } \mathcal{Y}_i(t) \text{ is the electron ionization collision frequency and is of course dependent upon the primary ionization coefficient } \alpha \text{ and electron mobility } \mu \text{ which, in turn, are governed by the laser beam intensity.}
\]

We note that over restricted ranges of electric field and gas pressure we can approximately express \( \mathcal{Y}_i(t) \) as follows.

\[
\mathcal{Y}_i/N = \alpha \mu E/N = \alpha \mathcal{W} - /N \quad \text{where } \mathcal{W}, \text{is the electron drift velocity,}
\]

but \( \alpha/N \propto E/N \) and \( \mathcal{W} - \propto E/N \)

thus \( \mathcal{Y}_i/N \propto (E/N)^2 \) i.e. \( \mathcal{Y}_i/N = k(E/N)^2 \)

where \( k \) is a "constant" which can be determined from the measurable dependence of \( \alpha, \mathcal{W} -, \text{and the excitation coefficient } \theta, \text{upon } E. \text{ Using Poyntings theorem, we can express this dependence as}

\[
\mathcal{Y}_i/N = \frac{377}{N^2} \left( \frac{\mathcal{Y}_C}{\mathcal{W}} \right)^2 k P(t)
\]

where \( P(t) \) is the laser beam power density expressed in watts cm\(^{-2}\).

Thus \( n(t) = n_0 \exp \left[ \frac{377}{N} \left( \frac{\mathcal{Y}_C}{\mathcal{W}} \right)^2 k \int_0^t P(t) dt \right] \)

where \( n_0 \) is the initial electron concentration.

If we approximate the true laser flash waveform as a triangle of height \( P_{\text{max}} \) and duration \( 2 \) then the amplification is

\[
n(2\tau)/n_0 = \exp \left[ \frac{377}{N} \left( \frac{\mathcal{Y}_C}{\mathcal{W}} \right)^2 k P_{\text{max}} \tau \right]
\]

In several gases of interest the momentum transfer collision frequency is approximately independent of electron energy and is proportional to the gas pressure. Thus we can write \( \mathcal{Y}_C = ap \) so that
\[
    n (2\tau) n_0 = \exp \left( \frac{377 p a^2 k P_{\text{max}\tau}}{3.56 \times 10^{16} \omega^2} \right) \\
    = \exp \left( 1.1 \times 10^{-16} p a^2 k P_{\text{max}} \frac{\tau}{\omega^2} \right)
\]

Approximate values of \(a\) and \(k\) obtained in studies of laser induced breakdown in gases carried out at Swansea \(14,15\) are given in the table below.

<table>
<thead>
<tr>
<th>Gas</th>
<th>(a)</th>
<th>(k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Helium</td>
<td>(2 \times 10^9)</td>
<td>(1.4 \times 10^{21})</td>
</tr>
<tr>
<td>Neon</td>
<td>(1.5 \times 10^9)</td>
<td>(1.8 \times 10^{21})</td>
</tr>
<tr>
<td>Argon</td>
<td>(2.5 \times 10^9)</td>
<td>(6.5 \times 10^{21})</td>
</tr>
<tr>
<td>Krypton</td>
<td>(4 \times 10^9)</td>
<td>(4.2 \times 10^{21})</td>
</tr>
</tbody>
</table>

Using these data we can obtain rough estimates of the maximum energy density and power density required to produce a given amplification say \(n (2\tau)/n_0 = e^\gamma\). Thus we obtain

\[
P_{\text{max}} \text{ joules/cm}^2 \sim 10^{16} \gamma \omega^2 / a^2 k p \quad (p \text{ in torr}).
\]

We see immediately the advantage of using long wavelength lasers since the energy required decreases with the inverse square of the radiation wavelength. The use of high gas pressure is also an advantage however, because of non-linear optical effects, there is a critical power and gas density which must be avoided in order to circumvent beam self-trapping and self-focussing, since the critical power for self trapping is proportional to the wavelength squared. The use of gases with low non-linear refractive indices will thus be advantageous.
Tabulated below are values of \( P_{\text{max}} \tau \) for ruby, neodymium and carbon dioxide lasers. These show that with presently available lasers, argon and krypton offer the best possibilities for the gas filling and that it is likely to prove difficult to obtain track detection in helium and neon.

<table>
<thead>
<tr>
<th>Laser</th>
<th>Ruby</th>
<th>Nd</th>
<th>CO₂</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gas Helium</td>
<td>( 2.7 \times 10^8 /p )</td>
<td>( 10^8 /p )</td>
<td>( 10^6 /p )</td>
</tr>
<tr>
<td>Neon</td>
<td>( 3.7 \times 10^8 /p )</td>
<td>( 1.6 \times 10^8 /p )</td>
<td>( 1.6 \times 10^6 /p )</td>
</tr>
<tr>
<td>Argon</td>
<td>( 3.7 \times 10^7 /p )</td>
<td>( 1.6 \times 10^7 /p )</td>
<td>( 1.6 \times 10^5 /p )</td>
</tr>
<tr>
<td>Krypton</td>
<td>( 2.3 \times 10^7 /p )</td>
<td>( 9 \times 10^6 /p )</td>
<td>( 9 \times 10^4 /p )</td>
</tr>
</tbody>
</table>

This view is supported by the very recent study at CERN 16) although the beam energy used was much lower than the theoretical values quoted in the table.

Apart from the difficulties associated with achieving sufficiently large uniform energy and power densities in large volumes to be of interest in track chamber work severe problems arise in connection with reflectivity and reliability of mirrors, reproducibility of laser light intensity and an adequate flash repetition rate. At present the repetition rate is far too low to be of interest in high energy nuclear physics applications. However, the tremendously rapid advances in laser technology makes the proposal 16) of a track chamber based on laser excitation an extremely interesting one.

**ACKNOWLEDGEMENTS**

I wish to offer my best thanks to Dr. Denys Nicholas of the Rutherford High Energy Laboratory and Messrs. E. Gygi, D.A.G. Neet, F. Schneider, G. Schneider and C.S. Taylor of CERN for many interesting discussions and helpful advice concerning the material on which these lectures are based.
REFERENCES

Secondary emission probe beam scanner

Fig. 3
Fig. 5
INJECTION

TRANSITION

F.E. 1 - 10.5 GeV/c

TARGET 6 - 16.4 GeV/c

TARGET 1

19.2 GeV/c

F.S.E. 58

PROTON BEAM: position and density distribution on horizontal plane

Fig. 6
Fig. 7 Field distribution in a single avalanche for the following times: 77.7, 79.9, 81.5, 82.9, 84.2, 85.3, 86.4, 87.4, 88.5, 90.5, 91.5, 92.5, 93.5 ns.
Fig. 8  Distribution of light intensity along the axis of a single avalanche at various times.
Fig. 9  Field lines and electron density contours during the development of track following spark.
Fig. 10 Data as for fig. 7 at time = 97.9 ns.
Fig. 11 Rotation of current vector into the track direction.
Fig. 12 Rotation of current vector into the track direction.
EVOLUTION OF SOME PARTICLE DETECTORS
BASED ON THE DISCHARGE IN GASES

G. Charpak
CERN, Geneva, Switzerland.

1. INTRODUCTION

In this year 1969, if we look into the experimental techniques used around high-energy accelerators we observe the following situation: the bubble chamber is still an important tool, absorbing a large fraction of the activity of the community of experimentalists. Its evolution is directed towards a greater efficiency and rapidity in the automatic evaluation of pictures, and towards the building of giant chambers.

What is it that keeps the bubble chamber surviving in the hostile surroundings of fast-growing counter techniques? Let us quote, for discussion, some qualities and defects of a typical large hydrogen bubble chamber, 2 metres long:

- Accuracy in localization
- Interaction length
- Minimum detectable momentum
- The target and the measuring media are identical, permitting the visualization of very complex configurations around the interaction point.
- Sensitive time
- Non-selective data-taking
- Maximum number of pictures
- Maximum number of beam particles per picture
- Maximum number of pictures analysed in one experiment

\[
\begin{align*}
70 \mu \\
10 \text{ m} \\
1200 \text{ GeV/c} \\
\text{milliseconds} \\
\text{several per beam burst} \\
20 \\
\sim 10^6 
\end{align*}
\]

As we shall see, it may well be that there is not any single property of the bubble chamber that cannot be equally well achieved by an electronic detector; or that could be achieved at least in the foreseeable future. However, there is not any one detector that incorporates all these properties at the same time.
The electronic detectors usually look at a very specific aspect of a reaction, and do it with the highest accuracy. These days one hears about wanted accuracies of $10^{-4}$ in symmetry problems, requiring at least 100 million events that are asked for in order to confront some theory with nature. In these two lectures, I am going to summarize the properties of some of the detectors that are commonly used in counter experiments to localize charged particles, and which are based on discharge in gases under the influence of electric fields. Since I also wish to underline the trends in the research being carried out on these detectors, I will refresh your memory about some basic facts of gaseous amplification in homogeneous and inhomogeneous fields.

2. FIELD DISTRIBUTION AROUND ELECTRODES MADE OF WIRE

Since I will have to discuss phenomena occurring in chambers where the electrodes are made of wires, let me start immediately with the properties of such structures. They are of general importance, since streamer chambers are often equipped with electrodes made of wires in order to have them transparent, and they are used in all types of wire chambers.

Let us consider two cases. In the first one, the electrode made of wires is facing a single electrode. In the second one, it is placed in the medium plane of two other electrodes.

In both cases, the field is very nearly uniform at distances from the wire greater than the wire spacing. This is why such electrodes are usually a good approximation for plane electrodes. However, as can be seen from the equipotentials (Fig. 1), a sizeable fraction of the potential can be lost in the gradient around the wire, and this is something of a nuisance when one aims only at obtaining a given uniform field at the lowest voltage. In the case of the symmetrical structure, the region around the wire is of great interest in the proportional detector, and it is quite easy to have a quantitative evaluation of the field distributions.

We assume an infinite assembly of wires, of diameter $d$, spacing $s$, distance from the wires to the external electrodes $L$. We centre the coordinate system on one wire, with $x$ in the plane of the wires and $y$ perpendicular to the plane. Then for infinitely thin wires, a straightfow-
ward calculation, done by summing up at one point the effects of all the wires, gives these very simple formulae for the field along the three symmetry lines:

\[
V(0,y) = 2q \ln \sinh \frac{n y}{s} \rightarrow E_y = \frac{2q n}{s} \coth \frac{n y}{s}
\]

\[
V(x,0) = 2q \ln \sin \frac{n x}{s} \rightarrow E_x = \frac{2q n}{s} \cot \frac{n x}{s}
\]

\[
V(s/2,y) = 2q \ln \cosh \frac{n y}{s} \rightarrow E_y = \frac{2q n}{s} \tanh \frac{n y}{s}
\]

At very small distances, such as 100 \( \mu \), sinh \( n y/s \) and sin \( n x/s \) are equal within \( 10^{-3} \). We thus see that the infinitely thin wire approximation is a very good approximation to the physical situation, and the wires of that thickness can be assimilated to an equipotential of the field distribution to a great accuracy. A rigorous treatment of these problems can be found in many textbooks\(^1\).

At a distance \( s/2 \) from the wires, the field is uniform within 10\%. At a distance 1.2 \( s \), it is uniform within \( 10^{-3} \).

In the vicinity of the wires, the field varies as \( 1/r \), as in a cylindrical chamber. We will see that this feature is one of the essential reasons why such electrodes can work like independent arrays of proportional counters.

3. EFFECT OF ELECTRIC FIELDS ON FREE ELECTRONS IN A GAS — MEMORY OF Chambers

If we apply electric fields to such structures, we meet different situations according to the field amplitude. Let us assume that the gap is filled, at atmospheric pressure, with noble gases where the electrons liberated by ionizing particles or by discharges are free from attachment to the heavy atoms. The mean collision free path of electrons is typically \( 10^{-5} \) cm in argon and \( 3 \times 10^{-5} \) in helium. Fields of considerable strength would be required in order to give to an electron enough energy between two collisions for it to reach the energies necessary to ionize the atoms of rare gases (A : 16 eV; Ne : 21 eV; He : 24 eV). But the electrons lose only a negligible fraction of their energy in one elastic collision, and they can capitalize the energy gained between two collisions.
The mean free path for ionization by electrons is strongly dependent on the gas composition, and is typically, for voltages of about 10 kV/cm, applied in spark chambers of the order of $10^{-3}$ cm; it is close to $10^{-4}$ cm in the region around the wire of proportional chambers where the field reaches $10^5$ V/cm.

At equilibrium, with no electric field, the average energy of the electrons is the same as that of the ions, so the random thermal velocity of the electrons is much higher than that of the ions. With an electric field, the energy of the electron is higher, and one defines a fictitious electron temperature corresponding to this increased energy. This temperature can be decreased by mixing small amounts of complicated organic gases. These molecules can be broken by the electrons in inelastic collision, as it requires only a few electron volts to do this. In the electric field, the electron distribution drifts towards the anode with a velocity that is dependent on the field and on the electron temperature. The lower the temperature, the higher the velocity. This is partly why organic additives are added in proportional chambers since, as we will see, this velocity controls the time resolution.

The field dependence of the drift velocities in a gas made of argon + organic additive is illustrated by Fig. 2. The velocities are in the range of $10^6$ to $5 \times 10^6$ cm/sec in the mixture of A + methane, widely used in proportional counters. In the neon-helium mixtures used in spark chambers, the drift velocity is also of the same order of magnitude.

If we apply a voltage of 4000 V to a structure made of wires of 20 μ, 2 mm apart, with a gap of 8 mm, we have a field of $2.2 \times 10^5$ V/cm at the wire surface. It decreases as $1/r$, and it is of only 3000 V/cm in the uniform region. Only the very limited region around the wire has enough field to have the electrons multiplying by inelastic collision. The electrons liberated in the gas outside this region will simply be drifted towards the wire where the amplification occurs.

If, however, we apply fields (10 kV/cm in spark chambers with neon filling, or 20 kV/cm in streamer chambers) that are such that even in the uniform region the mean free path for ionization is smaller than the gap length, then very fast phenomena occur as we will see, and the amplification at the wire has no time to play a role.
These predisruptive phenomena leading to the sparks are of basic importance for the understanding of spark chambers. Their description can be found elsewhere$^2$).

I will come back later to the case where the only region of amplification is concentrated around the wire. However, in spark chambers all the intermediate situations coexist to some extent. Between the application of high fields leading to break-down, low d.c. fields are applied to clear away electrons from old tracks, and it is this time of clearing that determines the "memory" of the chambers, or its time resolution. It is usually, at best, of the order of 300 nsec for small-gap chambers, and often of the order of 10 μsec for large-gap streamer chambers. Figure 3 shows the variation of the memory of a spark chamber as a function of the clearing field.

4. **GASEOUS AMPLIFICATION IN THE UNIFORM REGION**

If, in the region of uniform field, one electron makes $\alpha$ ionizing collisions per centimetre, then the number of electrons produced in the development of one avalanche along a distance $X$ is

$$N = \exp (\alpha X).$$

All exponential developments have to come to an end. When the field is maintained, different processes occur in succession, each of them corresponding to different types of detectors: the avalanche multiplication, the streamer process, the breakdown of a spark.

4.1 **Region of avalanche multiplication**

Let us take neon as the medium. The development occurs at about the same speed as the drift motion; in 10 nsec we have reached a dimension of about 1 mm. Because of diffusion this avalanche has also grown laterally to a dimension of about 1 mm.

The ideal visual gaseous detector would be the one for which we could interrupt the development at this stage and take pictures of it. However, nature has put a kind of universal limit on this growth. For $N = 10^8$, the electric field of the positive ions compensates the external field and the avalanche stops.

Is $10^8$ sufficient?
Experiments show that in neon there is about one visible photon per electron. With an optical system accepting $10^{-5}$ of the total solid angle, 1000 photons can be concentrated on one grain of 7 μ of the photographic emulsion and can trigger it. But this is not sufficient to make it stand out from the background.

If we were to use image intensifiers, then with modern photocathodes 10% of the photons can produce an electron giving a spot that has the brightness needed for photography. This is the technique used at CERN by F. Schneider, and with it he can easily see avalanches. Figure 4 shows one of his resulting pictures. He estimates that the accuracies that can be reached by this method are 0.28 mm in the direction orthogonal to the electric field and 0.4 mm in the direction of the field.

Since in the future such intensifiers are going to progress and become cheaper, this technique will have more applications. It should in principle make feasible the visualization of complex events around the vertex in dense gases, thus competing seriously with one of the main attractive features of the bubble chambers, and even with additional flexibility. For instance, one could imagine a pressurized helium avalanche chamber in which hypernuclei are formed, and in which such a rare and theoretically important process as the β-decay of the hyperfragment is studied. One of the most important parameters, the direction of the recoil proton, is out of reach with helium bubble chambers because the range is too low. In an avalanche chamber, the pressure could be adjusted to fit to the problem. So the progress in high-voltage pulses together with the progress in light amplifiers may open the way to a serious competitor for bubble chambers in the field where they are excellent.

4.2 Streamer chambers

For the time being, it would appear to be more practical and cheaper, and also more accurate, for large systems to apply the voltage for a longer time and to enter into the streamer region. If we keep applying the voltage after the critical size of $10^8$, we have secondary avalanches formed around the initial avalanche. Since the electric fields are higher in front of and behind an avalanche, these avalanches develop faster. This is the reason for the line of avalanches along the electric field, called streamers. The propagation speed is about $10^8$ cm/sec. In 10 nsec we
have a streamer length of 1 cm. If we look through transparent electrodes in the direction of the electric field, and if the depth of focus is higher than the length of the streamer, we can increase the amount of light in proportion to the number of avalanches, about a hundred. If we want to keep a stability of 10% in the streamer length, we need a stability of 1 nsec in the pulse length. Even like this, and using the best available films with demagnifications of 80, apertures of about f/2 are necessary in order to have a mediocre image. The accuracy reached with the present techniques is 0.5 mm in the directions orthogonal to the electric field, and about 2 mm in the direction of the field. This is attained with neon at atmospheric pressure at fields of 20 kV/cm. With helium, fields of 30 kV give good images. The use of hydrogen is still not practical. To study interactions in hydrogen, it is necessary to use hydrogen targets inside the chamber. One then loses the view of the vertex, but in many cases this is irrelevant; and recently published results of experiments show that such a technique is superior to bubble chambers for the study of very complex events such as photoproduction of resonances decaying with a high multiplicity. In such reactions the requirement of a production of hadrons in the reaction reduces by orders of magnitude the background due to electromagnetic interaction, which still represents 90% of the pictures.

4.3 Spark chambers

If we keep the voltage on while the streamers grow, then they touch the electrodes and the real spark occurs. It is a propagation of charges with a phase velocity that is huge. It can reach $10^{10}$ cm/sec. The light can be increased by three orders of magnitude if enough energy is delivered by the pulse.

4.3.1 Large-gap chambers

There are cases where the interaction between avalanches gives rise to an instant streamer, and the spark follows the line of the initial avalanches. Such avalanches are in use in some laboratories, and even at CERN you will see such an automatic chamber in the boson missing-mass experiment. One of its virtues is that it is 100% efficient for any number of particles, but this property is now shared by narrow-gap wire chambers built as transmission lines.
The only remaining virtue of a wide-gap chamber is to my knowledge, that in a strong background of X-rays it may lead to a reduction of the spurious tracks, since very curled tracks are suppressed or have a lower efficiency than nearly straight tracks connecting the electrodes. In the most general case, gaps of below 1 cm are used.

4.3.2 Properties of small gap chambers

I have already mentioned the memory as being one of the main characteristics. I should now mention the localization accuracy.

With optical chambers, the accuracy is a function of the angle. Figure 5 shows that the accuracy varies from 0.2 mm below 15° to 1 mm at 45°; this is because a spark may break down from any point along the trajectory. One can fight against this by reducing the gap width; but then there is a drop in efficiency. In neon the number of primary ion pairs per centimetre is 12. In one millimetre there is a strong probability of having no electron.

It was recently emphasized by Alvarez that there are great advantages in trying to use liquid argon as a medium\(^4\), and active research is being performed on this subject in several laboratories.

Since it has been demonstrated in the past that electron multiplication occurs in liquid or solid argon, one can hope to reduce the thickness of the gaps to 50 μ and to have accuracies of 5 μ.

This is of fundamental importance for the physics around the 300 GeV machine, where any gain in accuracy means a gain in length of the spectrometers or in the magnetic field strength.

For the time being, a more simple approach has been undertaken at CERN by C. Rubbia. At high pressures he uses gaps of 1 mm, and has shown that accuracies of ±30 μ can be reached with minimum ionizing particles.

I should point out that such accuracies have already been achieved by Fischer\(^5\), who also used a narrow gap, but with particles 10 times more ionizing than the minimum, which is equivalent to a higher pressure.

5. AUTOMATIC SPARK CHAMBERS

Up to now I have mentioned the two properties that gave incentive to the wide use of spark chambers: the memory and the accuracy. The accuracy
was poorer than that of bubble chambers, but the memory allowed the selection of events by additional counters, which resulted in a considerable increase in statistics for some phenomena.

However, the limitation came again from the number of pictures that one can normally handle with a decent budget, and within a decent time, and the automatic chamber provided an answer to it.

I am not going to discuss all the methods that have been invented and even used: the vidicon method, the sonic chamber, the current division method, the wire chambers with core read-out, with magnetostrictive read-out or with capacitive read-out. The description of some of them can be found in the literature\textsuperscript{2,6}.

I just wish to say some words about the methods that have been most widely used in large high-energy laboratories: the wire chambers.

5.1 Wire chambers with core read-out

The electrodes are made of wires spaced by a distance \( s \). When a spark occurs, the current will spread among the wires close to the spark. The problem is to read out the wire transporting the current.

The first method, put forward by Krienen and still popular, is to have each wire going through a memory core that gets flipped by the current. Two other wires also go through the centre of the cores: a read-out wire and a sense wire.

The technology of these read-out systems is well worked out, and at CERN several systems with 50,000 wires are in use. The accuracy reached is about \( \pm 0.3 \text{ mm} \) for 1 mm spacing for tracks orthogonal to the planes.

The weakness of this method lies in its cost and its sensitivity to magnetic fields. Even fields of 100 gauss can prevent the cores from flipping.

The main advantage is that any number of sparks can be handled, and recent progress in the construction has brought the efficiency close to unity for almost any number of sparks.
5.2 Wire chambers with magnetostrictive read-out

The second method, which is widely developed, is the magnetostrictive method.

A magnetostriction line is placed across the electrode wires, at a small distance from the electrodes. When the current passes through the wire, the magnetic field reorients the magnetic domains of the magnetostriction line. An elastic signal propagates along the line at a speed of about 5 mm/µsec. This signal can be read out with a pick-up coil placed at the end. By measuring the time of arrival of the signal, one knows the position of the wire responsible for the current signal.

The wire has to be slightly magnetized longitudinally by a field of about 100 gauss to give its best signal.

This method has several attractive features.

It can, in principle, work in a magnetic field. If the magnetostrictive lines are orthogonal or nearly orthogonal to a uniform magnetic field, they operate very well. If coordinates with several orientations are to be measured, then certain problems arise; however, these can be overcome by several methods. One method that has been described recently for a system of cylindrical wire chambers built into a large magnet for use at the Brookhaven AGS, is to have the wires supported by weightless rigid material and oriented in different directions, but all coming out parallel to the magnetic field at the place where the magnetostriction line measures the position.

Another advantage of the above-mentioned method is its low cost. The increase in the size of the chambers leads to almost no increase in the read-out system if one is ready to sacrifice reading speed. Systems with $10^5$ to $10^6$ wires are under construction, or are envisaged in the near future, using this technique.

Let me mention the double spectrometer of Lindenbaum\(^7\), where complex events with two V's are selected, and where chambers of 7 m are to be used. However, it should be stressed that in non-uniform magnetic fields, such as those often encountered inside large magnets, this method also fails because the signals become too small.
5.3 Wire chambers in magnetic fields

Different techniques have been designed to replace these two methods in case one wants the chambers inside strong inhomogeneous magnetic fields. One is the sparkostrictive method, the other one is the capacitive storage method.

In the first one, the current in the wire is used to produce an auxiliary spark in a line in which it produces a sound wave, the time of arrival of which is measured via piezo-electric transducers.

In the second one, the charge of a wire is fed to a large capacitor, typically $10^5$ pF, where it is stored for some milliseconds; this brings it to a voltage of, say, 15 V.

After an event, an electronic system reads out each capacitor and brings it to ground afterwards. The same read-out hardware as that used for cores can be employed.

Thus we see that in principle, even in the most general case, we can stuff the space with detectors giving typically an accuracy of $\pm 0.3$ mm to $\pm 1$ mm, depending on the direction, with a repetition rate of about 200 events/second, i.e. 20 to 50 events per machine burst. Rather great statistics can thus be accumulated.

5.4 The vidicon method

Finally, the vidicon system is now strongly advocated by some physicists. Their enthusiasm is based on the improvement of the properties of the commercial vidicons, and on the fact that for very complex arrangements with hundreds of gaps it is the only economical approach.

I refer you to the proceedings of the last Conference on Instrumentation for High-Energy Physics, held at Versailles in September 1968, where each of the methods I have mentioned is dealt with by several authors.

I now wish to speak of the most recent development, which makes use only of the amplification in the inhomogeneous part of the chamber.
6. THE MULTIWIRE PROPORTIONAL CHAMBERS\textsuperscript{9,10)

If an electric field is applied to the structure represented in Fig. 1, and if it is such that inelastic collisions start occurring in the proximity of the wire, then we have what is called the proportional amplification. The reason for this is that if a particle produces electrons in the region far from the wire, it is collected on the wire and the signal is proportional to the number of electrons. Each wire operates as in a normal cylindrical counter, and the old theory relative to these counters applies.

During many years, two factors limited the development of this technique. Before solid-state amplifiers became available the volume and the cost of the necessary electronics were too excessive. But the main factor was a widespread false appreciation of the electrostatic interaction between two neighbouring wires.

It was believed that because of the capacitive coupling, the wires next to the amplifying wires would receive a sizeable part of the signal, and for this reason many attempts were made to have each sensitive wire separated by a shielding wire. However, this was costing a factor of two in the spatial resolution, and was limiting the lower distance between wires since a high voltage had to be applied between them.

In fact, if it is true that when you send a negative pulse, with an external generator, on one wire, you receive a sizeable negative pulse on the neighbouring wire, then the situation is different when you detect a particle by proportional amplification on a wire. You have indeed a negative pulse on this wire, but positive pulses on the neighbouring ones. This effect is due to the mechanism generating these pulses, namely the motion of the positive ions in the strong fields around the wires. This effect is responsible for the perfect localization of the pulses on the sensitive wires, irrespective of the distance between the wires. It is sufficient to have amplifiers that are sensitive only to the good polarity, to avoid the spurious effect of capacitive coupling between wires.

6.1 Limits of proportional amplification

It was observed\textsuperscript{8}) that the proportional amplification ceases when the size of the avalanche exceeds a given value corresponding to about
3 \times 10^6 \text{ ion pairs}. In other words, with one electron we can reach a gain of about 10^6, which means that for the average energy loss corresponding to a minimum ionizing particle traversing 1 cm of argon, we can expect a maximum average gain of 10^6. In fact, since one is interested in detecting losses 10 times smaller than the average, one can push the gain to 10^5.

The capacity of wires of 20 µ spaced by 2 mm is about 10^{-1} \text{ pF/cm}, and the limit of 3 \times 10^6 ions corresponds to about 5 \times 10^{-13} \text{ Coulombs/cm}, while the charge per cm at 3 kV is 10^{-13} \times 3 \times 10^3 = 3 \times 10^{-10} \text{ Coulombs/cm}. However, over the avalanche length, which may be of the order of 10^{-1} to 10^{-2} mm, the local positive charge facing the avalanche is of the same order as the maximum observed charge. We can thus explain this effect of gain saturation by space charge, similar in a sense to the limit reached by the critical avalanche in a uniform field.

If we keep increasing the voltage, then photons emitted by the positive cloud start playing a role. The development of avalanches along the wire leads to the Geiger-Müller mechanism. I will not discuss this in these lectures, despite the fact that the use of multiwire chambers in the Geiger-Müller mode may have some future.

If we stop in the proportional region, what can we expect?

The capacity of a wire, because of its connections, is almost always larger than 20 pF. The maximum pulse-height that we can have is about 100 mV.

Because of the Landau fluctuations in the energy loss, we expect a large energy spread. In practice, it appears that in order to have 100% efficiency with minimum ionizing particles, we need to be sensitive at the level of 0.5 mV at least.

Such a method is clearly dependent on how well one can use such small pulses. Before discussing this, I wish to summarize the properties of these chambers, and to explain to you why a great effort is being made by several groups to develop them.

6.2 Spatial resolution

One may wonder how close to each other one can bring the wires and still keep them working independently.
As already mentioned, if one sends pulses to a wire with a generator, one finds induced pulses of the same sign on the neighbouring wire, and of a size increasing when the wire spacing is decreasing. The pleasant surprise with these chambers is that when one observes a negative pulse induced on a wire by an avalanche, the pulses induced on the neighbouring wires are of opposite sign.

A naive belief is that the collection of electrons is responsible for the negative pulses. This is not true. Since the most important part of the avalanche is produced at distances from the wires the order of microns, the effect of the collection of the negative charge $-Q$ is almost completely counterbalanced by the effect of the appearance of the positive charge $+Q$ so close to the wire. It is only when the positive ions move fast in the fields that reach several hundred kilovolts near the wire, that a negative pulse is induced. This motion induces a charge $-Q$ on the wire, and a charge $+Q = Q_1 + Q_2 + \ldots$ on the surrounding electrodes, like the neighbouring wires on the high-voltage electrodes. This is why we have an excellent localization on the wire collecting the avalanche.

Indeed, if tracks are inclined, it may well be that electrons liberated along a trail get amplified on different wires.

With wires of 20 $\mu$m diameter, distances of 2 mm between the wires give an easy operation. It is possible to detect a few ion pairs lost in the gas, by using electronics sensitive to 0.5 mV. If one wants to go to better resolutions, we should have in mind that the field around a wire is $2q/r$, where $q$ is the charge per unit length. By increasing the number of wires, we decrease the charge per wire and we have to compensate this by increasing the voltage. The relation between the charge and the different parameters of a chamber is:

$$q = V/2 \left[ \ln \sinh \frac{\pi L}{s} - \ln \sinh \frac{\pi d}{s} \right],$$

where $L$ is the distance grid-wire, $s$ is the distance between wires, $d$ is the wire diameter. For $L = 8$ mm, $d = 20$ $\mu$m, this charge varies in the ratios 1, 1.27, 2.2 when $s$ goes from 1 mm to 2 mm and 3 mm, respectively for a given voltage.

At too high voltages trouble occurs, such as corona effects on the external electrodes. If you consider that we are dealing with millivolt
pulses, whilst 10 kV may be necessary for 1 mm spacing, it is clear that the finest break-down is catastrophic.

An easier operation can be obtained by disentangling the region of amplification and the region of drift by means of an additional grid placed at a small distance from the wire. At a distance of 1.2 s, the field is already uniform within $10^{-3}$, so the addition of a metallic grid placed at this potential does not alter the field distribution. We observed that with 20 μ wires placed at 1 mm distance, with a grid at 2 mm, we have had perfect operation at voltages of 3 kV on the screen and 4 kV on the external electrodes, whilst 10 kV would have been necessary with a normal single-gap structure.

6.3 Time resolution

With chambers having 3 mm spacing and argon-isobutane filling we obtain a maximum jitter time of 36 nsec; with 2 mm spacing it goes down to 25 nsec; with 1 mm it reaches 18 nsec (Fig. 6), but there the electronics we used contributes in a non-negligible way.

Even there we observe a correlation between the position of the track between the wires and the time delay of the pulse arrival.

A repetition rate of $10^6$/wire is possible if the electronics on each wire can deal with it.

6.4 Operation problems

We are now faced with the problem of using these chambers.

Compared with spark chambers, we have gained a factor of 10 in time resolution and a huge factor in repetition rate. We are now facing the problem of using a hodoscope with thousands of elements. The difference with a scintillator hodoscope is in the slightly poorer resolution time, but there is a great decrease in the amount of matter (a factor of 100), and a gain in cost per element (a factor of 10).

We have lost an important quality of spark chambers, namely the memory. We could regain it by using the drift space of a modified chamber, and pulse it. The grid will shield the wires. We then lose the resolution time and just win some repetition rate with respect to a spark chamber, but at a very high cost.
The ideal circuit that we need with these chambers should have the following function: amplification from a level of 0.2 mV, shaping, delivery of undelayed pulses for fast decision-making logic, delivery of delayed pulses and transmission gates to control the admission in the memory, storage and read-out.

Since we are planning detectors with $10^5$ wires, it is clear that we are considerably dependent on the reliability and the cost of such a system.

There is room for much imagination and ingenuity in the development of the electronics systems connected to these chambers, in order to bring their cost to a level that justifies their use in all the cases where they are superior to other detectors in physical performance.

7. CONCLUSION

In these two lectures I wanted simply to give a rapid survey of some of the tools that are now in the hands of the experimentalists, and to give you some understanding of those techniques that are undergoing rapid evolution: the avalanche and streamer chambers for the visual techniques dealing with very complex configurations; and the proportional multivire chambers, which will probably, in most of their applications, be associated with large systems of wire spark chambers to act as a trigger hodoscope with many elements.
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Fig. 1 Equipotentials around electrodes made of wires:
   a) Two-electrode configuration.
   b) Three-electrode symmetrical configuration.

Fig. 2 Electron drift velocity in a mixture of 90% argon and 10% methane.
Fig. 3 Dependence of the memory time on the value of the clearing field I.I. [taken from Gromova et al. (1964)].

Fig. 4 Electron tracks in an avalanche chamber [F. Schneider, CERN]. Helium-neon; 10 cm length; field of 25 kV/cm; 10 nsec width. Image intensifier - equivalent aperture f/30.
Fig. 5 The percentage of sparks with deviations less than δ plotted as a function of δ in mm. The three curves represent the following angular intervals:

a) 0°–15°  
b) 15°–30°  
c) 30°–45°

[taken from J.C. Rutherglen et al. (1961)].

Fig. 6 Time resolution of a proportional multiwire chamber.
Distribution of the time interval between the passage of a particle and its detection on a wire.

- Wire spacing s = 1 mm.
- Distance between wires and outer electrodes: L = 2 mm.
- Argon-isobutane (80/20).
- HV = 3900 V.

The maximum time jitter is 18 nsec.
THE "OMEGA" PROJECT*)

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The Omega project consists of a large spectrometer magnet, a system of detectors (spark chambers, counters, etc.) operating inside the field volume, and a data acquisition system which is based on two on-line computers, 64 K 32 bits and 16 K 16 bits, respectively. The useful magnetic field of 18 kG extends over a volume of 3 m diameter × 2 m height.

This spectrometer is intended for high-energy experiments on interactions of a complicated pattern, where all charged secondaries \((n \geq 2)\) have to be detected and accurately measured over a large solid angle of acceptance. Examples of such interactions are:

**STRONG INTERACTIONS**

a) \(\text{meson} + \text{proton} \rightarrow \begin{cases} \text{meson} \\ \text{meson resonance} \end{cases} \begin{cases} \text{baryon} \\ \text{baryon resonance} \end{cases}\)

b) \(\pi + p \rightarrow Y + K\), etc.

In order to extract new useful informations from these experiments, the cross-sections that have to be measured are \(\lesssim 1 \mu b\) with a few hundred events, or \(\sim 100 \mu b\) with \(\geq 10^4\) events.

**WEAK INTERACTIONS**

Leptonic decays of hyperons:

c) \(\Lambda + p \rightarrow \begin{cases} e^- \\ \mu^- \end{cases} + \nu\)

\(\Xi^- \rightarrow \Lambda + \begin{cases} e^- \\ \mu^- \end{cases} + \nu\), etc.

The decay rates are of the order of \(10^{-3}\) or smaller. The required statistics are \(\geq 10^3\) per decay channel.

*) The Omega Project, CERN NP Internal Report 68-11, May 1968.
Until now, exploratory work on experiments of this type has been carried out at the CERN Proton Synchrotron (CPS), mostly with bubble chambers. It is believed that in future such an experimental program will be better performed by spectrometer magnets such as the Omega. It is worth recalling some of the general technical characteristics of the Omega as compared to the bubble chamber:

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Omega</th>
<th>H₂ Bubble chamber</th>
</tr>
</thead>
<tbody>
<tr>
<td>Acceptable beam intensity</td>
<td>( \sim 2 \times 10^5 ) particles/cycle</td>
<td>( \sim 10 ) particles/picture cycle</td>
</tr>
<tr>
<td>Recorded interactions per PS cycle</td>
<td>( \sim 100 ) max (digitized chambers)</td>
<td>5-10 max</td>
</tr>
<tr>
<td>Selection by trigger system</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>( \sim 4\pi ) solid angle</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>( \Delta p/p \lesssim 10^{-2} )</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

It is also worth mentioning that a development of magnet spark chamber spectrometers, to be used in experiments with detection of a great number of secondaries, was started a long time ago. The Omega follows this trend, as one example with a design optimized to get the best possible resolution and largest solid angle for experiments at the CPS.

An isometric view of the Omega magnet is shown in Fig. 1. The magnet consists of a rigid structure made of two horizontal (top and bottom) iron plates kept together by four vertical pillars mounted on the four corners of the plates. On the inner side of the two plates are mounted two superconducting coils to provide a vertical magnetic field of 18 kG. All other iron parts can be easily removed, namely: the two central cores of the horizontal plates and the side yoke made of modular slabs. This is in order to create for different experiments the necessary openings toward the outside of the magnet for photography (hole in the top plate) and for extraction of secondary particles originated at the target. Examples of two different arrangements of the side yoke are shown in Figs. 2 and 3.
Figure 4 shows a scheme of the optical spark chambers to be used in the first few years of operation of the Omega spectrometer magnet\(^*\)). The camera can look directly through the spark chamber plates, which are inclined to the proper angle for this purpose. The over-all dimensions of each spark chamber unit are \(\sim 200 \times 150 \times 35\) cm\(^3\). The spark chamber gap is 1 cm. The plates are made of Al 10 \(\mu\) thick, two plates per 2.5 cm, with a resulting radiation length of \(\sim 70\) metres (\(\sim 10\) m for liquid hydrogen). The expected space resolution is 200 \(\mu\). Two stereo views of the spark chambers are photographed on a frame of 70 \(\times\) 70 mm\(^2\) with a demagnification of 50. The frame can be moved by a camera in less than 50 msec.

Figure 5 shows a test coil of the superconducting type, which has been built and successfully operated by Dr. M. Morpurgo, with a view to testing it for use in the Omega magnet. The superconducting wire Nb-Ti is embedded in the metal of a copper pipe, which is then wound to form the coil pancakes. The superconductor is kept at 5\(^{\circ}\)K by a constant flow of liquid helium inside the pipe.

The behaviour of the magnetic field expected for the Omega magnet is illustrated in Fig. 6 (B at the centre of the magnet versus current) and Fig. 7 (B-uniformity). The curves represent the result of field measurements performed on a model scaled down to one-tenth the size of the Omega magnet.

The criterions for the choice of the dimension L and field strength B of the magnet can now be summarized briefly:

**Dimension L:**

The momentum resolution can be expressed as:

\[
\frac{\Delta p}{p} = \left\{ \frac{P_0 \sigma}{L^2} \right\}^2 + \left( \frac{D}{\sqrt{LL_0}} \right)^2 \frac{1}{B},
\]

where \(P_0\) is the maximum momentum to be measured in the spark chambers \((P_0 = 20\) GeV/c\), \(\sigma\) is the space resolution \((\sigma = 200\) \(\mu\)), \(L_0\) is the spark chamber radiation length \((L_0 = 70\) m\), A and D are constants. The term

\(^*\) It is hoped that later on a system of digitized chambers (for instance, wire chambers with capacitor read-out) of equally good resolution and efficiency will replace the optical chambers.
that contains the space resolution \( \sigma \) varies as \( L^{-2} \); the term that contains the radiation length \( L_0 \) (multiple scattering) varies as \( L^{-\frac{1}{2}} \) (see Fig. 8). It is therefore convenient to choose, as for the dimensions of the magnet, the value of \( L \) which makes the two terms equal. In fact, a further increase of \( L \) would decrease \( \Delta p/p \) very slowly (i.e. as \( L^{-\frac{1}{2}} \)).

Field strength \( B \):

In a similar way the choice of \( B \) can be based on the effective mass resolution \( \Delta M \) for the decay of a particle of mass \( M \) in two secondaries 1 and 2

\[
\Delta M = \left\{ \left( \Delta M \right)^2_{\Delta p_{1,2}} + \left( \Delta M \right)^2_{\Delta \Theta_{1,2}} \right\}^{\frac{1}{2}}.
\]

The term in \( \Delta \Theta_{1,2} \) (\( \Theta_{1,2} \) being the opening angle of particles 1 and 2) is determined by the track length. On the other hand, the term in \( \Delta t_{1,2} \) depends on \( B \). A convenient choice of the magnetic field is therefore the value of \( B \), which makes the term in \( \Delta \Theta_{1,2} \) equal to the term in \( \Delta p_{1,2} \). As can be seen from Fig. 9, this condition is well satisfied by \( B \leq 18 \) kG. It can be noticed that a practical advantage of this choice is that at 18 kG the iron yoke acts both as field shield by confining the field in a relatively well-defined volume, and as field shaper by making the field more uniform in the useful volume. In Fig. 10 are shown calculations of \( \Delta p/p \) expected in the case of the Omega (curve a) and based on tracks of various momenta emitted from the target at zero angle to the longitudinal axis of the magnet (see track configuration scheme in Fig. 10).

As an illustration of the future experimental programme with the Omega, we show some of the experiments performed with the CERN-ETH-IC magnet spectrometer, which has been in operation for many years at CERN\(^1\). Figure 11 illustrates the magnet-optical spark chamber system. The dimensions of the useful magnetic field are \( \approx 170 \times 90 \times 90 \) cm\(^3\) with maximum field of 10.5 kG. The optical system makes use of perspex prisms mounted above each gap to deflect the light coming from the sparks into the lens. A fast camera can move a frame of \( 70 \times 100 \) mm\(^2\) in \( \approx 30 \) msec. The camera over-all dead-time is \( \approx 50 \) msec.

The experiments performed so far with such a spectrometer are listed below:
A scheme of the experimental set-up for reactions (1) to (4) [neutral final-state trigger] is shown in Fig. 12. An anticounter made of scintillator-Pb plates rejects all interactions produced at the target associated to the production of charged particles or neutral \( \pi^0 \)'s. In Fig. 13 some details of the experimental set-up are shown. The long horizontal pipes are light guides of the anticounter, the photomultipliers being placed \( \sim 2 \text{ m} \) away from the magnet in a field-free region \( (B \leq 100 \text{ gauss}) \). One can also see a vertical cylinder, which is the liquid-hydrogen reservoir of the hydrogen target.

Figure 14 shows a typical event from reaction (2): two \( K_S^0 \) are observed decaying in the spark chamber. The analysis of the photographs is performed by digitizing the sparks on the CERN-HPD-1 \( ^9 \). The geometrical reconstruction and kinematical fits are made by a special version of THRESH and GRIND.

Recent results obtained from reaction (4) from 4 to 12 GeV/c are shown in Fig. 15 \( (d\sigma/du) \) and Fig. 16 \( [P(u)] \). The correlation of the break in \( d\sigma/du \) at \( u = 0.8 \) (GeV/c)\(^2\) with \( P(u) \) going through zero at the same value of \( u \) is very evident. This is an example of structure in the backward scattering
beside the backward $\pi^+ p \rightarrow \pi^+ p$. The curves of Figs. 15 and 16 represent a theoretical interpretation in terms of Regge exchange given by Barger, Cline and Matos\(^1\). Notice that the total cross-section for this backward peak goes from $\sim 1000$ to $\sim 50$ nanobarns in the region 4 to 12 GeV/c.

Figure 17 shows the scheme of the experimental set-up for the reactions (5) and (6). The trigger is provided by the measurement of the neutron missing-mass by means of time-of-flight with 14 neutron counters mounted at the back of the magnet. It is also required that at least one charged particle be emitted into the spark chambers. Notice that due to the "low" incident momentum, the hydrogen target is mounted well inside the region of uniform magnetic field in order to increase the angular acceptance of the two charged pions.

Finally, Fig. 18 shows the scheme of the experimental set-up for experiments (7) and (8). The target (usually heavy nucleus) is in this case mounted well outside the magnetic field in order to maximize the average track length of the forward three $\pi$'s. (Notice the high energy of the incident beam.) The trigger system requires at least two charged particles scattered in the forward direction and outside the beam region, and no particle emitted at large angle. For this experiment the optical beam spark chambers have been replaced by seven planes of proportional counter hodoscopes.

Some of the above experiments can obviously be repeated with Omega, with or without a polarized target, and improved by increasing considerably the angular acceptance and momentum and mass resolution.

Preliminary schemes of "new" experiments to be done with Omega are shown in Figs. 2 and 3.

Figure 2 represents a proton missing-mass experiment with complete detection of all associated charged secondaries. The protons emitted at $\sim 0^\circ$ scattering angle with low momentum are extracted sideways from the spark chamber and selected in the trigger system by time-of-flight with an array of scintillation counters.

Figure 3 shows the scheme of an experiment conceived to select mostly two-body or quasi-two-body reactions which imply the exchange of a baryon. For this reason, at the back of the magnet a scintillation counter hodoscope
associated to a large gas threshold Čerenkov counter select protons emitted at small angle with approximately the momentum of the incident particle. All charged particles produced in a wide angle and in association with the fast proton are detected simultaneously inside the spark chambers.

The above examples constitute some of the possible future applications for the Omega spectrometer.

The expected starting time for the operation of the Omega spectrometer is the beginning of 1972.
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9) J.C. Lassalle and P. Zanella, Proc. Int. Conf. on Advanced Data
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   ANL-7515.

Fig. 2  Missing-mass trigger
Fig. 3 Baryon exchange trigger on Omega
Fig. 4  Optical spark chamber arrangement
Fig. 5  The Nb-Ti superconductive coil tested at CERN in view of the Omega magnet
Fig. 6 Magnetization curve for model magnet
Fig. 7  Field slope in the median plane of model magnet
Fig. 8  Behaviour of the contributions to $\Delta p/p$ from multiple scattering and space resolution as a function of $L$.
$K^*(1420) \rightarrow K^+\pi$

$\sigma_y = 0.02\, \text{cm}$
$L_{\text{opt}} = 300\, \text{cm}$
$L_0 = 7 \times 10^3\, \text{cm}$

$M^2 \times (\Delta M)^2_{\Delta \theta}$

**Fig. 9** Behaviour of the contributions to the effective mass resolution $\Delta M$ from angular resolution $\Delta \theta_{1,2}$ and momentum resolution $\Delta p_{1,2}$ as a function of the field strength $B$
Fig. 10 $\Delta p/p$ as a function of $p$ for different choices of $L_0$ and $\sigma$.
Fig. 11  CERN-ETH-IC magnet spark chamber: section along beam direction
$S_{1,2,3,4,5,6}$, BEAM DEFINING COUNTERS
$R_{1,2}$, ROUND ANTICOINCIDENCE COUNTERS
$F_{1,2,3}$, FLAT ANTICOINCIDENCE COUNTERS
Č1, Č2, ČERENKOV COUNTERS
FOR SELECTION OF $\pi^-$, $K^-$, AND $\bar{p}$

BEAM MOMENTUM DEFINING MAGNET

BEAM DIRECTION SPARK CHAMBERS

TARGET

$\pi^-$ TRIGGER = $S_{1,2,3,4,5,6}, Č_1 Č_2, R_{1,2}, F_{1,2,3}$,
$K^-$ TRIGGER = $S_{1,2,3,4,5,6}, Č_1 Č_2, R_{1,2}, F_{1,2,3}$,
$\bar{p}$ TRIGGER = $S_{1,2,3,4,5,6}, Č_1 Č_2, R_{1,2}, F_{1,2,3}$,

Fig. 12 Schematic layout of the trigger system for reactions (1) to (4)
Fig. 13 View of a part of the trigger system for reactions (1) to (4). One can see the target-hydrogen reservoir, the photomultiplier light-guides, and the front edge of the magnet.
Fig. 14 Picture taken with the existing CERN-ETH-IC magnet spark chamber (CERN-ETH-IC group)
Fig. 15 Differential cross-section $d\sigma/du$ for $\pi^- + p \rightarrow \Lambda^0 + (K^0)$ from 4 to 12 GeV/c
Fig. 16  \( \Lambda \)-polarization as a function of momentum transfer \( u \) at 4 and 6.2 GeV/c
Fig. 17  Schematic layout of the apparatus for reactions (5) and (6). Neutron counters as seen from the target.
Fig. 18 Schematic layout of the coherent production experiment
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