RF Measurements and Tuning of the 750 MHz HF-RFQ

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Abstract

In the frame of the program on medical applications CERN has built a compact 750 MHz RFQ to be used as an injector for a hadron therapy linac. This RFQ was designed to accelerate protons to an energy of 5 MeV within only 2 m length. It is divided into four segments and equipped with 32 tuners in total. The length of the RFQ corresponds to 5 \( \lambda \) which is considered to be close to the limit for field adjustment using only piston tuners. Moreover the high frequency, which is about double the frequency of existing RFQs, results in a sensitive structure and requires careful tuning by means of the alignment of the pumping ports and fixed tuners. This note summarises the tuning procedure, RF and bead pull measurements of the RFQ.

Keywords: Radiofrequency, RFQ, Tuning.
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Introduction

The present document reports RF measurements as well as field and frequency tuning operations on the HF-RFQ (High Frequency - RFQ) for medical applications. This RFQ will be used as an injector for the LIGHT project Ref. [1], a linac based proton therapy facility. It accelerates protons from 50 keV to 5 MeV within 2 m and is designed to minimise beam losses above 1 MeV Ref. [2]. From an RF point of view the very compact 750 MHz 4-vane structure operates at about twice the frequency of existing RFQs Ref. [3]. It consists of four modules with a length of about half a meter.

The dipolar modes are detuned by means of dipole stabiliser rods at the end plates. The electrode voltage is designed to be constant along the RFQ that requires a constant longitudinal field distribution in terms of tuning. In order to compensate construction errors the structure is equipped with 32 tuners. The 12 pumping ports could be used as additional tuning devices if necessary. The 4 power couplers are placed in the two middle segments of the RFQ, one in each quadrant. Each power coupler will be fed by a 100 kW Inductive Output Tube (IOT) in order to maintain a nominal voltage of 67.6 kV. The Q-factor due to losses in copper is about 6440 according to 3D RF design simulations.

The bead pull system was previously used for tuning of the LINAC4 RFQ and was adjusted in size to fit the HF-RFQ. In order to debug this system and the data processing steps a simple aluminium mock up RFQ without vane modulation was built. After debugging, measurements on single modules of the HF-RFQ were executed to compare these results with simulations. For single module measurements boundary conditions on the modules extremities have been applied by means of cylindrical end tubes. All single modules have been measured without tuners, pumping ports, power couplers and antennas.

After full assembly of the RFQ, meaning all 4 modules, pumping ports, power couplers and tuners, reliability measurements have been done to assure comparable measurements results. Then all measurement in order to tune field and frequency of the cavity were executed followed by measurements of quality factors and antenna calibration after assembly of pickup antennas.
1 Measurement Equipment

1.1 Devices

For all RF measurements always the same vector network analyser (VNA), cables and calibration kit was used to ensure equal conditions for comparable results. The specifications are listed in the following

**VNA from Keysight (Agilent)**
- E5061B
- 2-ports
- 100 kHz - 3 GHz
- 50 ohm system impedance

**Calibration Kit (Keysight)**
- standard mechanical calibration kit
- DC to 9 GHz
- type-N
- 50 ohm

**Cables (Keysight)**
- Rugged phase-stable cable
- N-type (m)
- 50 ohm
- 3.6 m (12 ft)

**Temperature Measurement Gauges**
- Hanna Instruments HI98509 Checktemp®
- resolution 0.1 °C
- precision ±0.3 °C

1.2 Bead Pull System

The bead pull system, that means all parts necessary to make the bead travel through all the quadrants of the RFQ, was previously used for the tuning of the LINAC 4 RFQ. The pulleys, adjustment devices and the motor were assembled in a way to fit the HF-RFQs dimensions. The main difference to the LINAC4 setup was that in case of the LINAC 4 measurements the bead pull system was directly attached to the RFQ, while for the HF-RFQ the bead pull system was attached to a frame that also holds the RFQ. In the following several pictures show details of the bead pull setup.
For the LINAC4 measurements light sensors that identify the position of the bead were used together with a software that also could trigger the motor and network analyser. Some difficulties with the light sensors and Due to the fact that the bead used for the HF-RFQ was too small to be detected by the light sensors and some other difficulties with the light sensors, this software was not used for measurements of the HF-RFQ. Instead an external power supply for the motor that moves the bead was used that had to be triggered manually together with the network analyser.

1.3 Numbering and Conventions
The devices attached to the RFQ have engraved numbers to identify them, but anyway the numbering is explained in the following. Unless otherwise specified the numbering is done with the view in beam direction from low to high energy of the beam.

1.3.1 Quadrants Numbering
Quadrants \( q_1, \ldots, q_4 \) are numbered starting at the upper right quadrant continued in counter clockwise (CCW) order, while looking into beam direction from low to high energy of the beam, Figure 2 white numbers.

1.3.2 Tuner Numbering
Tuners \( T_1, \ldots, T_{32} \) are numbered following the same order as the quadrant numbering. There are four tuners in each vertical plane and eight per module. \( T_1 \) is on the upper right continued counter clockwise until \( T_4 \), then continued in a helix shaped way \( T_5 \) on the upper right again in the next vertical plane of tuners continued CCW order, and so on (Figure 2 black numbering).
1.3.3 Antenna Numbering

Antennas $a_1, \ldots, a_{16}$ are numbered following the same order as the quadrant and tuner numbering. There are 4 antennas in a vertical plane, one set of 4 antennas for each module. Antenna numbering starts as well on the upper right being continued in CCW direction in helix shaped was along the RFQ. The orientation of the antennas in terms of inner and outer connector of the loop is always the same in each quadrant and follows the same rule as the power couplers do (see Figure 3 (right)).

1.3.4 Power Coupler Numbering

For the power couplers $c_1, \ldots, c_4$ no logic behind the numbering is used. The way to identify the power couplers is shown in Figure 3.

The drawing on the right in Figure 3 also shows the orientation of the inner and outer connectors fo the power couplers. Power coupler $c_1$ and $c_4$ are located in module 2 in quadrant $q_2$ and $q_4$, while couplers $c_3$ and $c_4$ are attached to module 3 in quadrants $q_1$ and $q_3$.

1.4 Tuner Tooling

In order to allow a proper tuner adjustment while tuning, but also to guarantee about the same precision for reassembly after tuner cutting a special tuner tooling was developed. The tuner tooling is based on a cup-like body that is mounted to the outer surface of the tuner flanges. Figure 4 shows cross-sectional views of the tuner tooling.
Fig. 4: Cross sections of the tuner tooling.

The tuner itself is introduced using a tooling fixed to the tuners flange. This tooling has a fine thread with a thread pitch of 1 mm per turn connected to the cup. This cup guides the flange connected to the tuner stub and provides translational positioning in the layer of the orientation of the tuners backside. The rotational positioning is realised using a pin on the tooling held in place by a gap in the cup. In addition the play in the thread is suppressed by a spring providing continuous contact to one side of each of the external and internal thread. For precise alignment a coarse scaling with 1 mm steps using the pin as reference was provided, while a finer scaling on the screw-nut with $\frac{1}{100}$ mm division was used. Two pictures of the scaling at the tuner tooling is shown in Figure 5.

Fig. 5: Coarse and fine scaling of the tuner tooling to adjust the penetration depth of the a tuner.
2 RF Measurements

2.1 Bead Pull Measurements

For bead pull measurements on a 4-vane RFQ a bead has to travel along all 4 quadrants consecutively. The bead pull setup used for the HF-RFQ is shown in Fig. 6. It is based on the system which was previously used for the measurements of the LINAC4 RFQ at CERN and has been adapted to the dimensions of the HF-RFQ. For bead pulling a wire with 0.3 mm diameter was used and its tension was adjusted with a spring. The wire is one closed loop that enables consecutive measurements of all four quadrants while the bead can be pulled around the pulleys to travel between the quadrants. For each quadrant the wire can be adjusted in azimuthal direction using micrometer screws.

Fig. 6: Raw data processing steps visualised with measurements of module 3.

When a bead is entering a cavity, field is removed in the order of the volume of the bead. Since the energy is stored in the field the bead causes less stored energy in the cavity. For small changes according to Slater-Perturbation-Theory the relative change of energy is equal to the change of frequency or phase.

\[
\frac{\Delta W}{W} = \frac{\Delta \omega}{\omega} = \frac{\Delta \phi}{\phi}
\]

(1)

The frequency and phase shift caused by a bead is shown in Figure 7.
Since frequency deviation by a small bead is given by
\[
\frac{\Delta f}{f_0} = \frac{\pi \cdot r^3}{W_0} \left[ \frac{\epsilon_r - 1}{\epsilon_r + 2} \cdot \epsilon_0 \cdot E^2 + \frac{\mu_1 - 1}{\mu r + 2} \cdot \mu_0 \cdot H^2 \right]
\] (2)
the measured frequency or phase shift with a bead pull measurement is proportional to the squared fields and depends on the location of the bead. While the bead is pulled along the cavity with a constant speed the phase shift with time can be easily measured with a vector network analyser (VNA). The location of bead is given by the velocity of the bead and the measured time, displayed at the abscissa as sampling points of the VNA in Figure 8. The figure shows the phase shift depending on the beads position along modules length.

2.1.1 Bead Size
A comparison of beads was made between a titanium bead used for measurements at the LINAC4 RFQ at 352 MHz and an aluminium bead. Both bead have a cylindrical shape with rounded edges. The titanium bead has a length of 16 mm and a diameter of 8 mm, while the aluminium bead has only a length of 7 mm and a diameter of 4 mm. A picture of both beads is shown in Figure 9 together with a drawing of the dimensions of the aluminium bead.
Both bead have been entered into the cavity in order to check their influence on the phase between the input and output signal of the VNA of a S21 measurement. Figure 10 shows the phase shift of both bead against a certain frequency bandwidth. The titanium bead causes a large phase shift of about 60° that is not linear anymore.

In the case for the aluminium bead the phase changes only about 12° which is clearly in a linear range. The aluminium bead is giving a clear influence on the phase to be measured by the VNA, at the same time being is small enough to gain proper results for bead pull measurements. The influence of both bead sizes on bead pull measurements on a single module in chapter 3.1.1.

2.1.2 Phase Noise Ratio
A few tens of samples at the beginning and end of a module, later used to do the vertical alignment of the raw data, have been used to estimate the phase to noise ratio. These samples were taken from the straight line, see Figure 8 before the bead enters and after leaving the cavity. The phase noise variates between 0.006° and 0.01°. The phase shift due to the bead is about 10°. From this the phase to noise ratio of the squared fields is in the range of 1000 to 1700.

3 Raw Data Processing
In order to calculate the quadrupole and dipole components from the data acquired by the bead pull measurements as described in chapter 2.1 several steps of data processing were necessary as described
in the following. Figure 11 illustrates these step done for the case of single module 3. The same steps have been applied to measurements of the fully assembled RFQ.

**Raw Data**

The raw data data aquired from 4 bead pull measurements, one for each quadrant, is plotted as the phase of $S_{21}$ vs. the bead position in time. The phase is given in degrees and the bead position is plotted as sampling points of the VNA for practical reasons. Since the bead was guided in one closed loop, the first quadrant $q_1$ was measured in beam direction. Then the bead exits $q_1$, enters the second quadrant $q_2$ and travels through it against beam direction. The same way $q_3$ is measured in beam direction again and opposite direction for $q_4$. Also starting and stopping of each measurement could not aligned exactly leading to a horizontal shift of the single quadrants data. A phase drift of the VNA caused a vertical shift of this data not only for different measurement, but also within one measurement. The necessary steps to align the data are described in the following.

**Orientation**

From raw data to the proper orientation of all single quadrant measurements, the data taken for $q_2$ and $q_4$ was simply flipped for the abscissa. This step is shown in plot one and two of Figure 11.

**Vertical Alignment**

For the vertical alignment about 20 samples roughly within the first 200 sampling points were chosen at the beginning of the modules. An average value of the 20 chosen samples were set to $0^\circ$ of the phase as illustrated in plot 3 of Figure 11.

**Phase Drift**

In order to compensate the phase drift within one measurement also 20 sampling points at the end of the segment was chosen in addition to the points used for the vertical alignment. The phase difference between beginning and end of the module was compensated linearly over the RFQ quadrant length, shown in plot 4 in Figure 11.

**Smoothing**

To allow a precise horizontal alignment and in order to increase accuracy for the field level points later the measurement data was smoothed to average the single data points as shown in plot 5 of Figure 11.

**Horizontal Alignment**

For the horizontal alignment in plot 6 of Figure 11 an absolute value of the phase was taken at the rising and falling of the slope at the beginning and end of the module. From these points the points at the rising slope were used to align the beginning of the module. A possible slip of the wire at the motor was compensated by scaling the length between the two phase points to a common length for all measurements.

**Square Root of Phase and Quadrupole Sign**

After all alignment steps the square root of the phase is proportional to the field in the cavity. Since the field direction in adjacent quadrants changes to the opposite direction the proper quadrupole signing \{+,-,+,-\} was applied to the single quadrant measurements $q_1,\ldots,q_4$ shown in plot 7 of Figure 11.

**$Q$, $D_s$, $D_t$ Components**

Eventually the quadrupole and dipole components are calculated as defined by

$$Q = (q_1 - q_2 + q_3 - q_4)/4$$

$$D_s = (q_1 - q_3)/2$$

$$D_t = (q_2 - q_4)/2$$
The last plot in Figure 11 shows the quadrupole component $Q$ in blue and the dipole components $D_s$ and $D_t$ in red and yellow normalised to 100%.

Fig. 11: Raw data processing steps visualised with measurements of module 3.

3.1 Single Module Measurements

Measurements on single modules have been made in order to compare the longitudinal field distribution to simulation results and assess if there is any need for the use of the pumping ports as tuning features in addition to the tuners. In addition the single modules were used to debug and align the bead pull system to guarantee preparation for the full assembly measurements.

To match the boundary conditions the modules were closed with two extension parts. These parts are basically tubes that were adjusted in length and diameter by simulations to allow the field to decay within the tube. The diameter was matched to meet the frequency of the RFQ. Since the fringe fields of the RFQ decay exponentially with distance, the length of the tube was chosen to be as long as the frequency did not depend on the tube length anymore but only on its diameter. Simulations on the length and diameter of the end tubes are shown in Figure 12.
The best agreement on frequency was found with a tube diameter of 110 mm and a minimum length of 50 mm. A picture of the measurement set up of a single module using the end tubes is shown in Figure 13.

Fig. 12: Frequency of end tube depending on length and diameter.

Fig. 13: Measurement set up of a single module with the end extension tubes.

3.1.1 Error / Reliability Studies

3.1.1.1 Bead Size

The influence of the two beads with different sizes on the phase have been shown in chapter 2.1.1. Figure 14 compares simulations of the longitudinal field distribution of the quadrupole mode with bead pull measurements of the large titanium and the smaller aluminium bead on module 1.
Fig. 14: Comparison of aluminium and titanium bead with the simulation of Q components of module 1.

This plot confirms the larger inaccuracy of the measurement of the larger titanium bead caused by a nonlinear phase shift as shown before. It still gives an idea about the field distribution but it suggests a more flat field distribution than shown by the measurement with the aluminium bead and the simulation. The aluminium bead shows a overall good agreement with the simulations.

3.1.1.2 Wire Thickness
Measurements on module 1 have been made using 0.6 mm wire made of Vectran. Then a thinner fishing wire with diameter of 0.3 mm was used. In order to check the difference on measurements both wire have been compared on module 2, see Figure 15.

Fig. 15: Comparison of 0.3 mm and 0.6 mm wire for bead pulling at module 2.

The bead pull results did not show a significant difference between both wires. The wire and the bead has to go around may pulleys causing a stretching of the wire. The advantage of the thinner is that it provides a smoother movement of the whole system with less stuttering of the bead while it travels through the quadrants. Therefore for all further measurements the 0.3 mm wire was used.
3.1.1.3 RF Coupling

The measurements antennas in loop shape are not penetrating much into the cavity. But to check anyway if there is an influence of the loop configuration on the measurement results, the loop location was varied. Figure 16 illustrates the loop configuration and Figure 17 shows the quadrupole component of module 1 at these different loop configurations. Colour coded arrows correspond to same colours the plot.

Fig. 16: Schematic loop configurations of module 1.

Fig. 17: Comparison of different loop configurations on bead pull measurements of module 1.

The plot shows that the loop configuration does not have a significant influence on the bead pull measurements. For all modules and measurements loop in the middle of quadrant 2 and quadrant 4 were used, if not indicated otherwise.

3.1.1.4 Wire Position

A good position for the wire to do bead pull measurements would be close to the cavities outer walls, since the magnetic field gets stronger to the cavity walls. On the other hand the bead should not be too close to the tuners. It should not physically touch the tuner and there should be a certain distance because of local field perturbations due to the tuners. Towards the beam axis the electric field increases and the magnetic field decreases and gets more inhomogeneous in transverse direction which should be avoided in measurements.
The wire position used for all measurements unless indicated otherwise was at 34.2 mm distance from the beam axis at the bisecting line of two vanes. This position was also used in all simulations in order to obtain the longitudinal fields. This position is illustrated in Figure 18. Some wire displacement scenarios have been investigated and are explained in the following.

### 3.1.1.5 Wire Displacement

#### All Quadrants

Figure 20 and 19 show radial displacements of all 4 wire for the aluminium and titanium bead on module 1 without tuners or pumping ports. These plots show an increase of the local influence of the structure details like feedthroughs for the tuners, pumping ports and pickup antennas by coming closer to the cavity walls with the bead. Especially on $r = 40.7$ mm in Figure 20, one can clearly recognise the increasing field on the edges of the flange wholes. At the same position the curve for the titanium bead in Figure 19 is smoother since the larger bead averages about a larger longitudinal region.
**Fig. 20:** Wire displaced in all quadrants equally, measurements of module 1 with aluminium bead.

**Single Quadrant**

In Figure 21 only the wire in quadrant 1 was moved. This shows also the increase of field towards the cavity walls, but less than by moving all 4 wires at the same time. A wire displacement of 1 mm was considered as tolerable.

**Fig. 21:** Wire displacement of module 2. The plot shows the wire displacement in quadrant 1.

Also measurements of module 3 in Figure 22 confirm the same behaviour on displacement of a single or all wires. As one would expect the influence on the dipole components is stronger if only one wire is moved instead of all four equally.
Fig. 22: Wire displacement of module 3. Left plot shows wire displacement in quadrant 1. Right plot shows displacement of wire in all quadrants.

3.1.2 Comparison Measurements with Simulation

The bead pull and mode spectra measurements of the single modules have been compared to simulation results in order to assess the mechanical accuracy of the single modules. In Figure 23 the first two dipole modes $TE_{110}$, $TE_{111}$ and the operating quadruple mode $TE_{210}$ of all modules are shown. The measured spectra is the blue line and the cursors indicate the values from simulations. The agreement of the values from simulations and the measured values are well within a 0.5 MHz range.

In Figs. 24–27, the Q, Ds and Dt components of the single modules are compared to simulations at the actual frequency of the $TE_{210}$ mode of the modules. Since the dipole components of the $TE_{210}$ mode is zero in simulations it is not included in the plots. The Q component show a very good agreement between measurement and simulation for all modules. The difference is below the effect of the displacement of the wire in one quadrant of 1 mm. This confirms the precise machining and brazing within the specified tolerances.

The errors of the dipolar components, Ds and Dt, of the single modules are all below ±3%. A comparison of all dipolar components of the single modules is given in Table 1. To obtain these values the extremities of the modules were only estimated.
Table 1: Errors of the Ds and Dt components of the single modules.

<table>
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<tr>
<th>Module</th>
<th>Error Ds</th>
<th>Error Dt</th>
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<tr>
<td>1</td>
<td>± 2.8%</td>
<td>± 2.8%</td>
</tr>
<tr>
<td>2</td>
<td>± 0.5%</td>
<td>± 2.8%</td>
</tr>
<tr>
<td>3</td>
<td>± 2.2%</td>
<td>± 1.7%</td>
</tr>
<tr>
<td>4</td>
<td>± 1.4%</td>
<td>± 2.4%</td>
</tr>
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</table>

It is noticeable that module 1 and module 4 have slightly larger errors in both of the dipole components compared to the middle modules 2 and 3. This might be attributed to the more complicated geometry of the modules with the vane undercuts at the very beginning and end of the RFQ. Maximum errors for the quadrupole components are about ±2.6% by means of local deviations from simulations.

**Fig. 24:** Comparison of measured Q, Ds and Dt component with simulation of the Q component of module 1.

**Fig. 25:** Comparison of measured Q, Ds and Dt component with simulation of the Q component of module 2.
**Fig. 26:** Comparison of measured Q, Ds and Dt component with simulation of the Q component of module 3.

**Fig. 27:** Comparison of measured Q, Ds and Dt component with simulation of the Q component of module 4.
3.2 Field Tuning
The aim of field tuning is to find a mechanical setting for each tuner that provides a constant longitudinal field distributions for the quadrupole component (Q) and minimising the fraction of the dipole components Ds and Dt at the same time. Figure 28 shows the longitudinal field distribution for these components after full assembly of the RFQ and before tuning.

![Field Distribution Before Tuning](image)

**Fig. 28:** Q, Ds and Dt component of the field distribution before tuning.

The Q component shows an error of $\pm 10.8\%$, Ds $\pm 3.0\%$ and Dt $\pm 3.6\%$. The small bumps in Figure 28 are related to the tuners and the pumping ports. The field errors are mainly caused by manufacturing and assembly inaccuracies. A proper tuner setting will suppress these errors.

### 3.2.1 Reliability Measurements
Before the tuning process was started, several tuner movements were executed in order to learn about mechanical hysteresis effects of the tuner tooling and to determine reliability of the placements of the single tuners. The tuner movement should be reliable in such a way that a certain penetration depth of a tuner should always give the same results for frequency and field distribution.

Tuner number 5, 15, 19 and 26 have been moved with the steps 0, 1, 3, 5, 6 mm starting at 0 mm moving to 6 mm then going back to 0 mm and forth to 6 mm again. For each step a bead pull measurement of a single quadrant was executed and plotted (see Figure 29) in order to investigate the reliability of measurements of the same tuner penetration by movements in between these measurement. For comparison the numbering of the tuners and quadrants is explained in chapter 1.3.
Fig. 29: Tuner reliability check from top to bottom with tuner 15, tuner 5, tuner 19 and tuner 26.
By checking the field change due to tuner movements also the frequency was observed. Figure 30 shows the change of frequency due to the tuner movements of tuner 16 and tuner 26 in comparison to the field change in Figure 29.

![Fig. 30: Frequency shift due to tuner movements of tuners T15 and T26.](image)

Tuners have been moved starting at 0 mm to 6 mm in steps by 1 mm, then back to 0 mm and forth to 6 mm again. The plots show a linear change of frequency in this range and also quite comparable results within the three repetitions.

A bead pull measurements of all quadrants and calculation of the Q, Ds and Dt components was done for the movement of tuner 6 and tuner 25. A tuner in first and fourth module was chosen in order to estimate the possible field tuning range. Figure 31 shows a plot of all components of the movements of the two tuners.
Fig. 31: Tuners T6 and T25 at 3 mm and 5 mm. Maximum initial field of Q is normalised to 100%.

The quadrupole component could be compensated almost using only one tuner. From this plot it is also obvious that to compensate also the dipole components at the same time a more complex tuner configuration is necessary. The sensitivity of the tuners is well within the range of the initial field distribution.

This plot also shows that tuner 6 that is in quadrant $q_2$ has almost no influence on the Ds component that is calculated using quadrants $q_1$ and $q_3$. The same behaviour is true for tuner 25 in quadrant $q_1$ that does only barely affect the Dt component calculated using $q_2$ and $q_4$.

3.2.1.1 Conclusion

These reliability measurements show a learning curve of the handling of the tuner tooling. The plots of tuner 26, that were done at the latest, show a good comparability of the results of the repeated tuner movements several times and their influence on field and frequency. To approach a certain penetration depth of the tuner the thread of the tooling was always moved in a releasing manner. This means by moving the thread counter clockwise and releasing the tension of the spring. By doing so the tuner moves inside the cavity avoiding the spring to get stuck by minimising hysteresis like effects while doing mechanical movements.

3.2.2 Field Tuning Steps

After the reliability measurements the tuning process was started. First all tuners have been moved one by one by 3 mm inside the cavity. For each tuner movement a bead pull measurement was executed to obtain the influence of every single tuner to the longitudinal field distribution of all components. From these measurements 11 measurement points, shown in Figure 28 as purple crosses, along the length of the RFQ were taken to determine the vector $\vec{V}$ and the response matrix $M$, see Table 2.
Fig. 32: Tuner influences on the fields of Q, Ds and Dt components. Average of initial field distribution of Q is normalised to 100%.

After inversion of the matrix M and manipulation of the singular values, 32 solutions for tuner settings were found as described in chapter A. From comparison of \( \vec{V} \) with \( \vec{V}_{svd} \), an optimum solution for tuner settings \( \vec{T}_{svd}^{21} \) was found to start the tuning process. The predicted tuner displacements of \( \vec{T}_{svd}^{21} \) were applied to the 32 tuners followed by bead pull measurements. This procedure was repeated four times. The change of the quadrupole and dipole components within the first four tuning iterations can be found in Figure 33 and 34.
### Table 2: Response Matrix Entries in units of \( T/m \) Rounded to Two Positions after Decimal Point.

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<td>1.35</td>
<td>1.28</td>
<td>1.36</td>
</tr>
<tr>
<td>1.36</td>
<td>1.21</td>
<td>1.75</td>
<td>1.82</td>
<td>1.91</td>
<td>1.83</td>
<td>1.90</td>
<td>1.84</td>
<td>2.24</td>
<td>1.87</td>
<td>2.26</td>
</tr>
<tr>
<td>2.24</td>
<td>2.76</td>
<td>2.42</td>
<td>2.55</td>
<td>2.67</td>
<td>2.48</td>
<td>2.53</td>
<td>2.46</td>
<td>2.67</td>
<td>2.41</td>
<td>2.57</td>
</tr>
<tr>
<td>3.02</td>
<td>3.11</td>
<td>2.95</td>
<td>2.89</td>
<td>2.97</td>
<td>2.86</td>
<td>2.90</td>
<td>2.84</td>
<td>3.16</td>
<td>2.87</td>
<td>3.08</td>
</tr>
<tr>
<td>5.80</td>
<td>5.60</td>
<td>5.80</td>
<td>5.78</td>
<td>5.76</td>
<td>5.74</td>
<td>5.72</td>
<td>5.70</td>
<td>5.68</td>
<td>5.66</td>
<td>5.64</td>
</tr>
<tr>
<td>1.97</td>
<td>1.91</td>
<td>1.93</td>
<td>1.90</td>
<td>1.94</td>
<td>1.92</td>
<td>1.93</td>
<td>1.94</td>
<td>1.97</td>
<td>1.92</td>
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<tr>
<td>3.00</td>
<td>3.00</td>
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<td>2.90</td>
<td>2.85</td>
<td>2.80</td>
<td>2.75</td>
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<td>2.65</td>
<td>2.60</td>
<td>2.55</td>
</tr>
<tr>
<td>4.98</td>
<td>4.86</td>
<td>4.90</td>
<td>4.85</td>
<td>4.82</td>
<td>4.79</td>
<td>4.74</td>
<td>4.70</td>
<td>4.66</td>
<td>4.62</td>
<td>4.58</td>
</tr>
<tr>
<td>4.84</td>
<td>4.79</td>
<td>4.74</td>
<td>4.70</td>
<td>4.66</td>
<td>4.63</td>
<td>4.59</td>
<td>4.55</td>
<td>4.51</td>
<td>4.48</td>
<td>4.45</td>
</tr>
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<td>4.66</td>
<td>4.53</td>
<td>4.43</td>
<td>4.35</td>
<td>4.30</td>
<td>4.25</td>
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<td>4.05</td>
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<td>3.90</td>
<td>3.80</td>
<td>3.70</td>
<td>3.60</td>
<td>3.50</td>
<td>3.40</td>
<td>3.30</td>
<td>3.20</td>
<td>3.10</td>
<td>3.00</td>
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<tr>
<td>0.80</td>
<td>0.70</td>
<td>0.60</td>
<td>0.50</td>
<td>0.40</td>
<td>0.30</td>
<td>0.20</td>
<td>0.10</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Note:** The table entries represent the response matrix entries in units of \( T/m \) rounded to two positions after the decimal point.
Fig. 33: Q, Ds and Dt component for the initial state and for tuning iterations 1, 2 and 3.

Fig. 34: Q, Ds and Dt component during tuning iterations 3, 4, 5 and 6.

Figure 35 shows the predictions of the field compensation. The vector entries shown on the horizontal axis of this plot represent the 11 longitudinal measurement points for Q (1-11), Ds (12-23) and
Dt (24-33). On the vertical axis the relative deviation from the desired field components is shown.

![Figure 35: Predictions for field compensation.](image)

In this plot the blue line represents the initial field that needs to be compensated. The purple line is the prediction for the first tuning iteration and shows a very good agreement with the blue line at the first 11 vector entries representing the quadrupole component. After four tuning iterations the field to be compensated is given by the red line. The prediction to compensate this, see line in light blue, shows not a very good agreement with the red line. For this reason another solution from the 32 predictions was chosen. The prediction of the next tuning iteration using now $\vec{V}_{svd18}$ is given by the yellow line. This shows an acceptable agreement for the quadrupole mode, which is anyway very well compensated already, but a much better agreement with the two dipolar components (vector entries 12-33). After two more iterations, 5th and 6th iterations shown in Figure 34, with $\vec{V}_{svd18}$ the compensation of all modes was in a very acceptable range. All the field and frequency adjustment steps with the corresponding field errors is shown in Table 3 or in a more detailed version in Table 6. All tuner settings during iterations and after frequency adjustment are listed in Table 7.

### Table 3: Field and Frequency Tuning Steps and Tuner Cutting in Chronologic Order.

<table>
<thead>
<tr>
<th>Field Tuning</th>
<th>Frequency Tuning</th>
<th>Error Q</th>
<th>Error Ds</th>
<th>Error Dt</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial</td>
<td></td>
<td>± 10.78 %</td>
<td>± 3.04 %</td>
<td>± 3.57 %</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>± 6.87 %</td>
<td>± 4.75 %</td>
<td>± 2.25 %</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>± 2.69 %</td>
<td>± 3.28 %</td>
<td>± 2.46 %</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>± 1.63 %</td>
<td>± 3.16 %</td>
<td>± 2.64 %</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>± 1.33 %</td>
<td>± 2.60 %</td>
<td>± 2.80 %</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>± 1.29 %</td>
<td>± 2.55 %</td>
<td>± 2.39 %</td>
<td>$T-0.5$ mm</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>± 1.61 %</td>
<td>± 0.75 %</td>
<td>± 1.58 %</td>
<td>$T_{svd18}$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>± 1.32 %</td>
<td>± 1.03 %</td>
<td>± 1.72 %</td>
<td>$T_{svd18}$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>± 1.10 %</td>
<td>± 1.15 %</td>
<td>± 1.63 %</td>
<td>$T-0.2$ mm</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>± 0.94 %</td>
<td>± 1.07 %</td>
<td>± 1.37 %</td>
<td>$T-0.5$ mm</td>
</tr>
<tr>
<td>5</td>
<td></td>
<td>± 1.07 %</td>
<td>± 1.05 %</td>
<td>± 1.57 %</td>
<td>$T-0.03$ mm</td>
</tr>
<tr>
<td>6</td>
<td></td>
<td>± 1.02 %</td>
<td>± 0.80 %</td>
<td>± 1.65 %</td>
<td>$T-0.02$ mm</td>
</tr>
<tr>
<td>1st tuner cutting</td>
<td></td>
<td>± 1.38 %</td>
<td>± 1.07 %</td>
<td>± 1.72 %</td>
<td>fixed with springs</td>
</tr>
<tr>
<td>2nd tuner machining</td>
<td></td>
<td>± 1.04 %</td>
<td>± 1.04 %</td>
<td>± 1.71 %</td>
<td>final copper gasket</td>
</tr>
</tbody>
</table>

### 3.3 Frequency Tuning

A frequency tuning is not included in the field tuning algorithm. However in order to adjust the RFQ frequency to the operating frequency and simultaneously keeping the field distribution constant, all tuners
have to be moved equally by the same amount of length. A detailed description of this process is described in the following paragraphs.

### 3.3.1 Determination of the Target Frequency

Table 4 shows the HF-RFQ frequency parameters and the data used to calculate the compensation of dry nitrogen and the frequency dependency on temperature. This data was taken from a RF measurement report of Rolf Wegner about RF measurements of the 12SWV18026 − _01CSCC (TD26 CC N3) structure [7].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>749.48 MHz</td>
<td>subharmonic of S-band cavities</td>
</tr>
<tr>
<td>$T$</td>
<td>24 °C</td>
<td>working temperature of the RFQ</td>
</tr>
<tr>
<td>$\delta f$</td>
<td>0.01 $&lt;</td>
<td>\delta f</td>
</tr>
<tr>
<td>$\Delta f_{\text{wire}}$</td>
<td>+0.66 kHz</td>
<td>freq. change due to bead pull wire removal</td>
</tr>
</tbody>
</table>

**Example Cavity for Calculations**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_0$</td>
<td>11.994 GHz</td>
<td>design frequency</td>
</tr>
<tr>
<td>$\Delta f_{N_2}$</td>
<td>−3.48 MHz</td>
<td>frequency change due to vacuum $\Rightarrow N_2$</td>
</tr>
<tr>
<td>$\Delta f^T$</td>
<td>+1.79 MHz</td>
<td>freq. change due to temp. 30.0 °C $\Rightarrow 21.0$ °C</td>
</tr>
<tr>
<td>$\Delta T$</td>
<td>9 K</td>
<td>temperatur change from 30.0 °C $\Rightarrow 21.0$ °C</td>
</tr>
</tbody>
</table>

The example cavity has a frequency of 11.994 GHz at 21 °C. The change of frequency from vacuum to nitrogen is $-3.48$ MHz.

$$\frac{\Delta f}{f_0} = \frac{-3.48 \text{ MHz}}{11.994 \text{ GHz}} = -2.90 \cdot 10^{-4} \approx \frac{\epsilon_{N_2}}{\epsilon_0}$$  \quad (6)

The operation frequency of the HF-RFQ is a subharmonic of the LIGHT S-band cavities working at 2997.92 MHz.

$$f_0 = \frac{2997.92 \text{ MHz}}{4} = 749.48 \text{ MHz}$$  \quad (7)

This frequency is set for a working temperatur of 24 °C and under vacuum conditions. Due to the fact that $\epsilon_0$ of the air is depending on the air humidity the frequency of the RFQ is meant to be measured with dry nitrogen. Then the target frequency compensated for nitrogen is given by

$$f_0^{N_2} = 749.48 \text{ MHz} \cdot (1 - 2.90 \cdot 10^{-4}) = 749.262 \text{ MHz} \approx 749.263 \text{ MHz}$$  \quad (8)

The frequency of the RFQ in operation will be adjusted by the cooling water temperature. Therefore the frequency range for $\pm 1$ K and $\pm 5$ K is calculated. The thermal expansion coefficient of copper is given by

$$\frac{\Delta f}{f_0} = -1.66 \cdot 10^{-5} \frac{1}{K}$$  \quad (9)

From (9) follows for $f_0 = 749.263 \text{ MHz}$ $\Rightarrow \Delta f = f_0 \cdot -1.66 \cdot 10^{-5} \frac{1}{K}$

$$\Delta T = \pm 1 \text{ K} \quad \Rightarrow \quad \delta f = \pm 0.01244 \text{ MHz}$$  \quad (10)

$$\Delta T = \pm 5 \text{ K} \quad \Rightarrow \quad \delta f = \pm 0.0622 \text{ MHz}$$  \quad (11)

It would be desirable to have the frequency of the RFQ within the sufficient $\pm 1$ K range. An adjustment within the $\pm 5$ K range would be still acceptable but it is targeted to be as close to the $\pm 1$ K range as
reasonably possible. The calculated frequency shift for 1 K was confirmed by measurements at different temperatures.

During the frequency alignment steps after field tuning the field distribution was observed using bead pull measurements. Because of that the compensation of the wire was calculated as well. By removing the wire from the single modules the frequency changed by $+0.00066\text{ MHz}$. This is only a minor influence that lowers the the target frequency to

$$f_{N_{2}+\text{wire}} = 749.26234\text{ MHz} \quad (12)$$

### 3.3.2 Frequency Adjustment

Since the tuning algorithm is not able to adjust field and frequency at the same time the frequency had to be adjusted separately after the field tuning process. Therefore all tuners had to be moved equally to approach the target frequency of $749.48\text{ MHz}$. For the frequency adjustment the temperature in the RF lab was set to the working temperature of the RFQ and the measurements have been executed with dry nitrogen for which the scaling factor was calculated above, Equation (8). Also the wire was compensated, Equation (12) to find the proper target frequency in order to check the field distribution after frequency adjustment with bead pull measurements, Figure 36 shows the frequency at the different tuning and frequency adjustment steps.

Figure 36 shows the tuning iterations increase the RFQ frequency. In order to stay in a reasonable physical tuner range a first frequency adjustment was done after the fourth 4 field tuning iteration. Two more field tuning iterations increased the frequency further. Then the field errors were in an acceptable range and the tuners were moved all equally in order to adjust the frequency. After five frequency adjustment steps the target frequency was reached (step 6, Figure 36) and the tuners were cut to their proper individual length. After tuner machining and assembly (step 7) the frequency was unexpectedly too low. This is explained in more detail in the next paragraph. After a second tuner re-machining and assembly the target frequency was set (step 8). After the proper frequency was set the field distribution was confirmed without significant changes, Figure 37.
3.3.3 Tuner Cutting

The tuners were pre-machined with an additional length of 11 mm. The penetration depth of the tuners was measured using a scaling at the screw-nut using the turns of the thread and additionally with a caliber from the backside of the tuner flange to a reference surface at the tuner tooling. The agreement of both measurement methods was within a maximum error of $\pm 0.06$ mm.

During field tuning the each tuner was set to its individual penetration depth. This was measured and in order to insert them with their final length the tuners were re-machined. After assembly (step 7 in Figure 36) the frequency was unexpectedly lower. With a frequency of 749.175.81 MHz it was 0.086.53 MHz lower than the target frequency. That refers to approximately 100 µm less length of each tuner, which could not be explained by machining errors in the range of 20 µm.

This is explained by the following, the tuners have not been in the metrology before tuning. This would not have been a problem if the data given to the workshop would have been the $\Delta$ of the length the tuner needed to be cut. Even a systematical error of the tuners would have been cancelled out by using the $\Delta$ in length. This has been actually the original idea behind of using the real tuners and the tuner moving mechanism, but instead the final length was calculated using the nominal tuner length and given to the workshop resulting in the uncertainty of the pre-machined tuner length. This length of the tuners before cutting have been measured on three leftover spare tuners, see Table 5.

<table>
<thead>
<tr>
<th>Table 5: Metrology Results for the 3 Spare Tuners</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tuner</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

Indeed the errors are in a 100 µm range and could explain the frequency error if most of them would have been roughed 100 µm too short. In order to rise the frequency to compensate this error only a decrease of the working temperature would be possible, but still difficult to realise since the working temperature is already quite low.

To solve this problem, the flange surface facing towards the cavity has been machined increasing the penetration of the tuners into the cavity by about 100 µm. To approach the target frequency a reduction of 80 µm would have been enough but to be on the save side it was decided to cut the surface by 100 µm. A higher frequency could be compensated with a higher working temperature easily.
After the second tuner machining and assembly with the final copper gaskets, the frequency and field distribution was confirmed to be in the desired range. Frequency measurement in vacuum have given a frequency deviation of 0.00125 MHz above the target frequency. Figure 38 shows the bead pull measurement of the Q, Ds, Dt component after field and frequency tuning.

**Fig. 38:** Bead pull measurement of Q, Ds and Dt component after field and frequency tuning.
### Table 6: Field Error and Frequency during Field and Frequency Tuning Steps and Tuner Cutting in Chronological Order.

<table>
<thead>
<tr>
<th>Field Tuning Steps</th>
<th>Frequency Steps</th>
<th>Q [%]</th>
<th>Ds [%]</th>
<th>Dt [%]</th>
<th>Frequency [MHz]</th>
<th>Date</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial</td>
<td></td>
<td>10.78</td>
<td>3.04</td>
<td>3.75</td>
<td>749.23562</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>6.87</td>
<td>4.75</td>
<td>2.25</td>
<td>750.09437</td>
<td>03.07.2015</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>2.69</td>
<td>3.28</td>
<td>2.46</td>
<td>750.23562</td>
<td>04.07.2015</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>1.63</td>
<td>3.16</td>
<td>2.64</td>
<td>750.30312</td>
<td>04.07.2015</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>1.33</td>
<td>2.60</td>
<td>2.80</td>
<td>750.33437</td>
<td>04.07.2015</td>
<td>$T_{svd21}$</td>
</tr>
<tr>
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<td></td>
<td>1.29</td>
<td>2.55</td>
<td>2.39</td>
<td>749.80812</td>
<td>04.07.2015</td>
<td>−0.5 mm</td>
</tr>
<tr>
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<td></td>
<td>1.61</td>
<td>0.75</td>
<td>1.58</td>
<td>749.99500</td>
<td>05.07.2015</td>
<td>$T_{svd18}$</td>
</tr>
<tr>
<td>1st tuner cutting</td>
<td></td>
<td>1.32</td>
<td>1.03</td>
<td>1.72</td>
<td>750.19375</td>
<td>05.07.2015</td>
<td>$T_{svd18}$</td>
</tr>
<tr>
<td>2nd tuner machining</td>
<td></td>
<td>1.10</td>
<td>1.15</td>
<td>1.63</td>
<td>749.98312</td>
<td>05.07.2015</td>
<td>−0.2 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.94</td>
<td>1.07</td>
<td>1.37</td>
<td>749.47312</td>
<td>06.07.2015</td>
<td>−0.5 mm</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.02</td>
<td>1.00</td>
<td>1.65</td>
<td>749.23150</td>
<td>07.07.2015</td>
<td>−0.02 mm</td>
</tr>
<tr>
<td>1st tuner cutting</td>
<td></td>
<td>1.38</td>
<td>1.07</td>
<td>1.72</td>
<td>749.14294</td>
<td>29.07.2015</td>
<td>tuners fixed with springs only</td>
</tr>
<tr>
<td>2nd tuner machining</td>
<td></td>
<td>1.04</td>
<td>1.04</td>
<td>1.71</td>
<td>749.24531</td>
<td>22.08.2015</td>
<td>tuners fixed with final copper gasket</td>
</tr>
</tbody>
</table>

**1st tuner cutting**
- no bead pull measurement
- target frequency $N_2$
- target frequency vacuum

**2nd tuner machining**
- no bead pull measurement
- confirmation ($N_2$, 749.27750)
- confirmation ($N_2$, 749.27906)
- with wire
- without wire
- vacuum
### Table 7: Tuner Settings in mm for all Field Tuning Iterations.

<table>
<thead>
<tr>
<th>tuner iteration number</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
<th>4th</th>
<th>5th</th>
<th>6th</th>
<th>final</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-0.3190</td>
<td>0.2026</td>
<td>0.5113</td>
<td>0.6445</td>
<td>1.5979</td>
<td>1.9655</td>
<td>1.0155</td>
</tr>
<tr>
<td>2</td>
<td>2.9750</td>
<td>2.5871</td>
<td>2.6486</td>
<td>2.7739</td>
<td>3.2938</td>
<td>3.5153</td>
<td>2.5653</td>
</tr>
<tr>
<td>3</td>
<td>3.4296</td>
<td>2.1981</td>
<td>1.6877</td>
<td>1.5080</td>
<td>-0.3362</td>
<td>-0.5519</td>
<td>-1.5019</td>
</tr>
<tr>
<td>4</td>
<td>0.0083</td>
<td>0.1048</td>
<td>0.0881</td>
<td>0.0286</td>
<td>-0.5132</td>
<td>-0.2623</td>
<td>-1.2123</td>
</tr>
<tr>
<td>5</td>
<td>0.7361</td>
<td>0.5990</td>
<td>0.6152</td>
<td>0.6054</td>
<td>0.0346</td>
<td>-0.0653</td>
<td>-1.0153</td>
</tr>
<tr>
<td>6</td>
<td>0.3211</td>
<td>-0.5343</td>
<td>-0.6303</td>
<td>-0.5585</td>
<td>-1.3847</td>
<td>-1.5099</td>
<td>-2.4599</td>
</tr>
<tr>
<td>7</td>
<td>0.5635</td>
<td>0.0993</td>
<td>0.0447</td>
<td>0.0931</td>
<td>-0.9244</td>
<td>-1.0407</td>
<td>-1.9907</td>
</tr>
<tr>
<td>8</td>
<td>1.4213</td>
<td>1.3680</td>
<td>1.2744</td>
<td>1.2012</td>
<td>0.6049</td>
<td>0.6342</td>
<td>-0.3158</td>
</tr>
<tr>
<td>9</td>
<td>0.8884</td>
<td>1.1618</td>
<td>1.4314</td>
<td>1.5959</td>
<td>2.6229</td>
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<td>2.2925</td>
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<td>29</td>
<td>2.1673</td>
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<td>30</td>
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<td>0.0991</td>
<td>0.1409</td>
<td>2.0040</td>
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<td>1.6830</td>
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<td>1.5652</td>
<td>0.0124</td>
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<td>32</td>
<td>2.7725</td>
<td>3.1318</td>
<td>3.1672</td>
<td>3.1180</td>
<td>1.1304</td>
<td>0.9402</td>
<td>-0.0098</td>
</tr>
</tbody>
</table>
3.4 Power Couplers / Q-Values

For the determination of the different Q-values the S11 parameters of all power couplers have been measured as a function of frequency with a bandwidth of 1 MHz. Figure 3 shows the schematic power coupler configuration of the RFQ.

While one power coupler was measured the others where connected to a 50 Ω attenuator. This setting is indicated with a * at the Q-value. The definitions for the calculations are listen in Table 8. All measurements have been saved in the Re/Im format of a smith chart. Then $Q^*_0$ and $Q^*_L$ could be obtained from the data Ref. [5].

Table 8: Definitions for Q-factor Calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q^*_0_i$</td>
<td>effective $Q_0$ which account for the losses to the other couplers terminated by 50 Ω loads calculated from the S11 measurement taken from coupler $i$</td>
</tr>
<tr>
<td>$Q^*_L_i$</td>
<td>$Q_L$ calculated from the S11 measurement taken from coupler $i$</td>
</tr>
<tr>
<td>$Q_{ext_i}$</td>
<td>external Q-factor of couplers $i$ calculated from $Q^<em>_0_i$ and $Q^</em>_L_i$</td>
</tr>
<tr>
<td>$Q_{ext}$</td>
<td>total $Q_{ext}$ of all 4 power couplers</td>
</tr>
<tr>
<td>$Q_0_i$</td>
<td>$Q_0$ of the RFQ for each coupler measurement, they must be identical in case of no measurements errors</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>mean value of all $Q_0_i$</td>
</tr>
</tbody>
</table>

From the measured $Q^*_0_i$ and $Q^*_L_i$, $Q_{ext_i}$ were determined

$$Q_{ext_i} = \left( \frac{1}{Q^*_L_i} - \frac{1}{Q^*_0_i} \right)^{-1}$$

Then the total $Q_{ext}$ was calculated using

$$Q_{ext} = \left( \frac{1}{Q_{ext1}} + \frac{1}{Q_{ext2}} + \frac{1}{Q_{ext3}} + \frac{1}{Q_{ext4}} \right)^{-1}$$

to obtain $Q_0$, with the following formula

$$Q_0_i = \left( \frac{1}{Q^*_L_i} - \frac{1}{Q_{ext}} \right)^{-1}$$

The measured parameters are presented in Table 9 together with the design values for comparison.

Table 9: Comparison of the Designed and Measured Quality Factors

<table>
<thead>
<tr>
<th>Component</th>
<th>Design</th>
<th>Measurement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_0_1$</td>
<td>6440</td>
<td>6492</td>
</tr>
<tr>
<td>$Q_0_2$</td>
<td>6440</td>
<td>6492</td>
</tr>
<tr>
<td>$Q_0_3$</td>
<td>6440</td>
<td>6355</td>
</tr>
<tr>
<td>$Q_0_4$</td>
<td>6440</td>
<td>6944</td>
</tr>
<tr>
<td>$Q_{ext1}$</td>
<td>21900</td>
<td>26060</td>
</tr>
<tr>
<td>$Q_{ext2}$</td>
<td>21900</td>
<td>27878</td>
</tr>
<tr>
<td>$Q_{ext3}$</td>
<td>21900</td>
<td>27878</td>
</tr>
<tr>
<td>$Q_{ext4}$</td>
<td>21900</td>
<td>21410</td>
</tr>
<tr>
<td>$Q_0$</td>
<td>6440</td>
<td>6570</td>
</tr>
<tr>
<td>$Q_{ext}$</td>
<td>5475</td>
<td>6377</td>
</tr>
<tr>
<td>Coupling $\beta$</td>
<td>1.18</td>
<td>1.03</td>
</tr>
</tbody>
</table>
3.5 Antenna Calibration

For the antenna calibration $S_{21}$ parameters from all 4 power couplers to all 16 antennas have been measured at operating frequency. This data was used on the one hand to observe a possible change of longitudinal field distribution, for example due to transport, installation or during operation and on the other hand calculate the output power at the antennas at the nominal voltage of 67.6 kV.

3.5.1 Field Distribution

In order to be able to observe a change of the field distributions the relative field levels at the locations of the pickup antennas from bead pull measurements have been compared to the $S_{21}$ measurements from all power couplers to all antennas.

Each $S_{21}$ measurements is a function of the external Q of the coupler and of the antenna

$$S_{21}^{C1-A1}(Q^{C1}_{ext}; Q^{A1}_{ext})$$

which depends on the coupling and the field level at the location of the coupler and antenna

$$S_{21}^{C1-A1}((F_{C1}; \beta_{C1}); (F_{A1}; \beta_{A1}))$$

This means that the $S_{21}$ measurements are proportional to a function of the field and the coupling at every antenna

$$S_{21} \propto F \cdot \beta$$

Since only the relative field distribution is of interest each measurement can be normalised to a measurement of any other antenna. $S_{21}^{C1-Ai}$ is the $S_{21}$ measurement from coupler 1 to the i-th antenna that is normalised to the measurement from coupler 1 to antenna 1.

$$\frac{S_{21}^{C1-Ai}}{S_{21}^{C1-A1}} \propto \frac{f(F_{A1} \cdot \beta_{A1})}{f(F_{A1} \cdot \beta_{A1})}$$

Since the coupling is not changed $\beta_i/\beta_1$ is constant and the ratio in Equation (19) only depends on a possible change of field. By dividing the magnitude of the $S_{21}$ measurements of one power coupler to all antennas normalised to antenna 1 by the field level at the location of the antennas obtained from bead pull measurements also normalised to the field value at the location of antenna 1, one receives curves (shown in Figure 39) that should be equal for all 4 measurements for each power coupler. A change of field would result in a displacement of all four coupler to antenna measurements compared to the situation before. Figure 39 shows the measurements at CERN before the transport of the RFQ.

![Fig. 39: Antenna calibration measured at CERN.](image)

The plot shows a good agreement of all curves. Slight deviations are related to measurement errors. A displacement of a new measurements of all four curves to the four curves measured before would be a clear indication of a field change.
### 3.5.2 Power Level at the Antenna Output

Parameter definitions for the calculations in this paragraph are listed in Table 10.

#### Table 10: Parameter Definitions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_{0}^{\text{sim}}$</td>
<td>$Q_0$ design value</td>
</tr>
<tr>
<td>$P_{0}^{\text{sim}}$</td>
<td>design value of nominal power to obtain nominal voltage of 67.6 kV</td>
</tr>
<tr>
<td>$Q_{0}^{\text{meas}}$</td>
<td>measured $Q_0$ value</td>
</tr>
<tr>
<td>$S_{C_iA_j}^{21}$</td>
<td>magnitudes of $S_{21}$-measurement in dB from coupler $i = 1, \ldots, 4$ to antenna $j = 1, \ldots, 16$</td>
</tr>
<tr>
<td>$W_0$</td>
<td>energy stored in the RFQ</td>
</tr>
<tr>
<td>$\omega$</td>
<td>$2 \cdot \pi \cdot f$</td>
</tr>
<tr>
<td>$f$</td>
<td>nominal frequency of the RFQ</td>
</tr>
</tbody>
</table>

The $S_{21}$ measurements in dB have been converted into $S_{C_iA_j}^{21}$ magnitudes for calculations using

$$S_{C_iA_j}^{21} = 10^{\frac{\text{dB-value}}{20}}$$  \hspace{1cm} (20)

Table 11 lists the parameters used for the following calculations.

#### Table 11: Parameters used for Calculations

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
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<td></td>
</tr>
<tr>
<td>$Q_{0}^{\text{sim}}$</td>
<td>6440</td>
<td></td>
</tr>
<tr>
<td>$P_{0}^{\text{sim}}$</td>
<td>340 kW</td>
<td></td>
</tr>
<tr>
<td>$Q_{0}^{\text{meas}}$</td>
<td>6570</td>
<td></td>
</tr>
<tr>
<td>$S_{C_1A_1}^{21}$</td>
<td>-59.49 dB</td>
<td></td>
</tr>
<tr>
<td>$S_{C_2A_1}^{21}$</td>
<td>-59.67 dB</td>
<td></td>
</tr>
<tr>
<td>$S_{C_3A_1}^{21}$</td>
<td>-60.04 dB</td>
<td></td>
</tr>
<tr>
<td>$S_{C_4A_1}^{21}$</td>
<td>-58.71 dB</td>
<td></td>
</tr>
</tbody>
</table>

The design value of the energy stored in the RFQ at the nominal voltage of 67.6 kV is given by

$$W_0 = \frac{Q_{0}^{\text{sim}} \cdot P_{0}^{\text{sim}}}{\omega} = \frac{6440 \cdot 340 \cdot 10^3 W}{2 \cdot \pi \cdot 750 \cdot 10^6 Hz} = 0.465 J$$  \hspace{1cm} (21)

The power needed to achieve the nominal stored energy for the measured quality factor is given by

$$P_L = \frac{W_0 \cdot \omega}{Q_{0}^{\text{meas}}} = \frac{0.465 J \cdot 2 \cdot \pi \cdot 750 \cdot 10^6 Hz}{6570} = 334 \text{ kW}$$  \hspace{1cm} (22)

This agrees well with the design power of 340 kW. Then the power expected to be measured at the pickup antenna 1 for the total input power for all 4 couplers of 334 kW is calculated by

$$P_{0A1}^1 = \left[ (S_{C_1A_1}^{C_1A_1})^2 + (S_{C_2A_1}^{C_2A_1})^2 + (S_{C_3A_1}^{C_3A_1})^2 + (S_{C_4A_1}^{C_4A_1})^2 \right] \cdot P_L = 1.51 W$$  \hspace{1cm} (23)

This shows also a good agreement with the design value of 1.4 W. To calculate the power to any other antenna the $S_{21}$ measurements of all power couplers to all antennas are listed in Table 12.

---

35
Table 12: $S_{21}$ Measurements of all Power Couplers to all Antennas in dB

<table>
<thead>
<tr>
<th>dB</th>
<th>C1</th>
<th>C2</th>
<th>C3</th>
<th>C4</th>
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</thead>
<tbody>
<tr>
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<td>-59.49</td>
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<td>-58.71</td>
</tr>
<tr>
<td>A2</td>
<td>-60.17</td>
<td>-60.33</td>
<td>-60.68</td>
<td>-59.39</td>
</tr>
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<td>A3</td>
<td>-60.61</td>
<td>-60.81</td>
<td>-61.18</td>
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</tr>
<tr>
<td>A4</td>
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<td>-61.02</td>
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4 Acknowledgements

We would like to thank Sebastien Calvo, Yves Cuvet, Alessandra Lombardi, Serge Mathot, Eric Montesinos, Carlo Rossi, Maurizio Vretenar for their support.

5 References


Appendices

A Tuning Algorithm

The aim of tuning is to adjust the fields of the RFQ according to the requirements of the beam dynamics design. Therefore the RFQ is equipped with 32 tuners that can be moved precisely into the cavity to influence the fields. By adjusting the penetration depth of the tuners the magnetic field distribution in azimuthal and longitudinal direction can be influenced. The goal of the tuning process is to find a mechanical setting for each tuner that provides a constant longitudinal field distribution for the quadrupole mode (Q). The dipolar components (Ds, Dt) should be zero or minimised to a small fraction of the quadrupole component.

A.1 Response Matrix

The tuning algorithm is based on a response matrix that describes the influence of every tuner to every field component for all longitudinal locations. To obtain the field distribution the standard bead pull technique is used. The magnetic field distribution is described in terms of quadrupole (Q) and dipole (Ds, Dt) components. If \( q_1, \ldots, q_4 \) are the bead pull measurements of the longitudinal field distribution of the four quadrants, the transversal amplitudes are defined in Eq. 3, 4 and 5. The amplitudes are supposed to be

\[
Q = \text{const.} \quad (A.1)
\]

\[
Ds = Dt = 0 \quad (A.2)
\]

This implies that the magnetic flux in all quadrants is the same for a pure quadrupole mode. In longitudinal direction the quadrupole mode should be constant for the HF-RFQ and the dipole components should be as small as possible to guarantee a clean quadrupole mode. With the following definitions

\[
\vec{V} = \vec{V}_i - \vec{V}_{0i} \quad (A.3)
\]

\[
\vec{T} = \vec{T}_j - \vec{T}_{0j} \quad (A.4)
\]

where \( \vec{V} \) describes the difference between the actual \( \vec{V}_{0i} \) and desired longitudinal \( \vec{V}_i \) field distribution. The vector entries are the longitudinal measurements points along the RFQ for all components Q, Ds and Dt. The vector \( \vec{T} \) is defined as the difference between the actual \( \vec{T}_{0j} \) and desired \( \vec{T}_j \) tuner setting of all tuners. The index \( i \) represents the longitudinal measurement locations, while \( j \) is the tuner number used to adjust the fields. Now the derivative of every single longitudinal field component caused by a certain tuner movement is given by

\[
M_{i,j} = \sum_{j=1}^{32} \frac{\partial \vec{V}_i}{\partial T_j} \quad (A.5)
\]

This is defined as the response matrix (with 11 longitudinal measurements locations for each component \( i = 33 \) and a number of tuners \( j = 32 \)). The equation can be written as follows

\[
\vec{V} = \vec{M} \cdot \vec{T} \quad (A.6)
\]

where \( \vec{V} \) and \( \vec{M} \) are known from measurements and \( \vec{T} \) is the unknown desired tuner setting. The following equation shows the same equation again with colour coded components Q (blue), Ds (red) and Dt
All coloured entries are known by measurements and calculations. The initial tuner setting is zero for all tuners and the only unknown is the tuner setting obtained by inverting the Matrix $M$.

$$\vec{T} = M^{-1} \cdot \vec{V} \quad (A.7)$$

Due to errors in the tuner adjustment and the fact that the tuner movement does not influence field and frequency in a perfectly linear manner the procedure has to be repeated several times. Hence obtaining the desired field distribution is an iterative process. Therefore the calculated tuner settings $\vec{T}_j$ has to be applied to the tuners and the field has to be remeasured to obtain a new $\vec{V}_0$. This has to be repeated until an acceptable difference in $\vec{V}_i - \vec{V}_0$ is achieved Ref. [6].

The dimensions of the matrix are given by the number of tuners and by the number of measurement locations times three for three components $Q$, $D_s$ and $D_t$. For the HF-RFQ this was a $32 \times 33$ non-square matrix and hence not very simple to invert. In order to solve this the matrix was inverted using the singular value decomposition.

### A.2 Singular Values Decomposition

An introduction how the singular value decomposition (SVD) was used for the tuning algorithm is given in the following. More detailed information about SVD can be found for example in Ref. [8]. A singular value decomposition is a factorisation of normal or complex matrix into three matrices.

$$M = U S V^T \quad (A.8)$$

$U$ is a $m \times m$ and column orthogonal matrix whose columns are the eigenvectors of the $M M^T$ matrix (left eigenvectors)

$S$ is a $m \times n$ diagonal matrix whose diagonal elements (non-negative real values) are the singular values of $M$

$V$ is a $n \times n$ orthogonal matrix whose columns are the eigenvectors of the $M^T M$ matrix (right eigenvectors)

$$M = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & U & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \end{bmatrix} \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ \cdot \quad \cdot \quad \cdot \\ \end{bmatrix} \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & V^T & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \end{bmatrix}$$

$S = \text{diag}(\sigma_1, \sigma_2, ..., \sigma_n)$ is ordered so that $\sigma_1 \geq \sigma_2 \geq ... \geq \sigma_n$ (if $\sigma$ is an singular value of $M$ it’s square is an eigenvalue of $M^T M$)
Besides other applications SVD is a powerful tool to compute the inverse of a matrix, even if it is non-square and ill-conditioned.

If $M$ is a $n \times n$ matrix, its inverse is given by

$$M^{-1} = VS^{-1}U^T \quad (A.9)$$

where $S^{-1} = \text{diag} \left( \frac{1}{\sigma_1}, \frac{1}{\sigma_2}, ..., \frac{1}{\sigma_n} \right) \quad (A.10)$

If $M$ is singular or ill-conditioned, SVD can be used to approximate its inverse by the following

$$M^{-1} = (USV^T)^{-1} \approx VS_0^{-1}U^T \quad (A.11)$$

$$S_0^{-1} = \begin{cases} 
1/\sigma_i & \text{if } \sigma_i > t \\
0 & \text{otherwise}
\end{cases} \quad (A.12)$$

where $t$ is a small threshold.

If $M$ is a square matrix also $S$ is square and can be simple inverted by inverting each element of the matrix. If $M$ is non-square the also non-square diagonal matrix $S$ can be inverted by using the pseudo-inverse, inverting each element and then transposing it.

For the tuning process the following equation has to be solved.

$$\vec{V} = M \cdot \vec{T} \quad (A.13)$$

A solution of tuner settings is given by

$$\vec{T} = M^{-1} \cdot \vec{V} \quad (A.14)$$

since this set of equations is over-determined, $M$ is non-square and ill-conditioned and SVD is used for the inversion of $M$ and the pseudo-inverse is used to obtain $S_0^{-1}$. But still the solutions for $T$ might give some strange results. To solve this $M^{-1}$ is approximated by changing the threshold $t$ for $S_0^{-1}$ as given in Equation (A.12). To do so in a first step the largest $1/\sigma_i$ is set to zero. From now on the pseudo-inverse of $S$ is named $S_{\text{inv}}$ leading to a solution for a tuner setting

$$\vec{T}_{\text{svd0}} = (V \cdot S_{\text{inv}}0 \cdot U^T) \cdot \vec{V} \quad (A.15)$$

Since $M$ is a $33 \times 32$ matrix the number of singular values is $32$. As first step to obtain an approximation for $S_0^{-1}$, $S_{\text{inv}}1$ is calculated by setting the largest inverse singular value to zero (Equation A.16). In a second step the second largest inverse singular value is set to zero (Equation A.17), and so on.

$$1/\sigma_{32} = 0 \Rightarrow S_{\text{inv1}} \quad (A.16)$$

$$1/\sigma_{31} = 0 \Rightarrow S_{\text{inv2}} \quad (A.17)$$

$$\vdots$$

$$1/\sigma_{1} = 0 \Rightarrow S_{\text{inv32}} \quad (A.18)$$

From these different $S_{\text{inv}i}$ also different tuner settings can be obtained using Equation (A.15).

$$\vec{T}_{\text{svd1}} = (V \cdot S_{\text{inv1}} \cdot U^T) \cdot \vec{V} \quad (A.19)$$

$$\vec{T}_{\text{svd2}} = (V \cdot S_{\text{inv2}} \cdot U^T) \cdot \vec{V} \quad (A.20)$$

$$\vdots$$
\[
\vec{T}_{\text{svd}32} = (\vec{V} \cdot \text{S}\text{inv}_{32} \cdot \vec{U}^T) \cdot \vec{V}
\]

(A.21)

Now we found 32 possible solutions for tuner settings to compensate \(\vec{V}\) in the first iteration. But some of these solutions can not be used due to large tuner settings that are clearly out of the mechanical tuning range. Also the different \(\text{S}\text{inv}\_i\) with manipulated singular values are only approximations of \(\text{S}\_0^{-1}\) and one has to check the agreement of \(\vec{V}\). This agreements are also an indication for how good a certain solution for \(\vec{T}_{\text{svd}i}\) might compensate the field deviation \(\vec{V}\). These predictions are obtained by using the tuner settings \(\vec{T}_{\text{svd}i}\) and the original response matrix \(\vec{M}\).

\[
\begin{align*}
\vec{V}_{\text{svd}1} &= \vec{M} \cdot \vec{T}_{\text{svd}1} \\
\vec{V}_{\text{svd}2} &= \vec{M} \cdot \vec{T}_{\text{svd}2} \\
&\vdots \\
\vec{V}_{\text{svd}32} &= \vec{M} \cdot \vec{T}_{\text{svd}32}
\end{align*}
\]

(A.22)

(A.23)

(A.24)

These predicitons of which field deviation will be compensated \(\vec{V}_{\text{svd}i}\) can be compared to actual field deviation \(\vec{V}\) to find a best fitting tuner setting \(\vec{T}_{\text{svd}i}\) as shown in Figure A.1 on the top. The top graph shows the field that has to be compensated as blue dotted line. The other curves are a selection of different \(\vec{V}_{\text{svd}i}\) for different thresholds for the modification of the singular values. The plot on the bottom shows the corresponding \(\vec{T}_{\text{svd}i}\) to the \(\vec{V}_{\text{svd}i}\) above.

\[\text{Fig. A.1: Several predictions for field compensation (top) and corresponding tuner settings (bottom).}\]

This figure shows that many of the tuner setting would lead to a quite good compensation of the quadrupole component (vector entries 1-11). For the dipole components (vector entries Ds (12-22) and Dt (23-33)) the deviation is much larger for some curves. The plot also shows only a selection of \(\vec{V}_{\text{svd}17}\) to \(\vec{V}_{\text{svd}26}\). The other solutions are not very practical due to the large tuner settings in the range of up to 100 mm as well as poor agreement with \(\vec{V}\). Solutions for \(\vec{V}_{\text{svd}1}\) to \(\vec{V}_{\text{svd}15}\) show a good agreement in \(\vec{V}\) put too large tuner movement, while solutions for \(\vec{V}_{\text{svd}25}\) to \(\vec{V}_{\text{svd}32}\) are in a reasonable range for tuner movements but show poor agreement in \(\vec{V}\), that therefore were neglected for the tuning process.
B MATLAB Script

In the following the Matlab script to compute the response matrix and the calculation for the different solutions for $\vec{T}_{svd}^i$ and $\vec{V}_{svd}^i$ is given. Before this script can be used bead pull measurements of the initial field distribution as well as the field distribution for all tuner movements should be measured. Raw data should be processed and dipole and quadrupole components calculated. All components for all tuner settings should be in separated text files with file names as stated at the beginning of the script.

The script will also produce three plots. The first plot illustrates the Q, Ds and Dt components for all tuner movements using all sampling points for the x-axis and allows one to check if all measurements are aligned to each other properly. The second plot shows the same data as the first but for only 11 measurement points along the RFQ which are used for calculating the matrix and all further tuning steps. In the third plot the vector entries for all $\vec{T}_{svd}^i$ and $\vec{V}_{svd}^i$ are shown to compare different solution given by the SVD in terms of field compensation and tuning range.
% before running this script:
% bead pull measurements of initial field distribution as well as bead pull 
% measurements of the field distribution for all the tuner movements
% raw data processing and calculation of components
% data of components is a single column text file
% example for the initial and 'tuner number 7' field distribution
% Q.txt  Ds.txt  Dt.txt
% Q_T7.txt  Ds_T7.txt  Dt_T7.txt

% loading data
load Q.txt
load Ds.txt
load Dt.txt
load Q0.txt

% distribution for tuner movements
for i = 1:32
    load(['Q_T',int2str(i),'.txt']);
end
for i = 1:32
    load(['Ds_T',int2str(i),'.txt']);
end
for i = 1:32
    load(['Dt_T',int2str(i),'.txt']);
end

% assigning data to vectors
z=[1:1:1501]';
Q = Q(:,1);
Ds = Ds(:,1);
Dt = Dt(:,1);
Q0 = Q0(:,1);

% vectors for components of initial field distribution
for i=1:32
    eval(['Q_T',int2str(i),'.'] = Q_T',int2str(i),'.'](1,:));
end
for i=1:32
    eval(['Ds_T',int2str(i),'.'] = Ds_T',int2str(i),'.'](1,:));
end
for i=1:32
    eval(['Dt_T',int2str(i),'.'] = Dt_T',int2str(i),'.'](1,:));
end

% normalisation of components
Q_mean = mean(Q);
Q = 100 * Q / Q_mean;
Ds = 100 * Ds / Q_mean;
Dt = 100 * Dt / Q_mean;
for i=1:32
    eval(['Q_T',int2str(i),'.'] = 100 * Q_T',int2str(i),'.'] / Q_mean '])
end
for i=1:32
    eval(['Ds_T',int2str(i),'.'] = 100 * Ds_T',int2str(i),'.'] / Q_mean '])
end
for i=1:32
eval([' Dt_T',int2str(i),'] = 100 * Dt_T',int2str(i),'] / Q_mean ']);
end

%% plotting

figure
axes('position',[.08 .6 0.88 .38])
plot(z,Q,'o',z,Q_T1,z, Q_T2,z, Q_T3,z, Q_T4,z, Q_T5,z, Q_T6,z, Q_T7,z, Q_T8,...
z,Q_T9,z,Q_T10,z,Q_T11,z,Q_T12,z,Q_T13,z,Q_T14,z,Q_T15,z,Q_T16,...
z,Q_T17,z,Q_T18,z,Q_T19,z,Q_T20,z,Q_T21,z,Q_T22,z,Q_T23,z,Q_T24,...
z,Q_T25,z,Q_T26,z,Q_T27,z,Q_T28,z,Q_T29,z,Q_T30,z,Q_T31,z,Q_T32]
legend('initial', 'T1', 'T2', 'T3', 'T4', 'T5', 'T6', 'T7', 'T8', 'T9', 'T10', 'T11', ...
title('Q')
set(gca,'FontSize',12)
set(gca,'XTickLabel',[])}

axes('position',[.08 .34 0.88 .24])
plot(z,Ds,z,Ds_T1,z,Ds_T2,z,Ds_T3,z,Ds_T4,z,Ds_T5,z,Ds_T6,z,Ds_T7,z,Ds_T8,...
zs,Ds_T9,z,Ds_T10,z,Ds_T11,z,Ds_T12,z,Ds_T13,z,Ds_T14,z,Ds_T15,z,Ds_T16,...
zs,Ds_T17,z,Ds_T18,z,Ds_T19,z,Ds_T20,z,Ds_T21,z,Ds_T22,z,Ds_T23,z,Ds_T24,...
zs,Ds_T25,z,Ds_T26,z,Ds_T27,z,Ds_T28,z,Ds_T29,z,Ds_T30,z,Ds_T31,z,Ds_T32)
title('Ds')
set(gca,'FontSize',12)
yl = ylabel('relative field amplitude / arb. units')
set(yl,'FontSize',16')
set(gca,'XTickLabel',[])}

axes('position',[.08 .08 0.88 .24])
plot(z,Dt,z,Dt_T1,z,Dt_T2,z,Dt_T3,z,Dt_T4,z,Dt_T5,z,Dt_T6,z,Dt_T7,z,Dt_T8,...
zs,Dt_T9,z,Dt_T10,z,Dt_T11,z,Dt_T12,z,Dt_T13,z,Dt_T14,z,Dt_T15,z,Dt_T16,...
zs,Dt_T17,z,Dt_T18,z,Dt_T19,z,Dt_T20,z,Dt_T21,z,Dt_T22,z,Dt_T23,z,Dt_T24,...
zs,Dt_T25,z,Dt_T26,z,Dt_T27,z,Dt_T28,z,Dt_T29,z,Dt_T30,z,Dt_T31,z,Dt_T32)
title('Dt')
set(gca,'FontSize',12)
xl = xlabel('sampling points')
set(xl,'FontSize',16')

%% measurement locations %%

% define locations in z
p1 = 220
p2 = 297
p3 = 410
p4 = 530
p5 = 617
p6 = 757
p7 = 905
p8 = 990
p9 = 1107
p10 = 1221
p11 = 1297

Q_r = [Q(p1,:); Q(p2,:); Q(p3,:); Q(p4,:); Q(p5,:); Q(p6,:);...
Q(p7,:); Q(p8,:); Q(p9,:); Q(p10,:); Q(p11,:)]
Ds_r = [Ds(p1,:); Ds(p2,:); Ds(p3,:); Ds(p4,:); Ds(p5,:); Ds(p6,:);...
Ds(p7,:); Ds(p8,:); Ds(p9,:); Ds(p10,:); Ds(p11,:)]
Dt_r = [Dt(p1,:); Dt(p2,:); Dt(p3,:); Dt(p4,:); Dt(p5,:); Dt(p6,:);...
Dt(p7,:); Dt(p8,:); Dt(p9,:); Dt(p10,:); Dt(p11,:)]
for i=1:32
eval(['Q_T',\int2str(i),'_r = [Q_T',\int2str(i),',(p1,);Q_T',\int2str(i),',(p2,);Q_T'
       \int2str(i),',(p3,);Q_T',\int2str(i),',(p4,);Q_T',\int2str(i),',(p5,);Q_T'
       \int2str(i),',(p6,);Q_T',\int2str(i),',(p7,);Q_T',\int2str(i),',(p8,);Q_T'
       \int2str(i),',(p9,);Q_T',\int2str(i),',(p10,);Q_T',\int2str(i),',(p11,)]')
end
for i=1:32
eval(['Ds_T',\int2str(i),'_r = [Ds_T',\int2str(i),',
       (p1,);Ds_T',\int2str(i),',(p2,);Ds_T',\int2str(i),',(p3,);Ds_T',\int2str(i),',(p4,);Ds_T'
       \int2str(i),',(p5,);Ds_T',\int2str(i),',(p6,);Ds_T',\int2str(i),',(p7,);Ds_T',\int2str(i),',
       (p8,);Ds_T',\int2str(i),',(p9,);Ds_T',\int2str(i),',(p10,);Ds_T',\int2str(i),',(p11,)]')
end
for i=1:32
eval(['Dt_T',\int2str(i),'_r = [Dt_T',\int2str(i),',
       (p1,);Dt_T',\int2str(i),',(p2,);Dt_T',\int2str(i),',(p3,);Dt_T',\int2str(i),',(p4,);Dt_T'
       \int2str(i),',(p5,);Dt_T',\int2str(i),',(p6,);Dt_T',\int2str(i),',(p7,);Dt_T',\int2str(i),',
       (p8,);Dt_T',\int2str(i),',(p9,);Dt_T',\int2str(i),',(p10,);Dt_T',\int2str(i),',(p11,)]')
end

% normalisation
Q_r_mean = mean(Q_r)
Q_r = 100 * Q_r / Q_r_mean
Ds_r = 100 * Ds_r / Q_r_mean
Dt_r = 100 * Ds_r / Q_r_mean

for i=1:32
   eval(['Q_T',\int2str(i),'_r = 100 * Q_T',\int2str(i),',_r / Q_r_mean '])
end
for i=1:32
   eval(['Ds_T',\int2str(i),'_r = 100 * Ds_T',\int2str(i),',_r / Q_r_mean '])
end
for i=1:32
   eval(['Dt_T',\int2str(i),'_r = 100 * Dt_T',\int2str(i),',_r / Q_r_mean '])
end

% plot all tuner movements

z = [1:1:11]
figure
axes('position',[.08 .6 0.88 .38])
plot(z,Q_r,'o',z,Q_T1_r,z,Q_T2_r,z,Q_T3_r,z,Q_T4_r,z,Q_T5_r,z,Q_T6_r,z,Q_T7_r,...
z,Q_T8_r,z,Q_T9_r,z,Q_T10_r,z,Q_T11_r,z,Q_T12_r,z,Q_T13_r,z,Q_T14_r,...
z,Q_T15_r,z,Q_T16_r,z,Q_T17_r,z,Q_T18_r,z,Q_T19_r,z,Q_T20_r,z,Q_T21_r,...
z,Q_T22_r,z,Q_T23_r,z,Q_T24_r,z,Q_T25_r,z,Q_T26_r,z,Q_T27_r,z,Q_T28_r,...
z,Q_T29_r,z,Q_T30_r,z,Q_T31_r,z,Q_T32_r]
legend('initial', 'T1', 'T2', 'T3', 'T4', 'T5', 'T6', 'T7', 'T8', 'T9', 'T10', 'T11',...
title('Q')
set(gca,'FontSize',12)
set(gca,'XTickLabel',[])%axes('position',[.08 .34 0.88 .24])
plot(z,Ds_r,z,Ds_T1_r,z,Ds_T2_r,z,Ds_T3_r,z,Ds_T4_r,z,Ds_T5_r,z,Ds_T6_r,...
z,Ds_T7_r,z,Ds_T8_r,z,Ds_T9_r,z,Ds_T10_r,z,Ds_T11_r,z,Ds_T12_r,...
z,Ds_T13_r,z,Ds_T14_r,z,Ds_T15_r,z,Ds_T16_r,z,Ds_T17_r,z,Ds_T18_r,...
z,Ds_T19_r,z,Ds_T20_r,z,Ds_T21_r,z,Ds_T22_r,z,Ds_T23_r,z,Ds_T24_r,...
z,Ds_T25_r,z,Ds_T26_r,z,Ds_T27_r,z,Ds_T28_r,z,Ds_T29_r,z,Ds_T30_r,...
z,Ds_T31_r,z,Ds_T32_r]
title('Ds')
set(gca,'FontSize',12)
yl = ylabel('relative field amplitude / arb. units')
set(yl,'FontSize',16')
set(gca,'XTickLabel',[])
axes('position',[.08 .08 0.88 .24])
plot(z, Dt_r, z, Dt_T1_r, z, Dt_T2_r, z, Dt_T3_r, z, Dt_T4_r, z, Dt_T5_r, z, Dt_T6_r, ...
    z, Dt_T7_r, z, Dt_T8_r, z, Dt_T9_r, z, Dt_T10_r, z, Dt_T11_r, z, Dt_T12_r, ...
    z, Dt_T13_r, z, Dt_T14_r, z, Dt_T15_r, z, Dt_T16_r, z, Dt_T17_r, z, Dt_T18_r, ...
    z, Dt_T19_r, z, Dt_T20_r, z, Dt_T21_r, z, Dt_T22_r, z, Dt_T23_r, z, Dt_T24_r, ...
    z, Dt_T25_r, z, Dt_T26_r, z, Dt_T27_r, z, Dt_T28_r, z, Dt_T29_r, z, Dt_T30_r, ...
    z, Dt_T31_r, z, Dt_T32_r)

    title('Dt')
    set(gca, 'FontSize',12)
    xl = xlabel('sampling points')
    set(xl, 'FontSize',16')

% % difference between the desired and the actual longitudinal field distribution
% % desired Fields Q=100 Ds=Dt=0
% % example: delta_V1 = [Q_r-Q_T_r; Ds_r-Ds_T_r; Dt_r-Dt_T_r; Q0_r-Q0_T1_r]

for i=1:32
    eval(['delta_V',int2str(i),]' = [Q_r-Q_T_r, Ds_r-Ds_Tr, Dt_r-Dt_Tr, Q0_r-Q0_T1_r];
    % ,int2str(i), '_r'; Ds_r-Ds_T...'
    % ,int2str(i), '_r'; Dt_r-Dt_T...'
    % ,int2str(i), '_r']);
end

% tuner shift; all tuners were moved by +3 mm inside the cavity
% delta_T = T_actual - T_shift = 0 - (+3mm)

delta_T = -3

% % response matrix
M = [delta_V1/ delta_T, delta_V2/ delta_T, delta_V3/ delta_T, delta_V4/ delta_T, ...
    delta_V5/ delta_T, delta_V6/ delta_T, delta_V7/ delta_T, delta_V8/ delta_T, ...
    delta_V9/ delta_T, delta_V10/ delta_T, delta_V11/ delta_T, delta_V12/ delta_T, ...
    delta_V13/ delta_T, delta_V14/ delta_T, delta_V15/ delta_T, delta_V16/ delta_T, ...
    delta_V17/ delta_T, delta_V18/ delta_T, delta_V19/ delta_T, delta_V20/ delta_T, ...
    delta_V21/ delta_T, delta_V22/ delta_T, delta_V23/ delta_T, delta_V24/ delta_T, ...
    delta_V25/ delta_T, delta_V26/ delta_T, delta_V27/ delta_T, delta_V28/ delta_T, ...
    delta_V29/ delta_T, delta_V30/ delta_T, delta_V31/ delta_T, delta_V32/ delta_T]

% V-vector; desired - actual field
V1 = [100 - Q_r; 0 - Ds_r; 0 - Dt_r]

% T = M^-1 * V1

% % singular value decomposition
[U,S,V]=svd(M)

Minv_svd = V * pinv(S) * U' % should be the same as inv(M)
Sinv = pinv(S) % diagonal S matrix with inverted singular values,
% above threshold ones should be replaced by 0

% all singular values as initial
Sinv0 = Sinv
T_svd0 = (V * Sinv0 * U')*V1
V_svd0 = M*((V*Sinv0*U')*V1)

% % 32th singular value is set to zero
% Sinv1 = Sinv0
% Sinv1(32,32)=0 % preparation of Sinv; matrix entry at 32,32 is set to 0
% T_svd1 = (V * Sinv1 * U')*V1 % estimated tuner displacement
% V_svd1 = M*((V*Sinv1*U')*V1) % should be close to V1
%
% % also 31th singular value is set to zero
% Sinv2 = Sinv1 % preparation of Sinv; matrix entry at 31,31 is set to 0
% Sinv2(31,31)=0
% T_svd2 = (V * Sinv2 * U')*V1 % estimated tuner displacement
% V_svd2 = M*((V*Sinv2*U')*V1) % prediction for V1

% %
% %
% loop for the above commented

for i=1:32
    eval(['Sinv',int2str(i),'] = Sinv',int2str(i-1),'' ]);
    eval(['Sinv',int2str(i),'](33-i,33-i) = 0']);
    eval(['T_svd',int2str(i),'] = (V * Sinv',int2str(i),',', 'transpose(U)'),V1 ']);
    eval(['V_svd',int2str(i),'] = M*((V * Sinv',int2str(i),',transpose(U)'),V1 ']));
end

% plotting all predictions and tuner settings

% x-axis for vector entries
v = 1:33
t = 1:32

figure
subplot(2,1,1)
plot(v,V1, v,V_svd0, v,V_svd1, v,V_svd2, v,V_svd3, v,V_svd4, v,V_svd5, v,V_svd6, ...
    v,V_svd7, v,V_svd8, v,V_svd9, v,V_svd10,v,V_svd11,v,V_svd12,v,V_svd13, ...
    v,V_svd14,v,V_svd15,v,V_svd16,v,V_svd17,v,V_svd18,v,V_svd19,v,V_svd20, ...
    v,V_svd21,v,V_svd22,v,V_svd23,v,V_svd24,v,V_svd25,v,V_svd26,v,V_svd27, ...
    v,V_svd28,v,V_svd29,v,V_svd30,v,V_svd31)
legend('V1', 'V_svd0', 'V_svd1', 'V_svd2', 'V_svd3', 'V_svd4', 'V_svd5', 'V_svd6', ...
    'V_svd7', 'V_svd8', 'V_svd9', 'V_svd10', 'V_svd11', 'V_svd12', 'V_svd13', ...
    'V_svd14', 'V_svd15', 'V_svd16', 'V_svd17', 'V_svd18', 'V_svd19', 'V_svd20', ...
    'V_svd21', 'V_svd22', 'V_svd23', 'V_svd24', 'V_svd25', 'V_svd26', 'V_svd27', ...
    'V_svd28', 'V_svd29', 'V_svd30', 'V_svd31')
xlabel('# of vector entries (longitudinal measurements locations for Q, Ds, Dt)')
ylabel(sprintf('relative deviation from\nthe desired field component'))) subplot(2,1,2)
plot(t,T_svd0, t,T_svd1, t,T_svd2, t,T_svd3, t,T_svd4, t,T_svd5, t,T_svd6, t,T_svd7, ...
    t,T_svd8, t,T_svd9, t,T_svd10, t,T_svd11, t,T_svd12, t,T_svd13, t,T_svd14, t,T_svd15, ...
    t,T_svd16, t,T_svd17, t,T_svd18, t,T_svd19, t,T_svd20, t,T_svd21, t,T_svd22, t,T_svd23, ...
    t,T_svd24, t,T_svd25, t,T_svd26, t,T_svd27, t,T_svd28, t,T_svd29, t,T_svd30, t,T_svd31)
legend('T_svd0', 'T_svd1', 'T_svd2', 'T_svd3', 'T_svd4', 'T_svd5', 'T_svd6', 'T_svd7', ...
    'T_svd8', 'T_svd9', 'T_svd10', 'T_svd11', 'T_svd12', 'T_svd13', 'T_svd14', 'T_svd15', ...
    'T_svd16', 'T_svd17', 'T_svd18', 'T_svd19', 'T_svd20', 'T_svd21', 'T_svd22', 'T_svd23', ...
    'T_svd24', 'T_svd25', 'T_svd26', 'T_svd27', 'T_svd28', 'T_svd29', 'T_svd30', 'T_svd31')
xlabel('number of tuners')
ylabel('tuner settings / mm')