STATUS OF THE STANDARD MODEL

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At present the attention of the high energy physics community is mainly focused on the on-going LEP experiments. Accordingly, in the following I will discuss LEP physics¹ and its context both with respect to the electroweak and the QCD sectors of the Standard Model.

1. ELECTROWEAK INTERACTIONS

1.1. Introduction

The main goal of LEP 1 is to perform precision tests of the standard electroweak theory² at the Z peak. Theoretical predictions in the Standard Model for all relevant observables have been developed in detail¹. I refer the reader to my talk³ at the Stanford Conference for a concise summary and for many relevant discussions that I will not repeat here. One starts from the Standard Model Lagrangian and a conveniently chosen set of input parameters. The interesting quantities are computed in perturbation theory. The lowest-order formulae plus one-loop radiative corrections⁴, often improved by important renormalization group resummations, provide a sufficiently accurate approximation to match the precision of realistic experiments and to allow quite significant tests of the theory. For LEP physics, a self-imposing set of input parameters is given by $\alpha_s, \alpha, G_F, m_Z, m_f$ and m_H . Clearly the Fermi coupling $G_F = 1.166389(22) \times 10^{-5} \text{ GeV}^{-2}$ is conceptually more complicated than $\alpha_{weak} = \frac{g^2}{4\pi}$ (which would more naturally accompany $\alpha = 1/137.036$ and α_s) or $\sin^2 \theta_W$ or m_W , but is preferred for practical reasons because it

is known with all the desirable accuracy. Similarly, m_Z has now been measured (at LEP with remarkable precision). This preliminary task of LEP in view of precision tests of the Standard Model has already been accomplished to a nearly final degree of accuracy.

The LEP results on m_Z , as summarized in the talk by E. Fernandez⁵, are reported in Table 1. The resulting relative precision is impressive: $\delta m_Z/m_Z = 3.4 \times 10^{-4}$.

Experiment	m_Z (GeV)
ALEPH	91.186 ± 0.013
DELPHI	91.188 ± 0.013
L3	91.161 ± 0.013
OPAL	91.174 ± 0.011
AVERAGE	$91.177 \pm 0.006 \text{ (Stat.)} \pm 0.030 \text{ (LEP)}$ $\simeq 91.177 \pm 0.031$

TABLE 1

Among the quark and lepton masses, m_f , the main unknown is the top quark mass. Our ignorance of m_t is at present a serious limitation for precise tests of the electroweak theory because the radiative corrections are relatively large for large m_t and depend quadratically on m_t ^{3,4}. This fact can be used to put stringent constraints on m_t from the existing electroweak measurements, in particular an upper bound on m_t , to be discussed in detail later. As for lower bounds on m_t the best re-

sults arise from the failure to observe the t quark at e^+e^- and hadron colliders. LEP and SLC lead⁶ to a model-independent bound $m_t \gtrsim 45$ GeV. From CDF one learns⁷ that $m_t \gtrsim 89$ GeV, provided that the t quark semi-leptonic branching ratio is as predicted by the Standard Model.

The Higgs mass m_H is largely unknown. One of the most impressive performances of LEP up to now has been the dwarfing⁶ of all previous lower bounds on m_H . For the mass of the minimal Standard Model Higgs boson, ALEPH was able to establish the lower limit $m_H \gtrsim 41.6$ GeV. Less stringent but comparable limits were also obtained by the other LEP experiments. For the two-doublet Higgs sector of the minimal supersymmetric extension of the Standard Model^{8,9}, the corresponding limit is: $m_H \gtrsim 33$ GeV. The upper limit on m_H is mainly from theoretical arguments of consistency and is not equally clear. It is well known that for $m_H \gtrsim 0.8$ -1 TeV the Standard Model becomes affected by serious problems⁹ (e.g., Landau singularities moving down to energies of order 1 TeV) and the perturbative framework is no more reliable (weak interactions become strong). For this reason, most computations of radiative corrections are given for $m_H < 1$ TeV. The sensitivity of the radiative corrections to variations of m_H in the range $40 \text{ GeV} < m_H < 1 \text{ TeV}$ is not large. In a sense, this level of accuracy fixes the goal for precision tests of the Standard Model because the clarification of the symmetry breaking sector of the theory is the main target of present-day experiments.

Finally, for electroweak calculations involving hadrons, the value of the QCD coupling α_s must also be specified. The best value of α_s at the Z mass, obtained from experiments at energies lower than m_Z , is given by¹⁰ $\alpha_s(m_Z) = 0.11 \pm 0.01$. The QCD corrections to processes involving quarks are typically of order $\frac{\alpha_s}{\pi}$. As a consequence the stated error on α_s leads to a few

per mille relative uncertainty on the corresponding predictions.

1.2. Precision Tests of the Electroweak Theory

From the above discussion it is clear that the set of input parameters can be separated into two parts. On the one hand, $\alpha, G_F, m_Z, m_{f_{light}}$ are well known and the ambiguities associated with these quantities on the radiative corrections are quite small. We can add α_s to this class, in that, if it is true that the experimental error on α_s is relatively large, it only enters as a small correction to electroweak processes involving hadrons and is practically irrelevant for purely leptonic processes. On the other hand m_t and m_H are largely unknown. Thus, for each relevant observable, one can only express the prediction of the Standard Model as a function of m_t and m_H , obtained by using the best available calculations of radiative corrections, with $\alpha, \alpha_s, G_F, m_Z$ and $m_{f_{light}}$ fixed at their experimental values. By comparing this prediction with experiment one can check their mutual consistency and derive constraints on m_t and m_H .

Actually the sensitivity on m_H is so small that for all the measured quantities the ambiguity due to varying m_H in the range $40 \text{ GeV} < m_H < 1 \text{ TeV}$ is far below the present experimental error, so that for practical purposes, at the present stage of accuracy, the relevant predictions can be expressed as functions of m_t only (see Figs. 1.-5.). Note that from this point of view $\sin^2 \theta_W$ is not a primary quantity. It is not part of the set of input parameters. It is a derived quantity that one could even decide not to introduce at all. I stress this point in order to make clear that all disputes over which is the better definition of $\sin^2 \theta_W$ beyond the tree level are completely secondary. Not only it is always true that physical results are independent of definitions. Differences in physical results obtained from a different definition of input parameters (scheme dependence) can at most occur by terms of higher order, due to the

FIGURE 1

The value of $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$, computed for m_Z given by the LEP average in Table 1 and $m_H = 40-1000$ GeV (the central band is determined by δm_H , while the two narrow external bounds arise from adding δm_Z linearly), compared with s_W^2 measured by CDF and UA2 (the horizontal band) and with the data on R_N (the solid band refers to $m_H = 100$ GeV, while the dashed lines define the extended range according to δm_H). The combined value of s_W^2 , which follows from CDF/UA2 and R_N is also shown, together with the lower bounds on m_t from LEP (model independent) and CDF. (I am indebted with G.L. Fogli for providing the R_N curves.)

FIGURE 2

The prediction of the Standard Model for the total Z width Γ_T (obtained for $m_Z = 91.177 \pm 0.031$ GeV, $m_H = 40-1000$ GeV, $\alpha_s = 0.12^{+0.01}_{-0.02}$) as a function of m_t is compared with the LEP results. The central band is from δm_H . The two narrow intermediate bands are from δm_Z . The external bands are from $\delta \alpha_s$. All uncertainties are added linearly.

FIGURE 3

The hadronic width Γ_h (see Fig. 2 for a detailed explanation).

FIGURE 4

The leptonic width Γ_e (see Fig. 2 for a detailed explanation. Clearly Γ_e is independent of α_s).

FIGURE 5

The effective $\sin^2 \theta_W$ for on-shell Z decays $\bar{s}_W^2$ predicted from $m_Z = 91.177 \pm 0.031$ GeV, $m_H = 40$ -1000 GeV⁴¹, is compared with the experimental value obtained from Γ_e .

truncation of the perturbative series at a given order. But for $\sin^2 \theta_W$ its precise definition is only necessary to compute it from the input parameters, but cannot matter for the prediction of observables because, with the choice specified above, $\sin^2 \theta_W$ is not taken as an input parameter of the theory.

However, it certainly remains true that $\sin^2 \theta_W$ is an important observable of the electroweak theory and a useful reference quantity. The results of different experiments are often compared in terms of the values and the accuracies for $\sin^2 \theta_W$ that they correspond to. More important than that, with appropriate definitions of $\sin^2 \theta_W$, one can write simple improved Born approximations that include the main contributions of radiative corrections (e.g., large logarithms and terms of order $G_F m_t^2$). While for precision tests the use of as complete as possible radiative corrections is mandatory, these approximate formulae are very useful for our understanding of the pattern of radiative corrections and for everyday-life estimates of rates and experimental sensitivities.

One common definition¹¹ of $\sin^2 \theta_W$ is

$$\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2} \equiv s_W^2 \quad (1)$$

to all orders in the electroweak couplings. Clearly in this case the observables s_W^2 and m_W are directly equivalent given that m_Z is among the input parameters. In the Standard Model, s_W^2 can be computed from the input parameters by the relation

$$s_W^2 c_W^2 \equiv \left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W^2}{m_Z^2} = \frac{\pi \alpha}{\sqrt{2} G_F} \frac{1}{m_Z^2} \frac{1}{1 - \Delta r} \quad (2)$$

where $c_W^2 = 1 - s_W^2$ and $\Delta r \equiv \Delta r(\alpha, \alpha_s, G_F, m_Z, m_f, m_H)$ is the effect of radiative corrections. The quantity Δr as a function of the input parameters has been studied in great detail¹². The result for $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$, obtained starting from the average LEP value for m_Z (see Table 1), as a function of m_t , is plotted in Fig. 1.

where the uncertainties for $40 \text{ GeV} < m_H < 1 \text{ TeV}$ and $\delta m_Z = \pm 31 \text{ MeV}$ are also visible. We see that m_t is the main unknown in the calculation of $\frac{m_W}{m_Z}$ from m_Z , followed in importance by the ambiguity from varying the Higgs mass in the above range, while the remaining uncertainty from the experimental error on m_Z is very small.

When the available direct experimental information on $\frac{m_W}{m_Z}$ is added, the sensitivity of s_W^2 to m_t provides the best constraint that we have on m_t . $\frac{m_W}{m_Z}$ is directly measured at hadron colliders and can also be obtained (assuming the validity of the Standard Model) from the ratio $R_\nu = \sigma^{NC}/\sigma^{CC}$ of neutral current (NC) to charged current (CC) cross-sections in neutrino-nucleus deep inelastic scattering.

The measured values of $\frac{m_W}{m_Z}$ at hadron colliders have been discussed by Froidevaux in his talk¹³. From CDF and UA2 we have the results reported in Table 2.

Experiment	$\frac{m_W}{m_Z}$	$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$
CDF	0.8768 ± 0.0046	0.231 ± 0.008
UA2	0.8831 ± 0.0055	0.220 ± 0.010
AVERAGE	0.8794 ± 0.0035	0.2265 ± 0.006

TABLE 2

By combining $\frac{m_W}{m_Z}$ with the LEP value for m_Z one obtains $m_W = 80.19 \pm 0.32$. The corresponding average value of s_W^2 is also shown in Fig. 1 as horizontal band, obviously independent of m_t , in the $s_W^2 - m_t$ plane.

As is well known, the value of s_W^2 extracted from R_ν is also nearly independent of m_t in the interesting range of values for the top mass. This fact arises in the Standard Model from a largely accidental cancellation¹⁴, specific to this process, between two different sources of m_t dependence, as discussed in the following.

In general, at tree level, the four-fermion interaction from Z exchange is given by

$$M_{if} = \frac{\sqrt{2}G_F m_Z^2}{D(s)} \rho_{tree} (J_3^i - 2 \sin^2 \theta_W J_{em}^i) \cdot (J_3^f - 2 \sin^2 \theta_W J_{em}^f) \quad (3)$$

Where $D(s)$ is the Z propagator and $J_3^{i,f}, J_{em}^{i,f}$ are the weak isospin and electromagnetic currents for the fermion i or f . Excluding pure QED corrections, electroweak radiative corrections⁴ modify M_{if} according to

$$M_{if} = \frac{\sqrt{2}G_F m_Z^2}{D(s)} \rho_{i,f} (J_3^i - 2k_i \sin^2 \theta_W J_{em}^i) \cdot (J_3^f - 2k_f \sin^2 \theta_W J_{em}^f) + \dots \quad (4)$$

where $\rho_{i,f} = \rho_{tree}(1 + \delta\rho_{i,f})$, $k_a = 1 + \delta k_a$ ($a = i, f$) are different for different fermions and depend on the scheme adopted (for example δk_i depend on the definition of $\sin^2 \theta_W$). The ellipses indicate possible additional non-factorizable terms (for example from box diagrams). Let us call “large” radiative corrections those terms containing large logarithms, i.e., $\frac{\alpha}{\pi} \ln \frac{m_Z^2}{m_{light}^2}$ or quadratic dependences on m_t , i.e., $\sim G_F m_t^2$. For large enough m_t , the bulk of the contribution of electroweak radiative corrections arises from these terms^{3,4}. The “large” contributions to $\delta\rho_{i,f}$ and δk_f in Eqs. (4) are universal, i.e., they are the same at fixed q^2 for all i and f (except for b quarks). If for $\sin^2 \theta_W$ one adopts the definition $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$ one obtains^{2,4}:

$$1 - \Delta r = (1 - \Delta\alpha)(1 + \frac{c_W^2}{s_W^2} \delta\rho + \text{“small”}) \quad (5)$$

$$\rho \cong 1 + \delta\rho = 1 + \frac{3G_F m_t^2}{8\pi^2 \sqrt{2}} + \text{“small”} \quad (6)$$

$$k \cong 1 + \delta k = 1 + \frac{c_W^2}{s_W^2} \delta\rho + \text{“small”} \quad (7)$$

(for b quarks there are additional large terms) where Δr is defined in Eq. (2) and $\Delta\alpha$ arises from the running of the QED coupling:

$$\alpha(m_Z) = \frac{\alpha}{1 - \Delta\alpha} \quad (8)$$

$\Delta\alpha$ is dominated by large logs and its value is given by⁴

$$\Delta\alpha \simeq 0.0601 \pm 0.0009 \quad (9)$$

(or $\alpha_s^{-1}(m_Z) \simeq 128.8 \pm 0.1$). Note that both δk and Δr contain the large term $\delta\rho$ enhanced by the factor $\frac{c_W^2}{s_W^2}$. Logarithmic scale violations of order $\frac{\alpha}{\pi} \ln q^2/m_Z^2$ are included in the "small" terms (which is only appropriate for $q^2 \gg m_{light}^2$).

The ratio $R_\nu = \frac{\sigma^{NC}}{\sigma^{CC}}$ for $\nu - N$ scattering is given in terms of s_W^2 by:

$$R_\nu = \rho_{\nu N}^2 \left(\frac{1}{2} - k_{\nu N} s_W^2 + \frac{5}{9} (k_{\nu N} s_W^2)^2 (1+r) \right) + \dots \quad (10)$$

where $r = (\sigma^{\bar{\nu}}/\sigma^\nu)_{CC}$ is also measured. The tree approximation (with $\rho_{tree} = 1$) is recovered for $\rho_{\nu N} = k_{\nu N} = 1$. Some large logarithms from the radiative corrections to σ^{CC} are also included in $\rho_{\nu N}$. But for the sake of this argument we are only considering the $G_F m_t^2$ terms. For fixed $R_\nu \approx$ (the experimental value) and $s_W^2 \sim 0.23$ there is a strong cancellation in the Standard Model between the m_t dependence of $\rho_{\nu N} \simeq 1 + \delta\rho$ and of $k_{\nu N} \simeq 1 + \frac{c_W^2}{s_W^2} \delta\rho$, so that as a result $\delta s_W^2 \simeq 0.2\delta\rho$, where $\delta\rho$ is given in Eq. (6). For realistic values of m_t the resulting contribution of the quadratic m_t terms is no more dominant.

The most precise experimental results on R_ν were obtained by the CHARM¹⁵ and CDHS¹⁶ collaborations at CERN. The original results on $\sin^2\theta_W$ were given for fixed m_t and m_H . CHARM obtained $s_W^2 = 0.236 \pm 0.005$ (exp) ± 0.005 (th) for $m_t = 45$ GeV and $m_H = 100$ GeV, while the CDHS result was $s_W^2 = 0.2275 \pm 0.005$ (exp) ± 0.005 (th) for $m_t = 60$ GeV and m_H

$= 100$ GeV. The theoretical error arises from hadronic uncertainties and the effect of the charm threshold. An average at $m_t = 60$ GeV and $m_H = 100$ GeV gives $s_W^2 = 0.232 \pm 0.006$ (where the error 6×10^{-3} is obtained as $6 \times 10^{-3} = \sqrt{(\frac{5}{\sqrt{2}})^2 + 5^2} \times 10^{-3}$). The corresponding combined result at different values of m_t and m_H can also be obtained from the known form of the radiative corrections. The result is shown¹⁷ in Fig. 1.

There are many more less precise experimental results on s_W^2 from neutral current data most of them being well known^{3,18,19}. The new CHARM II result²⁰ on $(\bar{\nu}_\mu e)$ scattering will be discussed in Section 1.4. These additional data are all consistent among them and with the data in Fig. 1. But the resulting values of m_t and s_W^2 are essentially determined by the data in Fig. 1. From those data I obtain the results:

$$m_t = 140 \pm 45 \text{ GeV} \quad (11)$$

$$s_W^2 = 0.228 \pm 0.005 \quad (12)$$

These values are in agreement with other good analyses^{19,21-23} of the data on the electroweak theory. The quoted errors in Eqs. (11), (12) include **all** mentioned experimental and theoretical errors (on which I tend to be more conservative than others) and the effect of varying m_H in the whole range $40 \text{ GeV} < m_H < 1 \text{ TeV}$.

Given m_Z and s_W^2 from Eq. (12) (which also includes the information from R_ν) one immediately derives the corresponding value of m_W :

$$m_W = 80.1 \pm 0.3 \text{ GeV} \quad (13)$$

We shall see later that the LEP measurement of the Z partial widths adds little to the limits on m_t (so far at least). But LEP gives an important element for determining m_t by fixing m_Z . From Eq. (11) one obtains $m_t \lesssim 200$ GeV (90 % c.l.). The upper limit on m_t is

only slowly moving with time. A few years ago when $m_{W'}$ was better known than m_Z the data on $\nu - N$ combined with $m_{W'}$ favoured relatively large values of $s_{W'}^2$. Now that m_Z is precisely measured the upper limit on m_t has not improved by much because the recent data on $\frac{m_{W'}}{m_Z}$ from hadron colliders¹³ favour smaller values of $s_{W'}^2$.

In the minimal standard model, Δr and ρ are computable (given m_t and m_H). More in general by using Eq. (2) as a general definition of Δr one can obtain Δr from the data on m_Z and $\frac{m_{W'}}{m_Z}$. From Δr one can then derive a value for $\delta\rho$ from Eq. (5). By using the data on $\frac{m_{W'}}{m_Z}$ from both CDF/UA2 (Table 2) and $\nu - N$ scattering, one finds

$$\Delta r = 0.050 \pm 0.015 \quad (14)$$

From the approximate relation (Eq. (5))

$$\Delta r = \Delta\alpha - \frac{c_{W'}^2}{s_{W'}^2} \delta\rho \quad (15)$$

the previous result corresponds to

$$\delta\rho = 0.0030 \pm 0.0045 \quad (16)$$

Note that the derivation of $\delta\rho$ from an approximate relation (obtained by neglecting "small" terms) is adequate because a universal $\delta\rho$ (i.e., process independent) is only appropriate when "small" terms are neglected. It is perhaps safer to take $\frac{m_{W'}}{m_Z}$ from only CDF/UA2. In fact the indirect extraction of $\frac{m_{W'}}{m_Z}$ from $\nu - N$ data could be modified by some new physics, for example a new heavy Z' contribution. This more model-independent derivation of Δr leads to the results:

$$\Delta r = 0.045 \pm 0.018 \quad (17)$$

$$\Delta\rho = 0.0044 \pm 0.0056 \quad (18)$$

Note that the corresponding limit on $\delta\rho$:

$$\delta\rho \lesssim 0.014 \quad (95\%c.l.) \quad (19)$$

is a powerful constraint on all forms of non-standard physics which keep the relation (15) between Δr and $\delta\rho$:

$$\begin{aligned} \delta\rho = & \delta\rho_{\text{standard}} + \delta\rho_{\text{heavy loops}} + \delta\rho_{\text{non doublet Higgs}} \\ & + \delta\rho_{Z'} + \dots \end{aligned} \quad (20)$$

In particular one can address the question: how solid is the limit $m_t \lesssim 200$ GeV? I think it is quite reliable. It is true that one assumes no large cancellations of the m_t term in $\delta\rho$ with some other new physics contribution. But it is also true that $\delta\rho_{\text{heavy loops}}$ tends to be positive in all quantitative enough models, for example in the minimal supersymmetric extension of the Standard Model²⁴ (even if we only include the effect of the two Higgs doublets and no contribution from s -particles). Similarly $\delta\rho_{Z'} > 0$ in models with an extra $U(1)^{25-27}$ (see Section 1.5.). $\delta\rho$ from heavy gauge bosons can only be negative if there are extra charged W' with sufficiently low mass²⁸ (e.g., $m_{W'} < m_{Z'}$). But at low masses W' are more unlikely than Z' . For example, $m_{W'} \gtrsim 2$ TeV in left-right models²⁹ (with equal or complex conjugate CKM mixing matrix for left- and right-handed quarks). $\delta\rho_{\text{non-doublet Higgs}}$ could in fact be negative already at tree level. Note that we have always assumed $\rho_{\text{tree}} = 1$, so that possible deviations from this relation are included in $\delta\rho$. The general form of ρ_{tree} is given by³⁰:

$$\rho_{\text{tree}} = \frac{\sum_i v_i^2 [I_i(I_i + 1) - I_{3i}^2]}{\sum_i v_i^2 2I_{3i}^2} \quad (21)$$

where v_i , I_i and I_{3i} are the vacuum expectation value, the total weak isospin and its third component for the

Higgs multiplet i . For a doublet plus an additional non-doublet multiplet X one obtains:

$$\rho_{tree} = \frac{1 + \frac{v_x^2}{v_{1/2}^2} 2[I_x(I_x + 1) - I_{3x}^2]}{1 + \frac{v_x^2}{v_{1/2}^2} 4 I_{3x}^2} \quad (22)$$

We see that in order to obtain $\rho_{tree} < 1$ one needs I_{3x} to be large. This in turn implies charged Higgses with charge two or more. For example, for triplet Higgses, I_{3x} must be ± 1 . But recall that this is the I_3 of the neutral Higgs. By displacing I_{3x} by two units one then obtains the weak isospin of a doubly charged Higgs. In conclusion, $\delta\rho$ non-doublet Higgs can in principle be negative, but this possibility is actually associated with a somewhat baroque Higgs sector not really plausible. More in general the possibility of conspicuously evading the m_t upper limit by a cancellation of terms in $\delta\rho$, while in principle not excluded, is in practice difficult to implement.

Also note that in deriving the limit on m_t one always assumes $m_H < 1$ TeV. Formally if m_H is increased the upper limit on m_t is also increased. For $m_H \sim$ several TeV the perturbative expansion for $\delta\rho$ breaks down³¹ and in principle the radiative corrections become uncalculable. However, it is difficult to imagine that the m_t limit can be sizeably modified by this effect without at the same time observing other conspicuous deviations from the perturbative predictions.

1.3. The Z Line Shape

We now consider the implications for the standard electroweak theory of the LEP results on the Z partial widths. The relevant results are collected in Table 3 presented by E. Fernandez.

In Figs. 2-4, we compare the data on the Z widths with the predictions of the Standard Model, obtained by the programme ZSHAPE³² which includes a state of

	ALEPH	DELPHI	L3	OPAL	Average
Γ_Z MeV	2506. $\pm$ 26	2476. $\pm$ 28	2492. $\pm$ 25	2505. $\pm$ 20	2497. $\pm$ 15
Γ_ℓ MeV	84.2 $\pm$ 1.1	83.7 $\pm$ 1.5	84.0 $\pm$ 1.2	83.6 $\pm$ 1.0	83.9 $\pm$ 0.7
Γ_{had} MeV	1764. $\pm$ 23.	1756. $\pm$ 30.	1748. $\pm$ 35.	1778. $\pm$ 26.	1764. $\pm$ 16.
Γ_{inv} MeV	489. $\pm$ 22.	469. $\pm$ 29.	494. $\pm$ 32.	476. $\pm$ 25.	482. $\pm$ 16.
$R = \frac{\Gamma_{had}}{\Gamma_\ell}$	20.95 $\pm$ 0.31	21.00 $\pm$ 0.48	21.02 $\pm$ 0.62	21.26 $\pm$ 0.32	21.08 $\pm$ 0.20
σ_{had}^0 (nb)	41.78 $\pm$ 0.63	42.38 $\pm$ 1.02	41.38 $\pm$ 0.71	41.88 $\pm$ 0.74	41.78 $\pm$ 0.53
Γ_e MeV	84.9 $\pm$ 1.4	82.0 $\pm$ 1.9	84.3 $\pm$ 1.6	82.7 $\pm$ 1.3	83.6 $\pm$ 0.9
Γ_μ MeV	80.7 $\pm$ 2.2	87.2 $\pm$ 3.5	82.3 $\pm$ 2.9	85.9 $\pm$ 2.0	83.8 $\pm$ 1.2
Γ_{tau} MeV	81.8 $\pm$ 2.2	86.0 $\pm$ 4.1	83.5 $\pm$ 3.7	83.9 $\pm$ 2.3	83.3 $\pm$ 1.4

TABLE 3

Results from LEP. The average also includes systematic errors as given by E. Fernandez. The average value of Γ_{inv} corresponds to $N_\nu = 2.89 \pm 0.11$ which is the best current determination of the number of light neutrinos from LEP.

the art set of electroweak radiative corrections. Totally equivalent predictions are obtained by other complete calculations of the line shape^{33,34}. The predicted widths are plotted as functions of m_t . In all figures the uncertainties due to the errors on m_Z ($m_Z = 91.177 \pm 0.031$ GeV), on m_H ($m_H = 40-1000$ GeV) and α_s ($\alpha_s = 0.12^{+0.01}_{-0.02}$) are linearly summed. At the centre, the Higgs uncertainty for $m_Z = 91.177$ GeV and $\alpha_s = 0.12$ is shown. Then the effect of varying m_Z by $\pm 1\sigma$ is linearly added to enlarge the previous band and finally the same is done for α_s . Note that the range adopted here for α_s is different than the best value from all low energy experiments ($\alpha_s = 0.11 \pm 0.01$). The choice of α_s in Figs. 2-4 is more conservative and more or less corresponds¹⁰ to only taking PEP, PETRA, TRISTAN experiments into account (thereby comparing e^+e^- at LEP with e^+e^- at lower energy). Figures 2-4 contain all the information on the relation of the experimental values for the widths with the Standard Model predictions. Each width is predicted as a function of m_t given the input parameters $\alpha, G_F, m_Z, m_H, m_{f,igh}$ with their present error, by using a full-fledged set of radiative corrections. Two main conclusions are immediately derived. First, the observed widths are in perfect agreement with the Standard Model for m_t in the range indicated by previous experiments. Second, the additional information on m_t provided by the widths, does not very much improve the upper limit on m_t (the difference being of a few GeV)³⁵. The precision and sensitivity would be adequate but the central values are somewhat displaced toward the large m_t side.

For processes at the Z mass one can define an effective value of $\sin^2 \theta_W$ that makes improved Born approximations^{36,37} particularly simple. If $\sin^2 \theta_W$ is defined as $\sin^2 \theta_W \equiv s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$, then an approximation for the Z widths that takes all "large" terms into account, can be written down in the following form ($f \neq b$):

$$\Gamma(Z \rightarrow f\bar{f}) = N_c \frac{G_F m_Z^3 (1 + \delta\rho)}{24\pi\sqrt{2}} [1 + (1 - 4|Q_f|(1 + \delta k)s_W^2)^2] \quad (23)$$

where

$$N_c = 1 \quad \text{lepton} \\ = 3 \left[1 + \frac{\alpha_s(m_Z)}{\pi} + \dots \right] \quad \text{quarks} \quad (24)$$

and $\delta\rho$ and δk , given by Eqs. (6), (7), contain all "large" terms. We mentioned that the combination $(1 + \delta k)s_W^2$ will always appear in all neutral current processes when only "large" terms are included (and logarithmic scale violations are neglected). One is then naturally led to redefine $\sin^2 \theta_W$ in the following way⁴:

$$\bar{s}_W^2 \cong (1 + \delta k)s_W^2 \\ \cong s_W^2 + c_W^2 \delta\rho + \text{"small"} \quad (25)$$

Note that this relation is equivalent to

$$\bar{s}_W^2 \cong 1 - \frac{m_W^2}{\rho m_Z^2} \quad (26)$$

to first order in $\delta\rho$ with $\rho = 1 + \delta\rho + \text{"small"}$. Similarly we can go back to Eq. (2) and find how Δr is modified in the present case. We easily obtain the relations:

$$\bar{s}^2 \bar{c}^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F} \frac{1}{m_Z^2 \rho} + \text{"small"} \quad (27)$$

or

$$\bar{s}^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F} \frac{1}{m_W^2} + \text{"small"} \quad (28)$$

To within "small" corrections a whole class of $\sin^2 \theta_W$ coincides with $\bar{s}_W^2$: $\sin^2 \theta_{\overline{MS}}(m_Z^2)$ ³⁸ (computations of $\sin^2 \theta_W$ from grand-unified theories usually end up with a prediction for this quantity^{18,19}), $\sin^2 \theta^*(m_Z^2)$ ³⁷, $\sin^2 \theta_{on-shell}^4$ and so on. They are all equivalent for the present purposes in the sense that they lead to the same improved Born approximations valid when "small" terms are neglected.

In terms of $\bar{s}_{W'}^2$, the improved Born approximation for the width can be written in the form

$$\Gamma(Z \rightarrow f\bar{f}) = N_c \frac{G_F m_Z^3 \rho}{24\pi\sqrt{2}} (1 + (1 - 4|Q_f|\bar{s}_{W'}^2)^2) \quad (29)$$

For $\Gamma(Z \rightarrow b\bar{b})$ replace^{4,39,40} ρ by $\rho_b = \rho(1 - \frac{4}{3}\delta\rho)$ and $\bar{s}_{W'}^2$ by $\bar{s}_{W'}^2(1 + \frac{2}{3}\delta\rho)$. Note that by using Eq. (27) one can cast the previous formula into the form

$$\Gamma(Z \rightarrow f\bar{f}) = N_c \frac{\alpha_s(m_Z) m_Z}{48\bar{s}_{W'}^2 c_{W'}^2} [1 + (1 - 4|Q_f|\bar{s}_{W'}^2)^2] \quad (30)$$

This relation, valid up to “small” terms in the Standard Model, is less general than the previous one, where the effects of ρ and $\bar{s}_{W'}^2$ are kept separate. (In general, beyond the Standard Model, Δr , $\delta\rho$ and δk should be taken as independent parameters.) Equation (30) is interesting because it shows that a value of $\bar{s}_{W'}^2$ can be directly derived from the measured widths independent of m_t . This does not of course mean that the predictions for the widths do not depend on m_t . The dependence on m_t is hidden in $\bar{s}_{W'}^2$ when computed from the input parameters. In fact, it is practical for LEP experiments to define $\bar{s}_{W'}^2$ from a given simple Z process (for example $\Gamma(Z \rightarrow \ell^+\ell^-)$ with $\ell = e, \mu, \tau$) as given by Eq. (30), taken as exact (with $\alpha_s(m_Z)^{-1} = 128.8$). From the LEP average $\Gamma_{\ell\ell} = 83.9 \pm 0.7$ MeV one obtains

$$\bar{s}_{W'}^2 = 0.230 \pm 0.0025 \quad (31)$$

This value is to be compared (apart from “small” terms) with the result

$$\bar{s}_{W'}^2 = 0.232 \pm 0.002 \quad (32)$$

which is obtained⁴¹ (Fig. 5) from the input parameters $\alpha, G_F, m_{f_{light}}, m_Z = 91.177 \pm 0.031$ GeV, $m_H = 40$ -1000 GeV and $m_t = 140 \pm 45$ GeV (see Eq. (11)). Equation

(32) is the analogous of Eq. (12) which refers to s_W^2 . Both describe the conclusions of taking the LEP value for m_Z (and the bound on m_H) together with the whole of non-LEP results on neutral current processes and on $\frac{m_W}{m_Z}$. Note that the error on $\bar{s}_{W'}^2$ in Eq. (32) (± 0.002) is much smaller than that on s_W^2 which appears in Eq. (12) (± 0.005). This difference reflects the markedly milder dependence of $\bar{s}_{W'}^2$ on m_t with respect to s_W^2 .

1.4. Neutrino-Electron Scattering

The CHARM II collaboration has presented²⁰ new results on $\sin^2 \theta_{W'}$ measured from the ratio $R_\nu = \frac{\sigma_{\nu e}}{\sigma_{\nu e}}$ in $\nu_\mu \cdot e$ scattering. The resulting value of $\sin^2 \theta_{W'}$ is

$$\sin^2 \theta_{W'} = 0.240 \pm 0.012 \quad \text{CHARM II} \quad (33)$$

The corresponding accuracy is far better than that of previous experiments²⁰:

$$\sin^2 \theta_{W'} = 0.195 \pm 0.022 \quad \text{BNL} \quad (34)$$

$$\sin^2 \theta_{W'} = 0.211 \pm 0.037 \quad \text{CHARM I} \quad (35)$$

The present average value is thus:

$$\sin^2 \theta_{W'} = 0.228 \pm 0.010 \quad \text{AVERAGE} \quad (36)$$

What $\sin^2 \theta_{W'}$ is this one? The reported values are obtained from the Born expression for R_ν without non-QED corrections. As R_ν is in this case given by a pure Z exchange process, it is clear that the measured value of $\sin^2 \theta_{W'}$ refers to $\bar{s}_{W'}^2$ measured at q^2 small. The Z exchange diagram for $\nu_\mu - e$ scattering is just the crossed one of the LEP process $e^+e^- \rightarrow \nu_\mu \bar{\nu}_\mu$ via Z exchange. But the LEP widths measure the effective $\sin^2 \theta_{W'}$ entering in the Z couplings at $q^2 = m_Z^2$. Thus the effective $\sin^2 \theta_{W'}$ of LEP and of $\nu_\mu e$ are at different scales. The running of the effective $\sin^2 \theta_{W'}$ between q^2 small and

$q^2 \simeq m_Z^2$ can be accurately computed⁴². The leading logarithmic approximation^{43,44} is not good in this channel. In this approximation the effect of the change of scale in $\sin^2 \theta_W$ arises from a combination of the running of α and α_W plus the induced effects of charged $\nu_\mu - \mu$ currents via the relevant penguin diagram. While individual terms are large, there are strong cancellations among the different contributions. The resulting scale dependence⁴² for $q^2 \sim 0$ is small in comparison to the experimental errors in Eqs. (33)-(36)

$$\sin^2 \theta_W(q^2) - \sin^2 \theta_W(m_Z^2) \simeq +0.002 \quad (37)$$

In conclusion the result given in Eq. (36) for $\sin^2 \theta_W$ measured from R_ν in $\nu_\mu - e$ scattering is in good agreement with the values determined from LEP and from low energy neutral current processes.

The separate measurement of $\sigma^{\nu e}$ and $\sigma^{\bar{\nu} e}$ allows us to derive the values of both $g_V^2 + g_A^2$ and $g_V g_A$ (at the scale q^2) where g_V and g_A are defined in terms of the effective four-fermion couplings $\sim \bar{\nu} \gamma_\mu (1 - \gamma_5) \nu \bar{e} \gamma^\mu (g_V - g_A \gamma_5) e$ from Z exchange. CHARM II obtained the result $g_V = -0.020 \pm 0.024$ for the solution in the vicinity of $g_A \simeq -\frac{1}{2}$ (with their normalization, in the Standard Model $g_V = -\frac{1}{2} + 2 \sin^2 \theta_W$, $g_A = -\frac{1}{2}$). As g_V is predicted to be small in the Standard Model it could in principle be sensitive to, for example, Z' effects (to be discussed in the next section) which can be different at low energy and at LEP. At LEP by combining the widths and the asymmetry measurements one obtains $g_A = -0.501 \pm 0.0025$, $g_V = -0.045 \pm 0.006$ (the overall sign is determined from other experiments). Here g_A and g_V are defined including the $\rho^{\frac{1}{2}}$ factor. In fact, g_A is a good way to put limits on $\delta\rho$, in that $g_A \simeq -\frac{1}{2}\rho^{\frac{1}{2}}$, while $\frac{g_V}{g_A}$ measures $\hat{s}_W^2 = k s_W^2$.

1.5. Extended Gauge Models

The new LEP data impose important restrictions on extensions of the Standard Model with new heavy

Z' ²⁵⁻²⁹. We discuss here the simplest gauge extensions of the Standard Model, where only one extra $U(1)$ factor is added to the $SU(2) \otimes U(1)$ group of the standard electroweak theory. We follow the analysis of Refs. 27. The light and the heavy physical states Z_L and Z_H (Z_L is observed at LEP) are superpositions of the standard Z_S and of a new state $Z_N = Z'$

$$\begin{pmatrix} Z_L \\ Z_H \end{pmatrix} = \begin{pmatrix} \cos \xi_0 & \sin \xi_0 \\ -\sin \xi_0 & \cos \xi_0 \end{pmatrix} \begin{pmatrix} Z_S \\ Z_N \end{pmatrix} \quad (38)$$

At the tree level, even for doublet Higgses.

$$\cos^2 \theta_W = \frac{m_{W'}^2}{m_{Z_L}^2 \rho} \quad (39)$$

with $\rho = 1 + \delta\rho_M$, where $\delta\rho_M$ is due to the ξ mixing: the physical mass m_{Z_L} is pushed down with respect to m_{Z_S} . As a consequence, $\delta\rho_M \geq 0$. This important inequality holds in all extra $U(1)$ models. It could be violated if the $W^\pm$ were also mixed with some other heavy states^{28,29}. In particular, in a large class of models we have, for m_{Z_H} large:

$$tg\xi_0 = a \frac{m_{Z_L}^2}{m_{Z_H}^2} \quad (40)$$

with a being a constant, and one finds

$$\delta\rho_M = a^2 \frac{m_{Z_L}^2}{m_{Z_H}^2} \quad (41)$$

All effects of Z' at the Z_L peak arise because of mixing through $\delta\rho_M$ and ξ_0 . For example the partial widths, in the improved Born approximation become

$$\begin{aligned} \Gamma_f = N_c \frac{G_F m_{Z_L}^3 \rho}{6\pi\sqrt{2}} & [\cos^2 \xi_0 (v_f^2 + a_f^2) \\ & + 2 \cos \xi_0 \sin \xi_0 (v_f v_f' + a_f a_f') \\ & + \sin^2 \xi_0 (v_f'^2 + a_f'^2)] \end{aligned} \quad (42)$$

where $\rho = 1 + \delta\rho_M + \delta\rho_{top} + \dots$, $v_f = (T_{3f} - 2Q_f\bar{s}_{W'}^2)$, $a_f = -T_{3f}$, $\bar{s}_{W'}^2 = \bar{s}_W^2 - \frac{\bar{s}_W^2\epsilon_W^2}{\epsilon_W^2 - \bar{s}_W^2}\delta\rho_M$. Note that the effective $\sin^2\theta_{W'}$, $\bar{s}_{W'}^2$, differs from the standard value $\bar{s}_W^2$ because of $\delta\rho_M$. As we see there would be no effect on Γ_f if $\delta\rho_M$ and ξ_0 would vanish (given the relation Eq. (41), it is enough that $\xi_0 = 0$). For $\xi_0 = 0$, Z_N would decouple from Z_L and LEP 1 could not constrain its mass at all. However, Z_N could of course be produced and observed at hadron colliders. The effects of virtual Z_N exchange could be more apparent in the quantity g_V measured by CHARM II in $\nu_\mu - c$ scattering. Thus, there is an obvious complementarity between LEP experiments at the Z_L peak and other accelerator experiments.

As discussed in Section 1.2, a limit on $\delta\rho_M$ can be derived from the measured values of m_{Z_L} and of m_W/m_{Z_L} . Approximately, using Eq. (6), one finds from Eq. (29)

$$\delta\rho_M \lesssim 0.016 - 0.0025 \left[\frac{m_t}{90 \text{ GeV}} \right]^2 \quad (43)$$

so that the limit is stronger for larger m_t . In other words, because $\delta\rho_M \geq 0$, there is less space for a Z' if m_t is increased.

Bounds²⁷ from LEP data on $\delta\rho$ and ξ_0 , treated as independent quantities, valid for extra $U(1)$ models of the $E(6)$ type²⁵, are shown in Figs. 6a and 6b. The angle θ_2 describes²⁵⁻²⁷ the position of Z_N in $E(6)$ space. The allowed region in the $\xi_0 - \theta_2$ plane from neutral current experiments, taken from Ref. 18, is also shown for comparison. We see from Fig. 6 that LEP data have added much to the constraints on ξ_0 and $\delta\rho$.

1.6. Conclusion and Outlook

All the results of LEP presented at this Conference are in perfect agreement with the standard electroweak theory. So much that there is almost a sense of deception in the LEP community. One could in fact hope for a sensational discovery, e.g., the production of some new

FIGURE 6

(A) Bounds on ξ_0 obtained from LEP data in $E(6)$ -type extended gauge models (the allowed region is internal to the solid lines) where θ_2 is the angle describing the orientation in $E(6)$ space of the additional $U(1)$ generator. For comparison the region allowed by existing neutral current data is also shown (dashed lines). The overall allowed region is the intersection of the above two domains.

(B) Allowed region for $\Delta\rho = \Delta\rho_{top} + \Delta\rho_M + \dots$ from LEP data.

particle. Instead the limit on the Higgs has been set at $m_H > 41.6$ GeV, the number of light neutrinos has been fixed at $N_\nu \simeq 3$, no new charged particles have been observed and so on. All these limits are indeed quite impressive but are not as fulfilling as a real discovery. For precision tests of the electroweak theory we knew from the start that the widths cannot compete with asymmetries, which need a large integrated luminosity. The absolute error on $\bar{s}_W^2$ from the leptonic width is given by:

$$\delta \bar{s}_W^2 = \pm 0.27 \quad \bullet \quad \frac{\delta \Gamma_e}{\Gamma_e} \simeq \pm 0.0025 \quad (44)$$

for $\delta \Gamma_e / \Gamma_e \simeq 1\%$. The ultimate precision on $\frac{\delta \Gamma_e}{\Gamma_e}$ cannot be brought down by very much in the future. The precision on $\bar{s}_W^2$ expected from the asymmetries is reported⁴⁵ in Table 4.

No Polarization	$\int \mathcal{L} dt = 200 pb^{-1}$
Asymmetry	$\delta \bar{s}_W^2$
A_{FB}^μ	± 0.0017
A_{FB}^c	± 0.0015
A_{FB}^b	± 0.0009
A_{pol}^T	± 0.0014
Polarization $\langle P_L \rangle = 0.5$	$\int \mathcal{L} dt = 40 pb^{-1}$
A_{LR}	± 0.0004
A_{FB}^μ	± 0.0006

TABLE 4

Similarly m_W can be measured with $\delta m_W = \pm 100$ MeV at LEP 2 or at hadron colliders (given m_Z). $\delta m_W = \pm 100$ MeV corresponds to $\delta s_W^2 = \pm 0.002$ in terms of $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$. For $\delta \bar{s}_W^2$ one can use Eq. (28) to find $\delta \bar{s}_W^2 = \pm 0.0006$.

In spite of the fact that no striking discoveries have been found in the first few months of LEP there are all reasons to be satisfied. The big discoveries will presum-

ably occur in the next few years. LEP 1 plus LEP 2 have in fact very good chances to discover new physics. Precision tests of the Standard Model, the search for the Higgs and for signals of new physics remains a very promising and exciting programme for the future of LEP.

2. QCD

Extremely interesting results have also been obtained by LEP experiments on the QCD sector of the Standard Model. At LEP one can probe QCD at an extremely high value of the energy scale with large statistics, clean environment and very advanced detectors. The goal is to measure $\alpha_s(m_Z)$ and to quantitatively test the theory by comparing the renormalization group improved matrix elements of QCD with the experimental distributions of as many observables as possible. Measuring $\alpha_s(m_Z)$ is as important as measuring m_Z or $\sin^2 \theta_W$. It is well known that even a relatively loose determination of $\alpha_s(Q)$ at low Q leads to a very tight determination of $\alpha_s(Q)$ at large Q ¹⁰. As already mentioned from all low energy experiments on QCD, one derives¹⁰ a very sharp prediction of $\alpha_s(m_Z)$: $\alpha_s(m_Z) = 0.11 \pm 0.01$. Establishing at LEP that this prediction is experimentally true would be a very quantitative and accurate test of QCD.

The total hadronic width of the Z is conceptually equivalent to $R_{e^+e^-}$, the total hadronic e^+e^- annihilation cross-section which, together with the structure functions of deep inelastic scattering, have the cleanest theoretical standing. The QCD correction factor R_{QCD} :

$$\begin{aligned} \Gamma_h &= \Gamma_h^0 R_{QCD} \\ &= \Gamma_h^0 \left\{ 1 + \frac{\alpha_s(m_Z)}{\pi} + 1.411 \left(\frac{\alpha_s(m_Z)}{\pi} \right)^2 + \dots \right\} \end{aligned} \quad (45)$$

(where Γ_h^0 is independent of α_s) is essentially the same as for the virtual photon, because the difference between the vector coupling of the photon and the mixed

vector and axial couplings of the Z is numerically insignificant⁴⁶. The term of order $(\frac{\alpha_s(m_Z)}{\pi})^3$ was also computed⁴⁷ but we now have learnt that there was a bug in the calculation, as admitted by the authors themselves, so that we do not know its value anymore. This is fortunately not very crucial for LEP. In fact, R_{QCD} was measured at PEP, PETRA and TRISTAN. Thus its detailed α_s dependence is only useful to evaluate the small scale charge $Q \rightarrow m_Z$ and/or to obtain a precise value of $\alpha_s(m_Z)$ from R_{QCD} . From the measurements at lower energy one concludes that $R_{QCD}(m_Z) \simeq (70 \pm 7)$ MeV. The present experimental result for Γ_h , averaged over the LEP experiments is

$$(\Gamma_h)_{EXP} = 1764 \pm 16 \text{ MeV} \quad (46)$$

while the theoretical prediction (including R_{QCD}) is given by (Fig. 3):

$$(\Gamma_h)_{TH} = 1740 \pm 30 \text{ MeV} \quad (47)$$

We see that the QCD effect is important for bringing theory and experiment into agreement, but Γ_h is not precise enough to fix $\alpha_s(m_Z)$.

Much more significant tests of QCD arise from the analysis of hadronic final states, as was discussed in his talk by J. Drees⁴⁸. Indeed measurements of $\alpha_s(m_Z)$ with remarkable precision have already been obtained. These results are especially valuable for the unprecedented care on the evaluation of theoretical errors (for example those arising from hadronization, scale dependence and so on). This was also merit of the theorists' stimulus⁴⁶. First results confirm that LEP is a really sharp tool for QCD studies.

Determinations of $\alpha_s(m_Z)$ from jet multiplicities have been presented⁴⁸ by OPAL⁴⁹, DELPHI and L3, following earlier works⁵⁰ by MARK II, JADE, TASSO and AMY mainly prompted by Bethke⁵¹. A jet-finding

algorithm is defined, the same for hadrons (experiment) and partons (theory). For example, for each pair i, j , one defines

$$y_{ij} = \frac{M_{ij}^2}{E_{vis}^2} = \frac{2E_i E_j}{E_{vis}^2} (1 - \cos \theta_{ij}) \quad (48)$$

If $y_{ij} < y_{cut}$ then the two objects i and j are fused into a single "cluster" with $P_\mu = (P_i + P_j)_\mu$. The procedure is repeated until for all remaining "clusters" $y_{ij} > y_{cut}$. The jets are identified with the final "clusters". It is clear that many variants of the jet finding algorithm can be formulated. For example, all masses are neglected in Eq. (48) (not only for hadrons or for partons, but also for clusters). For a number of algorithms the corresponding theoretical calculations have also been performed. Thus one can follow different a priori equivalent procedures to obtain $\alpha_s(m_Z)$ and the related ambiguity can be studied (recombination scheme dependence). The hadronization effects are studied by Monte Carlo simulations and are apparently small for most of the recombination schemes. It is found that QCD event generators reproduce the data very well. In particular, when the recombination scheme is changed, the simulations correctly reproduce the different shapes of the curves for the jet fractions as functions of y_{cut} . The corresponding calculations at the parton level were performed by Kramer and Lampe⁵². The jet multiplicities are evidently of the form:

$$\begin{aligned} R_2 &= \frac{\sigma^2}{\sigma_{TOT}} = 1 + c_{21}\alpha_s(\mu) + c_{22}(\mu)\alpha_s^2(\mu) + \dots \\ R_3 &= \frac{\sigma_3}{\sigma_{TOT}} = c_{31}\alpha_s(\mu) + c_{32}(\mu)\alpha_s^2(\mu) + \dots \\ R_4 &= \frac{\sigma_4}{\sigma_{TOT}} = c_{42}\alpha_s^2(\mu) + \dots \end{aligned} \quad (49)$$

All the c_{ij} coefficients are obviously functions of y_{cut} while only the subleading ones depend on μ , the scale adopted for α_s . At a given order the μ dependence of c_{ij} is fixed by requiring that the result should be μ -

independent up to terms of higher order. All the coefficients c_{ij} shown here (and only those ones) are known at present. Non-leading terms (necessary for a meaningful determination of $\alpha_s(\mu)$) were only computed for R_2 and R_3 . A typical comparison of the data with the perturbative predictions is shown in Fig. 7. In this plot the same data enter many times for each bin of y_{cut} . Alternatively one can plot a differential distribution as in Fig. 8. In this case each event contributes to the bin of y_{cut} where it changes from two jets to three jets ($D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y}$). The data can be fitted in terms of μ and $\Lambda_{\overline{MS}}^{(5)}$. It is consistently found that small values of μ give a better fit at small y_{cut} . This is not very significant in my opinion. It is presumably only a reflection of the fact that the unknown non-leading corrections to R_4 are large and apparently positive. For a meaningful determination of $\alpha_s(\mu)$ one should actually take y_{cut} relatively large so that only R_2 and R_3 are important.

The results on $\alpha_s(m_Z)$ obtained by this method, quoted by Drees, are:

$$\begin{aligned} \alpha_s(m_Z) &= 0.116 \pm 0.016 & \text{OPAL} \\ \alpha_s(m_Z) &= 0.114 \pm 0.013 & \text{DELPHI} \\ \alpha_s(m_Z) &= 0.114 \pm 0.014 & \text{L3} \end{aligned} \quad (50)$$

- 0.008

Note that the error is predominantly of theoretical origin, and that the procedure of its evaluation is not identical for the three collaborations. In principle all errors from hadronization, μ -dependence and scheme dependence have been included. The results are in beautiful agreement with the value of $\alpha_s(m_Z)$ expected from QCD tests at lower energies. Comparison with jet multiplicities measured at PEP, PETRA and TRISTAN provide clear indications for the running of $\alpha_s(Q)$ at the predicted rate. But for this comparison one must beware of experiment-to-experiment systematics and rely on a

FIGURE 7

Two-, three-, four- and five-jet event rates observed by OPAL at $E_{cm} = 91$ GeV as a function of the jet resolution parameter y_{cut} , compared with two QCD fits.

FIGURE 8

The differential rates D_2 and D_3 (for D_2 each event is plotted at the value of y_0 where it changes from two- to three-jet and similarly for D_3) compared with a QCD fit.

Monte Carlo simulation on the energy independence of various effects, e.g., hadronization.

Other measurements of $\alpha_s(m_Z)$ are being performed. All preliminary results presented by Drees are consistent with the values in Eq. (50). Thus, it really seems that the rather precise prediction of $\alpha_s(m_Z)$ from low energy experiments is quite remarkably supported by the first results of LEP on QCD. Similarly, the predicted distributions of four-jet events as obtained from the QCD matrix element⁵³, are well reproduced by the data. There are direct indications of the presence of the three-gluon coupling. Actually DELPHI has fitted⁴⁸ the number of colours and found $N_c/c_F \simeq 2.05 \pm 0.40$ (in QCD $N_c = 3, c_F = 4/3, N_c/c_F = 2.25$). Finally, evidence⁴⁸ for coherence effects⁵⁴ is also found from the average multiplicity and the inclusive momentum distributions.

In conclusion, I think that it is fair to say that QCD is certainly also included when we say that the LEP results are in terrific agreement with the predictions of the Standard Model.

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REFERENCES

1. Z Physics at LEP I, eds. G. Altarelli, R. Kleiss and C. Verzegnassi, CERN Yellow Report 89-08 (1989).
2. S.L. Glashow, *Nucl.Phys.* **22** (1961), 579; S. Weinberg, *Phys.Rev.Lett.* **19** (1967), 1264; A. Salam, Proceedings of the 8th Nobel Symposium, Aspenäs garden, ed. N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.
3. G. Altarelli, Proceedings of the 1989 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, 1989, ed. M. Riordan (World Scientific), p. 286.
4. M. Consoli, W. Hollik and F. Jegerlehner, "Electroweak radiative corrections for Z physics", Ref. 1., Vol. 1, p. 7.
5. E. Fernandez, These Proceedings.
6. M. Oreglia, These Proceedings.
7. CDF: Presented at several Winter Conferences.
8. See, for example: H.P. Nilles, *Physics Reports* **C110** (1984), 1.
9. P.J. Franzini, P. Taxil et al., "Higgs search", in Ref. 1., Vol. 2, p. 59.
10. G. Altarelli, *Ann.Rev.Nucl.Part.Sci.* **39** (1989), 357. See also: G. Altarelli, Proceedings of the DPF Meeting of the APS, Houston, 1990, CERN Preprint TH. 5760 (1990).
11. A. Sirlin, *Phys.Rev.* **D22** (1980), 971; W.J. Marciano and A. Sirlin, *Phys.Rev.* **D22** (1980), 2659, *Phys.Rev.* **D29** (1984), 75, 945.
12. G. Burgers, F. Jegerlehner et al., " Δr or the relation between the electroweak couplings and the weak vector boson masses", Ref. 1., Vol. 1, p. 55.
13. D. Froidevaux, These Proceedings.
14. R.G. Stuart, *Z.Phys* **C34** (1987), 445; See also: D. Bardin and V. Khovansky, Contributed paper to this Conference, and A. Blondel, CERN Preprint EP/89-84 (1989).
15. J.V. Allaby et al., *Phys.Lett.* **B177** (1986), 446, *Z.Phys.* **C36** (1987), 611.
16. H. Abramowicz et al., *Phys.Rev.Lett.* **57** (1986), 298; A. Blondel et al., CERN Preprint EP/89-101 (1989).
17. G.L. Fogli, Private communication. I am grateful to G.L. Fogli for providing me with these curves.
18. U. Amaldi, A. Böhm et al., *Phys.Rev.* **D36** (1987), 1385; G. Costa, J. Ellis et al., *Nucl.Phys.* **B297** (1988), 244.
19. P. Langacker, in Review of Particle Properties, *Phys.Lett.* **239** (1990), 1.
20. P. Vilain, These Proceedings.
21. J. Ellis and G.L. Fogli, *Phys.Lett.* **B213** (1988), 526; **B231** (1988) 189; **B232** (1989) 139, CERN Preprint TH. 5817 (1990).
22. D. Haidt, DESY Preprint 89-073 (1989) (updated: private communication).
23. J.L. Rosner, EF1 90-18 (1990).
24. R. Barbieri et al., *Nucl.Phys.* **B341** (1990), 309; M. Drees and K. Hagiwara, CERN Preprint TH. 5649 (1990); B.W. Lynn, M. Peskin and R.G. Stuart, in Physics at LEP, eds. J. Ellis and R. Peccei, CERN Yellow Report 86-02 (1986), p. 90.

25. E. Witten, *Nucl.Phys.* **B258** (1985), 75;
M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi
and N. Seiberg, *Nucl.Phys.* **B259** (1985), 519;
S. Cecotti, J.P. Derendinger, S. Ferrara, L. Gi-
rardello and M. Roncadelli, *Phys.Lett.* **B156**
(1985), 318;
J.D. Breit, B.A. Ovrut and G. Segré, *Phys.Lett.*
B158 (1985), 33;
E. Cohen, J. Ellis, K. Enqvist and D.V. Nanopou-
los, *Phys.Lett.* **B165** (1985), 76;
J. Ellis, K. Enqvist, D.V. Nanopoulos and
F. Zwirner, *Nucl.Phys.* **B276** (1986), 14,
Mod.Phys.Lett. **A1** (1986), 57;
F. Del Aguila, G. Blair, M. Daniel and G.G. Ross,
Nucl.Phys. **272** (1986), -;
L. Ibáñez and J. Mas, *Nucl.Phys.* **B286** (1987),
107;
For a review of phenomenological implications of
low energy superstring inspired E_6 models, see:
F. Zwirner, *Int.J.Mod.Phys.* **A3** (1988), 49;
J.L. Hewett and T.G. Rizzo, *Physics Reports* **183**
(1989), 195.
26. M.C. Gonzales-Garcia and J.W.F. Valle, Preprint
FTUV/90-15 (1990);
F. Del Aguila, J.M. Moreno and M. Quiros, CERN
Preprint TH. 5646 (1990);
S.L. Glashow and U. Sarid, *Phys.Rev.Lett.* **64**
(1990), 725;
A.E. Faraggi and D.V. Nanopoulos, Preprint
CTP-TAMU-69/89 (1989);
J. Layssac, F.M. Renard and C. Verzegnassi,
Preprint LAPP-TH-290/90.
27. G. Altarelli, R. Casalbuoni, D. Dominici, F. Fer-
uglio and R. Gatto, *Mod.Phys.Lett.* **A5** (1990),
495, *Nucl.Phys.* **B342** (1990), 15;
G. Altarelli, R. Casalbuoni, F. Feruglio and R.
Gatto, *Phys.Lett.* **B245** (1990), 669.
28. H. Georgi, E.E. Jenkins and E.H. Simmons,
Phys.Rev.Lett. **62** (1989), 2789 [**E63** (1989) 1540].
29. J.C. Pati and A. Salam, *Phys.Rev.* **D10** (1974),
275;
R.N. Mohapatra and J.C. Pati, *Phys.Rev.* **D11**
(1975), 2558;
R.N. Mohapatra and G. Senjanovic, *Phys.Rev.*
D12 (1975), 1502, *Phys.Rev.Lett.* **44** (1980), 912,
Phys.Rev. **D23** (1981), 165;
G. Ecker and W. Grimus, *Z.Phys.* **C30** (1986),
293.
See also R. Decker and U. Turke, *Z.Phys.* **C26**
(1984), 117;
G. Beall, M. Bander and A. Soni, *Phys.Rev.Lett.*
48 (1982), 848;
G. Ecker, W. Grimus and H. Neufeld,
Phys.Lett. **B127** (1983), 365;
R.N. Mohapatra, G. Senjanovic and M.D. Tran,
Phys.Rev. **D28** (1983), 546;
F.J. Gilman and M.H. Reno, *Phys.Rev.* **D29**
(1984), 937.
30. See, for example: G. Altarelli, CERN Preprint
TH. 5590 (1989).
31. J. Van der Bij and M. Veltman, *Nucl.Phys.* **B231**
(1984), 205;
See also:
M. Veltman, *Acta Phys.Polon.* **B8** (1977), 475;
B.W. Lee, C. Quigg and H.B. Thacker,
Phys.Rev. **D16** (1979), 1519.
32. F.A. Berends et al., Ref. 1., Vol. 1, p. 89;
The authors of ZSHAPE are W. Beenakker, F.A.
Berends and S. Van der Marck.
33. D.C. Kennedy, B.W. Lynn, C.J.-C. Im and
R.G. Stuart, *Nucl.Phys.* **B321** (1989), 83 (EX-
POSTAR).
34. A. Borrelli, M. Consoli, L. Maiani and R. Sisto,
Nucl.Phys. **B333** (1990), 357.
35. S. Banerjee, S.N. Ganguli, A. Gurtu and K. Ma-
jumdar, L3 Note 796 (1990);
A. Borrelli, L. Maiani and R. Sisto, INFN Roma
Preprint Nr. 731 (1990);
M. Consoli, C. Dionisi and L. Ludovici, Proceed-
ings of "Les Rencontres de la Vallée d'Aoste", La
Thuile, 1990, eds. G. Bellettini and M. Greco;
R.D. Peccei, UCLA/90/TEP/11 (1990);
S.N. Ganguli and A. Gurtu, TIFR-EHEP 90/1.
36. See, for example, Ref. 4. and
F. Antonelli and L. Maiani, *Nucl.Phys.* **B186**
(1981), 269;
S. Bellucci, M. Lusignoli and L. Maiani,
Nucl.Phys. **B189** (1981), 329;
M. Consoli, S. Lopresti and L. Maiani,
Nucl.Phys. **B223** (1983), 474.
37. D.C. Kennedy and B.W. Lynn, *Nucl.Phys.* **B322**
(1989), 1;
D.C. Kennedy et al., *Nucl.Phys.* **B321** (1989), 83;
B.W. Lynn, SU-ITP-867 (1989).
38. A. Sirlin, CERN Preprint TH. 5506 (1989);
S. Fanchiotti and A. Sirlin, Preprint NYU (1989).
39. J.H. Kühn, P.M. Zerwas et al., in Ref. 2., Vol. 1,
p. 267.
40. A.A. Akhundov, D.Yu. Bardin and T. Riemann,
Nucl.Phys. **B276** (1988), 1;
F. Diakonov and W. Wetzel, Preprint HD-THEP-
88-21 (1988);
W. Beenakker and W. Hollik, *Z.Phys.* **C40** (1988),
141;
J. Bernabeu, A. Pich and A. Santamaria,
Phys.Lett. **B200** (1988), 569;
B.W. Lynn and R.G. Stuart, CERN Preprint TH.
5786 (1990).
41. I am grateful to B.W. Lynn for providing me the
points for the curve in Fig. 5. Precisely the curve
refers to $\sin^2 \theta_W^*$ as defined in Refs. 33. and 37.

42. S. Sarantakos, A. Sirlin and W.J. Marciano, *Nucl.Phys.* **B217** (1983), 84;
M.J. Marciano and S. Sirlin, *Phys.Rev.* **D22** (1980), 2695, **D29** (1984) 945, **D31** (1985) 213E;
D.Y. Bardin and O.M. Dokuchaeva, *Nucl.Phys.* **B246** (1984), 221, and Preprint JINR-E2-86-260 (1986).
43. See F. Antonelli et al. and S. Bellucci et al. quoted in Ref. 36. See also:
G. Altarelli, *Acta Phys. Austriaca, Suppl.* **XXIV** (1982), 229.
44. G. Gounaris and D. Schildknecht, *Z.Phys.* **C42** (1989), 107.
45. D. Treille, "Polarization at LEP", eds. G. Alexander et al., CERN Yellow Report 88-06 (1988), Vol. 1, p. 265;
D. Treille, Preprint CERN-EP/90-30.
46. Z. Kunszt, P. Nason et al., Ref. 2., Vol. 1, p. 373.
47. S.G. Gorishny, A.L. Kateev and S.A. Larin, *Phys.Lett.* **B212** (1988), 238.
48. J. Drees, These Proceedings.
49. OPAL Collaboration, M.Z. Akrawy et al., *Phys.Lett.* **B235** (1990), 389.
50. JADE Collaboration, W. Bartel et al., *Z.Phys.* **C33** (1986), 23;
JADE Collaboration, S. Bethke et al., *Phys.Lett.* **B213** (1988), 235;
TASSO Collaboration, W. Braunschweig et al., *Phys.Lett.* **B214** (1988), 286;
AMY Collaboration, I. Park et al., *Phys.Rev.Lett.* **62** (1989), 1713;
I. Park, Ph.D. Thesis, Rutgers University of New Jersey (1989);
MARK II Collaboration, S. Bethke et al., *Z.Phys.* **C43** (1989), 325.
51. S. Bethke, *Z.Phys.* **C43** (1989), 331.
52. G. Kramer and B. Lampe, *Z.Phys.* **C39** (1989), 101, and CERN Preprint TH. 5810 (1990).
53. O. Nachtmann and A. Reiter, *Z.Phys.* **C16** (1982), 45;
M. Bengtsson and P. Zerwas, *Phys.Lett.* **B208** (1988), 306;
M. Bengtsson, PITHA 88/12 (1988).
54. A.H. Mueller, *Phys.Lett.* **B104** (1981), 161;
A. Bassetto, M. Ciafaloni, G. Marchesini and A.H. Mueller, *Nucl.Phys.* **B207** (1982), 189;
B.I. Ermolaev and V.S. Fadin, *JETP Lett.* **33** (1981), 269;
Yu.L. Dokshitzer, V.S. Fadin and V.A. Khoze, *Phys.Lett.* **B115** (1982), 242;
yu.L. Dokshitzer, V.A. Khoze and S. Troyan, in *Perturbative Quantum Chromodynamics*, ed. A.H. Mueller, World Scientific, Singapore (1989), 241;
Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller and S.I. Troyan, *Rev.Mod.Phys.* **60** (1988), 373;
Yu.L. Dokshitzer, V.A. Khoze, G. Marchesini and B.R. Webber, CERN Preprint TH. 5738 (1990).