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Heavy Jet Mass Distribution in e^+e^- Annihilation*

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Abstract

Predictions are presented for the distribution of $\rho = M_h^2/s$ in e^+e^- annihilation, where s is the centre-of-mass energy squared and M_h is the heavy jet mass, defined as the greater of the invariant masses of two hemispheres separated by the plane normal to the thrust axis. The calculation includes resummation of leading and next-to-leading logarithms of ρ to all orders in QCD perturbation theory, together with non-logarithmic terms up to second order. This reduces the renormalization scale dependence relative to the fixed-order calculation and should permit the strong coupling constant to be measured accurately using a larger region of the distribution.

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Among the precision measurements of standard model parameters now being carried out by the LEP experiments, one of the most crucial is the determination of the strong coupling constant α_s . More accurate determinations of this quantity are essential, for example, in testing proposals for grand unification at higher energies.

The fitting of hadronic event shape distributions in e^+e^- annihilation is by now a well-established method for α_s determination [1-5]. A number of infrared-safe event shape variables have been defined and computed in full to second order in α_s [6]. The perturbative predictions can be compared with experimental data after correction for the non-perturbative hadronization process. However, there remain some serious uncertainties in how best to perform such comparisons:

- *Range to be fitted.* Near the kinematic limits, the perturbative expansions for event shapes involve large logarithmic terms, typically of the form $\alpha_s^m \ln^m \rho$ where $m \leq 2n$ and ρ is a small parameter. In such regions the real expansion parameter is the large effective coupling $\alpha_s \ln^2 \rho$ and any finite-order perturbative calculation becomes unreliable. In comparing data with fixed-order calculations one must therefore use only a restricted region well away from kinematic boundaries, whereas most of the data are close to the limit corresponding to a two-jet configuration.
- *Hadronization corrections.* The most successful hadronization models are those incorporated in parton-shower event simulation programs, which are based on the assumption that hadronization is a local process in phase space, involving a rather large number of final-state partons at a low momentum transfer scale. Such models are not suitable for estimating corrections to fixed-order perturbative calculations, which involve at most four partons in second order.
- *Renormalization scale dependence.* The result obtained for α_s depends on the renormalization scale μ^2 used in the perturbative calculation. This dependence represents a residual uncertainty due to orders higher than those computed. Intuitively, one would expect $\mu^2 \sim s$, the centre-of-mass energy squared, to be the natural scale, but in many cases $\mu^2 \ll s$ gives a better fit, with a substantially different value of $\alpha_s(s)$. The strong scale dependence, combined with the preference for such small scales, constitutes the biggest source of uncertainty in α_s determinations from event shapes at LEP.

In this paper we present the results of a calculation which reduces all of the above types of uncertainty for an important event shape variable, namely the *heavy jet mass* M_h , defined as the greater of the invariant masses of two hemispheres separated by the plane normal to the thrust axis. The calculation includes resummation of leading and next-to-leading logarithms of the quantity $\rho = M_h^2/s$ to all orders in QCD perturbation theory, together with all other terms up to second order. This will allow analyses of the heavy jet mass distribution to be extended to smaller values of ρ , where most of the data occur. Furthermore, the resummation corresponds closely to what is done, at a less precise level, in parton shower simulations, and so it can be combined more reliably with hadronization corrections estimated from models based on parton showers. Finally, we show that resummation reduces the renormalization scale dependence of the prediction, by taking into account an important class of higher-order corrections. Most importantly,

it reproduces, at a scale $\mu^2 \sim s$, the effect of using a scale $\mu^2 \ll s$ in the second-order calculation, and thus it removes the motivation for using a small scale.

Notice that our definition of the heavy jet mass corresponds to that used experimentally in Refs. [2,3], rather than the definition originally proposed by Clavelli [7]. The logarithmic terms resummed here are the same for both definitions. The difference affects only the non-logarithmic part of the prediction.

The calculation of the heavy jet mass distribution is closely related to that of the thrust distribution, presented in [8], and therefore we only give the result here. The details of both calculations will be provided in a longer paper [9].

The fraction of events with heavy jet mass-squared less than ρs , which we denote here by $R(\rho, \alpha_s(s))$, is an example of a quantity that *exponentiates* at small ρ . By this we mean that

$$R(\rho, \alpha_s(s)) = C(\alpha_s(s)) \Sigma(\rho, \alpha_s(s)) + F(\rho, \alpha_s(s)) \quad (1)$$

where

$$\begin{aligned} C(\alpha_s) &= 1 + \sum_{n=1}^{\infty} \alpha_s^n C_n, \\ \ln \Sigma(\rho, \alpha_s) &= \sum_{n=1}^{\infty} \alpha_s^n \sum_{m=1}^{n+1} G_{nm} \ln^m \rho, \\ F(\rho, \alpha_s) &= \sum_{n=1}^{\infty} \alpha_s^n F_n(\rho). \end{aligned} \quad (2)$$

Here C_n and G_{nm} are constants, while $F_n(\rho)$ are perturbatively computable functions that vanish at small ρ . Notice that although the perturbative expansion of $R(\rho, \alpha_s(s))$ itself contains terms of the form $\alpha_s^m \ln^m \rho$ for all values of m up to $2n$, its logarithm involves only $m \leq n+1$. We call this property exponentiation. In our calculation, the leading and next-to-leading terms in Σ , i.e. those with $m \geq n$, are summed to all orders, and sub-dominant terms with $m < n$, as well as C and F , are included to second order only.

If we now introduce a renormalization scale $\mu^2 \neq s$, then in place of $\alpha_s(s)$ in Eq. (1) we find $\alpha_s(\mu^2)$ together with an explicit s/μ^2 dependence of the next-to-leading logarithms and fixed-order terms. Thus we can study the scale dependence of the resummed expression and compare it with that of the fixed-order result.

When $\rho \ll 1$, the heavy jet mass cross section has a simple expression in terms of the distribution $J(s, k^2)$ of the mass-squared k^2 of a single jet produced at a scale s . We find

$$R(\rho, \alpha_s(s))_{\rho \ll 1} = \Sigma(\rho, \alpha_s(s)) = \left[\int_0^{\rho s} dk^2 J(s, k^2) \right]^2. \quad (3)$$

It can be shown [9] that kinematic correlations between the jet masses, and overlap between the jets, give only contributions that are sub-dominant in the sense defined above. The form of $J(s, k^2)$ was computed to next-to-leading logarithmic accuracy in Ref. [10,11]. The calculation of the heavy jet mass distribution from Eq. (3) now proceeds in a similar way to that outlined in [8] for the thrust. The final result for the exponentiated factor Σ in

Eq. (1), including the renormalization scale dependence, is

$$\Sigma(\rho, \alpha_S(\mu^2), s/\mu^2) = \frac{\exp[G(\ln(1/\rho), \alpha_S(\mu^2), s/\mu^2)]}{(\Gamma[S(\ln(1/\rho), \alpha_S(\mu^2))])^2} \quad (4)$$

where Γ represents the Euler gamma function,

$$\begin{aligned} G(L, \alpha_S, s/\mu^2) = & -\frac{A^{(1)}}{\pi\beta_0^3\alpha_S} \left[(1 - 2\beta_0\alpha_S L) \ln(1 - 2\beta_0\alpha_S L) - 2(1 - \beta_0\alpha_S L) \ln(1 - \beta_0\alpha_S L) \right] \\ & + \frac{1}{\pi\beta_0^2} \left[A^{(1)}\pi\beta_0 \ln(s/\mu^2) - A^{(2)} \right] \cdot [2\ln(1 - \beta_0\alpha_S L) - \ln(1 - 2\beta_0\alpha_S L)] \\ & + \frac{B^{(1)}}{\pi\beta_0} \ln(1 - \beta_0\alpha_S L) - \frac{2A^{(1)}\gamma_E}{\pi\beta_0} [\ln(1 - \beta_0\alpha_S L) - \ln(1 - 2\beta_0\alpha_S L)] \quad (5) \\ & - \frac{A^{(1)}\beta_1}{\pi\beta_0^3} [\ln(1 - 2\beta_0\alpha_S L) - 2\ln(1 - \beta_0\alpha_S L) + \frac{1}{2}\ln^2(1 - 2\beta_0\alpha_S L) \\ & - \ln^2(1 - \beta_0\alpha_S L)] + \mathcal{O}(\alpha_S^3 L^{n-1}), \end{aligned}$$

$$S(L, \alpha_S) = 1 + \frac{A^{(1)}}{\pi\beta_0} [\ln(1 - \beta_0\alpha_S L) - \ln(1 - 2\beta_0\alpha_S L)], \quad (6)$$

$$\begin{aligned} A^{(1)} &= C_F, \quad B^{(1)} = -\frac{3}{2}C_F, \\ A^{(2)} &= \frac{C_F}{2} \left(\frac{67}{6} - \frac{\pi^2}{2} - \frac{5}{9}N_f \right), \\ \beta_0 &= \frac{33 - 2N_f}{12\pi}, \quad \beta_1 = \frac{153 - 19N_f}{24\pi^2}. \end{aligned} \quad (7)$$

Here $\gamma_E = 0.5772\dots$ is the Euler constant, N_f is the number of flavours ($N_f = 5$ at LEP energies) and $C_F = 4/3$ for 3 colours.

Having evaluated Σ to next-to-leading accuracy, we can find the non-logarithmic coefficients C_1 and $F_1(\rho)$ in Eq. (2) by comparison with the one-loop analytical result for $R(\rho, \alpha_S)$. Similarly C_2 and $F_2(\rho)$ can be estimated by comparison with the numerical 2-loop result [6]. We have

$$\begin{aligned} \ln R = & G_{12}\alpha_S \ln^2 \rho + G_{11}\alpha_S \ln \rho + H_{10}\alpha_S + H_1(\rho)\alpha_S \\ & + G_{23}\alpha_S^2 \ln^3 \rho + G_{22}\alpha_S^2 \ln^2 \rho + G_{21}\alpha_S^2 \ln \rho + H_{20}\alpha_S^2 + H_2(\rho)\alpha_S^2 \quad (8) \\ & + G_{34}\alpha_S^3 \ln^4 \rho + G_{33}\alpha_S^3 \ln^3 \rho + \text{uncalculated} \\ & + \dots \end{aligned}$$

where $H_n(\rho) \xrightarrow{\rho \rightarrow 0} 0$ and the coefficients H_{nm} and $H_n(\rho)$ are straightforwardly related to C_n and $F_n(\rho)$. For $n = 1$ we find

$$\begin{aligned} C_1 = H_{10} &= \frac{C_F}{2\pi} \left[-\frac{5}{2} + \frac{\pi^2}{3} \right] \\ F_1(\rho) = H_1(\rho) &= \frac{C_F}{2\pi} \left[6\rho(1 + \ln \rho) + 4\ln \rho \ln(1 - \rho) - 2\ln^2(1 - \rho) \right. \quad (9) \\ &\quad \left. + 3(1 - 2\rho) \ln(1 - 2\rho) + \frac{9}{2}\rho^2 - 4\text{Li}_2\left(\frac{\rho}{1 - \rho}\right) \right], \end{aligned}$$

where

$$\text{Li}_2(z) = \sum_{n=1}^{\infty} \frac{z^n}{n^2}.$$

From Ref. [6] we obtain numerical values of $G_{21} \ln \rho + H_{20} + H_2(\rho)$ and hence of C_2 and $F_2(\rho)$ at specific values of ρ . Finally, the uncalculated terms in Eq. (8) are approximated by a constant such that $R = 1$ when $\rho = \frac{1}{2}$. Note that the non-logarithmic terms and the kinematic limit on ρ are different if one uses the definition of the heavy jet mass in Ref. [7], rather than the one described above.[†]

The resulting differential distribution of ρ at $\sqrt{s} = 91$ GeV is illustrated in Figs. 1-3. For definiteness we have assumed the (five-flavour) value $\Lambda_{\overline{\text{MS}}} = 200$ MeV, corresponding to $\alpha_S(s) = 0.116$.

Fig. 1 shows our full parton-level prediction, i.e. the resummed leading and next-to-leading logarithms plus finite corrections up to $\mathcal{O}(\alpha_S^5)$ (the first two columns and rows on the right-hand side of Eq. (8)), compared with the fixed-order result given in Ref. [6] (i.e. $R(\rho, \alpha_S)$ expanded to second order in α_S only), both for $\mu^2 = s$. The main effect of resummation is that the distribution is decreased at the lowest masses, as expected from the Sudakov suppression of real emission, and increased at higher values.

Fig. 2 shows the renormalization scale dependence of the results presented in Fig. 1, at various values of ρ . We see that resummation reduces the scale dependence in the region of physical scales, $0.1 \lesssim \mu^2/s \lesssim 1$.

As is suggested by Fig. 2 and shown in Fig. 3, the effect of resummation can be approximately simulated for $0.01 < \rho$ by a fixed-order calculation in which the renormalization scale is taken to be much smaller than s , which has been found to be favoured by the data on several event shape distributions [1]. In this way the large logarithms of ρ are simulated by logarithms of μ^2/s over a limited range of ρ . Fig. 3 suggests that there should be no need for such an unnatural choice of scale once the logarithms of ρ have been correctly resummed. It should also be noted from Eqs. (4), (5) that a choice of scale of the form $\mu^2 \propto \rho^p s$ for any power p would not suffice to resum even the leading logarithms to all orders.

High-statistics data on the heavy jet mass distribution from all four LEP experiments are in preparation, and therefore we have thought it best to present our predictions as soon as possible, rather than attempting detailed fits to existing data.[‡] Before comparing with data one must make hadronization corrections, which, as we argued above, can be estimated from parton shower simulations. To illustrate their magnitude, we show in Fig. 4 the corrections estimated from the model of Ref. [12]. It should be noted that the effect of hadronization is more uncertain at smaller ρ , making a detailed comparison with data more difficult in the region $\rho < 0.03$.

Also shown in Fig. 4 are the recent high-statistics data of Ref. [13]. From a second-order matrix element analysis of these data in the interval $0.08 < \rho < 0.20$, the DELPHI

[†]For the definition used here, the kinematic limit depends on the number of final-state particles. For 4 particles it is $(\sqrt{s} - 1)/(\sqrt{s} + 1) = 0.42$, and for a larger number it is closer to 0.5.

[‡]Fortran programs to compute the predictions for any range of ρ and any choice of renormalisation scale are available from the authors on request.

Collaboration obtained values of $\alpha_S(s)$ in the range 0.113–0.133 for scales ranging from $\mu^2/s = 0.002$ –1.0, with a preference for the lowest scales. For the reasons we have outlined, we expect the resummed predictions given in the present paper to be applicable over a wider range of ρ , to give results that are more stable with respect to scale variation in the physical range $\mu^2 \sim s$, and to disfavour scale choices outside this range.

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Figure Captions

- Figure 1: Predicted distribution of $\rho = M_h^2/s$ in e^+e^- annihilation at $\sqrt{s} = 91$ GeV. Solid: our prediction including resummation to all orders. Dashed: fixed $\mathcal{O}(\alpha_S^2)$ result [6].
- Figure 2: Dependence on renormalization scale μ of resummed (solid) and fixed-order (dashed) predictions at various values of ρ .
- Figure 3: Comparison between resummed result with $\mu^2 = s$ (solid) and fixed $\mathcal{O}(\alpha_S^2)$ prediction with $\mu^2 = 0.005s$ (dashed).
- Figure 4: Solid: resummed parton-level prediction as in Fig. 1. Dashed: prediction after correction for hadronization. Data: from Ref. [13].