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Vector Supersymmetry in the Universal Bundle

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Abstract

We present a vector supersymmetry for Witten-type topological gauge theories, and examine its algebra in terms of a superconnection formalism. When covariant constraints on the supercurvature are chosen, a correspondence is established with the universal bundle construction of Atiyah and Singer. The vector supersymmetry represents a certain shift operator in the curvature of the universal bundle, and can be used to generate the hierarchy of observables in these theories. This formalism should lead to the construction of vector supergravity theories, and perhaps to the gravitational analogue of the universal bundle.

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1 Introduction

In a previous publication [1], we discussed the presence of a vector supersymmetry (SUSY) in certain topological field theories [2]-[6] from the point of view of a superconnection formalism. An advantage of this formalism is that it allows one to consider all the symmetries of a given theory from a unifying perspective [7, 8]. For the cases studied (Chern-Simons and BF-type theories), the full symmetry algebra consisted of a scalar BRST symmetry, in addition to the vector SUSY. It was found that non-covariant curvature constraints represent the symmetry algebra present in those models. The consistency of a given set of constraints is established, within this framework, by verifying the corresponding Bianchi identities.

A natural question to pose at this point is whether there exist covariant solutions to this formalism. It transpires that one can indeed establish the consistency of a covariant solution. Furthermore, the SUSY in this case establishes an important connection with the universal bundle of Atiyah and Singer [9]; the SUSY represents a certain shift operator of dimension $(1, -1)$ in the curvature of the universal bundle. Here, the $(1, -1)$ indicates that the operator is a 1-form with ghost number -1 .

It is known that topological Yang-Mills theory (Donaldson theory)[10], as well as arbitrary dimensional super- BF systems [11]-[14] are built upon the geometry of the universal bundle [11, 15, 16], with the basic geometric multiplet of fields present being specified therein. In addition, the hierarchy of metric independent observables of Donaldson theory, for example, can be neatly cast in terms of the curvature of the universal bundle. The interesting feature of the SUSY operator is that, given the first basic observable, it generates the entire set.

The ability to obtain this covariant solution rests on the fact that we are now considering topological field theories of Witten-type. As mentioned above, these include Donaldson theory and super- BF systems. The BRST-SUSY algebra under study then corresponds to examining these models without a gauge fixing of the usual Yang-Mills invariance.

The outline of this article is as follows: We continue in the following section with a brief review of the bundle of frames approach to superspace, to set the stage and establish notation. The covariant curvature and torsion constraints of the solution are then presented, and this is followed by a study of the properties of the SUSY operator, in particular its use in generating the hierarchy of observables. We conclude with a discussion of how the present formalism naturally provides a framework for constructing vector supergravity theories. We present some ideas on how this might be achieved, and observe that if successful, this could possibly lead to the gravitational analogue of the universal bundle.

2 Superspace for Vector Supersymmetry

In this section, we review the relevant aspects of the bundle of frames approach to superspace which is tailored to the type of theories considered here. This should serve to fix our notation and conventions. For a more comprehensive introduction aimed at conventional supergravity, see [7, 8].

Superspace is locally parametrized by a set of even and odd coordinates; (x, θ) . In the case at hand, we will explicitly separate an additional odd coordinate θ which will be related to the BRST symmetry of the theory; $z^\Lambda = (x^\mu, \theta^\alpha, \theta)$. Notice that we use lower case Greek indices from the beginning of the alphabet to label the odd coordinates, while those from the middle label the even directions; a capital Greek index denotes generically any of the coordinates. Over a supermanifold, we can consider a fibre bundle together with a connection $h_\Lambda^{A_0}$ and form the covariant derivative

$$D_\Lambda = \partial_\Lambda - h_\Lambda^{A_0} D_{A_0} . \quad (1)$$

The labels $A_0, B_0, \dots$ are used to denote a vertical direction in the bundle, and the differential operators D_{A_0} represent the structure group

$$[D_{A_0}, D_{B_0}] = f_{B_0 A_0}^{C_0} D_{C_0} . \quad (2)$$

The structure group will be a product of $SO(n)$ for the local Lorentz transformations, and a compact Yang-Mills group G . It will be convenient to use $A_0 = [rs]$ when referring to a $SO(n)$ direction, and $I, J, K, \dots$ for the group G . It is also conventional to denote the spin and Yang-Mills connections respectively by $h_\Lambda^{[rs]} = \omega_\Lambda^{[rs]}$ and $h_\Lambda^I = A_\Lambda^I$. The equivariance of a connection is expressed through the relation $[D_\Lambda, D_{A_0}] = 0$, or equivalently

$$D_{A_0} h_\Lambda^{B_0} = f_{A_0 C_0}^{B_0} h_\Lambda^{C_0} . \quad (3)$$

The curvature is defined by the graded commutator

$$[D_\Lambda, D_\Sigma] = -R_{\Lambda\Sigma}^{A_0} D_{A_0} , \quad (4)$$

and it is usually conventional to write $R_{\Lambda\Sigma}^I = F_{\Lambda\Sigma}^I$ for the Yang-Mills curvature.

In addition to the bundle structure that we have so far considered, we will need to supplement this with a collection of local frame fields $h^{A_1} = dz^\Lambda h_\Lambda^{A_1}$ which generalize the notion of vielbein to the case of supermanifolds. An index $A_1, B_1, \dots$ is being used to denote the horizontal directions in the bundle, and objects with these labels transform in definite representations of the structure group,

$$D_{A_0} h^{A_1} = f_{A_0 B_1}^{A_1} h^{B_1} . \quad (5)$$

We will use a lower case Latin index from the beginning of the alphabet to denote an odd direction, and those near the middle ($i, j, k, \dots$) for an even direction. As in supergravity, the even labels transform as vectors under the

$SO(n)$ group, $f_{A_0 j}^i = f_{[rs]j}^i = \frac{1}{2}\delta_{[r}^i\delta_{s]j}$. However, instead of the h^a transforming as spinors, we will require them to also transform as vectors. This is simply because we seek to realize a Grassmann odd vector symmetry.

The addition of frame fields in the picture allows us to define torsion

$$T_{\Lambda\Sigma}^{A_1} = D_{\Lambda}h_{\Sigma}^{A_1} - (-1)^{\Lambda\Sigma}D_{\Sigma}h_{\Lambda}^{A_1} , \quad (6)$$

as well as relate objects with curved labels to those with flat labels, e.g.

$$D_{\Lambda} = h_{\Lambda}^{A_1}D_{A_1} = h_{\Lambda}^i D_i + h_{\Lambda}^a D_a + h_{\Lambda}^{\theta} D_{\theta} . \quad (7)$$

Notice that we also employ θ as an index to denote a “flat” BRST direction, and this naturally transforms as a scalar under $SO(n)$. It follows that the covariant derivatives with local Lorentz labels satisfy the algebra

$$[D_{A_0}, D_{A_1}] = -f_{A_0 A_1}^{B_1} D_{B_1} . \quad (8)$$

If we also define “flat” curvature and torsion components by

$$\begin{aligned} R_{\Lambda\Sigma}^{A_0} &= (-1)^{B_1\Sigma} h_{\Lambda}^{B_1} h_{\Sigma}^{C_1} R_{B_1 C_1}^{A_0} \\ T_{\Lambda\Sigma}^{A_1} &= (-1)^{B_1\Sigma} h_{\Lambda}^{B_1} h_{\Sigma}^{C_1} T_{B_1 C_1}^{A_1} , \end{aligned} \quad (9)$$

then the entire algebraic picture can be written very compactly as

$$[D_A, D_B] = F_{BA}^C D_C , \quad (10)$$

where $F_{B_1 A_1}^{C_1} = T_{B_1 A_1}^{C_1}$, $F_{B_1 A_1}^{C_0} = R_{B_1 A_1}^{C_0}$, $F_{B_0 A_1}^{B_1} = f_{B_0 A_1}^{B_1}$, and so on.

The Bianchi identities follow from the super Jacobi identity which represents the associativity of the differential operators, and with these conventions they take the form

$$\begin{aligned} 0 &= \sum_{(cyclic)} (-1)^{A_1 B_1} [D_{A_1} T_{B_1 C_1}^{D_1} - T_{C_1 A_1}^{E_1} T_{B_1 E_1}^{D_1} + R_{C_1 A_1}^{E_0} f_{E_0 B_1}^{D_1}] \\ 0 &= \sum_{(cyclic)} (-1)^{A_1 B_1} [D_{A_1} R_{B_1 C_1}^{D_0} - T_{C_1 A_1}^{E_1} R_{B_1 E_1}^{D_0}] . \end{aligned} \quad (11)$$

The sum here is over the three cyclic permutations of the lower indices.

The algebra which represents all of the local symmetries of the theory is given by

$$\begin{aligned} \delta h_{\Lambda}^A &= D_{\Lambda} \epsilon^A - h_{\Lambda}^{B_1} \epsilon^C F_{B_1 C}^A \\ \delta F_{BC}^A &= \epsilon^E D_E F_{BC}^A , \end{aligned} \quad (12)$$

where all of the parameters have a “flat” index. Note that $A, B, \dots$ with no subscript denotes either a vertical or horizontal direction.

3 Covariant Solution for Global BRST-SUSY

In Witten-type topological gauge theories, one is presented with a geometrical set of fields upon which the theory is built. An understanding of these fields finds a natural place within the universal bundle construction of Atiyah and Singer [9]. Before discussing this construct, however, we shall first address the problem of finding a covariant solution to the global Bianchi identities of the previous section. The symmetries under discussion consist of a topological BRST shift symmetry, in addition to a vector supersymmetry. The term vector supersymmetry simply indicates that the infinitesimal parameter is a Grassmann odd vector. To begin, let us specify the BRST symmetry transformations; we have

$$\begin{aligned}\delta A_\alpha &= \psi_\alpha , \\ \delta \psi_\alpha &= -D_\alpha \phi , \\ \delta \phi &= 0 .\end{aligned}\tag{13}$$

The statistics and ghost numbers of the multiplet of fields $(A_\alpha, \psi_\alpha, \phi)$ is given by $(+, -, +)$ and $(0, 1, 2)$, respectively. It is an easy task to verify that the corresponding algebra closes up to a Yang-Mills gauge transformation with gauge parameter 2ϕ , that is, we have

$$\begin{aligned}\{\delta, \delta\}A_\alpha &= -2D_\alpha \phi , \\ \{\delta, \delta\}\psi_\alpha &= 2[\phi, \psi_\alpha] , \\ \{\delta, \delta\}\phi &= 0 .\end{aligned}\tag{14}$$

In addition, however, we can also define the following vector supersymmetry transformations of this multiplet:

$$\begin{aligned}\delta_\mu A_\alpha &= 0 , \\ \delta_\mu \psi_\alpha &= F_{\mu\alpha} , \\ \delta_\mu \phi &= -\psi_\mu ,\end{aligned}\tag{15}$$

which obey the Abelian algebra

$$\{\delta_\mu, \delta_\nu\}X = 0 ,\tag{16}$$

for all X . Given these two symmetries, one can establish that the resulting algebra is given by

$$\begin{aligned}\{\delta, \delta_\mu\}A_\alpha &= F_{\mu\alpha} , \\ \{\delta, \delta_\mu\}\psi_\alpha &= D_\mu \psi_\alpha , \\ \{\delta, \delta_\mu\}\phi &= D_\mu \phi .\end{aligned}\tag{17}$$

It is clear at this point that all of the above analysis is covariant with respect to a Yang-Mills gauge invariance. Our task is therefore to establish whether this algebra can now be embedded in the superconnection framework of the previous section.

Since the symmetries on hand are of a global nature, we initiate our search by demanding that the vielbein describes a flat geometry, $h_\mu^i = \delta_\mu^i$. Furthermore, the potential gauge fields for the SUSY and BRST symmetries are taken to be trivial, $h_\mu^a = 0, h_\mu^\theta = 0$. It remains to determine the appropriate set of curvature and torsion constraints. The fundamental torsion constraints are immediately specified by the algebra of interest, namely (14), (16) and (17); we thus find one non-vanishing component

$$T_{a\theta}^i = \delta_a^i . \quad (18)$$

Moreover, the curvature constraints can also be read off from the known transformations of A_α . Re-writing (12) for the Yang-Mills connection as

$$\delta A_\mu^I = D_\mu \epsilon^I - h_\mu^i \epsilon^\theta F_{i\theta}^I - h_\mu^a \epsilon^a F_{ia}^I , \quad (19)$$

we ascertain that the appropriate curvature constraints are simply

$$F_{i\theta}^I = -\psi_i , F_{\theta\theta}^I = -2\phi , \quad (20)$$

with all other components taken to be zero, and $\psi_i \equiv \delta_i^\alpha \psi_\alpha$.

The consistency of this set can now be verified through an analysis of the Bianchi identities (11). In particular, we have the following non-trivial identities

$$\begin{aligned} (\theta ij)^I : 0 &= D_\theta F_{ij}^I + D_{[i} F_{j]\theta}^I , \\ (aij)^I : 0 &= D_a F_{ij}^I , \\ (\theta\theta i)^I : 0 &= 2D_\theta F_{i\theta}^I + D_i F_{\theta\theta}^I , \\ (\theta\theta a)^I : 0 &= D_a F_{\theta\theta}^I + 2\delta_a^i F_{i\theta}^I , \\ (\theta ai)^I : 0 &= D_a F_{i\theta}^I + \delta_a^j F_{ji}^I , \\ (\theta\theta\theta)^I : 0 &= D_\theta F_{\theta\theta}^I . \end{aligned} \quad (21)$$

It is straightforward to check that these identities reproduce the transformations and algebra of the fundamental fields $(A_\alpha, \psi_\alpha, \phi)$ given above. We have thus succeeded in obtaining a covariant solution to the superconnection formalism.

As already mentioned, this basic multiplet underpins all Witten-type topological gauge theories. However, in order to complete the construction of a given theory, one requires additional multiplets. The extra fields present are model dependent, and essentially consist of the various multiplier and ghost fields which enable the required gauge fixing procedure to be performed. The incorporation of these fields into the BRST-SUSY algebra outlined above is quite straightforward to achieve. For the purpose of illustration, we treat the case of Donaldson theory.

There are four new fields which have the transformations:

$$\delta \bar{\phi} = \eta ,$$

$$\begin{aligned}
\delta\eta &= [\phi, \bar{\phi}] , \\
\delta\chi_{\alpha\beta} &= B_{\alpha\beta} , \\
\delta B_{\alpha\beta} &= [\phi, \chi_{\alpha\beta}] , \\
\delta_\mu \bar{\phi} &= 0 , \\
\delta_\mu \eta &= D_\mu \bar{\phi} , \\
\delta_\mu \chi_{\alpha\beta} &= 0 , \\
\delta_\mu B_{\alpha\beta} &= D_\mu \chi_{\alpha\beta} .
\end{aligned} \tag{22}$$

Here, the statistics and ghost numbers of the fields $(\bar{\phi}, \eta, \chi_{\alpha\beta}, B_{\alpha\beta})$ are $(+, -, -, +)$ and $(-2, -1, -1, 0)$, where χ and B are self-dual 2-forms. The $(\bar{\phi}, \eta)$ multiplet is common to all Witten-type gauge theories, and is present to enforce a gauge fixing of the local symmetry present in the ψ_α field. The treatment of the arbitrary dimensional super-BF systems proceeds by analogy with the above description.

4 The Universal Bundle and Observables

The appropriate setting for a geometrical understanding of the (A, ψ, ϕ) fields and their transformations is provided by the ‘universal bundle with connection’ of Atiyah and Singer [9]. This has been illustrated by several authors [11, 15, 16]. Let us briefly recall the essential features of this bundle.

Given a principal bundle P , with structure group G , over a compact smooth manifold M , one can consider the affine space $\mathcal{A}$ of all connections on P . This space is acted on by the group $\mathcal{G}$ of pointed gauge transformations, and leads one to the principal $\mathcal{G}$ -bundle $P \times \mathcal{A}$ over $Q = (P \times \mathcal{A})/\mathcal{G}$. Since G acts freely on the base Q of this bundle, the following triplet

$$\mathcal{Q} = ((P \times \mathcal{A})/\mathcal{G}, G, Q/G = M \times \mathcal{A}/\mathcal{G}) , \tag{23}$$

defines the so-called ‘universal’ G -bundle over the base space $M \times \mathcal{A}/\mathcal{G}$.

Now, differential forms on $M \times \mathcal{A}/\mathcal{G}$ carry a natural bi-grading, a (p, q) -form referring to a p -form on M and a q -form on $\mathcal{A}/\mathcal{G}$. Previously, we have used the term ‘ghost number’ for the q degree. The connection on the universal bundle can, therefore, locally be written as

$$\hat{A} = A_{(1,0)} + A_{(0,1)} . \tag{24}$$

Similarly, the universal curvature, being a 2-form, decomposes as

$$\begin{aligned}
\mathcal{F} &= (d + \delta)\hat{A} + \frac{1}{2}[\hat{A}, \hat{A}] \\
&= \mathcal{F}_{(2,0)} + \mathcal{F}_{(1,1)} + \mathcal{F}_{(0,2)} ,
\end{aligned} \tag{25}$$

where d is the usual exterior derivative on M , and δ its $(0, 1)$ counterpart. It is thus clear that the components of the universal curvature have the correct

degree in order to allow them to be identified with the F, ψ, ϕ fields of our earlier discussion.

At this point, we can study the action of the SUSY operator δ_μ on the universal curvature. It is evident that it plays the role of a shift operator connecting successive components of the universal curvature, videlicet

$$\delta_\mu : \phi \rightarrow \psi \rightarrow F \rightarrow 0 . \quad (26)$$

Indeed, given its bi-grading of $(1, -1)$, it is naturally designed for this purpose.

One of the most important features of topological gauge theories is the construction of a family of metric independent observables. For the case of Donaldson theory, one begins by selecting the BRST invariant field ϕ , and forming the observable

$$W^{(0,4)} = \frac{1}{2} \text{tr } \phi^2 , \quad (27)$$

where the superscript denotes its bi-grading degree. A hierarchy of observables can then be constructed by successive applications of the exterior derivative d , leading to descent equations for a set of observables $W^{(p,q)}$ [10], which can be written as:

$$\begin{aligned} W_\mu^{(1,3)} &= \text{tr } \psi_\mu \phi , \\ W_{\mu\nu}^{(2,2)} &= \text{tr } (F_{\mu\nu} \phi - \psi_\mu \psi_\nu) , \\ W_{\mu\nu\rho}^{(3,1)} &= \text{tr } (\psi_\mu F_{\nu\rho} + \psi_\nu F_{\rho\mu} + \psi_\rho F_{\mu\nu}) , \\ W_{\mu\nu\rho\sigma}^{(4,0)} &= \text{tr } (F_{\mu\nu} F_{\rho\sigma} + F_{\mu\rho} F_{\sigma\nu} + F_{\mu\sigma} F_{\nu\rho}) . \end{aligned} \quad (28)$$

However, one can now see that the presence of the vector supersymmetry operator, δ_μ , provides an alternative route to the construction of these observables. Given the initial member of the hierarchy $W^{(0,4)}$, one simply acts successively with δ_μ , thereby immediately generating the remaining members in (28), as follows:

$$\begin{aligned} \delta_\mu W^{(0,4)} &= -W_\mu^{(1,3)} , \\ \delta_\mu W_\nu^{(1,3)} &= W_{\mu\nu}^{(2,2)} , \\ \delta_\mu W_{\nu\rho}^{(2,2)} &= -W_{\mu\nu\rho}^{(3,1)} , \\ \delta_\mu W_{\nu\rho\sigma}^{(3,1)} &= W_{\mu\nu\rho\sigma}^{(4,0)} , \\ \delta_\mu W_{\nu\rho\sigma\lambda}^{(4,0)} &= 0 . \end{aligned} \quad (29)$$

It is worth observing that, in the present instance, the SUSY operator δ_μ is metric independent, as can be seen from its action in (15). This is in contrast to the vector supersymmetry which is present in Chern-Simons and BF theories, which is explicitly metric dependent. Common to both, however, is the integrability constraint $[D_\mu, D_\nu] \epsilon_\lambda = R_{\mu\nu\lambda\sigma} \epsilon^\sigma = 0$ one encounters, due to the vector nature of the infinitesimal parameter ϵ^μ . Solutions to this constraint are provided in cases where either the Riemann curvature

vanishes, or has zero modes. The validity of the SUSY is thus restricted to certain manifolds. Nevertheless, it appears that the metric independence of δ_μ is sufficient to allow its use in constructing the above observable hierarchy.

5 Conclusion

It has become clear that vector supersymmetry is a generic feature of topological field theory, and in this paper we have initiated a study of covariant vector supersymmetry algebras. Our covariant solution to the Bianchi identities for global superspace has a natural interpretation in terms of the universal bundle of Atiyah and Singer; the supersymmetry operator is quite simply the map which connects different levels of the curvature of the universal connection.

The next course of action is naturally to power onwards towards a covariant solution which incorporates the gauge field h_μ^a for local supersymmetry, while still discarding the possible BRST gauge field h_μ^θ . One necessarily needs the usual vielbein h_μ^i in such a program, and it is clear that one is dealing with some formulation of topological gravity. We should stress from the outset that conventional topological gravity [17, 18] does not represent a solution. This is immediately apparent since a covariant solution to the Bianchi identities would incorporate a new gauge field and this local symmetry would have to be accounted for in any gauge fixing, and such ghosts are not present in the usual constructions. A glance at the BRST transformation of the vielbein (12) immediately shows that the vielbein has a shift symmetry,

$$\delta h_\mu^i = \epsilon^\theta h_\mu^a \delta_a^i + \dots \quad (30)$$

analogous to the shift symmetry in our global solution, as well as in topological gravity. A crucial difference, however, is that the ghost corresponding to this topological shift symmetry is now a gauge field for a new local symmetry, and not a covariant object like ψ_i which is a curvature component. In principle, one can completely analyze the Bianchi identities and conclusively prove whether a solution exists. While one believes that there is such a solution, it is easier to proceed by beginning with well motivated constraints and then checking their consistency.

Some constraints are essentially transparent. Since we have discarded the BRST gauge field, it is not difficult to see that the torsion $T_{A_1 B_1}^\theta = 0$. Beyond this, one needs some spirited guesswork. Perhaps the most promising course of action is to discard all torsion components that have negative or zero ghost number (aside from the piece $T_{a\theta}^i = \delta_a^i$), as is the case with our global solution. In order to ensure that the vielbein has a simple shift symmetry under BRST as above, one can enforce the vanishing of the ghost number one piece $T_{j\theta}^i$. One also finds the vielbein to be inert under SUSY. It is probable that the ghost number 2 torsion component $T_{\theta\theta}^i$ can be interpreted as the vector ghost in other topological gravity constructions, while the remaining

torsion components contain new fields needed to realize local supersymmetry. Whatever the solution, this program promises new insight into the universal bundle for gravity. All of this is under active investigation and we hope to report on this program in a future publication.

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