On the Universal Bundle for Gravity

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Abstract

We construct a supergravity type theory based on a superspace whose odd directions consist of a vector, together with a scalar representing a topological BRST shift symmetry. As such, the resulting theory is a theory of topological gravity. The gravitino is interpreted as a ghost field for this shift symmetry and plays the usual role of gauge field for local supersymmetry. Our construction is within the bundle of frames approach to superspace where covariant torsion constraints are analyzed, and we find that the resulting theory contains additional fields which are not present in existing theories of topological gravity. In particular, a minimal solution exists which contains a BRST invariant scalar field. Vector supergravity represents a field theoretic model of a universal bundle for gravity, and we show how the vector supersymmetry operator can be used to generate a hierarchy of observables.

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1 Introduction

It has become clear that vector supersymmetry is a generic feature of topological field theories, and in [1, 2] we have initiated a study of this feature from a superspace perspective, where supersymmetry is placed on an equal footing with BRST. Whereas the supersymmetry in Schwarz-type topological field theories [3]-[7] - these include Chern-Simons and BF theories - is present only after all gauge degrees of freedom have been fixed in a special way, it is possible to establish a supersymmetry in Witten-type theories at the level where only the topological shift symmetry has been gauge fixed. This we presented in [2], where a correspondence was established with the universal bundle construction of Atiyah and Singer [8]. Moreover, it was shown that the supersymmetry operator could be used to connect different levels of the curvature of the universal bundle, and that the observables in Donaldson theory [9] could be generated by successive applications of this operator to the basic observable with highest ghost degree. Our superspace formalism has a natural extension to vector supergravity, and in this paper we present the solution.

We construct vector supergravity in terms of an orthonormal bundle of frames over a superspace which has a vector and a scalar as its Grassmann odd directions. The odd scalar direction is interpreted in terms of a BRST topological shift symmetry. As in the global case [2], this construction is as generic as the universal bundle of Atiyah and Singer, and can be done in any number of dimensions. The fundamental fields in our solution consist of the vielbein h_μ^i , a gravitino ψ_μ^a (now a is a vector label; there are no spinors), together with an $O(n)$ scalar field ϕ^{ij} (antisymmetric) in the adjoint representation. This multiplet corresponds to the minimal solution, although there is the freedom to include an additional pair of fields, V^i and S^a , which are Grassmann even and odd with ghost numbers 2 and 3, respectively. The spin connection ω_μ^{ij} of course enters the picture, though it is not an independent field. Thus it should be stressed that we are not simply dealing here with an $O(n)$ version of the fundamental multiplet which appears in the gauge theory case.

The key to our analysis of the Bianchi identities is to discard all torsion components which have ghost degree less than or equal to zero. The remaining pieces can be quickly analyzed and we find representations based on the fields above. The field ϕ^{ij} appears as the ghost number 2 component of the curvature, and is not present in other formulations of topological gravity [10]-[15]. This field is BRST invariant, and can thus be used to construct an observable for the theory. As in the case of gauge theory, one can then generate a hierarchy of observables by successive applications of the vector supersymmetry operator. By analogy with the global case, the curvature of this model provides a field theoretic realization for the curvature of a universal bundle for gravity, and the observables can be neatly expressed in terms of the universal curvature.

2 Bundle of Frames on Superspace

Our superspace is locally parametrized by the coordinates $z^\Lambda = (x^\mu, \theta^\alpha, \theta)$, where x^μ is used to denote the Grassmann even coordinates, while θ^α denotes an equal number of odd coordinates and θ is reserved for an additional odd BRST coordinate. Notice that

we use lower case Greek indices from the beginning of the alphabet to label the odd coordinates, while those from the middle label the even directions; a capital Greek index denotes generically any of the coordinates.

Over a supermanifold, we can consider a bundle of orthonormal frames by analogy with the familiar construction in conventional geometry. We let $h^{A_1} = dz^A h_{\Lambda}^{A_1}$ denote such a frame which clearly generalizes the notion of vielbein. The structure group of this bundle is an orthogonal group $O(n)$, where n is the number of even coordinates. An index $A_1, B_1, \dots$ is used to denote the horizontal directions in the bundle, and objects with these labels transform in definite representations of the structure group,

$$D_{A_0} h^{A_1} = f_{A_0 B_1}^{A_1} h^{B_1} , \quad (1)$$

where D_{A_0} is a generator and satisfies the Lie algebra,

$$[D_{A_0}, D_{B_0}] = f_{B_0 A_0}^{C_0} D_{C_0} . \quad (2)$$

We will use a lower case Latin index from the beginning of the alphabet to denote an odd “flat” direction, and those near the middle ($i, j, k, \dots$) for an even direction. Notice also that the A_0 subscript labels vertical directions in the bundle. As in supergravity, the even labels transform as vectors under the $O(n)$ group, $f_{A_0 j}^i = f_{[rs]j}^i = \frac{1}{2} \delta_{[r}^i \delta_{s]j}$. However, instead of the h^a transforming as spinors, we will require them to also transform as vectors. This is simply because we seek to realize a Grassmann odd vector symmetry.

We will also need to introduce a spin connection in this bundle, which we denote in local coordinates by $h_{\Lambda}^{A_0}$, and form the covariant derivative

$$D_{\Lambda} = \partial_{\Lambda} - h_{\Lambda}^{A_0} D_{A_0} . \quad (3)$$

The spin connection, together with the frame fields, allows us to define both curvature and torsion, and in our conventions take the form

$$[D_{\Lambda}, D_{\Sigma}] = -R_{\Lambda\Sigma}^{A_0} D_{A_0} , \quad (4)$$

$$T_{\Lambda\Sigma}^{A_1} = D_{\Lambda} h_{\Sigma}^{A_1} - (-1)^{\Lambda\Sigma} D_{\Sigma} h_{\Lambda}^{A_1} . \quad (5)$$

It is straightforward to incorporate a Yang-Mills connection into this picture and consider a bundle whose structure group is a product of $O(n)$ with a compact group G . In this case, the label A_0 will denote both of the vertical directions, and when we need to specify more precisely, we will use $A_0 = [rs]$ when referring to a $O(n)$ direction, and $I, J, K, \dots$ for the group G . It is also conventional to denote the spin and Yang-Mills connections respectively by $h_{\Lambda}^{[rs]} = \omega_{\Lambda}^{[rs]}$ and $h_{\Lambda}^I = A_{\Lambda}^I$, and $R_{\Lambda\Sigma}^I = F_{\Lambda\Sigma}^I$ for the Yang-Mills curvature.

The supervielbein also allows one to relate objects with curved and flat indices (note that we reserve θ for a flat BRST index),

$$\begin{aligned} D_{\Lambda} &= h_{\Lambda}^{A_1} D_{A_1} = h_{\Lambda}^i D_i + h_{\Lambda}^a D_a + h_{\Lambda}^{\theta} D_{\theta} \\ R_{\Lambda\Sigma}^{A_0} &= (-1)^{B_1\Sigma} h_{\Lambda}^{B_1} h_{\Sigma}^{C_1} R_{B_1 C_1}^{A_0} \\ T_{\Lambda\Sigma}^{A_1} &= (-1)^{B_1\Sigma} h_{\Lambda}^{B_1} h_{\Sigma}^{C_1} T_{B_1 C_1}^{A_1} , \end{aligned} \quad (6)$$

and the entire algebraic picture can be cast very compactly as

$$[D_A, D_B] = F_{BA}^C D_C , \quad (7)$$

where $F_{B_1 A_1}^{C_1} = T_{B_1 A_1}^{C_1}$, $F_{B_1 A_1}^{C_0} = R_{B_1 A_1}^{C_0}$, $F_{B_0 A_1}^{B_1} = f_{B_0 A_1}^{B_1}$, and so on.

The Bianchi identities follow from the super Jacobi identity which represents the associativity of the differential operators, and with these conventions they take the form

$$\begin{aligned} 0 &= \sum_{(cyclic)} (-1)^{A_1 B_1} [D_{A_1} T_{B_1 C_1}^{D_1} - T_{C_1 A_1}^{E_1} T_{B_1 E_1}^{D_1} + R_{C_1 A_1}^{E_0} f_{E_0 B_1}^{D_1}] \\ 0 &= \sum_{(cyclic)} (-1)^{A_1 B_1} [D_{A_1} R_{B_1 C_1}^{D_0} - T_{C_1 A_1}^{E_1} R_{B_1 E_1}^{D_0}] . \end{aligned} \quad (8)$$

The sum here is over the three cyclic permutations of the lower indices.

The algebra which represents all of the local symmetries of the theory is given by

$$\begin{aligned} \delta h_\Lambda^A &= D_\Lambda \epsilon^A - h_\Lambda^{B_1} \epsilon^C F_{B_1 C}^A \\ \delta F_{BC}^A &= \epsilon^E D_E F_{BC}^A , \end{aligned} \quad (9)$$

where all of the parameters have a “flat” index. Note that $A, B, \dots$ with no subscript denotes either a vertical or horizontal direction. The gravitino field is sometimes more conventionally denoted by ψ_μ^a . The spin connection of course enters into this algebra, however, it is not an independent field as it can generically be expressed in terms of the vielbein and the torsion component $T_{\mu\nu}^i$. In our construction of vector supergravity, this torsion component will vanish.

The superspace we have constructed here mirrors the standard formulation used in conventional supergravity [16, 17]. Other essentially equivalent approaches can be found in numerous books dealing with supergravity, e.g. [18], and one will be able to quickly reproduce our results in any of these systems.

3 Construction of Vector Supergravity

The superspace formalism of the previous section provides a natural framework for constructing supergravity theories. The power of this technique has already been established in [2], where a covariant solution to the global Bianchi identities was obtained which yielded an important connection with the universal bundle of Atiyah and Singer [8]. Before presenting the solution to topological gravity, let us briefly recall the state of affairs in the globally supersymmetric case.

The global supersymmetry algebra which is present in Witten-type theories consists of a topological BRST shift operator, in addition to a vector supersymmetry operator. The algebra of interest is then modelled by choosing a non-vanishing torsion constraint $T_{a\theta}^i = \delta_a^i$, together with appropriate covariant curvature constraints. For the global algebra, one also specifies that the vielbein describes a flat geometry, $h_\mu^i = \delta_\mu^i$, and the potential gauge fields for the BRST and SUSY symmetries are taken to be trivial, $\psi_\mu^a = 0$, $h_\mu^\theta = 0$. The consistency of the torsion and curvature constraints can then be verified through an analysis of the Bianchi identities.

To construct a theory of vector supergravity, one begins by allowing the vielbein h_μ^i , and its gravitino ψ_μ^a , to be arbitrary; in the present analysis, we are maintaining the triviality of the BRST gauge field. Including the gravitino corresponds to localizing the

vector supersymmetry, and hence leads one to a theory of vector supergravity. The crucial step is then to decide on an appropriate set of covariant torsion constraints. The most promising road to take, and the one which yields the present solution, is to discard all torsion components which have negative or zero ghost number. We do, however, retain the $T_{a\theta}^i$ piece, in order to retrieve the global result in the limit of flat vielbein and vanishing gravitino. Furthermore, in order to ensure that the vielbein has a simple shift symmetry under BRST, we enforce the vanishing of the ghost number one component $T_{j\theta}^i$. One final piece $T_{\theta\theta}^a$, of ghost number one, can also be set to zero; this simplifies the analysis somewhat.

Without further ado, let us present the non-vanishing torsion components: we have the global piece

$$T_{a\theta}^i = \delta_a^i , \quad (10)$$

in addition to the set

$$T_{\theta\theta}^i, T_{\theta\theta}^a, T_{i\theta}^a, T_{ij}^a . \quad (11)$$

It should be noted that the final component in this set, T_{ij}^a , is not an independent field, and can be expressed in terms of derivatives of the gravitino. A slight note of warning is required in the use of (10); the i and a indices label different spaces and thus caution must be exercised accordingly.

At this juncture, one must simply analyze the resulting set of Bianchi identities. One can regard this formalism as providing a bootstrap approach to obtaining a solution. Given the above torsion constraints, one examines each of the identities in turn, and checks for consistency. The non-trivial identities are given by:

$$\begin{aligned} (abi)^j : 0 &= R_{ab}{}^i{}_j , \\ (a\theta i)^j : 0 &= R_{a\theta}{}^i{}_j , \\ (aij)^k : 0 &= R_{a[i}{}^k{}_{j]} , \\ (\theta ij)^k : 0 &= R_{\theta[i}{}^k{}_{j]} - T_{ij}^a \delta_a^k , \\ (\theta\theta i)^j : 0 &= D_i T_{\theta\theta}^j + 2T_{i\theta}^a \delta_a^j + R_{\theta\theta}{}^j{}_i , \\ (\theta\theta\theta)^i : 0 &= D_\theta T_{\theta\theta}^i - T_{\theta\theta}^a \delta_a^i , \\ (\theta\theta a)^i : 0 &= D_a T_{\theta\theta}^i , \\ (\theta\theta\theta)^a : 0 &= D_\theta T_{\theta\theta}^a + T_{\theta\theta}^i T_{i\theta}^a , \\ (\theta\theta a)^b : 0 &= D_a T_{\theta\theta}^b + \delta_i^b \delta_a^j R_{\theta\theta}{}^i{}_j + 2T_{i\theta}^b \delta_a^i , \\ (\theta\theta i)^a : 0 &= 2D_\theta T_{i\theta}^a + D_i T_{\theta\theta}^a - T_{\theta\theta}^j T_{ij}^a , \\ (a\theta i)^b : 0 &= D_a T_{i\theta}^b + \delta_j^b \delta_a^k R_{\theta i}{}^j{}_k - \delta_a^j T_{ij}^b , \\ (\theta ij)^a : 0 &= D_\theta T_{ij}^a + D_{[i} T_{j]\theta}^a , \\ (aij)^b : 0 &= D_a T_{ij}^b + \delta_k^b \delta_a^m R_{ij}{}^k{}_m . \end{aligned} \quad (12)$$

We can now simplify matters by letting

$$T_{\theta\theta}^i = V^i , T_{\theta\theta}^a = S^a , T_{i\theta}^a = \frac{1}{2} \delta^{aj} (\phi_{ij} - D_i V_j) , \quad (13)$$

where ϕ_{ij} is antisymmetric in its indices. It is then clear from (12) that the BRST and SUSY transformations of the three independent fields V^i, S^a, ϕ_{ij} are indeed consistently

specified to be:

$$\begin{aligned}
D_\theta V^i &= S^a \delta_a^i , \\
D_a V^i &= 0 , \\
D_\theta S^a &= -\frac{1}{2} \delta^{ai} V^j (\phi_{ji} - D_j V_i) , \\
D_b S^a &= \delta_b^i \delta^{aj} D_i V_j , \\
D_\theta \phi_{ij} &= V^k R_{\theta kij} , \\
D_a \phi_{ij} &= 2\delta_a^k R_{\theta kij} .
\end{aligned} \tag{14}$$

In addition, the non-zero components of curvature (besides R_{ijkl}) are given by

$$\begin{aligned}
R_{\theta kij} &= \frac{1}{2} (\delta_{ka} T_{ij}^a - \delta_{a[i} T_{j]k}^a) , \\
R_{\theta\theta ij} &= \phi_{ij} .
\end{aligned} \tag{15}$$

The transformations of the vielbein, gravitino and spin connection can be read off from (9):

$$\begin{aligned}
\delta h_\mu^i &= \epsilon \psi_\mu^a \delta_a^i , \\
\delta \psi_\mu^a &= D_\mu \epsilon^a - \epsilon h_\mu^i T_{i\theta}^a , \\
\delta \omega_\mu^{rs} &= -\epsilon h_\mu^i R_{i\theta}^{rs} .
\end{aligned} \tag{16}$$

Although the torsion and curvature components T_{ij}^a , $R_{i\theta}^{rs}$, and R_{ij}^{rs} are not independent fields, their transformations are simple expressions, and we record them here:

$$\begin{aligned}
D_\theta T_{ij}^a &= -\frac{1}{2} [R_{ij}^{rs} \delta_r^a V_s + D_{[i} \phi_{j]k} \delta^{ka}] , \\
D_b T_{ij}^a &= -R_{ij}^{rs} \delta_r^a \delta_{sb} , \\
D_\theta R_{\theta i}^{rs} &= \frac{1}{2} (D_i R_{\theta\theta}^{rs} - V^j R_{ij}^{rs}) , \\
D_a R_{\theta i}^{rs} &= -\delta_a^j R_{ij}^{rs} , \\
D_\theta R_{ij}^{rs} &= -D_{[i} R_{j]\theta}^{rs} , \\
D_a R_{ij}^{rs} &= 0 .
\end{aligned} \tag{17}$$

One can now proceed and check that the above multiplet of fields $h_\mu^i, \psi_\mu^a, \phi_{ij}, V^i, S^a$ provides a representation for a BRST-SUSY algebra, with the following closure properties:

$$[\delta_{SUSY}(\epsilon^a), \delta_{BRST}(\epsilon)] = \delta_{DIFF}(\epsilon^i = \delta_a^i \epsilon \epsilon^a) , \tag{18}$$

$$\begin{aligned}
[\delta_{BRST}(\epsilon_1), \delta_{BRST}(\epsilon_2)] &= \delta_{DIFF}(\epsilon^i = -\epsilon_1 \epsilon_2 V^i) + \delta_{SUSY}(\epsilon^a = -\epsilon_1 \epsilon_2 S^a) \\
&+ \delta_{O(n)}(\epsilon^{rs} = -\epsilon_1 \epsilon_2 \phi^{rs}) .
\end{aligned} \tag{19}$$

In addition, the $[\delta_{SUSY}, \delta_{SUSY}]$ algebra remains Abelian, as in the global case [2]. Indeed, given the non-zero torsion and curvature components above, this algebra is immediately specified from (7).

We have thus succeeded in constructing a theory of vector supergravity. Since the solution involves a localization of a vector supersymmetry, one is naturally led to include a vector gravitino ψ_μ^a , which then appears as the BRST topological shift partner for the vielbein. As such, the present theory thus represents a theory of topological gravity. It turns out, however, that the above solution does not represent the minimal multiplet. It is easy to check that we may consistently abandon the V^i, S^a fields, leaving us with the basic set $(h_\mu^i, \psi_\mu^a, \phi_{ij})$. It is then clear that the BRST algebra closes up to an $O(n)$ rotation. More importantly, the ϕ_{ij} field is now BRST invariant, and as such can be used to construct a basic observable for the theory. In the following section, we shall explicitly demonstrate this, and show how the vector supersymmetry operator can then be used to generate a hierarchy of observables.

We should remark, at this point, on the nature of the field content obtained in the above analysis. In addition to the gauge fields, we have three independent fields V^i, S^a, ϕ_{ij} . The first of these can clearly be identified with a similar field which appears in other topological gravity constructions. However, the introduction of the two fields S^a and ϕ_{ij} , are obviously a new feature. It should be stressed that the ϕ_{ij} field in fact corresponds to a scalar field in the adjoint representation of $O(n)$, and as such is the analogue of the BRST invariant $SU(n)$ -adjoint scalar field ϕ which appears in Witten-type gauge theory [9, 19]. We recall that the torsion component $T_{i\theta}^a$, of ghost number 1, was set to zero in the above analysis. It may be possible to relax this constraint, and still obtain a solution. If such a component can be regarded as an independent field, then perhaps it would play the role of a partner for the ϕ_{ij} field.

It is straightforward to consider the coupling of the above gravity multiplet to the gauge multiplet which is present in Witten-type models. The only modification to the algebra presented in [2] are in the BRST transformations of ϕ and ψ_i , and these become (suppressing the Yang-Mills labels),

$$\begin{aligned} D_\theta \phi &= -\frac{1}{2} V^i \psi_i , \\ D_\theta \psi_i &= -D_i \phi - \frac{1}{2} V^j F_{ij} . \end{aligned} \tag{20}$$

One then simply adds the term $\delta_{YM}(\epsilon^I = 2\epsilon_1 \epsilon_2 \phi^I)$ to the above commutator of two BRST transformations. In particular, we note that if we now restrict our attention to the minimal multiplet, then the BRST invariance of ϕ is maintained; the BRST transformation of ψ_i also assumes its global form.

4 The Universal Bundle for Gravity and Observables

Let us now concentrate our attention on the minimal solution consisting of the basic multiplet $(h_\mu^i, \psi_\mu^a, \phi^{\tau s})$. In the closing paragraphs of [8], the extension of the universal bundle program to the space of metrics was outlined. It is this mathematical framework for which vector supergravity provides a field theoretic representation. While this connection remains to be explored in detail, and the introduction of the supersymmetry gauge field

introduces a new twist into the picture, one can nevertheless consider the superspace curvature we computed in connection with characteristic classes and potential observables in topological gravity.

In [2], we showed that the SUSY operator connected different levels of the universal curvature; precisely the same situation transpires here. Under the action of the $(1, -1)$ operator δ_{SUSY} , one can simply read off the mapping which schematically is

$$\delta_{SUSY} : R_{\theta\theta} \rightarrow R_{\theta i} \rightarrow R_{ij} \rightarrow 0 . \quad (21)$$

This is immediately evident from the transformations given in (14) and (17). Here, the $(1, -1)$ refers to the fact that the SUSY operator is a 1-form, with ghost number -1 . As such, it is naturally designed for the above purpose.

One of the most important features of a topological field theory is the construction of a set of metric independent observables. In the present theory, we can begin with the BRST invariant scalar field ϕ^{rs} , and form the observable

$$W^{(0,4)} = \frac{1}{2} tr \phi^2 = \frac{1}{2} tr R_{\theta\theta}^2 , \quad (22)$$

where the superscript indicates that it is a zero-form of ghost number four, and the trace represents an $O(n)$ trace. As in the case of gauge theory, one can now successively apply the vector supersymmetry operator to $W^{(0,4)}$, thereby generating a complete hierarchy of observables. One obtains the following results:

$$\begin{aligned} D_a W^{(0,4)} &= \delta_a^i W_i^{(1,3)} , \\ D_a W_i^{(1,3)} &= \delta_a^j W_{ji}^{(2,2)} , \\ D_a W_{ij}^{(2,2)} &= \delta_a^k W_{kij}^{(3,1)} , \\ D_a W_{ijk}^{(3,1)} &= \delta_a^l W_{lijk}^{(4,0)} , \\ D_a W_{ijkl}^{(4,0)} &= 0 , \end{aligned} \quad (23)$$

where the observables W are explicitly given as:

$$\begin{aligned} W_i^{(1,3)} &= 2 tr R_{\theta i} R_{\theta\theta} , \\ W_{ij}^{(2,2)} &= 4 tr (R_{\theta i} R_{\theta j} + \frac{1}{2} R_{\theta\theta} R_{ij}) , \\ W_{ijk}^{(3,1)} &= 4 tr (R_{\theta i} R_{jk} + R_{\theta j} R_{ki} + R_{\theta k} R_{ij}) , \\ W_{ijkl}^{(4,0)} &= 4 tr (R_{ij} R_{kl} + R_{ik} R_{lj} + R_{il} R_{jk}) . \end{aligned} \quad (24)$$

It is straightforward to re-write these observables in curved indices, or in differential form notation.

One can construct a characteristic class by forming $tr R^2$, where R represents the entire curvature of superspace. By construction, the Bianchi identities ensure that this object is a closed 4-form. As we have seen, the present theory has three non-vanishing components of curvature, which are given above. In Donaldson theory, this characteristic class also represents the entire set of observables [20]-[22] beginning with the ghost number 4 piece proportional to $tr \phi^2$, and the Bianchi identity leads to the descent equations defining the observables. Clearly, a similar construction applies here.

5 Remarks

We have constructed a theory of vector supergravity based on a geometrical approach to superspace. The resulting theory provides a model for topological gravity in any dimension, and in addition a field theoretic representation of the universal bundle for gravity. The basic principle behind this construction is to localize the underlying supersymmetry which is present in Witten-type gauge theory. Clearly, many issues deserve further investigation.

We recall that the BRST charge of the minimal theory is nilpotent up to local $O(n)$ rotations. In a fully gauge fixed version of the theory, we will need to incorporate the ghosts for all of the local symmetries, including this $O(n)$ invariance. In the case of Donaldson theory, it has been shown that the basic observable $\text{tr } \phi^2$ is BRST exact when the Yang-Mills ghosts are included [23]. Nevertheless, the non-triviality of the observables is still maintained. The non-triviality of the observables presented above requires further investigation. In particular, it has been established in [24] that the characteristic class $\text{tr } R^2$ is non-trivial only in dimensions $4k - 2$. Apart from these considerations, however, the existence of a vector supersymmetry connecting different levels of curvature is certainly a new feature, and may possibly provide new insight into these constructions.

Another fruitful direction could be to incorporate an explicit conformal invariance into the model. One could also study the properties of the present model in two dimensions, and compare with the results of [25, 26], for example. In the same spirit that conventional supergravity can be regarded as a square root of Einstein gravity, it may be of interest to examine the possibility of a similar interpretation here. Recently, this question has been studied in relation to Chern-Simons gravity, in [27].

In many respects, BRST invariance is a fundamental symmetry in topological field theories. However, the existence of a supersymmetry in a model, together with its localization into a supergravity theory is always worthy of study, and in many cases leads to interesting structures. One may regard the present theory of vector supergravity as providing a paradigm for interesting topological gravity models.

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