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PHASE TRANSITION IN DENSE NUCLEAR MATTER WITH QUARK AND GLUON CONDENSATES

John Ellis

Theory Division, CERN,
23 Geneva, Switzerland

Joseph I. Kapusta and Keith A. Olive

School of Physics and Astronomy
University of Minnesota, Minneapolis, MN 55455

ABSTRACT

Nuclear matter is expected to modify the expectation values of the quark and gluon condensates. We utilize the chiral and scale symmetries of QCD to describe the interaction between these condensates and hadrons. We solve the resulting equations self-consistently in the relativistic mean field approximation. In order that these QCD condensates be driven towards zero at high density their coupling to sigma and vector mesons must be such that the masses of these mesons do not decrease with density. In this case a physically sensible phase transition to quark matter ensues.

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It is expected that a phase boundary exists between normal nuclear matter and quark-gluon matter. Furthermore, one expects that this boundary may be crossed at high temperatures and/or high baryon densities. High-temperature transitions are most relevant for cosmological studies, whilst high baryon density transitions are important for the structure and properties of neutron stars. Heavy-ion collisions in the laboratory may probe both extremes. Recently [1] an effective Lagrangian approach to the QCD phase transition has been explored utilizing the underlying scale symmetry of massless QCD. Scale (and chiral) symmetries are spontaneously broken by gluon (and quark) condensates in the vacuum. In ref. [1] it was argued that the finite-temperature behavior of the gluon condensate lent itself to the interpretation as an order parameter of the QCD phase transition.

This approach has been pursued in the description of nuclear matter at finite densities [2-4]. In ref. [2] a model of baryons interacting via the exchange of repulsive vector mesons in the relativistic mean field approximation was considered, with the burden of nuclear attraction given solely to scalar gluonium (fluctuation of the gluon condensate). Because the nuclear compressibility K came out too large (1600 MeV) compared to accepted values (200 - 300 MeV) this model was phenomenologically unsatisfactory. A subsequent attempt to incorporate the effects of broken scale symmetry included both the σ -field (representing attractive two-pion exchange) and scalar gluonium χ [3]. The model was successful from the point of view of having an acceptable range of values of compressibility, and led to a physically sensible phase transition to quark matter. However, it had the undesirable feature that the gluon condensate increased at high density instead of decreasing toward zero. In ref. [4] an additional scaling was imposed on the meson-nucleon coupling with the net effect that there was no χ -dependence in the single-particle energies. No attempt was made to fit the properties of ground state nuclear matter. The model considered did, however, have the desirable outcome that the gluon condensate decreased uniformly with density, and a sensible phase transition to quark matter occurred for a bag constant $B \geq 400 \text{ MeV/fm}^3$. None of the above papers [2-4] incorporated the effects of quark condensates and non-zero current-quark masses.

The purpose of this paper is to combine all the features mentioned above, namely, to fit the ground-state properties of nuclear matter, to include the effects of both scale and chiral symmetry, to examine under what conditions the condensates are driven to zero at high density, and finally to insure that a sensible phase transition to quark matter occurs at high density.

The Lagrangian we consider is

$$\begin{aligned} \mathcal{L} = & \bar{\psi}(i\gamma^\mu\partial_\mu - m_N^*(\chi) + g_\sigma\sigma - g_\omega\omega_\mu\gamma^\mu)\psi - \frac{1}{4}(\partial_\mu\omega_\nu - \partial_\nu\omega_\mu)^2 \\ & + \frac{1}{2}[m_\omega^*(\chi)]^2\omega_\mu\omega^\mu + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}[m_\sigma^*(\chi)]^2 - \frac{1}{3}bm_N^*(\chi)(g_\sigma\sigma)^3 \\ & - \frac{1}{4}c(g_\sigma\sigma)^4 + \frac{1}{2}(\partial_\mu\chi)^2 - V(\chi). \end{aligned} \quad (1)$$

This has nucleons (ψ) interacting via repulsive vector mesons (ω_μ) and an attractive scalar meson (σ) with nonlinear terms in the σ -potential. This scalar meson represents nuclear attraction really due to two-pion exchange: it is not the σ -meson of the σ -model [5]. Scalar gluonium χ has been introduced to reproduce the effects of the QCD trace anomaly [6]. The potential $V(\chi)$ and the χ -dependence of the nucleon and meson masses each merit more discussion.

The potential $V(\chi)$ is constructed [7] to mimic the explicit breaking of scale symmetry due to the quark masses and the QCD scale anomaly. To reproduce the divergence

$$\partial_\mu S^\mu = \sum_q m_q \bar{q}q - \frac{\beta(g)}{2g} F_{\mu\nu}^a F_a^{\mu\nu} \quad (2)$$

of the scale current S_μ , one chooses

$$V(\chi) = B_q \left[\left(\frac{\chi}{\chi_0} \right)^3 - \frac{1}{4} \right] + B_g \left[4 \left(\frac{\chi}{\chi_0} \right)^4 \ln \left(\frac{\chi}{\chi_0} \right) - \left(1 + \frac{3B_q}{4B_g} \right) \left(\frac{\chi}{\chi_0} \right)^4 + 1 \right]. \quad (3)$$

We take $B_g = 432 \text{ MeV/fm}^3$ as the contribution of the gluon condensate as extracted from QCD sum rules [8], whilst B_q is extracted from the quark condensates following [1]:

$$B_q = \sum_q m_q < 0 | \bar{q}q | 0 >. \quad (4)$$

As a first approximation we assume $< 0 | \bar{u}u | 0 > = < 0 | \bar{d}d | 0 > = < 0 | \bar{s}s | 0 >$ and use the sum rules

$$f_\pi^2 m_\pi^2 = (m_u + m_d) < 0 | \bar{q}q | 0 > \quad (5)$$

$$f_\pi^2 m_{K^+}^2 = (m_u + m_s) < 0 | \bar{q}q | 0 > \quad (6)$$

$$f_\pi^2 m_{K^0}^2 = (m_d + m_s) < 0 | \bar{q}q | 0 > \quad (7)$$

to find

$$B_q = \frac{1}{2} f_\pi^2 (m_\pi^2 + 2m_K^2) = 286 \text{ MeV/fm}^3. \quad (8)$$

The other constants in the potential (3) have been chosen so that $V(\chi_0) = 0$ and $V'(\chi_0) = 0$. The numerical value of χ_0 is not needed here, as only the ratio χ/χ_0 appears. The effective bag constant B is the energy density difference between the perturbative and true non-perturbative vacua:

$$B = V(0) - V(\chi_0) = B_g - \frac{1}{4} B_q = 360.5 \text{ MeV/fm}^3. \quad (9)$$

We see that the effects of the quark condensates on B are not negligible.

In previous works [2-4] it was assumed that $B_q = 0$, that is, the quark condensate had been neglected. In that case, the nucleon mass appearing in the Lagrangian was scaled,

$$m_N^*(\chi) = (\chi/\chi_0) m_N \quad (10)$$

where m_N is the nucleon mass in the usual non-perturbative vacuum. In fact, the chiral-symmetry-breaking contributions to the nucleon and other baryon masses m_B are also not negligible. Decomposing

$$m_B = < B | \mathcal{H}_{QCD} | B > = \sum_q m_q < B | \bar{q}q | B > + < B | \mathcal{H}_{QCD}^0 | B > \quad (11)$$

we expect that the SU(3)- and chiral-invariant piece $< B | \mathcal{H}_{QCD}^0 | B > \equiv m_0$ scales with χ as in (10)

$$m_0^*(\chi) = (\chi/\chi_0) m_0 \quad (12)$$

while the SU(3)- and chiral-symmetry breaking piece $\sum_q m_q < B | \bar{q}q | B > \equiv \delta m_B$ should be independent of χ in first approximation,

$$\delta m_B^*(\chi) = \delta m_B. \quad (13)$$

The behaviours (12,13) as functions of χ are motivated by the scaling properties of the chirally-symmetric and symmetry-breaking pieces of $\mathcal{H}_{QCD}$, respectively, and are consistent with our

understanding of the chiral Skyrme model of baryons [9]. The relative magnitudes of m_0 (12) and δm_B (13) are debatable. For definiteness, we use the estimates of Ref. [10],

$$\hat{m} < p|\bar{s}s|p > \simeq 15 \text{ MeV}, \quad \hat{m} \equiv \frac{1}{2}(m_u + m_d) \simeq \frac{1}{23}m_s \quad (14)$$

so that

$$m_s < p|\bar{s}s|p > \simeq 354 \text{ MeV} \quad (15)$$

and

$$\hat{m} < p|(\bar{u}u + \bar{d}d)|p > \simeq 57 \text{ MeV} \quad (16)$$

which implies, if $< p|\bar{u}u|p > \simeq < p|\bar{d}d|p >$, that

$$\delta m_p = \sum_q m_q < p|\bar{q}q|p > \simeq 400 \text{ MeV}. \quad (17)$$

It may well be that (17) is an overestimate, but we use it in eqs. (11-13) as a counterpoint to our previous assumption (10). Thus for the nucleons we assume

$$m_N^*(\chi) = 400 + (\chi/\chi_0) 540 \text{ MeV} \quad (18)$$

for the purposes of this paper.

The χ -dependence of the effective ω -mass is crucial to the high density behavior of the QCD condensates. It can be understood this way. In the relativistic mean-field approximation one allows the meson fields to acquire constant average values, $\bar{\sigma}$, $\bar{\omega}_0$, $\bar{\chi}$, and neglects fluctuations about these average values. The nucleon single-particle energy is

$$E = \sqrt{\mathbf{p}^2 + (m_N^*(\bar{\chi}) - g_\sigma \bar{\sigma})^2} + \left[\frac{g_\omega}{m_\omega^*(\bar{\chi})} \right]^2 n, \quad (19)$$

where n is the baryon density. In the limit of zero quark masses and perfect scaling the effective ω mass is proportional to $\bar{\chi}$. Then it is energetically impossible for the average value of χ to be driven to zero, as was found in ref. [3]. We thus discard perfect scaling. In the absence of any other information solidly based on QCD one would be tempted to scale the meson masses in the same way as the nucleon, namely [11]

$$m_{meson}^*(\bar{\chi})/m_{meson} = m_N^*(\bar{\chi})/m_N \quad (20)$$

and where $m_N^*(\bar{\chi})$ is given by eq. (18). We call this case 1. However, it is apparent from the nucleon single-particle energy that it would still be energetically unfavorable to develop a small average value of χ . Another possibility is that the ω and σ masses remain independent of χ to first approximation, which we call case 2. This possibility has some support in the literature. At finite temperature, where the most work on these problems has been done, lattice simulations [12], the Nambu-Jona-Lasinio model [13], and an effective interaction model based on the dilute instanton gas [14] all predict that the pion mass remains roughly temperature-independent. If so, one would expect the σ -mass which parametrizes the $\pi - \pi$ continuum also to be roughly constant. Moreover, QCD sum rule estimates predict approximately constant vector-meson masses at finite temperature, although what happens very near to the critical temperature is uncertain [15]. Here we take the point of view that nothing is settled, and we consider cases 1 and 2 as interesting possibilities. As we shall see, the results are quite different.

In the relativistic mean field approximation [16] nucleons move independently in the average meson fields $\bar{\sigma}$, $\bar{\omega}_0$ and $\bar{\chi}$. These average fields are determined self-consistently by minimizing

the energy density

$$\begin{aligned}\mathcal{E} = & \mathcal{E}_{FG}(m_N^{eff}, k_F) + \frac{1}{2} [m_\omega^*(\bar{\chi})]^2 \bar{\omega}_0^2 - \frac{1}{2} [m_\sigma^*(\bar{\chi})]^2 \bar{\sigma}^2 \\ & - \frac{1}{3} b m_N^*(\bar{\chi}) (g_\sigma \bar{\sigma})^3 - \frac{1}{4} c (g_\sigma \bar{\sigma})^4 - V(\bar{\chi})\end{aligned}\quad (21)$$

at fixed baryon density, subject to the constraint

$$[m_\omega^*(\bar{\chi})]^2 \bar{\omega}_0 = g_\omega n. \quad (22)$$

Here, $\mathcal{E}_{FG}$ is the noninteracting Fermi gas expression for nucleons with effective mass

$$m_N^{eff} = m_N^*(\bar{\chi}) - g_\sigma \bar{\sigma} \quad (23)$$

and Fermi momentum k_F determined by

$$n = \frac{2k_F^3}{3\pi^2}. \quad (24)$$

The only required parameters are b , g_ω/m_ω , g_σ/m_σ and c . The first three of these are adjusted to reproduce the ground state density ($n_0 = 0.153$ baryons/fm³), the binding energy (16.3 MeV per baryon) and the Landau mass ($0.83 m_N$), just as in ref. [2]. The parameter c controls the compressibility K . We fix it to be 0.0146, again as in ref. [2], giving $K = 287$ MeV. Smaller values of c give smaller values of K .

Before presenting the results of our calculations, it is important to keep in mind that we expect a phase transition to quark matter at a critical density $n_c < n_\chi$, the critical density for the scale symmetry transition from $\bar{\chi} \neq 0$ to $\bar{\chi} = 0$. Such a behavior was seen in ref. [4] only for $B \geq 400$ MeV/fm³. We recall that in Ref. 4 the single-particle constant [the coefficient of n in eq. (19)] was taken to be independent of χ , and that in ref. [3], where it varied as $\bar{\chi}^{-2}$, no transition to $\bar{\chi} = 0$ was observed. To this end, we will also calculate [2] the energy density of u , d and s quarks as a function of baryon density with an effective bag constant given by eq. (9).

In both the cases 1 and 2 introduced above, the nucleon and meson masses all remain nonzero as $\bar{\chi} \rightarrow 0$, thus avoiding singular interactions. In case 1 we find, similarly to ref. [3], no phase transition to $\bar{\chi} = 0$. The effective nucleon mass decreases continuously at high density. The gluon condensate $\bar{\chi}$ at first decreases, reaches a minimum value of about 0.97 at $3n_0$, and then increases steadily, going above 1 at higher densities. Thus, despite the presence of a nonvanishing term in the nucleon mass due to finite quark masses, there is no critical point in the hadron phase.

In case 2, however, we do get a transition to a phase where $\bar{\chi} = 0$. In Fig. 1, we show the behavior of the gluon condensate $\bar{\chi}$ as a function of baryon density. One sees clearly the discontinuity at $n = 15.5n_0$ indicating a first-order phase transition. We plot in Fig. 1 the position of the global minimum in the energy for fixed baryon density. The local minimum at $\bar{\chi} \neq 0$ actually disappears altogether at higher density, a possibility pointed out in Ref. [1]. Also shown in the figure is the behavior of the effective nucleon mass, which also shows a discontinuity at $n = 15.5n_0$.

In Fig. 2, we show the phase diagram, $\mathcal{E}$ vs. n , for nuclear matter (thick curve) and for quark matter (thin curve). The dashed line corresponds to the Maxwell construction for the first-order deconfinement phase transition. The slope of the dashed line gives the chemical potential and the intercept gives the negative of the pressure. The phase transition out of the nuclear phase occurs at about $n_{c1} = 8n_0 < n_\chi = 15.5n_0$, and goes into the quark phase at about $n_{c2} = 15n_0$.

The corresponding energy densities are $\mathcal{E}_{c1} = 1.53 \text{ GeV/fm}^3$ and $\mathcal{E}_{c2} = 3.42 \text{ GeV/fm}^3$. In this example we used $\alpha_s = 0.2$. The results change very little if we use $\alpha_s = 0$ or 0.4 instead.

The existence and properties of the expected phase boundary between normal nuclear matter and quark-gluon matter are of importance for cosmological and astrophysical applications, and the subject of exploration via heavy-ion collisions in the laboratory. There is a shortage of reliable theoretical models at high baryon densities. In this paper we have extended our earlier effective Lagrangian approach [1-4], including approximate chiral and scale symmetry, and fitting the ground-state properties of nuclear matter. We have found that the location and indeed existence of the expected phase boundary depend sensitively on the scaling properties of baryon and meson masses and couplings, which may well differ effectively from naive scaling laws. Within the uncertainties in these scaling properties, we find a physically sensible phase transition to quark matter if the effective meson masses remain independent of the gluon condensate to first approximation, while the nucleon mass decreases to about half its zero-density value, as suggested by models [10,11,12]. Specifically, we find a first-order transition such that in the Maxwell construction hadronic matter moves out of the pure nuclear phase at about $8n_0$ and becomes a pure quark phase at about $15n_0$. The numerical values of these densities are clearly sensitive to the detailed parametrization of the model, but we believe the satisfactory qualitative picture is a general one. It remains to establish on a theoretically sounder basis the scaling properties of nucleons and mesons at finite density. In this area, more QCD-based calculational schemes would be welcome.

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Figure 1: The behaviour (a) of the normalized gluon condensate parameter $\bar{\chi}$, and (b) of the effective nucleon mass m_N^{eff} , as functions of the density n , in the model described as case 2 in the text. Note the first-order transition at $n \simeq 15.5n_0$.

Figure 2: The $\mathcal{E}$ vs. n phase diagram, for nuclear matter (thick curve) and for quark matter (thin curve) in case 2 as described in the text. The dashed line corresponds to the Maxwell construction for the first-order deconfinement phase transition. The arrows indicate the transition baryon densities. The nuclear curve ends when the gluon condensate vanishes at $n \simeq 2.37 \text{ fm}^{-3}$.