LHC optics determination with proton tracks measured in the CT-PPS detectors in 2016, before TS2

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Abstract

Novel optics estimation methods have been developed for the CMS-TOTEM Precision Proton Spectrometer (CT-PPS), in order to determine the horizontal dispersion function $D_x$ and the orbit between IP5 and the Roman Pot detectors. The tools have been successfully applied at $\sqrt{s} = 13$ TeV collision energy and $\beta^* = 0.4$ m optics with $\alpha=370$ $\mu$rad horizontal crossing angle (settings valid in 2016 before Technical Stop 2 - TS2).
1 Introduction

The physics goals of the CMS-TOTEM Precision Proton Spectrometer (CT-PPS) detector require the reconstruction of the diffractive mass $M_X = \sqrt{s\xi_1 \xi_2}$ [1], where $s$ is the center-of-mass LHC energy and $\xi_1$ and $\xi_2$ are the fractional momentum losses of the two diffractively scattered protons. In Section 2.1 it is shown that the measurement of the fractional momentum loss $\xi$ means the determination of the so-called horizontal dispersion function $D_x$ of the LHC at the Roman Pot detectors (RPs).

The novel optics methods, developed for the CT-PPS detector, are the continuation of the TOTEM experiment’s optics estimation tools for the analysis of elastic scattering [2–4]. These tools have been applied for the CT-PPS alignment run at $\sqrt{s} = 13$ TeV collision energy and $\beta^* = 0.4$ m optics with $\alpha = 370$ µrad horizontal crossing angle. The results are described in Section 3.2. The most important outcome is the estimated value of the optical function $L_y$, the so-called vertical effective length.

The horizontal dispersion $D_x$ can be estimated using the point, where the vertical effective length $L_y = 0$ optical function vanishes, see Section 3.3.1. The difference of the dispersions in the two LHC beams $\Delta D_x$ can be also determined using the $\xi$-distributions of protons measured in the horizontal RPs; this is the subject of Section 3.3.2.

The measured constraints allow for the matching of the LHC optics model between IP5 and the RPs, described in Section 3.4. The obtained optics model provides the full optics information for the physics analysis. It describes the non-linearity of the proton transport, especially its dependence on the fractional proton momentum loss $\xi$, which is the main component of the inelastic kinematics reconstruction. The trajectory of the protons between IP5 and the RPs can be also derived, which is needed for the Monte Carlo and acceptance calculations.

2 LHC optics model

2.1 Proton transport description with matrices

Scattered protons are detected by the RPs after having traversed a segment of the LHC lattice containing 29 main and corrector magnets per beam.

The trajectory of protons produced with transverse vertex position$^1$ $(x^*, y^*)$ and angles $(\Theta^*_x, \Theta^*_y)$ at IP5 is described approximately by a linear formula

$$\vec{d}(s) = T(s, \xi) \cdot \vec{d}^*,$$

where $\vec{d} = (x, \Theta_x, y, \Theta_y, \xi = \Delta p/p)^T$, $p$ and $\Delta p$ denote the nominal beam momentum and the proton longitudinal momentum loss, respectively. The single-pass transport matrix

$$T = \begin{pmatrix}
  v_x & L_x & m_{13} & m_{14} & D_x \\
  \frac{dv_x}{ds} & \frac{dL_x}{ds} & m_{23} & m_{24} & \frac{dD_x}{ds} \\
  m_{31} & m_{32} & v_y & L_y & D_y \\
  \frac{dv_y}{ds} & \frac{dL_y}{ds} & \frac{dD_y}{ds} & 1 \\
  0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

is defined by the optical functions [5,6] and describes the proton transport in the vicinity of the so-called central orbit. The horizontal and vertical magnifications

$$v_{x,y} = \sqrt{\beta_{x,y}/\beta^* \cos \Delta \mu_{x,y}}$$

$^1$The ‘$^*$’ superscript indicates that the value is taken at the LHC Interaction Point 5.
and the effective lengths
\[ L_{x,y} = \sqrt{\beta_{x,y} \beta^* \sin \Delta \mu_{x,y}} \] (4)
are functions of the betatron amplitudes \( \beta_{x,y} \) and the relative phase advance
\[ \Delta \mu_{x,y} = \int_{\mathcal{P}} \frac{ds}{\beta_{x,y}} , \] (5)
and are of particular importance for proton kinematics reconstruction. The
\[ D_{x,y} = \frac{\partial x}{\partial \xi} \] (6)
and \( D_y \) matrix elements are the horizontal and vertical dispersions, respectively.

In this note the so-called beam based alignment of the detectors is always assumed, which means that
the beam appears at \( x = 0 \) [7]. The horizontal position of protons \( x(\xi) \) is a non-linear function of \( \xi \). According to Equation (6), for low-\( \xi \) values
\[ x \approx D_{x} \cdot \xi \] (7)
where the approximation takes into account the smearing due to the term \( \theta^*_x \cdot L_x \).

In case of the LHC nominal optics the coupling coefficients are, by design, equal to zero
\[ m_{13}, ..., m_{42} = 0 . \] (8)

For elastically scattered protons the interaction related contribution to longitudinal momentum loss \( \Delta p \) is 0. However, the beam protons are characterized by a momentum spread resulting from the beam longitudinal emittance and the RF configuration. For the LHC this spread is \( \delta p/p_0 = 1 \cdot 10^{-4} \) [8], which is not significant with respect to the beam momentum offset uncertainty of \( 1 \cdot 10^{-3} \) [9–12]. Therefore, the terms \( D_{x,y} \cdot (\Delta p/p) \) and \( dD_{x,y}/ds \cdot (\Delta p/p) \) of the transport equation can be neglected in the case of elastically scattered protons. Consequently, in case of elastic scattering only the upper left \( 4 \times 4 \) block of the matrix \( T \) is relevant and will be denoted with \( T_4 \).

Elastically scattered protons are relatively easy to distinguish due to their scattering angle correlations. In addition, these correlations are sensitive to the machine optics. Therefore, elastic proton-proton scattering is an ideal process to study the LHC optics.

Note also that formally the optical functions of the block matrix \( T_4 \) and the dispersion related terms appear in the same matrix (2). However, the dispersion terms describe the dependence on the momentum loss \( \xi \) and each \( \xi \) defines a new reference orbit [13, 14].

2.2 Real optics based on magnet and current databases

The proton transport matrix \( T(s; \mathcal{M}) \), calculated with MAD-X [13,14], is defined by the machine settings \( \mathcal{M} \), which are obtained on the basis of several data sources. The LHC sequence for Run 2 is used to describe the magnet lattice, while the nominal magnet strength file for a given beam optics is always updated using measured data: the currents of the magnet’s power converters are first retrieved using TIMBER, which is an application to extract data from heterogeneous databases containing information about the whole LHC infrastructure.

Then the currents are converted to magnet strengths with the LHC Software Architecture (LSA) [15] which applies for this purpose the conversion curves described by the Field Description for the LHC (FIDEL) [9].
The WISE database \cite{16} contains the imperfections (field harmonics, magnets displacement, rotations) included in $\mathcal{M}$, as well as statistical models describing the non-measured parameters' tolerances. Alignment uncertainties of the magnets are included in WISE based on measurements of the mechanical and magnetic axes. Other uncertainties for example relative and absolute measurement errors of hysteresis and power converters accuracy are also included for all magnets.

3 LHC optics estimation

3.1 Machine imperfections

The transport of protons in the vicinity of the central orbit, or any other reference orbit with a certain $\xi$, is mainly determined by the quadrupole fields of the FODO system of the LHC, while the position of the central orbit itself is determined by the distribution of the dipole fields (including the dipole fields created by misaligned quadrupole magnets).

The real LHC machine \cite{8} is subject to additional imperfections $\Delta \mathcal{M}$, which alter the transport matrix by $\Delta T$:

$$T (s; \mathcal{M}) \rightarrow T (s; \mathcal{M} + \Delta \mathcal{M}) = T (s; \mathcal{M}) + \Delta T. \quad (9)$$

In Section 3.1.1 the effect on elastic proton transport is analyzed, while its influence on the transport of inelastically scattered protons is the topic of Section 3.1.2.

3.1.1 Effect on elastic proton reconstruction

The most important imperfections are:

- magnet current–strength conversion error: $\sigma(k)/k \approx 10^{-3}$
- beam momentum offset: $\sigma(p)/p \approx 10^{-3}$.

Their impact on the most relevant optical functions $L_\gamma$ and $dL_\gamma/ds$ can be calculated with MAD-X, the results were described in \cite{2, 17}. From the results, it is clear that the imperfections of the inner triplet (the so-called MQXA and MQXB magnets) are of high influence on the transport matrix $T_\delta$ while the optics is less sensitive to the strength of the quadrupoles MQY and MQML.

Other imperfections that are of lower, but not negligible, significance:

- magnet rotations: $\delta \phi \approx 1$ mrad
- beam harmonics: $\delta B/B \approx 10^{-4}$
- power converter errors: $\delta I/I \approx 10^{-4}$
- magnet positions: $\delta x, \delta y \approx 100 \mu$m.

Generally, the low-$\beta^*$ optics sensitivity to the machine imperfections is significant. However, as it is shown in the following sections, $\Delta T$ can be determined well enough from the proton tracks in the RPs, by exploiting the properties of the optics and of elastic $pp$ scattering \cite{2, 17}.

3.1.2 Effect on inelastic proton reconstruction

The transport matrix $T$ of Equation (2) is a derivative matrix, which describes the transportation of the particle’s kinematical variables in the vicinity of a reference orbit (e.g. the LHC central orbit). This section discusses the LHC parameters, that influence the horizontal dispersion and the $x$-coordinate of the central orbit. The key parameters are:
– the horizontal crossing-angle
– the position of quadrupole magnets
– the kicker magnets’ strength.

According to the results of Table 1, the listed elements have negligible influence on the vertical effective length $L_y$, while the horizontal beam position $x$ and the horizontal dispersion $D_x$ are at least five orders of magnitude more sensitive.

<table>
<thead>
<tr>
<th>Perturbed element</th>
<th>Perturbation</th>
<th>$\delta L_y$ [%]</th>
<th>$\delta x$ [%]</th>
<th>$\delta D_x$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>horiz. crossing angle</td>
<td>5 %</td>
<td>$-5 \cdot 10^{-6}$</td>
<td>1.1</td>
<td>-3.7</td>
</tr>
<tr>
<td>quadrupoles’ x-position</td>
<td>1 mm</td>
<td>$3 \cdot 10^{-5}$</td>
<td>231</td>
<td>15.5</td>
</tr>
<tr>
<td>kicker strength (max)</td>
<td>$6 \cdot 10^{-5}$</td>
<td>$3 \cdot 10^{-5}$</td>
<td>257</td>
<td>18</td>
</tr>
</tbody>
</table>

Table 1: Sensitivity of the vertical effective length $L_y$, beam position $x$ and horizontal dispersion $D_x$ to machine perturbations at $\sqrt{s} = 13$ TeV with $\beta^*$ = 0.4 m optics in the 210-N-H of Beam 1, at 210 m from IP5. In the table the perturbation of the kickers’ strength is the maximum kick. However, the realistic field error is about 3 % for the kickers and an additional maximum 7 % due to residual magnetization. Finally, their influence on the orbit is an order of magnitude smaller than the value quoted in the table [16, 18].

As it was mentioned before, the transport matrix $T_4$ is a derivative matrix, which explains why the above parameters, until they remain within their nominal LHC tolerances, have negligible influence on the transport matrix elements of $T_4$: they shift only the coordinate system. This observation is the key for the dispersion measurement, see Section 3.3.1.

3.2 Optics estimation using elastic candidates

The standard optics estimation method of the TOTEM experiment has been applied to the CT-PPS alignment run, the details of the method are discussed in [2]. According to this method, the elastic candidate events have been selected and due to the small vertex size, $\sigma(x^*) \approx 15 \mu$m, the correlation between their measured vertical position and angle in both beams has been determined with 2.5 % uncertainty, see Figure 1.

These estimator values have been applied in the optics determination procedure, leading to an optics model for $T_4$ at the RPs with a confidence level of 12.2 %. The estimated vertical effective lengths at $\xi \approx 0 \%$ are $L_{y,\text{right,near}} = 16.45$ m and $L_{y,\text{left,far}} = 15.96$ m, with an uncertainty of 0.5 %.\(^2\)

3.3 Estimation of the LHC horizontal dispersion

3.3.1 The $L_y = 0$ method

The vertical effective length $L_y(\xi)$ is a function of the proton momentum loss $\xi$ and its dependence can be calculated with MAD-X at each RP location. The result shows that $L_y(\xi)$ is positive at $\xi = 0$, monotonically decreases with decreasing $\xi$ reaching large negative values and meanwhile it vanishes at $\xi \approx -4 \%$, see Figure 2.

According to Equations (1) and (2) at this $\xi_0$ value every proton is transported to the same vertical coordinate $y = 0$ irrespective of the particle’s vertical scattering angle $\theta_y^*$. At the same time these protons appear at the horizontal location $x_0 \approx D_x \cdot \xi_0$ according to Equation (7). Consequently, the proton’s measured coordinate distribution $(x, y)$ has to show a pinch at this horizontal location $x_0$. This point has been observed and measured with the horizontal RP CT-PPS detectors with large statistics, see Figures 3 and 4.

\(^2\)The convention is that right refers to beam1, while left to beam2.
Fig. 1: Correlation of the vertical coordinates of elastic candidate protons measured in the vertical RPs in the left arm, one of the distributions which serves as LHC optics estimator. The scatter plot (left) and its profile to determine the slope of the distribution’s principal eigenvector indicated with the red dashed line. A good fit is required, the $p$-value is 83% in the present case.
Fig. 2: The vertical effective length $L_y$ as a function of the proton momentum loss $\xi$ at different RPs. The curves have been calculated with the MAD-X software [13].

The dispersion is determined with

$$D_{x,0} \approx \frac{x_0}{\xi_0}.$$  \hspace{1cm} (10)

The horizontal dispersion $D_x(\xi)$ is a function of $\xi$, see Equation (6), hence the measured dispersion is an average dispersion in the range $(0, \xi_0)$. Since $x$ is approximately a quadratic function of $\xi$ at low-$\xi$, the Mean Value Theorem tells us that $D_{x,0}$ is the dispersion at $\xi \approx 0.5 \cdot \xi_0$.

The measured horizontal dispersion $D_x$ values are summarized in Table 2 at the different RP locations, together with the propagated uncertainties of Table 3. Note the asymmetry of the dispersions of the two LHC beams, which was not present in the nominal optics ($D_{x,\text{far,nominal}} \approx -7.7$ cm for both beams).

<table>
<thead>
<tr>
<th>RP210 horizontal</th>
<th>$D_x$ [cm]</th>
<th>$\xi_0/2$ [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left, far</td>
<td>-9.66 ± 0.42</td>
<td>-1.94</td>
</tr>
<tr>
<td>Left, near</td>
<td>-9.81 ± 0.38</td>
<td>-2.22</td>
</tr>
<tr>
<td>Right, near</td>
<td>-7.04 ± 0.37</td>
<td>-2.11</td>
</tr>
<tr>
<td>Right, far</td>
<td>-6.68 ± 0.41</td>
<td>-1.81</td>
</tr>
</tbody>
</table>

Table 2: Measured $D_x$ values in RP210 horizontals. The uncertainties are the total propagated uncertainties of Table 3.

<table>
<thead>
<tr>
<th>Description of uncertainty</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fit error of $x_0$ point</td>
<td>$\approx 0.1$ mm</td>
</tr>
<tr>
<td>$x$-alignment error</td>
<td>$\approx 0.1$ mm</td>
</tr>
<tr>
<td>Error of $\xi_0$ point after matching</td>
<td>&lt; 0.6 % (rel.)</td>
</tr>
<tr>
<td>Remaining uncertainty (from MC)</td>
<td>2 % (rel.)</td>
</tr>
</tbody>
</table>

Table 3: The uncertainties of the $L_y = 0$ zero method.
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![Track Distribution Diagram](image)

Fig. 3: Distribution of tracks in the 210-F horizontal RPs; note the pinch where the vertical effective length $L_y = 0$ vanishes. The figures show that the $x_0$ position is larger for Beam 2, which indicates that the dispersion $D_x$ of this beam is larger.

From the estimated dispersion values the difference of the dispersions in the two arms can be also derived, and the obtained value can be compared with the data, see the next Section. To be in the center of the measured data the difference of the dispersions $\Delta D_x$ is calculated at $\xi = -0.08$:

$$\Delta D_x = (3.7 \pm 0.6) \text{ cm} \quad (11)$$

where the $D_x$ extrapolation from $\xi_0$ to $\xi = -8 \%$ has been made with the matched LHC lattice model, see Section 3.4. As a cross-check an alternative method has been investigated in the next section.

### 3.3.2 Dispersion $D_x$ difference from matching $\xi$ distributions

The measured $x$-position of the tracks in the horizontal RPs allow for the determination of the difference of the dispersion in the two LHC beams $\Delta D_x$. The idea here is that the protons scattered at IP5 follow the same $\xi$-distribution due to left-right physics symmetry. The measured $\xi$-distributions of the two beams, if they disagree, can be scaled until they overlap in a selected representative $\xi$-range by scaling the LHC nominal dispersion. The overlap cannot extend to large $\xi$-range, since the dispersions are $\xi$-dependent quantities.

The measured value is $\Delta D_x = (4.0 \pm 0.2) \text{ cm}$ in a selected $\xi$-range with mean $\xi = -0.08$; this value is in good agreement with the result of Equation (11). The propagated uncertainties of this method are shown in Table 4.
3.4 Matching of the LHC lattice

According to Section 3.1.2 the following machine parameters have to be matched per LHC beam to obtain the LHC orbit model for the proton reconstruction:

- Horizontal crossing-angle
- Quadrupole positions ($\sigma_x = 0.5 \text{ mm}$, 6 parameter)
- Kicker strength ($\sigma_k \approx 3 \%$, 3 parameter)

The starting point of the lattice matching is the result of Section 3.2, which means that the quadrupole strengths have been already estimated. In this way the matched LHC model remain consistent with the measured elastic constraints.

The following additional measured constraints have been applied to constrain the orbit of both LHC beams:

- 3 BPMs (at $s = 22 \text{ m}, 58 \text{ m}, 199 \text{ m}$): $\sigma_{x,\text{absolute}} \approx 0.43 \text{ mm}$
- RP 210, near, vertical, beam position: $\sigma_x = 0.5 \text{ mm}$
- 2 measured dispersions $D_x$, with their errors, see Table 2
Fig. 5: The $\xi$-distribution of tracks measured in the far horizontal RPs in the left and right arm using nominal dispersion (top); note the gap between the histograms. The difference of $\xi$-distributions after matching the dispersions (bottom).
With this procedure a confidence level of 44.6% and 76.5% has been achieved for the lattice model of Beam 1 and Beam 2, respectively. The matched LHC lattice leads to a good agreement between model and measurement.

As a consequence of this procedure a full simulation of the LHC lattice between the IP5 and the RPs can be calculated. At first stage the correlation between the horizontal coordinate $x$ and proton fractional momentum loss $\xi$ is derived for the physics analysis, see Fig. 6.

![Graph](image)

**Fig. 6:** The $\xi(x)$ curves calculated from the matched lattice model for Beam 1. These functions allow for a preliminary $\xi$ reconstruction of the measured protons in the horizontal RPs.

### References


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