Observable Hierarchies from Vector Supersymmetry in Cohomological Field Theories

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Abstract

We examine the underlying algebraic structure of cohomological field theories. The pattern of observable hierarchies is shown to be universally derived from the vector supersymmetry algebra. As such, vector supersymmetry plays a fundamental role in all topological field theories of this type. Within this algebraic approach, we present observable hierarchies for topological gravity, gauge theory and sigma models, together with their coupling.

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1 Introduction

It has recently been shown [1, 2] that the presence of a vector supersymmetry plays a crucial role in cohomological field theories. This class of field theories has been introduced by Witten, and includes topological gauge theory [3], topological sigma models [4], and topological gravity [5]. While the construction of observables for the first two met with no difficulty [3, 4], the situation for the case of topological gravity was far less clear [5]-[10]. Indeed, for the case of gauge theory, the naturally occurring hierarchy of observables found a neat expression [11]-[13] in terms of the curvature of the universal bundle of Atiyah and Singer [14]. In [1], it was shown that the presence of a vector supersymmetry played an important role in the construction of these observables. Given the basic observable with highest ghost degree, the remaining members of the hierarchy can be generated by successive applications of the vector supersymmetry operator.

Since vector supersymmetry is an important ingredient in defining topological gauge theory, one is naturally led to ask whether it can play a similar role in topological gravity. Indeed, in [2], it has been established that by localizing this vector supersymmetry one obtains a theory of topological gravity. Furthermore, the theory can be defined by a minimal multiplet of fields which consists of the vielbein and a gravitino (an odd vector), together with a BRST invariant scalar field in the adjoint representation of $O(n)$. Given this BRST invariant field, one can then generate an entire hierarchy of observables by successive applications of the vector supersymmetry operator. In addition, the observables can then be expressed in terms of the curvature of the universal bundle for gravity [14, 15]. The construction of this universal bundle is a generic object which is not specific to a particular dimension. Hence, the minimal multiplet given above provides the basic field content necessary to construct a theory of topological gravity in any dimension.

In the present paper, we shall examine more closely the meaning of this vector supersymmetry. The underlying algebraic structure of cohomological field theories is based upon the BRST and vector supersymmetry operators, together with a covariant derivative (where covariance is with respect to a Yang-Mills connection, or a spin connection, for gauge and gravity theory, respectively). We shall show that the pattern of observable hierarchies in all these theories is a simple consequence of this algebraic structure. Since it is the defining equations of the hierarchy which ensure the metric independence of each member of the fold, this algebra forms the basis of cohomological field theory. In the same way that BRST invariance plays a fundamental role in defining gauge theory, or conventional spinor supersymmetry forms the basis of supersymmetric field theories, one may regard the vector supersymmetry algebra as defining cohomological field theory.

A prerequisite for constructing observables is to define a minimal multiplet of fields which satisfy the vector supersymmetry algebra. Given these fields, the observables correspond to certain composite operators; their hierarchical relations are in turn defined by the same algebra. In the following, we shall first discuss the essential features of this algebra in a general setting. The minimal multiplets for gauge theory and sigma models have been specified in [3, 4]; in addition to these we present the gravity multiplet of [2], and show how, within this algebraic setting, the coupling of these theories follows straightforwardly.

2 The Algebraic Structure of Cohomological Field Theory

It has been known for some time that BRST invariance plays an important role in topological field theory [12, 6, 11]; indeed, the observables in cohomological models correspond to BRST cohomology classes [3, 4]. Let us for definiteness consider a theory defined on an n -dimensional manifold M . In this case, the observables are defined via a set of operators $W^{(i)} (i = 0, \dots, n)$, which satisfy the following descent equations:

$$\begin{aligned} \delta W^{(0)} &= 0 , \\ \delta W^{(1)} &= dW^{(0)} , \\ &\vdots \\ \delta W^{(n)} &= dW^{(n-1)} , \\ dW^{(n)} &= 0 . \end{aligned} \tag{1}$$

Here, i denotes the differential form degree, δ is the BRST operator, and d is the exterior derivative on M . As a consequence of these defining relations, one can now construct a set of observables by integrating each of the operators $W^{(i)}$ over an appropriate homology i -cycle on M . The integrated operators, which are referred to as observables, are thus topological invariants, by virtue of their BRST invariance. In practice, the above hierarchy of equations is initiated by choosing a BRST invariant operator $W^{(0)}$. However, one may wonder whether the ensuing relations in this hierarchy are merely fortuitous, or whether they are a result of some more fundamental property of cohomological field theory.

It has become clear that vector supersymmetry is a generic feature of cohomological field theories [1, 2]. In the same way that BRST symmetry is accompanied by an infinitesimal Grassmann odd scalar parameter, one defines vector supersymmetry as corresponding to a symmetry with a Grassmann odd vector parameter. Thus, the underlying algebra of these theories contains structure beyond BRST invariance. It transpires that the above hierarchical relations are a simple and direct consequence of this more powerful underlying algebraic structure.

Let us begin by specifying the fundamental algebra of cohomological field theory. We have the following relations [1, 2]:

$$\{D_\theta, D_a\} = \delta_a^i D_i , \tag{2}$$

$$[D_a, D_i] = 0 , \tag{3}$$

$$\{D_a, D_b\} = 0 , \tag{4}$$

$$\{D_\theta, D_\theta\} = R_{\theta\theta}^{C_0} D_{C_0} . \tag{5}$$

Before studying this algebra, we require some notational explanation.

The theory is defined on a manifold M , and contains both a BRST and a vector supersymmetry. Thus, we can consider a superspace which is locally parametrized by the coordinates $z^\Lambda = (x^\mu, \theta^\alpha, \theta)$, where x^μ is used to denote the Grassmann even coordinates, θ^α denotes an equal number of odd coordinates, and θ is reserved for the additional odd BRST coordinate. Over such a supermanifold, we can consider a bundle of orthonormal

frames: $h^{A_1} = dz^\Lambda h_\Lambda^{A_1}$, where the flat indices $A_1 = (i, a, \theta)$ denote the horizontal directions. The structure group of this bundle can, in general, be taken to be a product of $O(n)$ and a compact group G ; the label A_0 shall be used to denote both of the vertical directions, with $A_0 = [rs]$ and I , for $O(n)$ and G , respectively.

One can also introduce a connection in this bundle, which we denote in local coordinates by $h_\Lambda^{A_0} = (\omega_\Lambda^{[rs]}, A_\Lambda^I)$, and form the covariant derivative

$$D_\Lambda = \partial_\Lambda - h_\Lambda^{A_0} D_{A_0} . \quad (6)$$

This connection, together with the frame fields, allows us to define both curvature and torsion, $R_{\Lambda\Sigma}^{A_0}$ and $T_{\Lambda\Sigma}^{A_1}$, where $R_{\Lambda\Sigma}^I = F_{\Lambda\Sigma}^I$ is the Yang-Mills curvature.

The supervielbein also allows one to relate objects with curved and flat indices (where we reserve θ for a flat BRST index),

$$D_\Lambda = h_\Lambda^{A_1} D_{A_1} = h_\Lambda^i D_i + h_\Lambda^a D_a + h_\Lambda^\theta D_\theta . \quad (7)$$

We should point out that the potential gauge field for the BRST symmetry h_μ^θ is set to zero in the sequel. The entire algebraic picture is specified by the commutation relations

$$[D_A, D_B] = F_{BA}^C D_C , \quad (8)$$

where $F_{B_1 A_1}^{C_1} = T_{B_1 A_1}^{C_1}$, $F_{B_1 A_1}^{C_0} = R_{B_1 A_1}^{C_0}$, $F_{B_0 A_1}^{B_1} = f_{B_0 A_1}^{B_1}$, and so on.

The algebra which represents all of the local symmetries of the theory is given by

$$\begin{aligned} \delta h_\Lambda^A &= D_\Lambda \epsilon^A - h_\Lambda^{B_1} \epsilon^C F_{B_1 C}^A \\ \delta F_{BC}^A &= \epsilon^E D_E F_{BC}^A , \end{aligned} \quad (9)$$

where all of the parameters have a “flat” index, and the labels A denote either a vertical or horizontal direction. For more complete details of the relevant notational conventions, we refer to [1, 2].

We are now in a position to understand the structure of the algebra (2)-(5). It is easy to see that the algebra is based upon the fundamental torsion constraint

$$T_{a\theta}^i = \delta_a^i . \quad (10)$$

In addition, the curvature component $R_{\theta\theta}^{C_0}$ is non-vanishing, which means that the BRST symmetry is nilpotent up to both local $O(n)$ rotations, and Yang-Mills gauge transformations. We also note that the vector supersymmetry part of the algebra is Abelian. We should stress that (2)-(5) only represents the part of the algebra which will concern us in the following. The full algebra is consistently specified by a complete set of covariant curvature and torsion constraints, which have been derived in [1, 2]. Suffice it to mention here that the only non-vanishing components of curvature are given by $R_{\theta\theta}^{A_0}$, $R_{\theta i}^{A_0}$, $R_{ij}^{A_0}$.

Consider a BRST invariant object $W^{(0)}$; $D_\theta W^{(0)} = 0$, which is also covariant with respect to $O(n)$ rotations, and gauge transformations. A superscript will indicate the differential form degree; however, in the following we shall present our results in terms of components. We define

$$W_a^{(1)} = D_a W^{(0)} , W_i^{(1)} = \delta_i^a W_a^{(1)} . \quad (11)$$

If we now act on $W^{(0)}$ with the algebraic relation (2), we obtain the following result:

$$D_\theta W_i^{(1)} = D_i W^{(0)} . \quad (12)$$

It is immediately evident that such a relation is precisely the second step in the hierarchy of (1). The reason the covariant derivative D_i appears, rather than the exterior derivative as in (1), is simply because our present discussion is more general. We are first considering covariant objects $W^{(i)}$; for the purposes of constructing observables one then specializes to objects which are invariant with respect to $O(n)$ rotations, and Yang-Mills transformations. The covariant derivative acting on such objects is equivalent to the exterior derivative. We thus conclude that such a pattern is not simply a twist of good fortune, but is a direct consequence of the underlying vector supersymmetry algebra. This analysis can be continued, and one verifies that the construction of the entire hierarchy (1) is based solely on the three relations (2)-(4). The k th level is obtained by defining

$$\begin{aligned} W_{a_1 \dots a_k}^{(k)} &= D_{a_1} W_{a_2 \dots a_k}^{(k-1)} , \\ W_{i_1 \dots i_k}^{(k)} &= \delta_{i_1}^{a_1} \dots \delta_{i_k}^{a_k} W_{a_1 \dots a_k}^{(k)} , \end{aligned} \quad (13)$$

and using (2)-(4) one finds:

$$D_\theta W_{i_1 \dots i_k}^{(k)} = \frac{1}{(k-1)!} D_{[i_1} W_{i_2 \dots i_k]}^{(k-1)} . \quad (14)$$

For the purpose of illumination, we have chosen to endow the objects W with flat indices. It is quite straightforward to adopt a curved index notation, or write these objects as differential forms. For example, let us define

$$W_\mu^{(1)} = h_\mu^i W_i^{(1)} . \quad (15)$$

A little extra care is needed here as the operators D_θ, D_a do not represent the symmetries when acting on non-covariant objects such as the vielbein h_μ^i . In this case, one uses the relation

$$[\delta_{SUSY}(\epsilon^a), \delta_{BRST}(\epsilon)] = \delta_{DIFF}(\epsilon^i = \delta_a^i \epsilon \epsilon^a) , \quad (16)$$

and acting on $W^{(0)}$, one finds

$$\delta_{BRST} W_\mu^{(1)} = D_\mu W^{(0)} = (\partial_\mu - \omega_\mu^{[rs]} D_{rs} - A_\mu^I D_I) W^{(0)} . \quad (17)$$

Similarly, the general result is given by

$$\delta_{BRST} W_{\mu_1 \dots \mu_k}^{(k)} = \frac{1}{(k-1)!} D_{[\mu_1} W_{\mu_2 \dots \mu_k]}^{(k-1)} . \quad (18)$$

The above derivation of the descent equations which define a cohomological field theory clearly demonstrates that they have their origin in the vector supersymmetry algebra. The knowledge that BRST plays a role in these theories is not a sufficient guarantee for this structure. As we have seen, the BRST operator is nilpotent up to local $O(n)$ rotations, and Yang-Mills gauge transformations. In order to construct an observable, one must therefore ensure that the objects W are invariant with respect to these local symmetries. In practice, this simply involves taking an appropriate group trace. Clearly, these group

invariant objects also satisfy the same hierarchy pattern, and can in turn be integrated over appropriate homology cycles to define observables. Thus, the chain of events which eventually leads to the presence of metric independent observables in a cohomological field theory find its natural explanation within the above algebraic structure.

In order to actually begin this program of observable construction, one must first obtain a minimal multiplet of fields which also satisfies this algebra, and contains a BRST invariant member. The observables then correspond to group invariant composites of this basic multiplet. In the next section, we shall recall the minimal multiplets for cohomological gauge theory and sigma models [3, 4], and show that they obey this algebra. Thus the pattern of observables obtained by Witten in these cases follows immediately.

The application of these ideas to topological gravity is, however, more interesting. There have been many attempts to establish an appropriate minimal multiplet and candidate observables for such a theory; however, in general these attempts have been based solely on a BRST symmetry [5]-[10]. In [2], it was shown that the global vector supersymmetry algebra which is present in cohomological gauge theory finds a natural localization within the bundle of frames approach to superspace. A minimal topological gravity multiplet satisfying the vector supersymmetry algebra is obtained; this multiplet includes a BRST invariant field and hence, by an application of the vector supersymmetry algebra, one is led to a hierarchy of observables.

3 Observable Hierarchies and Their Coupling

In the previous section, we examined the origin of the hierarchical equations from a general standpoint. We shall now discuss some specific examples, including gauge theory, gravity, and sigma models, together with their coupling.

Let us begin with the case of cohomological gauge theory. The minimal multiplet of fields which defines such a theory is given by: $(A_\mu^I, \psi_\mu^I, \phi^I)$, where A_μ^I is a Yang-Mills gauge connection, ψ_μ^I is a Grassmann odd 1-form, and ϕ^I is a scalar field in the adjoint representation of the gauge group G . In [1], it was shown that this multiplet obeyed the vector supersymmetry algebra of eqns. (2) -(5), with the identification:

$$F_{\theta\theta}^I = \phi^I, F_{\theta i}^I = \psi_i^I. \quad (19)$$

We note that for ease of comparison with the case of gravity, we have rescaled the ϕ field of ref. [1]. The BRST and vector supersymmetry transformations of this multiplet are then consistently defined through an analysis of the resulting Bianchi identities. One obtains the following results:

$$\begin{aligned} D_\theta F_{\theta\theta}^I &= 0, \quad D_a F_{\theta\theta}^I = 2\delta_a^i F_{\theta i}^I, \\ D_\theta F_{\theta i}^I &= \frac{1}{2} D_i F_{\theta\theta}^I, \quad D_a F_{\theta i}^I = \delta_a^j F_{ji}^I, \\ D_\theta F_{ij}^I &= -D_{[i} F_{j]\theta}^I, \quad D_a F_{ij}^I = 0. \end{aligned} \quad (20)$$

In addition to these, we also have the transformation of the gauge connection

$$\delta A_\mu^I = D_\mu \epsilon^I + \epsilon^\theta h_\mu^i \psi_i^I. \quad (21)$$

One can now explicitly check that any gauge covariant and BRST- invariant object, such as $F_{\theta\theta}^I$ for example, will lead to a hierarchy of relations as outlined above. In particular, for the purposes of constructing observables, one must choose a BRST and gauge invariant operator; in this case a relevant operator being

$$W^{(0,4)} = \frac{1}{2} \text{tr} F_{\theta\theta}^2 , \quad (22)$$

where we now also include a superscript to indicate the ghost number of the operator. Applying the defining algebraic relations (2)-(4) to this operator, one immediately obtains the descent equations of Witten [3], with

$$\begin{aligned} W_i^{(1,3)} &= 2 \text{tr} F_{\theta i} F_{\theta\theta} , \\ W_{ij}^{(2,2)} &= 4 \text{tr} (F_{\theta i} F_{\theta j} + \frac{1}{2} F_{\theta\theta} F_{ij}) , \\ W_{ijk}^{(3,1)} &= 4 \text{tr} (F_{\theta i} F_{jk} + F_{\theta j} F_{ki} + F_{\theta k} F_{ij}) , \\ W_{ijkl}^{(4,0)} &= 4 \text{tr} (F_{ij} F_{kl} + F_{ik} F_{lj} + F_{il} F_{jk}) . \end{aligned} \quad (23)$$

The extension to the case of topological gravity turns out to be quite straightforward, the crucial step being the construction of an appropriate minimal multiplet which satisfies the vector supersymmetry algebra. This construction has been presented in [2]. As for the case of gauge theory, the minimal multiplet contains three fields: $(h_\mu^i, \psi_\mu^a, \phi^{rs})$, where h_μ^i is the vielbein, ψ_μ^a is the gravitino gauge field for local vector supersymmetry, and ϕ^{rs} is a scalar field in the adjoint representation of $O(n)$. Again, this set of fields provides the basis for a consistent solution to the Bianchi identities with the curvature component identification:

$$R_{\theta\theta}{}^{rs} = \phi^{rs} , R_{\theta i}{}^{rs} = \frac{1}{2} (\delta_{ia} T_{jk}^a - \delta_{aj} T_{ki}^a) \delta^{jr} \delta^{ks} . \quad (24)$$

The transformations of the curvature components are identical to the Yang-Mills case, with the replacements:

$$F_{\theta\theta}^I \rightarrow R_{\theta\theta}{}^{rs} , F_{\theta i}^I \rightarrow R_{\theta i}{}^{rs} , F_{ij}^I \rightarrow R_{ij}{}^{rs} . \quad (25)$$

We also have the gauge field transformations

$$\begin{aligned} \delta h_\mu^i &= \epsilon \delta_\mu^a \psi_\mu^a , \\ \delta \psi_\mu^a &= D_\mu \epsilon^a - \frac{1}{2} \epsilon \delta_\mu^a h_\mu^i \delta_{ir} \phi^{rs} , \\ \delta \omega_\mu^{rs} &= \epsilon h_\mu^i R_{\theta i}{}^{rs} . \end{aligned} \quad (26)$$

The important element in this construction is the presence of the BRST invariant scalar field ϕ^{rs} . We can now simply call on the general arguments of the previous section to guarantee the existence of a hierarchy of relations defining a set of observables for topological gravity. Another important ingredient in this construction is the fact that the BRST operator for the case of gravity is nilpotent up to local $O(n)$ rotations, as compared to local Yang-Mills transformations for gauge theory; the trace for the group G is then replaced by an $O(n)$ trace.

Thus, one set of observables is defined by making the replacement (25) in (22) and (23). At a first glance, therefore, it may appear that the gravity situation is simply an $O(n)$ version of the gauge theory scenario. It must be stressed that this is not the case; the minimal multiplet for gravity does not include the $O(n)$ spin connection as an independent field. The independent fields in this construction are as stated above: the vielbein, the gravitino for vector supersymmetry, and a scalar field in the adjoint representation.

However, another set of observables immediately suggest themselves. Let us consider a four-dimensional situation. Then, using the $SO(4)$ invariant tensor $\epsilon_{mnr s}$, we can form the $SO(4)$ invariant operator

$$W^{(0,4)} = \frac{1}{2} \epsilon_{mnr s} \phi^{mn} \phi^{rs} . \quad (27)$$

One now proceeds as before, and a new hierarchy of observables is generated. In this case, the top form is given by the Euler characteristic, $\int \epsilon^{ijkl} R_{ij} \wedge R_{kl}$, whereas the previous example leads one to the signature, $\int \text{tr } R \wedge R$.

We now turn our attention to the coupling of these theories. Clearly, a coupling of the gauge and gravity multiplets follows without any difficulty. The point to mention here is that in a coupled system the BRST operator is nilpotent up to both Yang-Mills gauge transformations, and local $O(n)$ rotations, as seen from (5). However, since the gauge multiplet is inert under local $O(n)$ rotations, and the gravity multiplet is inert under Yang-Mills transformations, the coupling is achieved automatically.

It is also instructive to consider the situation in two dimensions. A minimal multiplet for topological sigma models has been presented by Witten [4]; it is given by $(u^{i'}, C^{i'})$, where $u^{i'}$ ($i' = 1, \dots, N$) are the coordinates of an almost complex target space M , and $C^{i'}$ is a Grassmann odd target space vector. The primed index i' is used to provide a distinction between target space indices, and flat indices i for the base manifold, which in this case is a Riemann surface Σ . The relevant part of the vector supersymmetry algebra takes the form [16, 17]

$$\{D_\theta, D_a\} = \delta_a^i \partial_i , \quad (28)$$

$$[D_a, \partial_i] = 0 , \quad (29)$$

$$\{D_a, D_b\} = 0 , \quad (30)$$

$$\{D_\theta, D_\theta\} = 0 , \quad (31)$$

with the transformations

$$\begin{aligned} D_\theta u^{i'} &= C^{i'} , D_a u^{i'} = 0 , \\ D_\theta C^{i'} &= 0 , D_a C^{i'} = \delta_a^i \partial_i u^{i'} . \end{aligned} \quad (32)$$

We note here that the covariant derivative which is intrinsic to the Riemann surface is simply the exterior derivative, and it is this object which appears in the vector supersymmetry algebra. To construct a hierarchy of observables for topological sigma models, one begins by choosing a closed p -form $A = A_{i'_1 \dots i'_p} du^{i'_1} \wedge \dots \wedge du^{i'_p}$ on M , and forming the composite

$$W^{(0)} = A_{i'_1 \dots i'_p} C^{i'_1} \dots C^{i'_p} . \quad (33)$$

By virtue of the BRST invariance of $C^{i'}$, this represents an observable [4]. Once more, a simple application of the vector supersymmetry algebra (28)-(30) yields the hierarchy

$$\begin{aligned} D_\theta W_i^{(1)} &= \partial_i W^{(0)} , \\ D_\theta W_{ij}^{(2)} &= \partial_{[i} W_{j]}^{(1)} , \\ \partial_{[i} W_{jk]}^{(2)} &= 0 , \end{aligned} \tag{34}$$

where

$$\begin{aligned} W_i^{(1)} &= p A_{i'_1 \dots i'_p} \partial_i u^{i'_1} C^{i'_2} \dots C^{i'_p} , \\ W_{ij}^{(2)} &= -p(p-1) A_{i'_1 \dots i'_p} \partial_i u^{i'_1} \partial_j u^{i'_2} C^{i'_3} \dots C^{i'_p} . \end{aligned} \tag{35}$$

Finally, we would now like to consider the coupling of topological sigma models to the two-dimensional topological gravity multiplet. We have seen that for the case of gravity, the BRST operator is nilpotent up to local $O(2)$ rotations, while for sigma models it is nilpotent. The coupled multiplet should then satisfy the weaker nilpotency condition. In general, one would expect the sigma model transformations to be modified. Fortunately, in the present instance one sees that both $u^{i'}$ and $C^{i'}$ are scalars with respect to $O(2)$ rotations; thus, no modification is required. The importance of this lies in the fact that the hierarchy of sigma model observables depends on the BRST invariance of $C^{i'}$. Since this invariance requires no modification when dealing with the coupling to gravity, the observable structure is maintained, and the coupling follows straightforwardly. Similarly, since the sigma multiplet is a scalar under gauge transformations, one may couple to the two-dimensional gauge theory multiplet. Previous attempts in this direction met with some difficulty [4], primarily due to the fact that the gravity multiplet was endowed with a BRST operator which was nilpotent up to diffeomorphisms, local $O(2)$ rotations, and Weyl scalings. In particular, the BRST transformation of $C^{i'}$ required modification, thereby spoiling its invariance. The observable hierarchy was thus weakened, and its coupling to the proposed gravity multiplet was unclear.

Thus, it should again be stressed that the construction of an appropriate gravity multiplet, based on the vector supersymmetry algebra, leads in a direct fashion to observable hierarchies for both gravity, and its coupling to gauge theory and sigma models.

4 Remarks

While traditional BRST techniques have been used in the construction of cohomological field theories, the algebraic structure that BRST alone provides is quite weak. Clearly, we now see that the underlying algebraic picture is far richer, and more akin to the situation in spinor supersymmetry. It would be of interest to construct all realizations of this more general algebra. One approach would be to develop further the twist procedure of Witten [3, 4]. It turns out that in addition to providing the BRST operator, the twisting of $N = 2$ supersymmetric Yang-Mills, as presented in [3], see also [18], produces a vector supersymmetry operator which can be identified with that given in [1].

The most interesting application is to topological gravity, where this construction leads to new observable candidates. A more thorough investigation of these observables

is certainly warranted. In particular, it remains to be seen whether new observables are obtained for two-dimensional topological gravity [19, 20], and, if so, whether they play any role in the matrix model approach.

Finally, we recall that the minimal gravity multiplet presented in [2] provides a field theoretic realization of the universal bundle for gravity [14, 15]. From this point of view, one would naturally expect these observables to be interesting and non-trivial.

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