Non-Perturbative Subtractions in the Heavy Quark Effective Field Theory

L.Maiani^{1,2}, G.Martinelli¹ and C.T.Sachrajda³

¹ Dip. di Fisica, Università di Roma 'La Sapienza',
I-00185 Roma, Italy

and
INFN, Sezione di Roma, Italy

² CERN, Geneva, Switzerland,

³ Dept. of Physics, The University,
Southampton SO9 5NH, UK

Abstract

We demonstrate the presence of ultra-violet power divergences in the $O(1/m_h)$ corrections to matrix elements of hadronic operators containing a heavy quark field, (where m_h is the mass of the heavy quark). These power divergences must be subtracted non-perturbatively. The implications for lattice computations are discussed in detail.

1 Introduction

Recently there has been considerable interest in the use of an expansion in inverse powers of the heavy quark mass (m_h) for the study of the physics of hadrons containing a charm or bottom quark [1, 2, 3, 4]. Using this expansion it is possible to derive an effective field theory in which the heavy quark is represented by a two component spinor.¹ In the lowest order of this $1/m_h$ expansion, this formalism provides a powerful tool for the study of heavy quark physics, and the symmetries present in the effective field theory allow one to relate different operator matrix elements [1].

Our work is motivated by the use of the $1/m_h$ expansion in numerical simulations using the lattice formulation of QCD. Indeed this expansion allows one to study heavy quark physics on the lattice, even though the Compton wavelength of the heavy quark is smaller than the lattice spacing [4]. Using this expansion, and keeping only the leading term, the decay constant of the B -meson has been computed [5, 6], the result ($f_B = 310 \pm 25 \pm 50 \text{ MeV}$ [5]) is larger than that obtained in most model calculations. A comparison of the lattice results for the decay constant of heavy mesons obtained using the $1/m_h$ expansion with those obtained with a fully propagating heavy quark (for which the mass of the heavy quark is necessarily limited to be less than about that of the charm quark, since typical lattice spacings are currently in the region 2-3 GeV) suggests that the $O(1/m_h)$ corrections are large for the D -meson and still significant for the B -meson. It is therefore very important to calculate these $O(1/m_h)$ corrections to verify this conclusion explicitly, and to develop a truly quantitative method for heavy quark physics. In this paper however, we show that the presence of power divergences, i.e. ultraviolet divergences which behave like inverse powers of the lattice spacing, complicates considerably the evaluation of non-leading contributions to matrix elements of operators containing a heavy quark field. Although it may be possible to circumvent this difficulty by enforcing a set of non-perturbative normalization conditions at the level of quark and gluon Green functions, the necessary technology has still to be developed.

The problem of power divergences in the $1/m_h$ expansion is the following.

¹In this paper however, we will use the equivalent four component formalism.

At order $1/m_h$ the effective action contains a term proportional to:

$$\frac{1}{2m_h} \bar{Q}(x) \vec{D}^2 Q(x) \quad (1)$$

where Q represents the field of the heavy quark, and $\vec{D}$ the spatial components of the covariant derivative. The effective theory is invariant under the three dimensional cubic group. Under renormalisation the dimension 5 operator in eq.(1) mixes with the dimension 3 operator $\bar{Q}(x)Q(x)$, with a coefficient which diverges like $1/a^2$ where a is the lattice spacing². Although this quadratic divergence is the most severe one, it does not present a difficulty in the evaluation of matrix elements since it is absorbed into the renormalisation of the heavy quark mass. The difficulty arises because the operator in eq.(1) also mixes with the dimension 4 operator, $\bar{Q}(x)\gamma^4 D^4 Q(x)$ (which is part of the effective action to leading order in $1/m_h$), with a coefficient which diverges linearly, i.e. like $1/a$. Thus the contribution to the wave-function renormalisation constant, and hence to the renormalisation constant of any local operator containing the heavy quark field, contains a power divergence. This divergence must be subtracted non-perturbatively, as will be explained below.

We stress that although most of our presentation is in the context of lattice field theory, our arguments are more general. They apply to any regularization which breaks scale invariance. Recent perturbative studies of the $1/m_h$ corrections in the continuum effective theory [7, 8] have used dimensional regularisation and hence the power divergences did not appear explicitly, (having been cancelled by 'infrared' effects in some artificial number of dimensions). Thus the absence of explicit power divergences in this case gives a misleading impression, and non-perturbative techniques for the subtraction of the ultra-violet power divergences have also to be used in continuum calculations.

The standard procedure for obtaining continuum operator matrix elements from those computed in lattice simulations is to perform a perturbative calculation relating the renormalisation constants of the operator in the lattice and continuum renormalisation schemes being used (see [9] and references therein). However if power divergences are present this procedure

²or an equivalent quadratically divergent expression in continuum regularisation schemes.

cannot be used. The principal reason for this is that non-perturbative effects, e.g. factors of the form

$$\exp\left(-\int^{g_0(a)} \frac{dg'}{\beta(g')}\right) = a\Lambda_{QCD}, \quad (2)$$

combined with factors which diverge as inverse powers of the lattice spacing can give finite (i.e. non-vanishing) contributions. Indeed it has been found several times that, when power divergences are present, the perturbative evaluation of the term to be subtracted gives an underestimate (e.g. the calculation of the critical value of the quark mass, i.e. the value for which the pion is massless [10, 11], the evaluation of the 'eye' diagram present in the $\Delta I = \frac{1}{2}$ amplitude for kaon decays [12, 13], and the computation of the second moment of the pion's structure function [14]). Thus the power divergences must be subtracted using a non-perturbative subtraction procedure³.

In the remainder of this paper we explain in some detail the statements made in the preceding paragraphs. In the next section we exhibit the heavy quark propagator up to terms of $O(1/m_h)$. We choose to work in the formalism of the full theory, with four component spinors and an expansion of the heavy quark propagator in powers of $1/m_h$. This formalism is equivalent to the effective theory with two-component spinors, frequently used in the literature. Section 3 contains an explicit demonstration that the power divergences mentioned above, not only are allowed by the symmetries of the theory, but do indeed appear in perturbation theory. The implications of these divergences are also discussed. In section 4 we discuss the presence of these power divergences in the matrix elements of a general local operator and advance some conjectures as to possible procedures for subtracting them without losing the predictive power of the theory. Finally in section 5 we present our conclusions.

2 The Heavy Quark Propagator.

In this section we present the expression for the propagator of the heavy quark in the lattice theory, keeping the first two terms in the $1/m_h$ expansion. The

³This is true even if one accepts that part of the underestimate in the perturbative evaluation of the subtraction is due to the bare coupling constant g_0 being a poor expansion parameter [15].

starting point is the expression for the propagator in the continuum Euclidean theory, which can be obtained following the procedure used in ref.[16] for the theory in Minkowski space.

In Euclidean space, the propagator of the heavy quark, $S_h(x, y)$, satisfies the equation

$$(\gamma \cdot D + m_h)S_h(x, y) = \delta_{x, y} \quad (3)$$

In the following we will restrict the discussion to the case for which $x^4 > y^4$; the corresponding expressions for the case where $y^4 > x^4$ can be readily obtained using

$$S_h(y, x) = \gamma^5 S_h^\dagger(x, y) \gamma^5 \quad (4)$$

We write the solution in the form:

$$S_h(x, y) = \exp(-m_h(x^4 - y^4)) \left\{ S_h^0(x, y) + \frac{1}{2m_h} S_h^1(x, y) + O\left(\frac{1}{m_h^2}\right) \right\} \quad (5)$$

For the computations described in this paper we need to use both S_h^0 and S_h^1 .

The leading term S_h^0 is given by

$$S_h^0(x, y) = \frac{1 + \gamma^4}{2} \delta(\vec{x} - \vec{y}) \mathcal{P}_x(x^4, y^4) \quad (6)$$

where $\mathcal{P}_x(x^4, y^4)$ is the path ordered exponential from $(\vec{x}, x^4)$ to $(\vec{x}, y^4)$ along a path whose position in space is constant.

It is convenient to distinguish the various contributions to S_h^1 , so we write

$$S_h^1 = S_h^{1a} + S_h^{1b} + S_h^{1c} + S_h^{1d} \quad (7)$$

where the explicit expressions for the four terms $S_h^{1a} - S_h^{1d}$ are as follows:

$$S_h^{1a}(x, y) = \frac{1 + \gamma^4}{2} \int_{y^4}^{x^4} dw^4 \mathcal{P}_x(x^4, w^4) \vec{D}^2(\vec{x}, w^4) \mathcal{P}_y(w^4, y^4) \delta(\vec{x} - \vec{y}) \quad (8)$$

$$S_h^{1b}(x, y) = \frac{1 + \gamma^4}{2} \sum_{i,j} \int_{y^4}^{x^4} dw^4 \mathcal{P}_x(x^4, w^4) \sigma_{ij} F_{ij}(\vec{x}, w^4) \mathcal{P}_y(w^4, y^4) \delta(\vec{x} - \vec{y}) \quad (9)$$

where $i, j = 1, 2, 3$ and $F_{\mu\nu} \equiv [D_\mu, D_\nu]$,

$$S_h^{lc}(x, y) = -\frac{1+\gamma^4}{2} \mathcal{P}_x(x^4, y^4) (\vec{\gamma} \cdot \vec{D}(\vec{x}, y^4)) \delta(\vec{x} - \vec{y}) \quad (10)$$

and finally

$$S_h^{ld}(x, y) = -\frac{1-\gamma^4}{2} (\vec{\gamma} \cdot \vec{D}(\vec{x}, x^4)) \mathcal{P}_y(x^4, y^4) \delta(\vec{x} - \vec{y}) \quad (11)$$

In order to compute the $O(1/m_h)$ corrections to various physical quantities we take discretised forms of eqs.(6)-(11). There are many possible choices for the discretisations which are equivalent up to $O(a)$ corrections; in this paragraph we present the ones which were used in the perturbative calculations described below (as well as in numerical work currently in progress). For the path ordered exponentials in these expressions we take

$$\mathcal{P}_x(x^4, y^4) \equiv U_4^\dagger(\vec{x}, x^4 - a) U_4^\dagger(\vec{x}, x^4 - 2a) \cdots U_4^\dagger(\vec{x}, y^4 + a) U_4^\dagger(\vec{x}, y^4) \quad (12)$$

For $\vec{D}^2$ in S_h^{la} we take

$$\vec{D}^2(\vec{z}, z^4) f(z) = \frac{1}{a^2} \sum_{j=1}^3 \left(U_j(z) f(z + a\hat{j}) + U_j^\dagger(z - a\hat{j}) f(z - a\hat{j}) - 2f(z) \right) \quad (13)$$

where $\hat{j}$ is the unit vector in the j direction. Elsewhere for the covariant derivative we take

$$D_\mu(z) f(z) = \frac{1}{2a} (U_\mu(z) f(z + a\hat{\mu}) - U_\mu^\dagger(z - a\hat{\mu}) f(z - a\hat{\mu})) \quad (14)$$

For the $F_{\mu\nu}$'s in S_h^{lb} we take the definition given in ref.[17]

$$F_{\mu\nu}(x) \equiv \frac{1}{4a^2} \sum_P \frac{1}{2} (U_P - U_P^\dagger) \quad (15)$$

where the sum is over the four plaquettes P in the $\mu\nu$ plane starting and ending at x . Finally in eqs.(8) and (9) we replace the integrals by summations over w^4 , keeping half the contributions at each end point.

Figure 1: One loop diagrams contributing to the renormalisation of the leading term in the heavy quark propagator. The dots represent lattice points.

3 Power Divergences in Perturbation Theory

In this section we demonstrate that the power divergences which were discussed in the introduction do indeed occur in perturbation theory. We start with a discussion of the calculation of matrix elements in the lowest order of the $1/m_h$ expansion. Although in this case the only power divergence can be absorbed into the mass renormalisation, and does not appear in the renormalisation constants of the operators whose matrix elements are being evaluated, it is nevertheless instructive to see how this works explicitly.

3.1 Leading Order in the $1/m_h$ Expansion.

In leading order in $1/m_h$ the heavy quark propagator is just the product of links given by eqs.(6) and (12). At tree level all the links are equal to the unit matrix. Typical one loop corrections to the propagator are drawn in Fig. 1. They can be readily evaluated [18, 19] with the result that:

$$S_h^0(x, y) = O_0(x, y) \left(1 + g_0^2 \left(X \frac{t}{a} + Y \right) + O(g_0^4) \right) \quad (16)$$

where $t = x^4 - y^4$, and

$$O_0(x, y) \equiv \frac{1 + \gamma^4}{2} \delta(\vec{x} - \vec{y}) \quad (17)$$

X and Y are given by the integrals⁴

$$X = -\frac{C_F}{4} \int_{-\pi}^{\pi} \frac{d^2 k}{(2\pi)^3} \frac{1}{A} \quad (18)$$

$$Y = \frac{C_F}{4} \int_{-\pi}^{\pi} \frac{d^2 k}{(2\pi)^3} \frac{1}{A \sqrt{(1+A)^2 - 1}} \quad (19)$$

where

$$A = \sum_{i=1}^3 (1 - \cos k_i) \quad (20)$$

The linear power divergence (i.e. the term proportional to $1/a$ in eq.(16)) is absorbed into the renormalisation of the heavy quark mass,

$$1 + g_0^2 X \frac{t}{a} \simeq \exp(-\Delta m_h t) \quad (21)$$

where $\Delta m_h = -g_0^2 \frac{X}{a}$. Y is logarithmically divergent and contributes to the wave-function renormalisation of the heavy quark, and hence to the renormalisation constant of any local operator containing the heavy quark field. Thus in this case the power divergence does not enter into the renormalisation of the operators. It does however imply that in correlation functions of operators containing a heavy quark, which at large times are proportional to $\exp(-\mathcal{E}t)$, the exponent $\mathcal{E}$ itself has no physical significance. It is only possible to calculate mass differences by taking ratios of correlation functions, for which the power divergences cancel, (e.g. it is possible to calculate the mass differences of B_s and B_d , of B^* and B_d , and of Λ_B and B_d [20]).

We mention in passing that in [18] the authors rewrite the $O(g_0^2)$ term as follows:

$$1 + g_0^2 \left(X \frac{t}{a} + Y \right) = 1 + g_0^2 \left(X \frac{t+a}{a} + (Y - X) \right) \simeq \exp(-\Delta m_h(t+a)) + g_0^2(Y - X) \quad (22)$$

and interpret $Y - X$ as the one loop contribution to the wave-function renormalisation of the heavy quark. This corresponds to the imposition on the one particle irreducible Green function in Fourier space

$$\Sigma(k^0) = (1 + g_0^2 Y)(1 - e^{-ik^0}) + g_0^2(X/a)e^{-ik^0} \quad (23)$$

⁴ X is gauge invariant and Y is presented in the Feynman gauge

of the following renormalization conditions

$$\begin{aligned}\Delta m_h &= -\Sigma(k^0=0) = -g_0^2 X/a \\ Z^{-1} &= -i \frac{\partial \Sigma}{\partial k^0}(k^0=0) = 1 + g_0^2(Y - X)\end{aligned}\quad (24)$$

When using this prescription (i.e. when taking $Y - X$ to be the wave function renormalisation [18]) one fits to a correlation function of the form $c_1 \exp(-\Delta E(t+a))$ where c_1 is a constant. On the other hand when Y is taken to be the wave function renormalisation [19] one fits instead to $c_2 \exp(-\Delta E t)$. The matrix elements are obtained from the constants c_1 and c_2 , and it is clear that these two procedures are equivalent up to $O(g_0^4)$ ⁵. The difference between ref.[19] and ref.[18] is only due to the use of different renormalization prescriptions and not, as incorrectly stated in ref.[18], to the use of the effective action for the heavy quark on the lattice. In this paper we follow the procedure of keeping Y as the one-loop perturbative contribution to the wave-function renormalisation constant of the heavy quark.

3.2 $O(1/m_h)$ corrections

We now show that once $O(1/m_h)$ corrections are included the power divergences will appear not only in the mass but also in the renormalisation of the heavy quark wave function. Consider the contribution to the quark propagator which is proportional to S_h^{1a} , (see eqs.(5),(7) and (8)). The tree level contribution to S_h^{1a} is obtained from eq.(8):

$$S_h^{1a}(x, y) = O_{D^1}(x, y) + O(g_0^2) \quad (25)$$

where

$$O_{D^1}(x, y) \equiv \frac{1+\gamma^4}{2a^2} \sum_{w^4=y^4} \sum_{j=1}^3 \{ \delta(\vec{x} + a\vec{j} - \vec{y}) + \delta(\vec{x} - a\vec{j} - \vec{y}) - 2\delta(\vec{x} - \vec{y}) \} \quad (26)$$

where $\vec{j}$ is the unit vector in the j direction. We have approximated the integral in eq.(8) by a summation over w^4 , in which we keep 1/2 of the contribution at each end-point (this choice does not affect the remainder of

⁵The difference may nevertheless not be negligible numerically in current simulations.

[illegible]

Figure 2: Pictorial representation of the tree level contribution to $S_1^{1a}(x, y)$. The horizontal (vertical) direction represents time (space). Each of the three terms starts at y^4 and ends at x^4 .

the discussion). In this case the summation over w^4 is trivial, giving a factor of $x^4 - y^4$, but it is instructive to exhibit it explicitly. We represent the right hand side of eq.(26) by the diagram in fig.2.

We now evaluate the one-loop corrections to the three terms of Fig.2. These arise from the expansion of the links in the covariant derivatives and path ordered exponentials in eq.(8) to $O(g_0^2)$. A sample set of one-loop diagrams which correct the first term in Fig.2 are shown in Fig.3. The diagrams can be readily evaluated. Combining the results with those of eq.(16) we find:

$$S_h(x, y) = \exp(-m_h(x^4 - y^4)) \left\{ 1 + g_0^2 \left(\frac{X}{a} + \frac{W}{2m_h a^2} \right) t + g_0^2 \left(Y + \frac{2X}{2m_h a} \right) \right\} \\ \times \left(O_0(x, y) + \frac{Z_{D^2}}{2m_h} O_{D^2}(x, y) \right) \quad (27)$$

where X and Y are defined in eqs.(18) and (19) respectively,

$$W = -C_F \int_{-\pi}^{\pi} \frac{d^3 k}{(2\pi)^3} \frac{1}{\sqrt{(1+A)^2 - 1}} \quad (28)$$

and

$$Z_{D^2} = 1 + \frac{g_0^2}{6} W \quad (29)$$

Figure 3: Sample one loop diagrams contributing to the renormalisation of the S_H^{1a} term in the heavy quark propagator.

It should be remembered that the right hand side of eq.(27) has only been calculated to $O(1/m_h)$ and $O(g_0^2)$.

The result in (27) is incomplete in that we have not yet evaluated the contributions from $S_h^{1b} - S_h^{1d}$ (see (7)). It is straightforward to check that these do not lead to any power divergences. In order to have such power divergences the expansion of S_h^{1b} , S_h^{1c} or S_h^{1d} to $O(g_0^2)$ must have a term proportional to $O_0(x, y)$ whose spinor structure is the matrix $(1 + \gamma_4)/2$. An inspection of the spinor structure of the diagrams readily shows that this is not possible. Thus, although it is necessary to calculate the one loop corrections to all the terms in the propagator in order to extract physical quantities from lattice computations, in this paper, since we are only discussing the implications of the power divergences we need only consider the terms in eq.(27).

The most severe ultra-violet divergence on the right hand side of eq.(27) is a quadratic one. This term however, is proportional to t and hence is absorbed into the renormalisation of the heavy quark mass. The real problem for the evaluation of the matrix elements is the linearly divergent term:

$$g_0^2 \frac{2X}{2m_h a} O_0(x, y) \quad (30)$$

This term contributes to the renormalisation of the wave function of the heavy quark, (it is proportional to $O_0(x, y)$) and hence to the renormalisation of all operators containing the heavy quark field. It cannot be subtracted by imposing perturbative matching conditions. Separating explicitly the mass renormalization, and recalling that $\gamma^4 D^4 = O_0^{-1}$, we may rewrite eq.(27) in the form:

$$S_h(x, y) = \exp\left(-(m_h + \Delta m_h)(x^4 - y^4)\right) \left(1 + g_0^2 Y\right) \cdot \left\{ O_0(x, y) + \frac{Z_{D^2}}{2m_h} O_{D^2+\delta D^2}(x, y) \right\} \quad (31)$$

$$D^2 + \delta D^2 = \bar{D}_{\text{corr}}^2 = \bar{D}_{\text{bare}}^2 + \frac{2g_0^2 X}{a} \gamma^4 D^4 \quad (32)$$

which shows that the problem originates from a linearly UV-divergent mixing of $\bar{D}^2$ with the lower dimension operator $\gamma^4 D^4$, made possible by the simultaneous breaking of Lorentz and scale invariance.

4 The Extent of the Disease and Possible Cures

The appearance of power divergences in perturbation theory shows that they are not forbidden by some unexpected symmetry. We stress that power divergences at order $1/m_h$ occur in the matrix elements of any operator, even those which cannot mix with lower dimensional ones. From this point of view, the result for the propagator in the previous section, is just the tip of the iceberg. As an example consider the heavy light axial current:

$$A_\mu(x) = \bar{Q}(x)\gamma_\mu\gamma_5 q(x) \quad (33)$$

The $O(1/m_h)$ corrections to its matrix elements include the insertion of the operator (13):

$$\langle \delta A_\mu(x) \rangle = \frac{1}{m_h} \langle \int d^4y \bar{Q}(y) \bar{D}^2 Q(y) A_\mu(x) \rangle \quad (34)$$

It is evident from eq.(34) that the $1/m_h$ term in the action acts as a non-renormalizable interaction. Even though $A_\mu(x)$ does not mix with any lower dimensional operator, the correction to its normalization factor will be linearly divergent, to compensate for the "external" coupling $1/m_h$.

Of course the presence of power divergences in lattice computations is not a new phenomenon. They appear in several important matrix elements of operators which do not contain heavy quarks and for which non-perturbative subtractions have to be performed. In some cases (e.g. in the evaluation of the second moment of hadronic structure functions [14]) it is possible to make use of lattice symmetries to determine the subtraction coefficients, with successful consequences. In other cases, most notably that of the matrix elements relevant for the $\Delta I = \frac{1}{2}$ rule [12, 13], the subtraction coefficients themselves had to be determined numerically, with the result that the errors on the matrix elements were overwhelming. We remark again that the appearance of power divergences in the $1/m_h$ expansion, is not a lattice artefact and it would be present in any continuum regularization which breaks scale invariance, as for example in the Pauli-Villars regularization scheme.

The subtraction constants needed to remove the power divergences must all be determined non-perturbatively. How this can be done, without loss

of predictive power, has not been worked out yet. We will however advance some conjectures.

A possibility which can be envisaged is to impose renormalisation conditions on quark and gluon Green functions. This involves calculating these Green functions with a given gauge fixing condition [22, 23]. In the case of the heavy quark propagator, see eq.(13), a counter-term, proportional to $\gamma^4 D^4$ has to be subtracted from the kinetic term, $\bar{D}^2/2m_h$, precisely to cancel the power divergences arising from quantum corrections to the $O(1/m_h)$ term in the lowest order propagator. The subtraction coefficient can be determined non-perturbatively, via a numerical calculation, and one may hope that the subtraction will not result in a significant loss of precision. Otherwise it will not be possible to use the $1/m_h$ expansion to compute operator matrix elements (e.g. f_B) beyond the leading order in the heavy quark mass. Having restored scale invariance by the subtraction of the power divergences, the matching of the continuum and lattice results can be computed in the usual way using perturbation theory.

Similarly, one could require, as a non-perturbative condition, that the $1/m_h$ correction to the matrix element of the axial current A_μ vanish at a given subtraction point. The renormalization conditions can be imposed in such a way as to force the axial vector and vector currents to obey the Ward identities of the full theory [21]. At this point a further renormalization condition is imposed to match the effective current to the current in the full theory. The latter condition can be taken from perturbation theory.

The imposition of non-perturbative renormalisation conditions on quark and gluon Green functions clearly deserves further study. It requires a better understanding of the rôle of Gribov copies in gauge-fixed lattice calculations [24, 25].

5 Conclusions

In this paper we have demonstrated explicitly the presence of ultra-violet power divergences in the $O(1/m_h)$ corrections to matrix elements of operators containing a heavy quark field. We have argued that the elimination of these power divergences requires a non-perturbative subtraction procedure. This in turn requires the development of the corresponding techniques in order to make the $1/m_h$ expansion and lattice QCD a truly quantitative tool for

heavy quark physics. Until this is done we are restricted to those quantities, such as the B^*-B mass difference[20], which only has a contribution from S_A^{1b} .

Acknowledgements

We thank M.Testa for interesting and helpful discussions. CTS thanks Prof.J.Langer and the staff of the Institute for Theoretical Physics in Santa Barbara for their hospitality during the early stages of this work,(supported in part by NSF Grant No. Phy89-04035), and acknowledges the support of the SERC. LM and GM acknowledge the partial support of the M.U.R.S.T., Italy, and INFN.

References

- [1] N.Isgur and M.Wise, Phys.Lett. 232B (1989) 113; Phys.Lett. 237B (1990) 527
- [2] A.F.Falk, H.Georgi, B.Grinstein and M.B.Wise, Nucl.Phys. B343 (1990) 1
- [3] J.D.Bjorken, SLAC preprint, SLAC-PUB-5362 (1990), to be published in the proceedings of the XXth Conference on High Energy Physics, Singapore, August 1990.
- [4] E.Eichten, in Field Theory on the Lattice, Nucl.Phys. B (Proc.Suppl.) 4 (1988) 170
- [5] C.R.Allton et al., Nucl.Phys. B349 (1991) 598
- [6] C.Alexandrou et al., Phys.Lett. B256 (1991) 60
- [7] E.Eichten and B.Hill, Phys.Lett. B243 (1990) 1
- [8] A.F.Falk, B.Grinstein and M.E.Luke, Harvard University preprint, HUTP-90/A044 (1990)

- [9] C.T.Sachrajda, proceedings of the 1988 Conference on Lattice Field Theory, Nucl.Phys. B (Proc.Suppl.)9 (1989) 121
- [10] A.Gonzalez-Arroyo, F.Yndurain and G.Martinelli, Phys.Lett. 117B (1982) 437
- [11] R.Groot, J.Hoek and J.Smit, Nucl.Phys. B237 (1984) 111
- [12] L.Maiani, G.Martinelli, G.C.Rossi and M.Testa, Phys.Lett. B176 (1986) 445; Nucl.Phys. B249 (1987) 505
- [13] M.B.Gavela et al., Phys.Lett. B211 (1988) 139
- [14] G.Martinelli and C.T.Sachrajda, Nucl.Phys. B306 (1988) 865
- [15] G.P.Lepage and P.Mackenzie, proceedings of the 1990 Conference on Lattice Field Theory, Nucl.Phys. B (Proc.Suppl.)20 (1991) 173
- [16] E.Eichten and F.Feinberg, Phys.Rev. D23 (1981) 2724
- [17] J.Mandula, G.Zweig and J.Goeverts, Nucl.Phys. B228 (1983) 109
- [18] E.Eichten and Hill, Phys.Lett. B240 (1990) 193
- [19] Ph.Boucaud, O.Pene and Liu, Phys.Rev. D40 (1991) 1529
- [20] M.Bohicchio et al., Rome Prep. no. 809, submitted to Nucl.Phys.B.
- [21] M.Bohicchio et al., Nucl.Phys. B262 (1985) 331
- [22] C.Bernard, A.Soni and K.Yee, proceedings of the 1990 Conference on Lattice Field Theory, Nucl.Phys. B (Proc.Suppl.)20 (1991) 410
- [23] J.E. Mandula, M. Ogilvie Phys.Lett. B185 (1987) 127.
- [24] E.Marinari, C.Parrinello and R.Ricci Prep.ROM2F-90-61(1990)
- [25] C.Parrinello,S.Petrarca and A.Vladikas, in preparation.