CERN-TH. 6195/91

TKP91-10

The τ decay library TAUOLA, update with exact $O(\alpha)$ QED corrections in $\tau \rightarrow \mu(e)\nu\bar{\nu}$ decay modes.

M. Jeżabek¹ and Z. Wąs

Institute of Nuclear Physics, Kraków, ul. Kawiorów 26a, Poland

S. Jadach

*CERN, Theory Division, Geneva 23, Switzerland,
and*

*Institute of Physics, Jagellonian Univ., Kraków, ul. Reymonta 4, Poland
and*

J. H. Kühn

*Institut für Theoretische Kernphysik, Universität Karlsruhe,
W-7500 Karlsruhe, Germany*

ABSTRACT

A new algorithm for the generation of the complete $O(\alpha)$ QED corrections for τ decays into leptons is presented. It provides an update to the τ decay library TAUOLA and can be easily combined with this library and any other bigger packages like KORALB or KORALZ which contain TAUOLA.

Submitted to Computer Physics Communications

CERN-TH. 6195/91

TKP91-10

August, 1991

¹Alexander von Humboldt Foundation Fellow.

NEW VERSION SUMMARY

Title of the program: TAUOLA, version: 2.1

Reference to original program: Comput. Phys. Commun. **64** (1991) 275.

Authors of original program: S. Jadach J. H. Kühn and Z. Was

Computer: IBM 3090; *Installation:* CERN

Operating system: VM/CMS

Programing language used: FORTRAN 77

High speed storage required: 45000 words

No. of bits in a word: 32

Peripherals used: Line printer

No. of cards in combined program and test deck without TAUOLA library: 1193

Keywords: radiative corrections, heavy lepton τ , Monte Carlo simulation, quantum electrodynamics, spin polarization, electro-weak effects

Comparison with the previous version, and reasons for the new version: The powerful τ decay library [1] is now available. This program, so far did not included QED radiative corrections in any of the decay modes. This update includes QED corrections for the decay modes $\tau \rightarrow \mu(e)\nu\bar{\nu}$.

Typical running time: 37 events per (IBM 3090 168 units) sec., in case of $\tau \rightarrow e\nu\bar{\nu}(\gamma)$ decay.

Unusual features of the manual: This is not an independent document. Only the differences with Ref. [1] are documented.

Limitations of the solution: For QED corrections V-A current for τ and vanishing mass of ν_τ are assumed.

References:

1. S. Jadach, J. H. Kühn, Z. Was, Comput. Phys. Commun. **64** (1991) 275.

UPDATE DESCRIPTION

Introduction

The present upgrade of the TAUOLA [1] package which adds the QED corrections in leptonic decays is strongly constrained due to very different environments in which the package is used. In particular we had to obey the following conditions:

1. Generated events have to be filled into the LUND common block in a way that allows for easy adaptability to the two presently available standards of LUND (versions 6.3 and 7.2) [2, 3, 4].
2. Implementation of the program into KORALZ and KORALB [5] environments has to be straightforward – both in the case when the QED corrections are already added in the PHOTOS [6] approximation and in the case when they are not.
3. The input parameters specific to QED corrections in decay have to be transmitted through the common blocks already defined in KORALZ [7], even at the price of keeping some entries dummy.

All above requirements are fulfilled in the present upgrade.

Let us comment briefly on one aspect of the QED matrix element used in the present calculation for the τ decay into electron or muon. Strictly speaking it is valid only for pure $V - A$ coupling of the τ to the W . In any case, part of the matrix element which is proportional to α_{QED} is small and small variation of the appropriate input parameters GV , GA will not matter – for those terms we simply assume $GV=1$, $GA=-1$. In addition, this constraint is also a consequence of ultraviolet divergences arising for QED corrections in the case of arbitrary four-fermion interactions [8]. Summarizing, since $V + A$ admixture is expected to be small it can be incorporated without including radiative corrections. The same remark applies to a non-vanishing neutrino mass. The QED correction (with $V - A$ couplings), the $V+A$ contribution and finite τ mass are treated as *independent* corrections to Born distribution.

In chapter 2 matrix element used for radiative decays of τ is presented. In chapter 3 all modifications necessary to run TAUOLA, KORALB or KORALZ programs including QED radiative corrections in leptonic decays are listed. An extract from the demonstration output of the modified TAUOLA concludes the paper. All routines of the program are collected in the file RADEKCPC, except the double precision FUNCTION DILOG(X), calculating dilogarithm, which has to be added.

Differential rates for leptonic τ decays

The differential rate for a leptonic decay of τ

$$d\Gamma_l = d\Gamma_3 + d\Gamma_4; \quad l = e, \mu \quad (1)$$

is written as a sum of contributions from a 3-body channel

$$\tau^-(P, s) \longrightarrow \nu_\tau(N) l^-(q_1) \bar{\nu}_l(q_2) \quad (2)$$

and a 4-body one

$$\tau^-(P, s) \longrightarrow \nu_\tau(N) l^-(q_1) \bar{\nu}_l(q_2) \gamma(q_3). \quad (3)$$

The 3-body channel includes contributions from Born, virtual and soft photons corrections. The soft photons are defined as those which in the τ rest frame have energies smaller than² $k_0/2$. Thus, the 4-body channel contains only photons of energies above $k_0/2$.

The 3-body contribution reads:

$$d\Gamma_3(k_0) = \frac{1}{2M} \mathcal{N}_3 (1 + s^\mu h_\mu) dLips(P; q_1, q_2, N), \quad (4)$$

where s^μ gives the polarization of the decaying τ , h_μ is the polarimeter vector, and $\mathcal{N}_3$ denotes the matrix element squared for the unpolarized case. We express $\mathcal{N}_3$ and h_μ in terms of the zero order (Born) expressions and the $O(\alpha)$ corrections:

$$\mathcal{N}_3 = \mathcal{N}_3^{(0)} + \mathcal{N}_3^{(1)}, \quad (5)$$

$$h_\mu = (H_\mu^{(0)} + H_\mu^{(1)})/\mathcal{N}_3. \quad (6)$$

²Note that program input parameter $k_0^{decay} = k_0/M$.

In the Born approximation we consider a general $v + a\gamma_5$ coupling for the τ weak charged current and allow a non-zero mass for ν_τ , i.e. $P^2 = M^2$, $N^2 = m^2$, $q_1^2 = \mu^2$ and $q_2^2 = 0$. Then,

$$\mathcal{N}_3^{(0)} = \left(\frac{G_F}{\sqrt{2}}\right)^2 32B \quad (7)$$

and

$$H_\mu^{(0)} = \left(\frac{G_F}{\sqrt{2}}\right)^2 32H^\mu, \quad (8)$$

where, see eq. (3.4) from [1]:

$$\begin{aligned} B &= (v+a)^2 P \cdot q_1 N \cdot q_2 + (v-a)^2 P \cdot q_2 N \cdot q_1 - Mm(v^2 - a^2) q_1 \cdot q_2, \\ H^\mu &= (v+a)^2 M q_1^\mu N \cdot q_2 + (v-a)^2 M q_2^\mu N \cdot q_1 \\ &+ (v^2 - a^2) m q_1^\mu P \cdot q_2 - (v^2 - a^2) m q_2^\mu P \cdot q_1. \end{aligned} \quad (9)$$

For the first order correction we assume the $(V-A)$ form for the τ weak current ($v=1, a=-1$) and neglect the mass of ν_τ (i.e. we put $m=0$). We evaluate $\mathcal{N}_3^{(1)}$ and $H_\mu^{(1)}$ from the following formula [9]:

$$\begin{aligned} \mathcal{N}_3^{(1)} + s^\mu H_\mu^{(1)} &= - \frac{\alpha}{2\pi} 64 G_F^2 \{ F_0 N \cdot q_1 R \cdot q_2 + F_+ \epsilon^2 P \cdot N R \cdot q_2 \\ &+ F_- N \cdot q_1 [q_1 \cdot q_2 + (R \cdot q_2 P \cdot q_1 - P \cdot q_2 R \cdot q_1)/M^2] \\ &+ F_3 (M^2 N \cdot q_2 + P \cdot N R \cdot q_2 - R \cdot N P \cdot q_2) \}, \end{aligned} \quad (10)$$

where

$$\epsilon = \frac{\mu}{M}, \quad R^\mu = P^\mu - M s^\mu, \quad (11)$$

and [10]:

$$\begin{aligned} F_0(k_0) &= 2 \frac{u_0}{u_3} \left[\text{Li}_2 \left(1 - \frac{u_- w_-}{u_+ w_+} \right) - \text{Li}_2 \left(1 - \frac{w_-}{w_+} \right) \right. \\ &+ \left. \text{Li}_2 \left(1 - \frac{u_-}{u_+} \right) - 2Y_u + 2 \ln u_+ (Y_u + Y_w) \right] \\ &+ \frac{1}{y} [2u_3 Y_u + (1 - \epsilon^2 - 2y) \ln \epsilon] + 2 \\ &- 4 \left(\frac{u_0}{u_3} Y_u - 1 \right) \ln \frac{k_0}{M}, \end{aligned} \quad (12)$$

$$F_\pm = \frac{Y_u}{2u_3} \left(1 \pm \frac{1 - \epsilon^2}{y} \right) \pm \frac{\ln \epsilon}{y}, \quad (13)$$

$$F_3 = \frac{1}{2} \epsilon^2 (F_+ + F_-), \quad (14)$$

where

$$\begin{aligned} u_0 &= q_{1,0}/M, & w_0 &= (N_0 + q_{2,0})/M, & u_3 &= w_3 = \sqrt{u_0^2 - \epsilon^2}, \\ u_\pm &= u_0 \pm u_3, & w_\pm &= w_0 \pm w_3, & y &= w_0^2 - w_3^2, \\ Y_u &= \frac{1}{2} \ln \frac{u_+}{u_-}, & Y_w &= \frac{1}{2} \ln \frac{w_+}{w_-}. \end{aligned} \quad (15)$$

The 4-body contribution reads:

$$d\Gamma_4(k_0) = \frac{1}{2M} \mathcal{N}_4 (1 + s^\mu h'_\mu) dLips(P; q_1, q_2, q_3, N) \Theta(2q_{3,4} - k_0), \quad (16)$$

where h'_μ is the polarimeter vector and $\mathcal{N}_4$ denotes the matrix element squared for unpolarized τ . We evaluate them from the formula:

$$\begin{aligned} \mathcal{N}_4(1 + s^\mu h'_\mu) = & 256\alpha\pi G_F^2 \left\{ N \cdot q_1 \ R \cdot q_2 \left[-\frac{\mu^2 \mathcal{A}}{(q_1 \cdot q_3)^2} + \frac{2 P \cdot q_1 \mathcal{B}}{q_1 \cdot q_3 \ P \cdot q_3} - \frac{M^2}{(P \cdot q_3)^2} \right] \right. \\ & + \frac{N \cdot q_1}{(P \cdot q_3)^2} [M^2 q_2 \cdot q_3 - P \cdot q_2 \ R \cdot q_3 + q_2 \cdot q_3 \ R \cdot q_3] \\ & \left. - \frac{R \cdot q_2 \ P \cdot N}{P \cdot q_3} - N \cdot q_1 \frac{R \cdot q_1 \ q_2 \cdot q_3 - R \cdot q_3 \ q_1 \cdot q_2}{q_1 \cdot q_3 \ P \cdot q_3} \right\}, \end{aligned} \quad (17)$$

where

$$\begin{aligned} \mathcal{A} &= 1 + x, & \mathcal{B} &= 1 + x(1 + z)/2 + z/2, \\ x &= N \cdot q_3 / N \cdot q_1, & z &= P \cdot q_3 / P \cdot q_1. \end{aligned} \quad (18)$$

Modifications of the TAUOLA library

The case of the radiative τ decay into lepton did not require development of a new Monte Carlo algorithm. The program for the generation of the hard photon plus $\nu_\tau \bar{\nu} e$ is nearly identical with the one for generating $\nu_\tau 3\pi$ (the a_1 case described in chapter 3.5 of ref. [1]). Only the masses of final state particles had to be changed and importance sampling was introduced to balance collinear and infrared singularities in the generation of Q^2 and $\cos \theta_3$ from formula (3.52) of reference [1]. In the soft photon case, the generation of the photon variables is simply skipped. The hard photon configurations are combined with the rest of cross-section according to the classical solution of separating the phase space into the hard photon and Born+virtual+soft photon part as described in [11]. We refer the reader to those references for more information concerning details [1] and principles [11] of the construction of our program's update.

The following modifications of the TAUOLA library have to be done depending on the environment. If TAUOLA is used separately, all routines of the RADEKCPC file have to be added to TAUOLA file. Every routine in TAUOLA with the same name as in RADEKCPC has to be replaced by the routines from RADEKCPC (on most installations linking editor does it automatically). In the case when TAUOLA is used not as a stand-alone program but is interfaced to other program (KORALZ, KORALB) one has to proceed slightly differently:

1. All routines from the demonstration deck of the RADEK and the routine DWLUPH from RADEKCPC have to be rejected – only the rest of RADEKCPC has to be included in TAUOLA.
2. Ensure that variables XKODEC, ITDKRC from common block /TAURAD/ and ALFINV from common block /QEDPRM/ are respectively filled with k_0^{decay} and $1/\alpha$ (it is done automatically in KORALZ). Note that k_0^{decay} is the usual dimensionless soft photon cut parameter [11] and it should be kept in the range of about $10^{-4} - 10^{-3}$ while $1/\alpha = 137.0...$ is the inverse of the QED coupling constant. ITDKRC is the QED correction switch, for ITDKRC=1 the QED corrections are switched on, whereas for ITDKRC=0 they should be excluded.
3. In the case of TAUOLA interfaced with the KORALZ or KORALB which include the approximate leading-logarithmic treatment of the QED bremsstrahlung in τ decays of the PHOTOS type³ one has to modify routines DECAY1, DECAY2, DEXAY1, DEXAY2 as follows: (i) CALL RADCOR in the case of JAK=1 or 2 has to be enabled, also in case of JAK=1,2 the CALL DWLUPH has to be replaced by the CALL DWRPH(KTO).

So far the following tests of the program have been performed (all tests done for small electron and neutrinos masses):

1. Using the KORALZ [7] environment, we have checked for a sample of $2 \cdot 10^5$ events, that the electron spectrum from the τ pair produced at the top of the Z peak is nearly indistinguishable from the one generated with the help of the algorithm given in Ref. [6]. Small differences (at the level of 2-3% of the bin content) are located in the soft and hard corners of the energy distribution.

³The PHOTOS-like package is included in KORALZ [7] and KORALB [5] and it is not part of TAUOLA [1]. It is a temporary solution for QED bremsstrahlung in τ decays. For important τ decays like leptonic and π we are in the process (this paper) of replacing the corresponding subprograms in TAUOLA by the version including QED bremsstrahlung. For the other decays the temporary package will be replaced by the PHOTOS itself.

2. For a sample of $2 \cdot 10^6$ events and artificially increased electromagnetic coupling by a factor 7, we have checked that the branching ratio and spectra of all final state fermions ($e, \nu_\tau, \bar{\nu}_e$), agree within statistical errors with the theoretical results, given e.g. in Refs. [12, 10].
3. We have also checked, that within statistical errors for the samples listed above, the answers do not depend on the infrared regulator XKODEC for XKODEC smaller than 10^{-3} .

Acknowledgements

One of the authors (ZW) is very grateful for the warm hospitality extended to him by Prof. R. Settels from the Max-Planck-Institut für Physik, München, where part of this work was performed, and by Prof. D. Schlatter from PPE division CERN, Geneva where this work was completed. Another author (MJ) would like to thank Physics Department of Karlsruhe University for support. Work supported by the Polish Government Grants CPBP.01.09, CPBP.01.03 and RRI.14.1.

References

- [1] S. Jadach, J. H. Kühn, Z. Wąs, Comput. Phys. Commun. **64** (1991) 275.
- [2] T. Sjöstrand, Comput. Phys. Commun. **27** (1982) 243; **28** (1983) 229,
M. Bengtsson and T. Sjöstrand, Comput. Phys. Commun. **43** (1987) 367.
- [3] QCD event generators subgroup of the 1989 LEP physics workshop, B. Bambah et al., CERN-TH.5466/89 (1989), and T. Sjöstrand et al. QCD generators, in: Z-Physics at LEP 1, eds G. Altarelli, R. Kleiss and C. Verzegnassi, vol. 3, CERN, Geneva, 1989, p. 147.
- [4] M. Aguilar-Benitez et al. (Particle Data Group), 'Review of Particle Properties', Phys. Lett. **B239** (1990) 1.
- [5] S. Jadach, Z. Wąs, Comput. Phys. Commun. **64** (1991) 267.
- [6] E. Barberio, B. van Eijk, Z. Wąs, PHOTOS - A universal Monte Carlo for QED radiative corrections in decays, CERN preprint, CERN TH-5857/90, Comput. Phys. Commun. (1991) in press.
- [7] S. Jadach, B. F. L. Ward and Z. Wąs, The Monte Carlo program KORALZ, version 3.8, for the lepton or quark pair production at LEP/SLC energies, CERN preprint, CERN TH-5994/91, Comput. Phys. Commun. (1991) in press.
- [8] T. Kinoshita, A. Sirlin, Phys. Rev. **B113** (1959) 1652.
- [9] A. Czarnecki, M. Jezabek, J. H. Kühn, Nucl. Phys. **B351** (1991) 70.
- [10] M. Jezabek, J. H. Kühn, Nucl. Phys. **B320** (1989) 20.
- [11] F. A. Berends, S. Jadach, R. Kleiss, Comput. Phys. Commun. **29** (1983) 185.
- [12] E. M. Lifshitz, L. P. Pitajevskii, Relativistic Quantum Theory, vol. 4 part 2., Pergamon Press, Oxford, 1974.

=====

DEMONSTRATION PROGRAM
TESTS OF TAU DECAY ROUTINES
INTERFACE OF THE KORAL-B TYPE

=====

JAK = 1 KEY DEFINING DECAY TYPE
IDFF = -11 LUND IDENTIFIER FOR FIRST TAU
POL(3)= -1.00 THIRD COMPONENT OF TAU POLARIZ.
PTAU = 0.00 THIRD COMPONENT OF TAU MOM. GEV

=====

*****TAUOLA LIBRARY: VERSION 1.5 *****
*****AUGUST 1990*****
AUTHORS: S.JADACH, J.H.KUEHN, Z.WAS
AVAILABLE FROM: WASH AT CERNVM ***
**TO BE PUBLISHED IN COMP. PHYS. COMM.*
*****CERN-TE-5856 SEPTEMBER 1990*****
****DEKAY ROUTINE: INITIALIZATION*****
1 JAK1 = DECAY MODE TAU+
1 JAK2 = DECAY MODE TAU-

=====

===== EVENT NO. 1 =====

POLARIMETRIC VECTOR: HH(1) HH(2) HH(3) HH(4)
-0.18907315 -0.41422462 -0.89031965 1.00000000

EVENT LISTING (EXTENDED)

I	K(I,1)	K(I,2)	PART/JET	P(I,1)	P(I,2)	P(I,3)	P(I,4)	P(I,5)
1	4	12	NUTAO	-0.15128	-0.33119	-0.71144	0.79920	0.00053
2	20004	-3	W B- D	0.00000	0.00000	0.00000	1.78420	1.78420
3	2	7	E -	0.23837	0.27057	0.11409	0.37821	0.00053
4	2	-8	NUE B	-0.08710	0.06062	0.59735	0.60679	0.01001
SUM:				-1.00000	0.00000	0.00000	0.00000	1.78420

===== EVENT NO. 2 =====

POLARIMETRIC VECTOR: HH(1) HH(2) HH(3) HH(4)
-0.79466707 -0.60486490 0.05140948 1.00000000

EVENT LISTING (EXTENDED)

I	K(I,1)	K(I,2)	PART/JET	P(I,1)	P(I,2)	P(I,3)	P(I,4)	P(I,5)
1	4	12	NUTAO	-0.25106	-0.19129	0.01613	0.31605	0.00024
2	20004	-3	W B- D	0.00000	0.00000	0.00000	1.78420	1.78420
3	2	7	E -	-0.34036	0.43657	0.41873	0.69410	0.00069
4	2	-8	NUE B	0.59142	-0.24528	-0.43487	0.77405	0.01001
SUM:				-1.00000	0.00000	0.00000	0.00000	1.78419

===== EVENT NO. 3 =====

POLARIMETRIC VECTOR: HH(1) HH(2) HH(3) HH(4)
-0.45832717 0.69076073 0.55927247 1.00000000

EVENT LISTING (EXTENDED)

I	K(I,1)	K(I,2)	PART/JET	P(I,1)	P(I,2)	P(I,3)	P(I,4)	P(I,5)
1	4	12	NUTAO	-0.06950	0.10462	0.08473	0.15151	0.00023
2	20004	-3	W B- D	0.00000	0.00000	0.00000	1.78420	1.78420
3	2	7	E -	0.79046	-0.16603	-0.32437	0.87040	0.00073
4	2	-8	NUE B	-0.72096	0.06141	0.23964	0.76228	0.01000
SUM:				-1.00000	0.00000	0.00000	0.00000	1.78419

===== EVENT NO. 4 =====

POLARIMETRIC VECTOR: HH(1) HH(2) HH(3) HH(4)
 0.47849816 -0.47119969 -0.74095207 1.00000000

EVENT LISTING (EXTENDED)

I	K(I,1)	K(I,2)	PART/JET	P(I,1)	P(I,2)	P(I,3)	P(I,4)	P(I,5)
1	4	12	WUTAO	0.31626	-0.31147	-0.48978	0.66100	0.00049
2	20004	-3	W B- D	0.00193	0.00155	-0.00116	1.78146	1.78146
3	2	7	E -	-0.33219	-0.26719	0.17746	0.46177	0.00060
4	2	-8	WUE B	0.01786	0.58021	0.31116	0.65870	0.01001
5	4	1	GAMM	-0.00193	-0.00155	0.00116	0.00273	0.00000
SUM:			-1.00000	0.00000	0.00000	0.00000	1.78419	1.78419

***** DADMEL FINAL REPORT *****
 *
 * 40417 NEVRAW = NO. OF EL DECAYS TOTAL *
 * 10000 NEVACC = NO. OF EL DECS. ACCEPTED *
 * 0 NEVGVR = NO. OF OVERWEIGHTED EVENTS *
 * 0.41588E-12 PARTIAL WTDTH (ELECTRON) IN GEV UNITS *
 * 1.006550790 IN UNITS GFERMI**2*MASS**5/192/PI**3 *
 * 0.004617881 RELATIVE ERROR OF PARTIAL WIDTH *
 * COMPLETE QED CORRECTIONS INCLUDED *
 * BUT ONLY V-A COUPLINGS *

 *****TAUOLA LIBRARY: VERSION 1.5 *****
 *
 * *****AUGUST 1990*****
 * **AUTHORS: S.JADACH, J.H.KUEHN, Z.WAS**
 * **AVAILABLE FROM: WASH AT CERNVM *****
 * **TO BE PUBLISHED IN COMP. PHYS. COMM.*
 * *****CERN-TH-5856 SEPTEMBER 1990*****
 * *****DEKAY ROUTINE: FINAL REPORT*****
 * 0 NEV1 = NO. OF TAU+ DECS. ACCEPTED *
 * 10000 NEV2 = NO. OF TAU- DECS. ACCEPTED *
 * 10000 NEVTOT = SUM *
 *
 * NOEVTS PART.WIDTH ERROR ROUTINE DECAY MODE *
 * 10000 1.0065508 0.0046179 DADMEL ELECTRON *
 * 0 0.0000000 0.0000000 DADMMU MUON *
 * 0 0.0000000 0.0000000 DADMPI PION *
 * 0 0.0000000 0.0000000 DADMRO RHO (->2PI) *
 * 0 0.0000000 0.0000000 DADMAA A1 (->3PI) *
 * 0 0.0000000 0.0000000 DADMKK KAON *
 * 0 0.0000000 0.0000000 DADMKS K* *
 * 0 0.0000000 0.0000000 DADNPI PI- 2PI+ PIO *
 * 0 0.0000000 0.0000000 DADNPI PI- 3PIO *
 * 0 0.0000000 0.0000000 DADNPI 3PI- 2PI+ *
 * 0 0.0000000 0.0000000 DADNPI 3PI- 2PI+ PIO *
 * THE ERROR IS RELATIVE AND PART.WIDTH *
 * IN UNITS GFERMI**2*MASS**5/192/PI**3 *