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Detailed fermion mass and mixing angle predictions from
a class of three-family models *

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Abstract

We explore in detail a pattern of standard fermion mass matrices motivated by a previous analysis of a class of three-family superstring models. These matrices can accommodate, assuming the present experimental accuracy, the observed fermion masses and mixing angles. We provide a numerical analysis, including renormalization effects, of the allowed regions for the CKM angles $\theta_{ub,cb}$, the CP violating phase δ_{ub} and the top mass m_t . We find $m_t < 156 \text{ GeV}$ but no bound on δ_{ub} . We also find that the ratio of the two Higgs vacuum expectation values in the minimal supersymmetric standard model, $\tan\beta$, can vary little near 1.

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In this paper we examine a pattern of quark and charged lepton mass matrices motivated by a definite model derived from a class of three-family models based on the $E_8 \times E'_8$ superstring compactified on the Calabi-Yau manifold of Tian and Yau. We only refer to the origin of these mass matrices to introduce their structure and the constraints on some of their entries [1]. The quark mass matrices depend on 10 parameters. This is also the number of physical quantities in the quark mass sector: 6 quark masses ($m_{u,c,t}$, $m_{d,s,b}$), 3 Cabibbo-Kobayashi-Maskawa (CKM) mixing angles and 1 CP violating phase [2, 3]. We shall follow the notation of Ref. [4],

$$CKM = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{13}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{13}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{13}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{13}} & -c_{13}s_{13}e^{i\delta_{13}} & c_{23}c_{13} \end{pmatrix}, \quad (1)$$

where $\theta_{ub} = \theta_{13}$, $\theta_{cb} = \theta_{23}$, $\delta_{ub} = \delta_{13}$. Nevertheless, to have the same number of parameters as of physical quantities does not guarantee that the experimental values of these quantities can be reproduced, as it is well-known for practitioners. The dependence on the 10 parameters of the $2 \times 2 \times 3 \times 3 = 36$ real numbers fixing the up and down mass matrices must be appropriate to obtain a set of definite mass eigenvalues and eigenvectors, even more when some of these parameters are ordered hierarchically. As a matter of fact, it is non-trivial that these matrices do accommodate the presently allowed range of quark masses and mixing angles. In particular, they predict $m_t < 156 \text{ GeV}$, $\theta_{ub} > 1.4 \times 10^{-3}$ for $m_t > 90 \text{ GeV}$ and $\theta_{cb} > 0.03$, and $\theta_{cb} < 0.06$ for $m_t > 100 \text{ GeV}$ and $\theta_{ub} < 7 \times 10^{-3}$. They also imply that the relative size of the up and down quark mass matrices or the ratio of Higgs vacuum expectation values, $\tan\beta$, is order 1, a prediction which could be tested in the Higgs sector but we do not discuss it here [5]. There is no prediction for the charged lepton masses but once the mass eigenstates are identified, the proton lifetime can be estimated in the unified model. We find that this is within experimental limits.

Mass matrices motivated by superstrings have been studied by other authors [6]. In particular, a realistic generalized Fritzsch ansatz [7] has been argued to emerge for the quark mass matrices from orbifold compactification [8]. Our quark mass matrices are not of the same type, although they are also parametrized by 8 moduli and 2 phases. (Other patterns of mass matrices have been also proposed in the literature by other motivations [9].)

Charged fermion mass matrices at very large scales. The field theory which motivates the pattern of mass matrices we are interested in is defined near the Planck scale, at the compactification/unification scale M_c . At M_c the matter content of the model is very rich and the gauge group is $SU(3)_C \times SU(3)_L \times SU(3)_R$ [1, 10]. At M_1

near M_c the gauge symmetry reduces to $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$, and at M_2 a few orders of magnitude below to $SU(3)_C \times SU(2)_L \times U(1)_Y$ [11]. At this scale we are left with the standard gauge group and an almost minimal matter content. Our starting point is the pattern of charged Yukawa couplings at M_2 , which can be written

$$h^0 u_j^c \begin{pmatrix} & u_i \\ D & . & B \\ . & A & C \\ B & -C & . \end{pmatrix}, \quad (2)$$

$$h'^0 d_j^c \begin{pmatrix} & d_i \\ \epsilon_1 D & . & \epsilon_1 B \\ . & \epsilon_2 A & \epsilon_2 C \\ B & -C & F \end{pmatrix}, \quad (3)$$

$$h'^0 e_j^c \begin{pmatrix} & e_i \\ Q & . & P \\ G & . & . \\ . & H & . \end{pmatrix}, \quad (4)$$

where h^0, h'^0 are the two neutral Higgses and u, d and e stand for the up and down quarks and charged leptons, respectively. A, B and G are the only Yukawa couplings resulting from renormalizable terms, and then the only entries possibly large. Hence, A and/or B give mass to the top. All the other entries result from non-renormalizable terms and mixing, and then they are relatively small. The dots correspond to negligible Yukawa couplings. The factors $\epsilon_{1,2}$, which appear due to the mixing of the right-handed down quarks, are small numbers. They allow for the different size of the top and bottom quark masses. In the following we shall study the pattern of mass matrices implied by (2-4), assuming no constraint on the entries except for the hierarchy just mentioned.

Quark mass matrices at the electroweak scale. The Yukawa couplings in (2,3) are fixed at M_2 , which we take to be equal to 10^{14} GeV . This is the order of magnitude in the unified model. (A small change in M_2 is of no significance for the present analysis, however.) These couplings must be evolved down to the electroweak scale to compare with experiment. The renormalization group equations can be found in Ref. [12]. (See also Ref. [13], and for the supersymmetric case Refs. [14,15].) Although we shall present numerical results, a simple approximate analytical treatment is also possible [16, 17].

All entries in (2,3) are complex. With a proper phase redefinition of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplets (2,3) can be written

$$h^0 u_j^c \begin{matrix} u_i \\ \left(\begin{array}{ccc} D & . & B \\ . & Ae^{i\alpha} & C \\ B & -C & . \end{array} \right), \end{matrix} \quad (5)$$

$$h'^0 d_j^c \begin{matrix} d_i \\ \left(\begin{array}{ccc} \epsilon_1 D & . & \epsilon_1 B \\ . & \epsilon_2 Ae^{i\alpha} & \epsilon_2 C \\ B & -C & Fe^{i\gamma} \end{array} \right), \end{matrix} \quad (6)$$

where all the parameters are now real and positive, and α and γ can vary from 0 to 2π . (The presence of non-renormalizable terms avoids the possibility of a CP conserving theory [18].) Hence the up and down quark Yukawa matrices depend on 9 parameters, 7 moduli and 2 phases. Their evolution down to the electroweak scale is due to gauge and to Yukawa interactions. The Yukawa contribution, however, is only important for large m_t . To the extent that the other Yukawa couplings are small (and that m_t is mainly proportional to a as we argue below), the evolution of (5,6) stands only for a multiplicative renormalization of the moduli but not of the phases. In this approximation the Yukawa matrices can be written at low energy

$$h^0 u_j^c \begin{matrix} u_i \\ \left(\begin{array}{ccc} d & . & b \\ . & ae^{i\alpha} & c \\ b & -c & . \end{array} \right), \end{matrix} \quad (7)$$

$$h'^0 d_j^c \begin{matrix} d_i \\ \left(\begin{array}{ccc} \epsilon_1 d & . & \epsilon_1 b \\ . & \epsilon_2 ae^{i\alpha} & \epsilon'_2 c \\ b' & -c' & fe^{i\gamma} \end{array} \right), \end{matrix} \quad (8)$$

where the change from capital to small letters takes into account the renormalization. The primes stand for different renormalization factors. $\epsilon_1, \epsilon_2^{(')}$ are redefined to absorb the renormalization not included in the small letters. At any rate, the order of magnitude of the entries is unchanged. Thus, prime and unprime entries are of the same order. At the electroweak scale h^0 and h'^0 develop vacuum expectation values v and v' , respectively, with

$$M_W^2 = \frac{g_L^2}{2}(v^2 + v'^2), \quad (9)$$

where M_W is the W mass (80 GeV [2]) and g_L the $SU(2)_L$ gauge coupling. Reabsorbing v' in the definition of the entries in (7,8) the down quark mass matrix M^d is equal to (8), whereas the up quark mass matrix M^u is equal to (7) except for a global multiplicative factor, $\tan\beta = \frac{v}{v'}$, which becomes the tenth parameter.

Because it is a good approximation and for simplicity, in the numerical evolution we only distinguish two low scales. Thus, we have evolved the Yukawa couplings from M_2 , Eqs. (5,6), down to $M_3 = 1 \text{ TeV}$, where we decouple the supersymmetric partners, and from M_3 down to the Z mass scale (M_Z). In this way we identify the electroweak, the top and the Z mass scales. The gauge evolution is fixed by requiring [2]

$$\begin{aligned} g_C^2(M_Z) &= 4\pi(0.12), \\ g_L^2(M_Z) &= 4\pi(0.23 \times 128)^{-1}, \\ g_Y^2(M_Z) &= \frac{5}{3}4\pi(0.77 \times 128)^{-1}. \end{aligned} \tag{10}$$

Quark masses and the CKM matrix. The up and down quark mass matrices $M^{u,d}$ are evaluated at M_Z . Diagonalizing them,

$$\begin{aligned} U_R^u M^u U_L^{u\dagger} &= D^u, \\ U_R^d M^d U_L^{d\dagger} &= D^d, \end{aligned} \tag{11}$$

we obtain the CKM matrix

$$CKM = U_L^u U_L^{d\dagger}. \tag{12}$$

To compare the mass eigenvalues $D^{u,d}$ with the light quark masses we have to evolve these up to M_Z . We take $m_{u,d,s,c} = 5.6, 9.9, 199, 1350 \text{ MeV}$ at 1 GeV and evolve them up to $m_b = 5 \text{ GeV}$, where we introduce the bottom quark [2]. Then, we evolve the five light masses up to M_Z . In this renormalization the Yukawa contributions are neglected. The phenomenological relevance of the proposed pattern of mass matrices can be now discussed. We allow for arbitrary values of the 7 moduli and 2 phases at M_2 . Then after evolving them down to M_Z and including $\tan\beta$ (for which we also allow an arbitrary value), we calculate the mass eigenvalues and the CKM matrix. We require that the five light quark masses and the Cabibbo angle be equal to their central experimental values ($s_{12} = 0.221$ [2]) and plot the allowed regions for $s_{13,23}, \delta_{13}$ and m_t . It is non-trivial that the predicted regions include the regions experimentally favoured. In Fig. 1 we plot $s_{23,13}$ and δ_{13} as a function of m_t . The frame is fixed by the presently allowed intervals [2]. The dash line gives the boundary of the allowed region. Inside we plot the regions of fixed $\tan\beta$ for several illustrative values. We also plot δ_{13} as a function of s_{13} . As can be seen, this pattern of mass

matrices predicts

$$\begin{aligned}
m_t &< 156 \text{ GeV}, \\
\theta_{13} &> 1.4 \times 10^{-3} \text{ for } m_t > 90 \text{ GeV and } \theta_{23} > 0.03, \\
\theta_{23} &< 0.06 \text{ for } m_t > 100 \text{ GeV and } \theta_{13} < 7 \times 10^{-3},
\end{aligned} \tag{13}$$

but no bound on δ_{13} . To better illustrate the correlation between the different quantities we also plot a particular case (cross). The $\tan\beta$ value for this point is given in parentheses. Since the sign of δ_{13} is not fixed, the corresponding plots show a sign degeneracy. Fig. 2 is the same as Fig. 1 but interchanging m_t and $\tan\beta$.

From these Figures it can be observed that a very precise θ_{23} and θ_{13} measurement would fix m_t and $\tan\beta$ but not δ_{13} . The CP violating phase δ_{13} is not constrained by this pattern of mass matrices. The δ_{13} plots have a symmetry (or sign ambiguity). As all the constraints are unchanged under complex conjugation, if a set of parameters reproduce the light quark masses and the Cabibbo angle so does the complex conjugate set. Complex conjugation reverses the sign of α and γ , as well as the sign of δ_{13} . An implicit constraint, but which is of no relevance for the m_t upper bound because this is relatively low, is the requirement that no Yukawa coupling must explode (or have a Landau pole) below M_2 . This implies $\frac{m_t}{v} < 1.1$ (which translates into $\tan\beta > 0.54$ for $m_t > 90 \text{ GeV}$). This is a prediction of the minimal supersymmetric standard model (and then it is on the same footing as the gauge unification near the Planck scale [19]). In Fig. 3 we plot the allowed region for $\frac{m_t}{v}$ and $\tan\beta$. In particular $\frac{m_t}{v} > 0.84$, implying a very narrow $\frac{m_t}{v}$ variation.

In our numerical analysis we use approximate expressions to optimize the scanning of the allowed regions in parameter space. They also allow for a better understanding of the different constraints. For instance, m_t is mainly proportional to A in (5), or after renormalization to a in (7). This follows from the observation that A and B in Eq. (5) are the only Yukawa couplings coming from renormalizable terms in the unified model and then the only possibly large [1]. However if $m_t \sim b$, then $m_c \sim m_t$ because b appears in M^u in two different lines and columns (note that a (double-)prime stands for a correction order one). Hence $m_t \sim a$ and $a^2 \gg b^2, c^2, d^2$. Although one would then expect $m_c \sim b$, this can not be so for it would imply $m_u \sim m_c$. Thus $m_c \sim d$ and $d^2 \gg b^2$. This, in turn, requires a large cancellation to implement $m_u \sim 0$. These constraints completely fix the approximate solution when realistic values for the *CKM* mixing angles are also required. We must emphasize that f in M^d is an entry somewhat suppressed in the unified model and then it can not be proportional to m_b . All this is explicit in the

approximate perturbative diagonalization of $M^{u,d}$,

$$\begin{aligned} m_u &\sim \left| \frac{c^2}{a} - \frac{b^2}{d} e^{i\alpha} \right| \tan\beta, \\ m_c &\sim d \tan\beta, \\ m_t &\sim a \tan\beta, \end{aligned} \quad (14)$$

with

$$U_L^u \sim \begin{pmatrix} . & . & 1 \\ 1 & . & . \\ . & 1 & . \end{pmatrix} \times \begin{pmatrix} 1 & . & \frac{b}{d} \\ . & 1 & \frac{c}{a} e^{-i\alpha} \\ -\frac{b}{d} & -\frac{c}{a} e^{i\alpha} & 1 \end{pmatrix}, \quad (15)$$

and

$$\begin{aligned} m_d &\sim \left| \epsilon_1 b - \epsilon_1 d \frac{f}{b'} e^{i\gamma} - \frac{\epsilon_1 d \epsilon_2' c c'}{\epsilon_2 a b'} e^{-i\alpha} \right|, \\ m_s &\sim \frac{\epsilon_2 a b'}{\sqrt{\epsilon_2^2 a^2 + c'^2}}, \\ m_b &\sim \sqrt{\epsilon_2^2 a^2 + c'^2}, \end{aligned} \quad (16)$$

with

$$U_L^d \sim \begin{pmatrix} . & . & 1 \\ 1 & . & . \\ . & 1 & . \end{pmatrix} \times \begin{pmatrix} 1 & \frac{c' b'}{\epsilon_2^2 a^2 + c'^2} & \frac{\epsilon_2' c c' e^{-i\alpha} + \epsilon_2 a f e^{i\gamma}}{\epsilon_2 a b'} \\ -\frac{c' b'}{\epsilon_2^2 a^2 + c'^2} & 1 & \frac{\epsilon_2 a \epsilon_2' c e^{-i\alpha} - c' f e^{i\gamma}}{\epsilon_2^2 a^2 + c'^2} \\ -\frac{\epsilon_2' c c' e^{i\alpha} + \epsilon_2 a f e^{-i\gamma}}{\epsilon_2 a b'} & -\frac{\epsilon_2 a \epsilon_2' c e^{i\alpha} - c' f e^{-i\gamma}}{\epsilon_2^2 a^2 + c'^2} & 1 \end{pmatrix}. \quad (17)$$

(A phase redefinition is necessary to compare the corresponding *CKM* matrix to (1).) For obtaining the last equation we use $b^2 \gg \epsilon_1^2 d^2$ and $\epsilon_2^2 c^2 \gg \epsilon_1^4 d^2$. The large cancellation required to explain the smallness of m_u implies in particular $\alpha \sim 0$ in (14). All moduli in this expression are positive and relatively large, then the phase must be near zero to cancel the whole expression. This reduces effectively the number of parameters to 9 and allows for writing m_t as a function of the other masses and of the *CKM* angles. Inverting the former equations the parameters of the model can be also determined as a function of the quark masses and the *CKM* angles.

Charged lepton masses and proton decay. Similarly to the quark case the charged lepton Yukawa matrix (4) must be renormalized down to low energies to confront experiment. In this case, however, the Yukawa contributions to the renormalization group equations are small. The gauge contributions are also smaller for they do not depend on the strong gauge coupling. After evolving (4) and reabsorbing the Higgs vacuum expectation value v' , the charged lepton mass matrix reads (neglecting the smaller renormalization effects)

$$M^e = \begin{pmatrix} q & . & p \\ g & . & . \\ . & h & . \end{pmatrix}, \quad (18)$$

with no special relation among the different entries. Then, the lepton masses $m_{e,\mu,\tau}$ [2] can be fitted. It is better motivated, however, that m_τ be mainly g in (18) because G in (4) is the only Yukawa coupling coming from a renormalizable term in the unified theory and then possibly large. Due to their suppression at M_c it is also better motivated

$$\begin{aligned} m_\mu &\sim h, \\ m_e &\sim p. \end{aligned} \tag{19}$$

Once the quark and charged lepton eigenstates and mass eigenvalues have been determined, experiment fixes the arbitrary parameters (couplings and normalization factors) in the unified model. Then the proton lifetime can be estimated. In our case the dominant decay is into a muon and it is mediated by a dimension 5 operator [20]. (This class of unified models incorporates a matter parity [21].) The predicted rate is within experimental limits if the complex structure σ , which defines the Yukawa couplings at M_c , is large ($|\sigma| > 10^{1.5}$). It is remarkable that there is a choice of parameters at the unification scale that, to the extent we know, is not in contradiction with low energy data (even could predict the top mass with a more precise determination of the CKM mixing angles) and gives the correct pattern of gauge symmetry breaking [11]. It may happen, however, that when more theoretical constraints on these parameters are known, no such choice exist and this pattern of masses loses its motivation. This pattern can be also ruled out by more precise data on $\theta_{ub,cb}$ and m_t (an eventually on δ_{ub}). We have not yet analysed in detail the neutrino sector, which is more involved.

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Fig. 1

Fig. 2

Fig. 3