CERN-TH-6239/91

CP VIOLATION WITH ADDITIONAL B DECAYS

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ABSTRACT

The observation of CP violation and the clean extraction of the fundamental CKM phases form the subject of this chapter. After reviewing the classic CP eigenmodes, where CP asymmetries are straightforwardly related to the CKM phases, more detailed methods are discussed. They allow the extraction of the phases from many additional modes, which are no longer CP eigenstates. The methods covered in this review are angular correlations, time-dependent studies of non-CP eigenmodes, isospin analyses, and detailed studies of B modes with a D^0 .

To be published in "B Decays," edited by S. Stone

CERN-TH-6239/91
October 1991

1. Introduction

This chapter shows that B-modes other than the pure CP eigenmodes--that is, decay modes that are CP eigenstates--have large CP violation effects. A careful study of those additional modes allows us to determine the fundamental Cabibbo-Kobayashi-Maskawa¹ (CKM) phases. As CP non-invariance is still a mystery even 27 years after its discovery, it is imperative to probe it in as many ways as possible. B mesons appear promising, because the current Standard Model--where CP violation is believed to arise from the complex values of the 3×3 CKM matrix--predicts large effects for some of their modes.²⁻⁵

Although the effects that arise when CP eigenmodes are considered are well known,²⁻⁵ Section 2 reviews them again, because they are the standard against which other methods are compared. What makes those modes so special is that when the amplitude of a pure B^0 is dominated by one weak phase, the asymmetry can be related immediately to the fundamental CKM parameters.

One group of CP eigenmodes is not as well known as it deserves to be. It consists of CP eigenmodes of B_d , governed by the quark subprocess $\bar{b} \rightarrow \bar{c} + u \bar{d}$, which are predicted to exhibit the same asymmetry as the $B_d \rightarrow \psi' K_S$ mode.⁶ No matter how complicated the final-state interactions, the CP asymmetry does not depend on them, and measures the CKM phase, just as is the case for the classic $\psi' K_S$ mode.⁷

Even though the asymmetry for a few of the CP eigenmodes is predicted to be large, their tiny branching rates make it highly desirable to search for additional modes that could complement them in the search for CP violation.

Section 3 demonstrates how angular correlations of the decay products add many new modes.⁸⁻¹³ The particle content of all the modes in this section is such that states with a definite CP parity can be constructed from a superposition of helicity states, without invoking a different particle content. The CP asymmetry could suffer from a partial cancellation, due to an admixture of CP-even and -odd parities. A detailed study of their angular correlations, however, eliminates this partial cancellation and probes the fundamental CP asymmetry.

That even non-CP eigenmodes lead to an extraction of the CKM phase is shown in Section 4. Naively, one would think that the strong interactions might cause additional phases which would make the extraction of the CKM phase impossible. Such is not the case, and a careful time-dependent study reveals the underlying CKM phase, in which four distinct processes are involved.¹⁴ They supply sufficient information to construct a quantity that is insensitive to the final-state phases and that measures the CKM phase up to some n -fold ambiguity. The drawback of this method is that large data samples may be required. The analysis can be simplified for a few modes where it seems likely that no strong final-state phase differences occur.

Whereas all sections assume the dominance of one weak phase in the decay amplitude, Section 5 discusses how to extract the CKM phase when this assumption is relaxed. In principle, an isospin analysis helps in disentangling the various contributions.¹⁵⁻¹⁷

The weak phases α and β can be determined from many B_d modes through the methods reviewed in Sections 2-5. In contrast, the conventional way to probe the phase γ uses only a handful of processes, such as $B_s \rightarrow \rho^0 K_S$, with predicted tiny rates and inherent theoretical uncertainties. Section 6 shows an alternative. Large CP violating effects can occur in B modes with a D^0 . The effects do not require any $B^0 - \bar{B}^0$ mixing, but happen because of interference effects between two B amplitudes with the weak phase difference γ . In the literature, the interferences are accomplished by requiring the D^0 to be seen in a CP eigenmode.¹⁸⁻²¹ Because this is really not essential, additional CP violating asymmetries can be sought. The extraction of the weak phase γ is made possible by removing the final-state phase difference. The removal is accomplished by determining separately the modulus of each individual amplitude--of which there are six.¹⁹⁻²¹

Prospects and conclusions are found in Sections 7 and 8, respectively. Other avenues which further enrich the data samples for CP violation studies, such as Dalitz plot analyses, will have to be relegated to a later review.

2. CP Eigenmodes

The B decay asymmetries can be graphically related to the CKM parameters in an elegant way. The unitarity of the 3 X 3 CKM matrix yields:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0. \quad (2.1)$$

The unitarity triangle is a geometrical presentation of this equation in the complex plane. The angles α, β, γ of this triangle do not depend on the choice of the CKM parametrization and have an important physical content.²² They are related to the rate asymmetries of B decays into CP eigenmodes.

A CP eigenmode (or eigenstate), f , is a state that has definite transformation properties under CP,

$$\bar{f} \equiv CP f = \pm f. \quad (2.2)$$

As for the rate asymmetries, consider a time-evolved, initially pure B^0 , which decays into f , $B^0_{\text{phys}} \rightarrow f$. In our case, it has two amplitudes, either

- (a) direct, when B^0 decays into the mode f , or
- (b) induced by mixing, when B^0 first mixes into $\bar{B}^0$, which then decays into the same mode.

The rate is thus sensitive to an interference. The CP conjugated process, $\bar{B}^0_{\text{phys}} \rightarrow \bar{f}$, involves two analogous amplitudes and is also sensitive to interference. Even when the magnitudes are equal for the direct amplitudes of $B^0_{\text{phys}} \rightarrow f$ and $\bar{B}^0_{\text{phys}} \rightarrow \bar{f}$, and the same would hold true for those induced by mixing to excellent approximation, large rate differences could still appear. This CP violation effect occurs precisely because of the above-mentioned interferences.

To quantify what was said above, the interference is measured by

$$\lambda = \frac{q < f | \bar{B}^0 >}{p < f | B^0 >}. \quad (2.3)$$

The coefficients q and p relate the eigenstates of the B-mass matrix to the B^0 and $\bar{B}^0$ states,

$$|B_{L(H)} > = p |B^0 > + (-) q | \bar{B}^0 >, \quad (2.4)$$

the light (heavy) B^0 being denoted by $B_{L(H)}$. The Standard Model predicts that⁵

$$\frac{q}{p} \approx \frac{V_{td}V_{tb}^*}{V_{td}V_{tb}^*}, \quad (2.4)$$

and $\sigma = d$ or s for the B_d or B_s system. The time-dependent rates are

$$\Gamma(B^0_{\text{phys}}(t) \rightarrow f) = |<f|B^0>|^2 e^{-\Gamma t} \{1 - \text{Im} \lambda \sin \Delta m t\}, \quad (2.5)$$

$$\Gamma(\bar{B}^0_{\text{phys}}(t) \rightarrow f) = |<f|\bar{B}^0>|^2 e^{-\Gamma t} \{1 + \text{Im} \lambda \sin \Delta m t\}. \quad (2.6)$$

Any difference between those two rates--that is, $\text{Im} \lambda \neq 0$ --signals CP violation. Information on $\text{Im} \lambda$ is obtained from the rate asymmetry,

$$\text{Asym}(B^0 \rightarrow f) \equiv \frac{\Gamma(B^0_{\text{phys}}(t) \rightarrow f) - \Gamma(\bar{B}^0_{\text{phys}}(t) \rightarrow f)}{\Gamma(B^0_{\text{phys}}(t) \rightarrow f) + \Gamma(\bar{B}^0_{\text{phys}}(t) \rightarrow f)} = -\text{Im} \lambda \sin \Delta m t. \quad (2.7)$$

CP eigenmodes of the B meson have the nice feature that all hadronic uncertainties cancel and a measurement of CP violation is a measurement of

$$\sin 2\phi = \pm \text{Im} \lambda, \quad (2.8)$$

where ϕ is one of the unitarity angles α, β or γ . Thus, from a measurement, one obtains information about fundamental CKM elements.

The implicit assumption throughout is that only one weak phase dominates the direct amplitude of $B^0 \rightarrow f$. Its validity for various modes is discussed below. The only other place in this chapter where this assumption is questioned is Section 5. In principle, a more involved analysis can there overcome the complications that arise when more than one weak phase dominates the direct amplitude. The clean interpretation as given in Eq. (2.8) is, however, lost.

To prove Eq. (2.8) for the process $B_d \rightarrow D^+ D^-$, the following theorem will be used.

Theorem²⁻³ If the direct amplitude of a B^0 (not time-evolved!) into any mode g (not necessarily a CP eigenmode) is dominated by one weak phase, and parametrized as $<g|B^0> \equiv G |a| e^{i\delta}$, the CP conjugated direct amplitude is then given by $<\bar{g}|\bar{B}^0> \equiv G^* |a| e^{i\delta}$. Here G is the relevant weak phase, $|a|$ the strength of the amplitude, and δ the strong interaction phase. The phase convention is used where $|\bar{B}^0> = CP |B^0>$.

Two immediate consequences are:

$$(1) \quad |\langle \bar{g} | \bar{B}^0 \rangle| = |\langle g | B^0 \rangle|. \quad (2.9)$$

(2) If g is a CP eigenmode, then $\text{Im } \lambda(B^0 \rightarrow g)$ flips sign according to whether g is CP-even or -odd:

$$\lambda(B^0 \rightarrow \text{CP-even}) = -\lambda(B^0 \rightarrow \text{CP-odd}). \quad (2.10)$$

The direct amplitude is dominated by one weak phase and can be written as

$$\langle D^+ D^- | B_d \rangle \approx \frac{V_{cd} V_{cb}^*}{|V_{cd} V_{cb}|} |a| e^{i\delta}. \quad (2.11)$$

The theorem yields for the CP conjugated direct amplitude,

$$\langle D^+ D^- | \bar{B}_d \rangle \approx \frac{V_{cb} V_{cd}^*}{|V_{cd} V_{cb}|} |a| e^{i\delta}. \quad (2.12)$$

All hadronic uncertainties cancel when the interference term is formed,

$$\lambda = \frac{q \langle D^+ D^- | \bar{B}_d \rangle}{p \langle D^+ D^- | B_d \rangle} = \frac{q V_{cb} V_{cd}^* |a| e^{i\delta}}{p V_{cd} V_{cb} |a| e^{i\delta}} = \frac{q V_{cb} V_{cd}^*}{p V_{cd} V_{cb}}, \quad (2.13)$$

and so $\text{Im } \lambda = -\sin 2\beta$. Q.E.D.

For a given quark subprocess, Table 1 lists a few representative CP eigenmodes and the relevant interference term, $\text{Im } \lambda$, responsible for CP violation. This interference term, quoted always for a CP-even eigenmode, is obtained by assuming that the quark subprocess of column 1 is dominant. As shown in column 4, that assumption is better in some cases than in others.

Table 1

Decay modes and interference terms for various classes. The interference term (third column) assumes the quark subprocess (first column) to be dominant. The fourth column examines this assumption and notes whether contributions with different weak phases exist.

Quark subprocess (class)	Decay mode	$\text{Im } \lambda$	Different weak phase
$\bar{b} \rightarrow \bar{c} + c\bar{s}$ (i)	$B_d \rightarrow \psi K_s, \chi K_s, \eta_c K_s, \psi K_L, \dots$	$-\sin 2\beta$	Totally negligible
$\bar{b} \rightarrow \bar{c} + u\bar{d}$ (i)	$B_d \rightarrow f_{D^*} X^0$	$-\sin 2\beta$	Totally negligible
$\bar{b} \rightarrow \bar{c} + c\bar{d}$ (i)	$B_d \rightarrow D^+ D^-, D^0 \bar{D}^0, \dots$	$-\sin 2\beta$	Negligible
$\bar{b} \rightarrow \bar{u} + u\bar{d}$ (ii)	$B_d \rightarrow \pi^+ \pi^-, \rho^0 \pi^0, \omega \pi^0, \pi^0 \pi^0$	$\sin 2\alpha$	Possible
$\bar{b} \rightarrow \bar{u} + u\bar{d}$ (iii)	$B_s \rightarrow \rho^0 K_s, \omega K_s, \rho^0 K_L, \omega K_L$	$-\sin 2\gamma$	Possible
$\bar{b} \rightarrow \bar{c} + c\bar{s}, \bar{c} + c\bar{d}$	$B_s \rightarrow \psi \phi, \eta_c \phi, \psi K_s$	$\frac{V_{ub} V_{ub}^*}{2 V_{ud} V_{cb} } \sin \gamma$	Negligible

¹ Angular correlations are required to measure $\text{Im } \lambda$ for this mode, see Section 3.

We focus on three classes of measurements.⁴

(i) *Measuring $\sin(2\beta)$ in B_d decays.*

This class has the advantage that different quark sub-processes, $\bar{b} \rightarrow \bar{c} + c\bar{s}$, $\bar{b} \rightarrow \bar{c} + u\bar{d}$, $\bar{b} \rightarrow \bar{c} + c\bar{d}$, yield the same interference term, $\text{Im } \lambda = -\sin(2\beta)$. The CP eigenmodes of those three quark subprocesses have negligible contributions from different weak phases. This is especially true for the former two, and still holds for the $\bar{b} \rightarrow \bar{c} + c\bar{d}$ process. Here CP violation indeed measures $\text{Im } \lambda = -\sin(2\beta)$.

Because the CP eigenmodes that are governed by the $\bar{b} \rightarrow \bar{c} + u \bar{d}$ transition are more involved, we discuss them in some detail.^{6,7} They are denoted by $f_{D^{(*)}}$, X^0 , and arise from the decay chain $B_d \rightarrow D^{(*)0} X^0 \rightarrow f_{D^{(*)}} X^0$. Here $f_{D^{(*)}}$ is a CP eigenmode of the $D^{(*)0}$ meson. The X^0 symbolizes any flavour neutral meson, such as π^0 , ρ^0 , ω , a_1^0 . Whereas the state f_D has a definite CP parity for any X^0 , the state f_{D^*} has a definite one only for spinless X^0 . Angular correlation studies overcome this last restriction and allow the extraction of $\sin(2\beta)$ from the $f_{D^*} X^0$ modes, when X^0 has any spin.¹²

In addition to the X^0 , the other important element to obtain CP eigenmodes of B_d driven by $\bar{b} \rightarrow \bar{c} + u \bar{d}$, is that the intermediate D or D^* decay themselves to modes with definite CP parities. Examples of CP eigenmodes of the D^0 are

$$D^0 \rightarrow K_S \pi^0, K^+ K^-, \pi^+ \pi^-, K_S \rho^0, K_S \phi, \rho^0 \bar{K}^{*0}, \pi^0 \pi^0.$$

As for the D^* decays, they involve the D^{*0} , $BR(D^{*0} \rightarrow D^0 \pi^0) = BR(D^{*0} \rightarrow D^0 \gamma) = 50\%$. As long as this subsequent D^0 is seen in a CP eigenmode, the whole D^* mode has definite CP parity. The CP parity changes sign, depending whether a photon or π^0 was emitted in the D^{*0} decay, which causes the asymmetry to flip sign. Care is thus required in adding those modes.

(ii) Measuring $\sin(2\alpha)$ in B_d decays.

The relevant quark sub-process is $\bar{b} \rightarrow \bar{u} + u \bar{d}$. Possible two body decay modes are $B_d \rightarrow \pi^+ \pi^-, \rho \pi^0, \omega \pi^0, \pi^0 \pi^0$. These modes may suffer from additional contributions, which involve different weak phases, either from virtual intermediate states in the form of penguin diagrams at the quark level (see Fig. 1), or from real intermediate states, i.e., rescattering effects at the hadron level. For example, $B_d \rightarrow D^+ D^- \rightarrow \pi^+ \pi^-$ may upset the identification of $B_d \rightarrow \pi^+ \pi^-$ as a pure $\bar{b} \rightarrow \bar{u} + u \bar{d}$ transition. These additional contributions were estimated to be small,²⁴ and will be neglected in this section. They will be discussed in Section 5.

(iii) Measuring $\sin(2\gamma)$ in B_s decays:

The relevant quark sub-process is again $\bar{b} \rightarrow \bar{u} + u \bar{d}$, the same as that in class (ii). The interference term measures a different angle of the unitarity triangle, because it depends not only on the quark sub-process, but also on the B^0 . $\bar{B}^0$ mixing, which involves different CKM elements for the B_d and B_s systems. Possible modes are $B_s \rightarrow \rho K_S, \omega K_S$, although again rescattering effects may be important.

Figure 1: A penguin diagram for the process $B_d^0 \rightarrow \pi^+ \pi^-$. The internal quark, $\bar{q} = \bar{u}, \bar{c}, \bar{t}$ couples to gluon[s].

All of the above-mentioned modes have tiny detectable branching rates. To increase statistics, all those modes sensitive to the same interference term ought to be added. This addition must be performed carefully, because the asymmetry for the CP-even modes is opposite to the CP-odd ones [see Eqs. (2.7) and (2.10)]. Despite this addition and the prediction by the Standard Model that CP violation is large for several of those modes, the observation of CP non-invariance requires large data samples.

The chances to observe CP violation with the neutral B mesons are enhanced when additional decay modes are considered. The CP violation effects of those new modes ought to be easily related to the fundamental angles of the unitarity triangle, just as was the case for the CP eigenmodes.

With the aid of angular correlations, many additional modes can be used to extract those angles. That is the topic of the next section. Furthermore, Section 4 shows that non-CP eigenmodes allow a measurement of the fundamental unitarity angles.

3. Angular Correlations

This section treats modes that are mixtures of CP-even and -odd parities because of orbital angular momentum, such as $\Psi\rho^0$, $D^{*+}D^{*-}$, $\rho^+\rho^-$, $\omega\rho^0$, $\Psi\phi$, $\Psi\pi^0K_S$, $\Psi\rho^0K_S$, $\Psi\omega K_S$, $\Psi\rho^0\omega$. The modes suffer a partial cancellation of CP violation.

That partial cancellation occurs because of the consequences of the theorem stated in the last section, which yield²⁻³

$$\text{Asym}(B^0 \rightarrow \text{CP-even}) = -\text{Asym}(B^0 \rightarrow \text{CP-odd}). \quad (3.1)$$

The time-dependent (or time-integrated) decay rates are then

$$\Gamma \equiv \Gamma(B_{\text{phys}}^0 \rightarrow f) = \Gamma_+(1+a) + \Gamma_-(1-a), \quad (3.2)$$

$$\bar{\Gamma} \equiv \Gamma(\bar{B}_{\text{phys}}^0 \rightarrow \bar{f}) = \Gamma_+(1-a) + \Gamma_-(1+a). \quad (3.3)$$

The CP-even and CP-odd rates are parametrized by the widths Γ_+ and Γ_- , respectively. The fundamental asymmetry, a , is proportional to $\text{Im } \lambda$, and would be the asymmetry if the CP-even state dominated,

$$a \sim \text{Im } \lambda = -\sin 2\phi, \quad \text{where } \phi = -\alpha, \beta, \gamma. \quad (3.4)$$

The total rate asymmetry,

$$\text{Asym} \equiv \frac{\Gamma(B_{\text{phys}}^0 \rightarrow f) - \Gamma(\bar{B}_{\text{phys}}^0 \rightarrow \bar{f})}{\Gamma(B_{\text{phys}}^0 \rightarrow f) + \Gamma(\bar{B}_{\text{phys}}^0 \rightarrow \bar{f})} = a \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-}. \quad (3.5)$$

is indeed proportional to the fundamental asymmetry. It suffers, however, from a partial cancellation, described by the last term in the equation above.

Although the fundamental asymmetry can be very large, the measured rate asymmetry is zero when the CP-even and -odd rates are equal.

A study of the modes' angular correlations saves the situation, because the CP-even and CP-odd components of the same mode differ in their angular dependence.⁸⁻¹³ Once a definite CP component is extracted, the simple formulas for CP eigenstates apply. The rate asymmetry for the projected CP component depends on the fundamental asymmetry a , without any cancellation.

Angular studies are useful even when the partial cancellation is so small as to be negligible. Suppose that a single CP channel dominates and that the CP asymmetry is approximately measured without any angular analysis. An angular analysis can still be performed, with negligible loss in statistical accuracy and without any error from the small opposite CP contribution, and will yield a more precise measurement of the fundamental CP asymmetry.

Various detailed approaches of angular analysis are available.²⁵ One approach, developed by Kayser, Kuroda, Peccei and Sanda, considers $B^0 \rightarrow VV$ (vector vector) modes, where one vector decays into two spinless particles. The helicity zero component can be extracted. Its rate asymmetry suffers no cancellation, because it is a CP eigenstate.¹¹

Pursuing the work of Kayser et al., a later study identified further possible approaches.^{12,13} The second approach analyzes events in terms of a quantity called transversity, which characterizes the spin projections of a three-body intermediate state in a direction transverse to the plane of the three-body system. Moments of the data with respect to a single polar angle can achieve the required separation. This method has the advantage that it allows the summing of resonant and non-resonant contributions to certain final states, whereas the more detailed angular analysis requires reconstruction of a specific two-body parent system for the three-body state. To show how it is applied, see the appendix. The first two approaches require the minimum amount of angular analysis to reach definite CP quantities.

The third approach uses a complete angular analysis and forms all possible independent angular moments of the data. That allows the study of additional channels not amenable to the transversity treatment. Like the transversity moment analysis, it has the advantage of allowing asymmetries to

be extracted without *a priori* knowledge of the relative strengths of the different helicity contributions. In both cases that can be done by combining results from both B^0 and $\bar{B}^0$ decays.

The remaining two methods do not extract any definite CP components. They use a maximum-likelihood fit to the angular structure of the CP-violating decay and to a set of isospin-related channels that are not influenced by CP violating effects. The fit can be achieved either using only the transversity polar angle distributions or using the full angular distributions. For a transversity analysis of this type, one needs to know the relative strengths of the contributions for each possible absolute value of the transversity of each of the three particles. The strengths can frequently be determined from isospin-related channels. For the full angular analysis one needs to determine the full set of helicity amplitudes and their relative strong-interaction phases. Again, data from isospin-related channels may make this possible. This method will provide the most accurate measure of the asymmetry for those modes where sufficient data are available in all related channels to determine all the necessary quantities.

4. Non-CP eigenmodes.

This section deals with modes, such as $\rho^\pm \pi^\mp$, $a_1^\pm \pi^\mp$, $D^{*\pm} D^\mp$, where the particle content is not CP-self-conjugate. They have neither a definite CP parity, nor are they admixtures of states with different CP parities.

To appreciate the difficulties associated with such modes, we compare them to CP eigenmodes. The interference term of the latter measures the fundamental CKM parameters, i.e. weak phases, because the final-state phases cancel. The cancellation occurs because a CP transformation relates $\langle f | \bar{B}^0 \rangle$ to $\langle f | B^0 \rangle$, where the final-state phases are left unchanged. The final-state phases factor out in the interference term,

$$\lambda = \frac{q \langle f | \bar{B}^0 \rangle}{p \langle f | B^0 \rangle} . \quad (4.1)$$

Non-CP eigenmodes do not display this nice feature. That is because a CP transformation no longer relates the denominator to the numerator of Eq. (4.1)–

as is the case for CP eigenmodes–but rather $\langle f | B^0 \rangle \rightarrow \langle \bar{f} | \bar{B}^0 \rangle$. The interference term contains not only the desired weak phase, but also strong phases, which obscure its extraction. In further contrast to the CP eigenmodes, the magnitude $|\lambda|$ differs in general from 1.

All predictive power appears to be lost with non-CP eigenmodes, but that is not, in fact, the case. Salvaging the CKM phase is the subject of this section. We first show that a study of the four relevant time-dependent decay rates can extract the fundamental CKM phase. The extraction is accomplished by removing the dependence on final-state phases.¹⁴ Because that may require large data samples, we then discuss assumptions that are made for a few modes, which allow the determination of the weak phase with fewer statistics. We will show an argument for the assumption that no final-state phases occur when B^0 is seen in modes of two charged light particles. Assuming factorization and the Isgur-Wise symmetry, the B_q -modes $D^{*\pm} D^\mp$ have no final-state phase difference and satisfy $|\lambda| = 1$. The CKM phase can thus be easily recovered for modes like $\rho^\pm \pi^\mp$, $a_1^\pm \pi^\mp$, $D^{*\pm} D^\mp$.

We begin with the extraction of the weak phase, even when no cancellation of final-state phases occurs. Consider final states f , such as $\rho^\pm \pi^\mp$, $D^{*+} D^-$, etc.. The four direct amplitudes are

$$\langle f | B^0 \rangle = M e^{i\phi_w} e^{i\delta} , \quad (4.2)$$

$$\langle f | \bar{B}^0 \rangle = M' e^{-i\phi_w} e^{i\delta'} , \quad (4.3)$$

$$\langle \bar{f} | \bar{B}^0 \rangle = M e^{-i\phi_w} e^{i\delta} , \quad (4.4)$$

$$\langle \bar{f} | B^0 \rangle = M' e^{i\phi_w} e^{i\delta'} . \quad (4.5)$$

M and M' are magnitudes, ϕ_w is the weak phase, and δ, δ' are strong phases. Eqs. (4.4)–(4.5) use the theorem of Section 2, because it is likely that one weak phase dominates the direct amplitudes for the modes of interest.

The weak phase ϕ_w is not an observable, for the $B^0 \bar{B}^0$ mixing has yet to be incorporated. The observables are

$$\lambda(f) \equiv \frac{q \langle f | \bar{B}^0 \rangle}{p \langle f | B^0 \rangle} , \quad \bar{\lambda}(f) \equiv \frac{p \langle \bar{f} | B^0 \rangle}{q \langle \bar{f} | \bar{B}^0 \rangle} , \quad \lambda(\bar{f}), \quad \bar{\lambda}(\bar{f}). \quad (4.6)$$

They are rephase invariant.²⁶

The four time-dependent rates become

$$\Gamma(B_{\text{phys}}^0(t) \rightarrow f) = \frac{A^2}{2} e^{-\Gamma t} \{ 1 - R \cos \Delta m t - D \sin(\phi_\lambda - \Delta\delta) \sin \Delta m t \}, \quad (4.7)$$

$$\Gamma(B_{\text{phys}}^0(t) \rightarrow \bar{f}) = \frac{A^2}{2} e^{-\Gamma t} \{ 1 + R \cos \Delta m t - D \sin(\phi_\lambda + \Delta\delta) \sin \Delta m t \}, \quad (4.8)$$

where

$$\rho \equiv \frac{M'}{M}, \quad R \equiv \frac{\rho^2 - 1}{\rho^2 + 1}, \quad D \equiv \frac{2\rho}{\rho^2 + 1} = \sqrt{1 - R^2}, \quad A \equiv M'^2 + M^2, \quad \Delta\delta \equiv \delta - \delta', \quad (4.9)$$

Here ϕ_λ is the weak phase of the rephase invariants. It denotes the relevant unitarity angle, $\phi_\lambda = 2\alpha, -2\beta$, for $f = (p^+\pi^- \text{ or } a_1^+\pi^-)$, $D^{*+}D^-$, respectively.

Following the time-dependence, the three quantities ρ , $\sin(\phi_\lambda - \Delta\delta)$ and $\sin(\phi_\lambda + \Delta\delta)$ are measured. A simple trigonometric exercise recovers the relevant unitarity angle, ϕ_λ , from the latter two quantities up to some discrete ambiguity. Irrespective of the final-state phase difference $\Delta\delta$, the relevant unitarity angle ϕ_λ can thus be extracted.

The extraction of weak phases from non-CP eigenstates was used to find $\phi_\lambda = -\gamma, -(2\beta+\gamma)$, for $f = D^0\phi, D^0K_S$, respectively.¹⁹ A study of all the weak phases that can be extracted via this method has been completed. It exhaustively considers non-CP eigenstates, which are two-body or quasi two-body.²⁷

Although the method above has many applications, its drawback is the need for large data samples. For some of the modes it addresses, however, simplifications can be made. No final-state phases occur for B^0 modes into two charged light particles. In the factorization approximation, the $\bar{q}q$ system from the virtual W is created at the same space-time point. Because of its pointlike creation, the $\bar{q}q$ starts out as a colour singlet without any colour dipole moment. It is a light system and flies off with a large Lorentz boost.⁹ Only after having covered the distance of a few fermis from its point of creation does the $\bar{q}q$ system develop a sizeable colour dipole moment, through which strong

interaction could be felt. By that time this $\bar{q}q$ system is too distant to interact appreciably with the other light particle. Furthermore, the other light particle has also travelled a sizeable distance in the opposite direction from the point of creation. In short, no final-state interactions are expected, and thus no final-state phases.

Throughout we assumed that a single weak phase dominates the direct amplitude. The following section shows how to handle complications that arise when this is no longer the case—for instance, when $B_d \rightarrow p^+\pi^-$ is not dominated by the $\bar{b} \rightarrow \bar{u}u \bar{d}$ transition.

5. isospin Analysis.

Up to now we have studied the physics when the direct amplitude is dominated by one weak phase. For the majority of the modes in this chapter, this is a good assumption (see Table 1). The modes driven by $\bar{b} \rightarrow \bar{u}u \bar{d}$ are suspect, however. Arguments in the literature claim that modes governed by the $\bar{b} \rightarrow \bar{u}u \bar{d}$ transition, consisting of two light charged particles, have small contributions from different weak phases.²⁴

Because those arguments involve unknown hadronic matrix elements and QCD effects, we must disentangle the tree diagrams from contributions with different weak phases, without making any assumptions about their relative strengths and phases. Gronau and London show how an isospin analysis can eliminate the contributions with the different weak phases and thus determine the angle α within a discrete ambiguity.¹⁵

Their method requires high statistics measurements of a full set of isospin related channels. To succeed with such an analysis, the rates of the following six processes must be measured,

$$B^+ \rightarrow \pi^+\pi^0, B^- \rightarrow \pi^-\pi^0, B_d^+ \rightarrow \pi^+\pi^-, B_d^- \rightarrow \pi^+\pi^-, \bar{B}_d^+ \rightarrow \pi^+\pi^0, \bar{B}_d^- \rightarrow \pi^0\pi^0. \quad (5.1)$$

An interference term that signals CP violation, for example the one for $B_d \rightarrow \pi^+\pi^-$, must also be observed. This isospin idea has been applied to more involved processes, such as the $p\pi, p\rho$ modes.^{16, 17} Even for the simplest example, the $B \rightarrow \pi\pi$, the method requires large data samples and

superb detectors. In principle, the isospin method allows extraction of the weak phase α by removing the uncertainty on different weak ones. Whether this method is feasible, is left for our experimental colleagues to decide.

We present the idea of isospin analysis for the simplest case, $B \rightarrow \pi\pi$.^{15, 17} The $B \rightarrow \pi\pi$ decays have both "tree" and "penguin" contributions. The tree operator, $\bar{b} \rightarrow \bar{u} + u \bar{d}$, is a mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$, whereas the penguin operator is only $\Delta I = 1/2$. An isospin decomposition yields a triangle relation between the pure (not time-evolved) amplitudes,

$$\frac{1}{\sqrt{2}} A^{+-} + A^{00} = A^{+0}. \quad (5.2)$$

The CP conjugated processes have similar relations,

$$\frac{1}{\sqrt{2}} \bar{A}^{+-} + \bar{A}^{00} = \bar{A}^{+0}. \quad (5.3)$$

The pure amplitudes $A(B \rightarrow \pi\pi)$ are defined by the charges of the corresponding pions, A^{+-}, A^{00}, A^{+0} . The CP conjugated amplitudes are denoted by $\bar{A}^{+-}, \bar{A}^{00}, \bar{A}^{+0}$, and are obtained from the pure $A(B \rightarrow \pi\pi)$ amplitudes, by charge conjugating the weak phase and leaving the strong phases unchanged. Because of Bose statistics, the two pion states cannot be in an $l = 1$ state. That forces the $B^{+0} \rightarrow \pi^{+}\pi^0$ mode to be in the $l = 2$ state and to receive contributions only from the tree diagram. Because only the $l = 2$ strong interaction phase is involved, no CP violation occurs in the A^{+0} versus $\bar{A}^{+0}$ modes. Furthermore, the weak phase for those processes is given in the standard model by $\exp(i\phi_1) = V_{ud}V_{ub}^* / |V_{ub}V_{ud}|$ and its complex conjugate, respectively. In contrast, the $B_d^0 \rightarrow \pi^{+}\pi^{-}$ or $\pi^0\pi^0$ amplitudes involve in addition to the $l = 2$ strong interaction phase, δ_2 , also the $l = 0$ one, δ_0 . CP violation might occur for the pure $B_d^0 \rightarrow \pi\pi$ modes.

It is convenient to define $\tilde{A} = \exp(2i\phi_1) \bar{A}$, so that $\tilde{A}^{+0} = A^{+0}$.¹⁶ The two triangles, Eqs. (5.2)-(5.3), are shown in Figure 2. They have a common base, A^{+0} , but the lengths of the other sides may differ. This difference signals CP violation, because its conservation requires equality of all the corresponding pairs of sides.

Figure 2: The two triangles of Eqs. (5.2)-(5.3) for the $B \rightarrow \pi\pi$ processes.

The six sides of the triangles can be measured. Whereas $|A^{+0}|$ and $|\tilde{A}^{+0}|$ are obtained from the charged B rate without complications, the other four sides are extracted from time-dependent decay rates of the B_d meson, which are more demanding. The time-dependent rates also reveal information about the interference terms. Explicitly,

$$\Gamma(B_{\text{phys}}^0(t) \rightarrow f) = |A_f|^2 e^{-\Gamma t} \left\{ \cos^2\left(\frac{\Delta m t}{2}\right) + |\zeta_f|^2 \sin^2\left(\frac{\Delta m t}{2}\right) - \text{Im}\zeta_f \sin \Delta m t \right\}, \quad (5.4)$$

$$\Gamma(\bar{B}_{\text{phys}}^0(t) \rightarrow f) = |A_f|^2 e^{-\Gamma t} \left\{ |\zeta_f|^2 \cos^2\left(\frac{\Delta m t}{2}\right) + \sin^2\left(\frac{\Delta m t}{2}\right) + \text{Im}\zeta_f \sin \Delta m t \right\}, \quad (5.5)$$

where $A_f \equiv A(B^0 \rightarrow f)$, $\tilde{A}_f \equiv A(\bar{B}^0 \rightarrow f)$, $\zeta_f = \exp(-2i\phi_M) \frac{\tilde{A}_f}{A_f}$.

The phase of the B^0 - $\bar{B}^0$ mixing amplitude is $2\phi_M = 2\beta$. The coefficients of the first two terms in Eqs. (5.4)-(5.5) give the other four sides of the two triangles, $|A^{+-}|$, $|A^{00}|$, $|\tilde{A}^{+-}|$, $|\tilde{A}^{00}|$. This determines the two triangles within a four-fold

ambiguity. The coefficient of the $\sin(\Delta m t)$ term in Eqs. (5.4)-(5.5) determines the interference term,

$$\text{Im } \zeta_{+-}(\infty) = \left| \frac{\tilde{A}_{+-}(\infty)}{A_{+-}(\infty)} \right| \sin(2\alpha + \theta_{+-}(\infty)). \quad (5.6)$$

where $\theta_{+-}(\infty)$ is the angle from $A_{+-}(\infty)$ to $\tilde{A}_{+-}(\infty)$. The unitarity angle α can be extracted from the six length measurements and from the interference term $\text{Im } \zeta_{+-}$ within a discrete ambiguity. This ambiguity could be reduced if additional information were available on the $\pi^0\pi^0$ interference term, $\text{Im } \zeta_{00}$. This interference term is very difficult to measure, however.

A rather successful model by Bauer, Stech and Wirbel predicts the $\pi^0\pi^0$ rates to be much suppressed relative to the $\pi^+\pi^-$ ones.²⁸ If this turned out to be the case, first generation experiments could come up with only upper bounds on the $\pi^0\pi^0$ rates, whereas the $\pi^+\pi^-$ modes could be observed. Even such a scenario still provides important information by restricting the shape of the triangles in Fig. 2. The weak phase α can still be extracted with an accuracy dependent upon the various relevant measurements.

6. Extracting γ with B mesons

The determination of the weak phases β and α can be pursued with many B_d modes. In contrast, the phase γ is conventionally probed by only a handful of processes, such as $B_s \rightarrow \rho^0 K_S, \omega K_S$. Their predicted rates are tiny.

Not only is the B_s meson harder to produce than the B_d one, but it is expected to oscillate rapidly in time. Time-dependent measurements are thus necessary to extract CP violating effects in the $B_s - \bar{B}_s$ system. Even if the technological challenges of time-dependent measurements of the B_s modes were to be overcome, and a CP violating asymmetry extracted, it would still not be clear that $\sin 2\gamma$ had been observed, because those B_s -modes could have large contributions with different weak CKM phases. In fact, those contributions are predicted to affect the B_s -modes more than the $B_d \rightarrow \pi^+\pi^-$, $\rho^+\pi^-$, $a_1^+\pi^-$ ones, because the former are colour suppressed, whereas the latter are colour allowed.

There exists an alternative method for measuring this angle γ . It is based on the observation that the exclusive B-mode, $B \rightarrow D^0 X$, interferes with $B \rightarrow \bar{D}^0 X$, when D^0 is seen in a mode that is accessible to both a D^0 and $\bar{D}^0$. This interference can be accomplished for D^0 modes that have either

- (a) a definite CP parity, as discussed in the literature,¹⁸ or
- (b) no CP parity, as we proceed to show.

CP violation can then occur. The extraction of the weak phase γ (up to a discrete ambiguity) is made possible by removing the final-state phase difference, because the modulus of each individual amplitude--of which there are six--can be determined separately.¹⁹⁻²¹

Originally this method was applied to time-dependent measurements of six B^0 (six B_d or six B_s) modes, which are grouped into two triangles--such as the six modes, $B_d \rightarrow D^0 (\bar{D}^0, D_{CP}^0) K_S$ and $\bar{B}_d \rightarrow D^0 (\bar{D}^0, D_{CP}^0) K_S$.¹⁹ The D_{CP}^0 denotes a CP eigenmode of a D^0 or $\bar{D}^0$. The method extracts, among others, the angle γ . Applying it to six charged B decay modes, the angle γ can again be extracted.²⁰ This angle can also be determined from six self-tagging B_d modes: $B_d \rightarrow D^0 (\bar{D}^0, D_{CP}^0) K^{*0}$, $\bar{B}_d \rightarrow D^0 (\bar{D}^0, D_{CP}^0) \bar{K}^{*0}$.²¹ The K^{*0} must be seen in its $K^+\pi^-$ mode, which tags the beauty flavour. A merit of the charged B modes and the self-tagging B_d ones is that no time-dependent measurements are required. CP violating effects, when the D^0 is no longer seen in a CP eigenmode, will be taken up later in this section.

Let us take the charged B decays as an example of how to extract γ .²⁰ The B_u modes of interest are $B_u^+ \rightarrow D_{1(2)}^0 K^+$ and $\bar{B}_u^- \rightarrow D_{1(2)}^0 K^-$, where $D_{1(2)}^0$ is the CP-even (-odd) state, $D_{1(2)}^0 \equiv (D^0 + (-) \bar{D}^0) / \sqrt{2}$.²⁹ The state $D_{1(2)}^0$ can be identified by its CP-even eigenmodes, $\pi^+\pi^-$, K^+K^- , etc., and $D_{1(2)}^0$ by its CP-odd ones, $K_S \pi^0$, $K_S \rho^0$, $K_S \omega$, $K_S \phi$, etc.⁷ In the D^0 - $\bar{D}^0$ system, the CKM model predicts tiny CP violation effects that are neglected here.

Two interfering amplitudes contribute to the process $B_u^+ \rightarrow D_{1(2)}^0 K^+$, with different weak phases, and with probably different final-state phases,

$$\begin{aligned}
A(B_U^+ \rightarrow D_{1(2)}^0 K^+) &= \frac{1}{\sqrt{2}} (A(B_U^+ \rightarrow D^0 K^+) + (-) A(B_U^+ \rightarrow \bar{D}^0 K^+)) \\
&= \frac{1}{\sqrt{2}} (|A| e^{i\gamma} e^{i\delta} + (-) |B| e^{i\tau}), \quad (6.1)
\end{aligned}$$

Here $|A|$ and $|B|$ are the magnitudes, δ and τ the final-state phases, and γ and 0 the weak phases for the amplitudes $B_U^+ \rightarrow D^0 K^+$ and $B_U^+ \rightarrow \bar{D}^0 K^+$, respectively. The CP conjugated process leaves the final-state phases unchanged and complex conjugates the weak phases,

$$\begin{aligned}
A(B_U^- \rightarrow D_{1(2)}^0 K^-) &= \frac{1}{\sqrt{2}} (A(B_U^- \rightarrow \bar{D}^0 K^-) + (-) A(B_U^- \rightarrow D^0 K^-)) \\
&= \frac{1}{\sqrt{2}} (|A| e^{-i\gamma} e^{i\delta} + (-) |B| e^{i\tau}). \quad (6.2)
\end{aligned}$$

The final-state phases probably differ because the final state from $B_U^+ \rightarrow \bar{D}^0 K^+$ has pure isospin 1, whereas that from $B_U^+ \rightarrow D^0 K^+$ is a superposition of isospin 0 and 1. They differ in their charm quantum numbers as well. The rates differ in general and CP violation occurs,

$$\begin{aligned}
&|A(B_U^+ \rightarrow D_1^0 K^+)|^2 - |A(B_U^- \rightarrow D_1^0 K^-)|^2 = \\
&= -2 |A(B_U^+ \rightarrow D^0 K^+)| |A(B_U^+ \rightarrow \bar{D}^0 K^+)| \sin \Delta \sin \gamma. \quad (6.3)
\end{aligned}$$

Here the final-state phase difference is denoted by $\Delta \equiv \delta - \tau$.

The weak phase γ is simple to obtain. One measures the following six rates. The first two are likely to show CP violation,

$$\Gamma(B_U^+ \rightarrow D_1^0 K^+) \neq \Gamma(B_U^- \rightarrow D_1^0 K^-). \quad (6.4)$$

The four remaining rates are grouped into two pairs of equal magnitudes,

$$\Gamma(B_U^+ \rightarrow D^0 K^+) = \Gamma(B_U^- \rightarrow \bar{D}^0 K^-), \quad \Gamma(B_U^+ \rightarrow \bar{D}^0 K^+) = \Gamma(B_U^- \rightarrow D^0 K^-). \quad (6.5)$$

The flavour of the D^0 and $\bar{D}^0$ is obtained by the charge of the decay kaon or lepton. In the D^0 - $\bar{D}^0$ system, the CKM model predicts negligible mixing effects which we disregard. Measurement of the six rates determines the sides of the two triangles (see Figure 3). The angle from $A(B_U^- \rightarrow \bar{D}^0 K^-)$ to

$A(B_U^+ \rightarrow D^0 K^+)$ is 2γ . To increase statistics, the whole analysis ought to be repeated with D_2^0 .

Figure 3: The six processes of Eqs. (6.1) and (6.2) arranged into two triangles.

The angle from $A(B_U^- \rightarrow \bar{D}^0 K^-)$ to $A(B_U^+ \rightarrow D^0 K^+)$ is 2γ .

Measuring the weak phase γ is not as simple as that, for it involves a discrete ambiguity. The cosine theorem for planar triangles determines unambiguously $\cos(\Delta + \gamma)$ and $\cos(\Delta - \gamma)$ from the B_U^+ and B_U^- triangles, respectively. A trivial trigonometric exercise reveals that $\sin^2 \gamma$ satisfies a quadratic equation with coefficients that depend on the observables, $\cos(\Delta + \gamma)$ and $\cos(\Delta - \gamma)$. In general, a two-fold ambiguity in $|\sin \gamma|$ occurs. It arises because the final-state phase difference Δ might be confused with the weak phase γ . The additional sign ambiguity in $\sin \gamma$ is removed by using the known CP violation constraint of the neutral kaon system—that is, the ϵ parameter. That constraint allows only the range⁴

$$0.1 \leq \sin \gamma \leq 1. \quad (6.6)$$

The quantity $\sin \gamma$ is never zero, always positive, and larger than about 0.1.

The remaining ambiguity in the magnitude can also be removed.^{20, 21}

We wish to mention a remarkable property of this method. In principle, even for a vanishing final-state phase difference, the weak phase γ can be

determined. In comparison, for the usual charged B rate asymmetries not to vanish, not only must there exist a weak phase difference, but also a non-vanishing final-state phase difference. When those asymmetries vanish, nothing can be learnt about the weak phases. Although the method advocated here also encounters a vanishing asymmetry,

$$\frac{\Gamma(B_u^+ \rightarrow D_1^0 K^+) - \Gamma(B_u^- \rightarrow D_1^0 K^-)}{\Gamma(B_u^+ \rightarrow D_1^0 K^+) + \Gamma(B_u^- \rightarrow D_1^0 K^-)} \quad (6.7)$$

when $\Delta = 0$, it is still able to extract the weak phase γ . The extraction is possible, because the magnitudes of all six relevant amplitudes are observables. This completes the description of the method.

The determination of γ from the method requires great efforts by the experimental community. A real step forward would be a non-zero rate asymmetry, such as Eq. (6.7) or

$$\frac{\Gamma(B_d \rightarrow D_1^0 K^{*0}) - \Gamma(\bar{B}_d \rightarrow D_1^0 \bar{K}^{*0})}{\Gamma(B_d \rightarrow D_1^0 K^{*0}) + \Gamma(\bar{B}_d \rightarrow D_1^0 \bar{K}^{*0})}, \quad \text{where } K^{*0} \rightarrow K^+ \pi^-. \quad (6.8)$$

We conclude the section with a few modes that may harbour large CP violation effects, but which require excellent charged K and π discrimination. We remind the reader that for the method to work, all that is required is that D^0 and $\bar{D}^0$ interfere. This interference is accomplished by looking at CP eigenmodes of D^0 . There is no need, however, to restrict ourselves to CP eigenmodes. Any mode that can occur both from a D^0 and $\bar{D}^0$ suffices. Large CP violation effects could occur in additional B_u, B_s and B_d modes.

Consider an exclusive B-mode where a $\bar{D}^0$ is made, driven by the CKM favoured $\bar{b} \rightarrow \bar{c} + u \bar{d}$ transition. Have the $\bar{D}^0$ decay into a doubly Cabibbo suppressed mode, denoted by g. (This g is then a Cabibbo allowed mode for D^0 .) Another amplitude interferes with the one just mentioned. It consists of the doubly suppressed CKM transition $\bar{b} \rightarrow \bar{u} + c \bar{d}$, followed by the Cabibbo allowed mode $D^0 \rightarrow g$. Not only do the two interfering amplitudes have a weak phase difference γ , but they even have comparable magnitudes. Large CP violation can occur, since there is no reason for the final-state phase difference to vanish.

For example, the following asymmetries could be searched for,³⁰

$$\frac{\Gamma(B_u^+ \rightarrow (K^- \pi^+)_{D^0} X^+) - \Gamma(B_u^- \rightarrow (K^+ \pi^-)_{D^0} X^-)}{\Gamma(B_u^+ \rightarrow (K^- \pi^+)_{D^0} X^+) + \Gamma(B_u^- \rightarrow (K^+ \pi^-)_{D^0} X^-)}, \quad \text{where } X^\pm = \pi^\pm, \rho^\pm, a_1^\pm, \text{ etc.} \quad (6.9)$$

$$\frac{\Gamma(B_s \rightarrow (K^- \pi^+)_{D^0} \bar{K}^{*0}) - \Gamma(\bar{B}_s \rightarrow (K^+ \pi^-)_{D^0} K^{*0})}{\Gamma(B_s \rightarrow (K^- \pi^+)_{D^0} \bar{K}^{*0}) + \Gamma(\bar{B}_s \rightarrow (K^+ \pi^-)_{D^0} K^{*0})}, \quad \text{where } K^{*0} \rightarrow K^+ \pi^-. \quad (6.10)$$

The asymmetries do not require time-dependent measurements, because the B_s modes are self-tagging, when the K^{*0} is seen in its $K^+ \pi^-$ mode. To succeed in measuring those last two asymmetries, superb K^+ and π^+ discrimination must be available. Otherwise the CKM allowed background would swamp the signal. To increase statistics for the B_d and B_s asymmetries [Eqs. (6.8) and (6.10)], one could replace the K^{*0} by any $K^+ \pi^-$ —not only the ones that originate from a K^{*0} .³¹ Other B^0 (B_s or B_d) modes, which are fed from both the B^0 and $\bar{B}^0$, demand even more experimental ingenuity and will be mentioned only in the footnote.³²

If such an ideal detector could be envisioned, one could even try to extract the phase γ from a set of six related processes. The extraction would follow along analogous lines to the six $B_u^+ \rightarrow D_1^0 K^+$ processes discussed above. Care must be taken because of doubly Cabibbo suppressed modes which will be discussed for the example of Eq. (6.9). The magnitudes of the interfering amplitudes are

$$|A(B_u^+ \rightarrow D^0 X^+ \rightarrow (K^- \pi^+)_{D^0} X^+)| \sim \sqrt{B(B_u^+ \rightarrow D^0 X^+) B(D^0 \rightarrow K^- \pi^+)}, \quad (6.11)$$

$$|A(B_u^+ \rightarrow \bar{D}^0 X^+ \rightarrow (K^- \pi^+)_{D^0} X^+)| \sim \sqrt{B(B_u^+ \rightarrow \bar{D}^0 X^+) B(\bar{D}^0 \rightarrow K^- \pi^+)}. \quad (6.12)$$

The measurement of the $B_u^+ \rightarrow D^0 X^+$ rate of Eq. (6.11) requires the D^0 to be seen in modes that are not accessible to the $\bar{D}^0$ at all, such as the semileptonic modes. In contrast, the $B_u^+ \rightarrow \bar{D}^0 X^+$ rate measurement of Eq. (6.12), may use the Cabibbo allowed $\bar{D}^0$ modes, in addition to the ones that are not accessible

to the D^0 . Here $D^0 - \bar{D}^0$ mixing is neglected, but if observed would be possible to incorporate.³³

In conclusion, the message of this section is that large CP violating effects can occur in B modes with a D^0 . The effects do not require any $B^0 - \bar{B}^0$ mixing, but happen because of interference effects between two B amplitudes with the weak phase difference γ . The first major challenge is to measure an asymmetry at all. A more detailed experimental program has the chance of actually extracting the phase γ from such modes.

7. Feasibility

We now wish to compare the prospects for the various methods, in order to show that CP violation can be observed at a B factory or at a hadron machine. The feasibility of measuring the three angles α , β and γ with the standard set of three CP eigenmodes is discussed. Whereas the extraction of β (and probably of α , too) is straightforward, that of γ has problems, which are emphasized. The usefulness of angular correlations is demonstrated. Non-CP eigenmodes are mentioned next, followed by a brief note on the isospin analysis. All the methods require tagging--that is, the distinction of a B^0 from a $\bar{B}^0$. In contrast, an alternative way of determining γ , described in Section 6, does not require tagging. Results from a preliminary feasibility study for this method are quoted.

The Standard Model predicts large, non-vanishing CP asymmetries for class (i) processes,⁴

$$0.1 \leq \sin 2\beta \leq 1. \quad (7.1)$$

Furthermore, the $B_d - \bar{B}_d$ mixing has been observed to approach its optimal value for CP violation experiments,³⁴

$$(\Delta m/\Gamma)_{B_d} \approx 0.7, \quad (7.2)$$

and the branching rate for a class (i) mode has been measured,³⁴

$$B(B_d \rightarrow \Psi K_S) \approx 3 \times 10^{-4}. \quad (7.3)$$

The observation of CP violation in class (i) is **guaranteed** within the Standard Model, provided a tagging scheme is implemented and a large enough data sample accumulated.

Although class (i) processes probe the one independent weak phase of the CKM matrix, it is important to over-constrain the unitarity triangle. The angle α is measured by class (ii) processes, such as $B_d \rightarrow \pi^+ \pi^-$, with a predicted branching rate of $O(10^{-5})$.²⁸

The last angle γ is more difficult to extract. The conventional wisdom is to use class (iii) processes, which are CP eigenmodes of B_s mesons, such as $B_s \rightarrow \rho^0 K_S$, ωK_S . The B_s mesons are harder to produce than the B_d ones, however, and they are expected to oscillate rapidly in time. Not only tagging, but also time-dependent measurements are thus necessary to extract CP violating effects in the $B_s - \bar{B}_s$ system. Also disadvantageous is the fact that the branching rates of the class (iii) processes are predicted to be tiny,²⁸

$$B(B_s \rightarrow \rho^0 K_S) \sim O(10^{-6}). \quad (7.4)$$

Even if the experimental challenges were to be overcome, and a CP violating asymmetry extracted, it would still not be clear that $\sin 2\gamma$ had been observed, because those B_s -modes could have large contributions with different weak CKM phases. In fact, those contributions are predicted to affect the B_s -modes more than the $B_d \rightarrow \pi^+ \pi^-$ one, because the former are colour suppressed, whereas the latter is colour allowed. That is why a whole section was devoted to an alternate extraction of γ .

Angular correlations are helpful, as can be seen from the $B_d \rightarrow \Psi K^{*0} \rightarrow \Psi(K_S \pi^0)_{K^*}$ example. A study of the angular distribution finds this mode to have mainly a helicity 0 component, which is a CP eigenstate.^{11,34} This is useful information, because the branching rate is of the same order of magnitude as that for the classic ΨK_S ,

$$B(B_d \rightarrow \Psi K^{*0} \rightarrow \Psi(K_S \pi^0)_{K^*}) \approx (1.2 \times 10^{-3}) \times \frac{1}{3} \times \frac{1}{2} \approx 2 \times 10^{-4}. \quad (7.5)$$

A reduction in the luminosity requirement can thus be achieved from many new modes that are now accessible through detailed angular correlation studies.

Non-CP eigenmodes are an important complement to CP eigenmodes. For instance, an investigation¹⁴ shows that the error on $\sin 2\alpha$ as measured in

the $B_d \rightarrow \rho^\pm \pi^\mp$ channels is smaller than the one obtained from the $B_d \rightarrow \pi^+ \pi^-$ one as long as

$$\frac{\Gamma(B_d \rightarrow \rho^- \pi^+)}{\Gamma(B_d \rightarrow \rho^+ \pi^-)} \geq \frac{1}{7}. \quad (7.6)$$

This follows partly because, under the very plausible factorization assumption,

$$\frac{\Gamma(B_d \rightarrow \rho^+ \pi^-)}{\Gamma(B_d \rightarrow \pi^+ \pi^-)} \approx 2. \quad (7.7)$$

Because all extant estimates for the $B_d \rightarrow \pi\pi$ rates are unreliable due to unknown hadronic matrix elements and QCD effects, we cannot predict much about isospin analysis. The model of Bauer, Stech and Wirbel expects that

$$\frac{\Gamma(B_d \rightarrow \pi^0 \pi^0)}{\Gamma(B_d \rightarrow \pi^+ \pi^-)} \sim \frac{1}{50}. \quad (7.8)$$

Whether the $\pi^0 \pi^0$ mode is indeed as suppressed as that ought to be tested in a time-integrated manner and without tagging. If this turns out to be the case, the CP violation effects, which were the subject of Section 5, can be neglected. The mode $B_d \rightarrow \pi^+ \pi^-$ would then essentially measure the angle α , as discussed in Section 2.

As for the angle γ , the alternative method of Section 6 can extract it from six related rate measurements. Although only order of magnitude estimates can be made for any of the rates, the ratio of rates is better known for the case of the self-tagging B_d modes, $B_d \rightarrow D^0 (\bar{D}^0, D_{CP}^0) K^{*0}$ and $\bar{B}_d \rightarrow D^0 (\bar{D}^0, D_{CP}^0) \bar{K}^{*0}$. Here the K^{*0} must be seen in its charged $K^+ \pi^-$ mode. Whereas the CP violating effects depend on the relative branching rates, their observability depends on the absolute branching rate. We predict for the absolute rate³⁵

$$B(B_d \rightarrow \bar{D}^0 K^{*0}) \sim O(5 \times 10^{-5}), \quad (7.9)$$

whereas for the ratio, we get³⁵

$$\frac{\Gamma(B_d \rightarrow D^0 K^{*0})}{\Gamma(B_d \rightarrow \bar{D}^0 K^{*0})} \approx \left| \frac{V_{ub} V_{cs}}{V_{cb} V_{cd}} \right|^2 \approx \frac{1}{4}. \quad (7.10)$$

A preliminary feasibility study finds that even in the worst case scenario—that is, when the final-state phase difference vanishes— $\cos \gamma$ can be determined from realistic data samples.²¹

We demonstrated that large CP asymmetries occur for class (i) processes, such as $B_d \rightarrow \psi K_S$. More detailed methods exist, which are not mere theoretical curiosities, but actually aid in the search for CP violation. With sufficient data the phenomenon can be observed or the Standard Model ruled out. It is therefore important to test the predictions.

8. Conclusions.

We hope to have convinced the reader that promising CP violating experiments can be conducted with many new modes, in addition to the classic CP eigenstate ones. That the fundamental CKM phase can be straightforwardly extracted from CP asymmetries, because the hadronic uncertainties cancel, has become common knowledge about many CP eigenmodes.

This clean extraction is by no means limited to the CP eigenmodes. With more detailed methods, many modes—from which the weak phases could previously not be extracted—can be used for this purpose. For modes that are mixtures of CP-even and -odd parities due to angular momentum, uncalculable cancellations in the asymmetries occur. Various angular analyses allow us to circumvent the cancellation and thereby to extract the CKM phase with precision. Also, such extractions can be accomplished with non-CP eigenmodes, by studying time-dependences of four related decay rates. Isospin analyses isolate the tree-diagrams from contributions with different weak phases and allow the clean measurement of fundamental CKM phases.

Large CP violating effects can occur in B modes with a $\bar{D}^0$. The D^0 must decay into modes that can also be fed from a $\bar{D}^0$. Two interfering amplitudes exist with a weak phase difference γ . As there is no reason for the final-state phase difference to vanish, large CP non-invariance effects may occur. The weak phase γ can be determined if all six related rates can be measured. In

principle, the determination can be made even when the final-state phase difference vanishes.

It is remarkable how much has been learnt theoretically about different CP search strategies since the large $B_d - \bar{B}_d$ mixing was discovered. Many new avenues will continue to open, as long as we continue to probe those issues. Research will be greatly aided by B experiments at hadronic colliders and the construction of a B factory.

Acknowledgements

I am grateful to Roy Aleksan, Jean-Rene Cudell, Boris Kayser, Helen Quinn, Tony Pich, Jon Rosner, Sheldon Stone and G. Valencia for many useful suggestions, and am delighted to thank James D. Bjorken for many fun discussions concerning CP violation. I am grateful to R. Aleksan, W. B. Atwood, J. D. Bjorken, C. O. Dib, D. Du, F. J. Gilman, B. Kayser, F. Le Diberder, H. J. Lipkin, O. W. Greenberg, P. Grosse-Wiesmann, S. Matsuda, T. Nakada, Y. Nir, H. Quinn, J. L. Rosner, R. G. Sachs, A. I. Sanda, A. Snyder, W. Toki and Dan-di Wu for collaboration on some of the topics discussed here. I am thankful to Alexandra, my wife, for the many hours spent editing the manuscript, and to Michèle Jouhet, Guy Martin and Jeanne Rostant for their editorial help. I acknowledge the hospitality of the Aspen Center for Physics.

Appendix: CP Projections from Transversity

A more detailed presentation of the transversity analysis is useful because it is a simple way to extract the fundamental asymmetry a , without any partial cancellation.¹² The relationship between transversity and CP parity will be established. Then the polar angular dependence is shown to disentangle the various transversities, and thus the various CP parities. Moments from the B^0 and $\bar{B}^0$ data samples can be constructed, which yield the whole fundamental asymmetry, a . The partial cancellation, as described in Eq. (3.5), is avoided.

For illustrative purposes, consider the decay of a neutral B^0 particle into unstable ones, A and C, which have arbitrary spin. Particles A and C must be unstable so that spin information can be learnt from their subsequent decay. Particle C is subsequently seen in its two body mode, $C \rightarrow C_1 C_2$, where C_1 and C_2 stand for two spinless particles,

In contrast, we allow the decay products of A to have spin. Consider the rest frame of A. The $\hat{z}$ -axis (transverse axis) is defined as normal to the plane containing the C_1 and C_2 particles. The CP-parity of the mode $AC_1 C_2$ can be classified by the spin projection of the particle A along this transverse axis, which is denoted as the transversity,

$$t \equiv \vec{S}_A \cdot \hat{z}. \quad (A.1)$$

It is found that the CP parity of the $AC_1 C_2$ mode is proportional to the transversity,

$$CP = \zeta (-1)^t. \quad (A.2)$$

Table 2 lists three categories of B^0 modes where a transversity analysis is possible, and relates CP parity to transversity. A transversity analysis may be

performed for Category (1) even when C_1 and C_2 have arbitrary spins.¹² The symbol $\eta(D^{\ast 0})$, for the example of Category (2), requires the $D^{\ast 0}$ to be seen in a CP eigenmode, where the value of $\eta(D^{\ast 0})$ denotes the CP parity of that mode.

Table 2

The CP parity for the mode AC_1C_2 with transversity τ , where C_1 and C_2 are spinless. The first column defines the categories. The CP-conjugated particle of X is denoted by $\bar{X}$. The symbol $X \leftrightarrow \bar{X}$ denotes that particle X is either a CP eigenstate or is observed via its decay into a CP eigenstate. The symbol $\eta(X)$ denotes the intrinsic CP parity of particle X , and s_C the spin of particle C .

Category	Example AC_1C_2	CP Parity $= \zeta(-1)^\tau$
(1) $A \leftrightarrow \bar{A}$ and $C_1 \leftrightarrow \bar{C}_1, C_2 \leftrightarrow \bar{C}_2$	$\psi K_S \pi^0$	$\eta(A) \eta(C_1) \eta(C_2) (-1)^\tau$
(2) $A \leftrightarrow \bar{A}, C \leftrightarrow \bar{C}$ and $C_2 = \bar{C}_1$	$D^{\ast 0} (\pi^+ \pi^-)_\rho$	$\eta(A) (-1)^{s_C + \tau}$
(3) $C = \bar{A}$	$D^{\ast+} D^{\ast-}$	$(-1)^{s_C + \tau}$

Even though a full angular analysis will generally obtain better accuracy on the fundamental asymmetry than a transversity analysis, there might be a few Category (1) modes in which the transversity analysis could be superior to the full angular one. Consider the example $\psi K_S \pi^0$ quoted in Table 2. The origin of the $K_S \pi^0$ from either a K^* or an S wave background is irrelevant to the

transversity analysis, while it is a distinction that must be made by a full angular one.

As for the measurement of transversity from the subsequent A decay, Table 3 presents two readily analyzed groups of possible decays. For simplicity, we assume particle A to have spin 1. Group (a) includes all decays of a spin-1 particle into two spinless particles and also decays of a vector particle into three pseudoscalar ones. Group (b) includes the decay of an axial-vector particle into three pseudoscalars, the decay of a spin-1 particle into a photon plus a spinless particle, or the decay of a spin-1 particle into a pair of negligible mass spin-1/2 particles via a vector or axial-vector coupling. The second column presents examples of decays of particle A . We assume that the decay of A proceeds through parity conserving interactions.

Columns 3 and 4 present the angular distributions for each τ , $r_{\tau 1}(\theta)$, normalized so that

$$\int_{-1}^1 d\cos\theta r_{\tau}(\theta) = 1. \quad (A.3)$$

Here θ is an angle that describes the distribution of the decaying particle A , in the rest frame of A , relative to the transverse axis. When A decays into two particles, angle θ is the polar angle for one of them. When A decays into three spinless particles, angle θ is the polar angle of the normal to the plane containing the three decay products.³⁶ In all cases, other decay angles have been integrated out.

Table 3

Polar angle structure of the decay of particle A

Group	Example	$r_1(\theta)$	$r_0(\theta)$
(a) $1 \rightarrow 0 + 0$ or $1^- \rightarrow 3(0^-)$	$D^* \rightarrow \pi D$ $\omega \rightarrow \pi^+ \pi^- \pi^0$	$\frac{3}{4} \sin^2 \theta$	$\frac{3}{2} \cos^2 \theta$
(b) $1^+ \rightarrow 3(0^-)$ or $1 \rightarrow \gamma + 0$ or $1 \rightarrow \frac{1}{2} + \frac{1}{2}$	$f_1 \rightarrow \eta \pi \pi$ $D^* \rightarrow \gamma D^0$ $\psi \rightarrow e^- e^+$	$\frac{3}{8} (1 + \cos^2 \theta)$	$\frac{3}{4} \sin^2 \theta$

The θ -dependent decay rates are given by

$$\Gamma(\theta) \equiv \Gamma(B_{\text{phys}}^0 \rightarrow f_\theta) = \Gamma_+ r_+(\theta) (1 + a) + \Gamma_- r_-(\theta) (1 - a), \quad (\text{A.4})$$

$$\bar{\Gamma}(\bar{\theta}) \equiv \Gamma(\bar{B}_{\text{phys}}^0 \rightarrow \bar{f}_{\bar{\theta}}) = \Gamma_+ r_+(\bar{\theta}) (1 - a) + \Gamma_- r_-(\bar{\theta}) (1 + a). \quad (\text{A.5})$$

The quantity $r_{\pm}(\theta)$ denotes the polar angle dependence of a CP-even (-odd) state and is related to the $r_{\text{pl}}(\theta)$ by

$$r_{\pm}(\theta) = r_0(\theta) (1 \pm \zeta) / 2 + r_1(\theta) (1 \mp \zeta) / 2, \quad (\text{A.6})$$

where ζ is given in Table 2. The state $|\bar{f}_{\bar{\theta}}\rangle \equiv \text{CP} | f_\theta \rangle$. The angle $\bar{\theta}$ is sometimes $\pi - \theta$ and sometimes θ depending on the particle content of the state. This leads to no complications, because $r_{\text{pl}}(\pi - \theta) = r_{\text{pl}}(\theta)$.

Whereas a total rate asymmetry suffers from a partial cancellation (see Eq. (3.5)), a transversity analysis is able to extract the whole fundamental

asymmetry a , because it separates CP-even from CP-odd components. The extraction is accomplished using the method of moments.

Four moments are constructed from the two θ -dependent rates of Eqs. (A.4)-(A.5). The first two moments are just the total rates obtained by an integration over the polar angle,

$$\stackrel{(-)}{M}_0 \equiv \int_{-1}^1 d\cos\theta \stackrel{(-)}{\Gamma}(\theta) = \Gamma_+ + \Gamma_- + a(\Gamma_+ - \Gamma_-). \quad (\text{A.7})$$

The dependence on a cancels for the sum

$$M_0 + \bar{M}_0 = 2(\Gamma_+ + \Gamma_-). \quad (\text{A.8})$$

The last two moments ought to have the property that

$$\stackrel{(-)}{M}_2 = \dots + a(\Gamma_+ + \Gamma_-), \quad (\text{A.9})$$

where the ellipses denote the same quantity. Then

$$M_2 - \bar{M}_2 = 2a(\Gamma_+ + \Gamma_-). \quad (\text{A.10})$$

That such moments can always be constructed from the data follows from the fact that the polar dependences $r_{\pm}(\theta)$ are known. They, in turn, determine the weight $g_2(\theta)$ for which Eq. (A.9) holds,

$$\stackrel{(-)}{M}_2 \equiv \int_{-1}^1 d\cos\theta g_2(\theta) \stackrel{(-)}{\Gamma}(\theta) = \dots + a(\Gamma_+ + \Gamma_-). \quad (\text{A.11})$$

For instance, the mode $\Psi K_S \pi^0$, has $r_+(\theta) = 3/8 (1 + \cos^2 \theta)$, $r_-(\theta) = 3/4 \sin^2 \theta$, and the weighting turns out to be $g_2(\theta) = -3 + 10 \cos^2 \theta$. A measurement of the whole fundamental asymmetry is made possible by constructing the ratio

$$\frac{M_2 - \bar{M}_2}{M_0 + \bar{M}_0} = a \frac{\Gamma_+ + \Gamma_-}{\Gamma_+ + \Gamma_-} = a. \quad (\text{A.12})$$

The presence of both CP-even and CP-odd components does not cause a cancellation or partial cancellation. Independent of the Γ_+/Γ_- ratio, the quantity $(M_2 - \bar{M}_2) / (M_0 + \bar{M}_0)$ always measures the fundamental asymmetry.

Contrast this to the total rate asymmetry, Eq. (3.5), where a partial cancellation occurs when both components are present,

$$\frac{M_0 - \bar{M}_0}{M_0 + \bar{M}_0} = a \frac{\Gamma_+ - \Gamma_-}{\Gamma_+ + \Gamma_-} \quad (\text{A.13})$$

There exists an infinite number of moments which measure a without any cancellation. For any two real numbers α and β , there exist weights $\tilde{g}_{0,2}(\theta)$, such that

$$\stackrel{(-)}{M}_0 \equiv \int_{-1}^1 d\cos\theta \tilde{g}_0(\theta) \stackrel{(-)}{\Gamma}(\theta) = (\alpha \Gamma_+ + \beta \Gamma_-) \stackrel{(+)}{a} \quad (\text{A.14})$$

$$\stackrel{(-)}{M}_2 \equiv \int_{-1}^1 d\cos\theta \tilde{g}_2(\theta) \stackrel{(-)}{\Gamma}(\theta) = \dots + a (\alpha \Gamma_+ + \beta \Gamma_-) \quad (\text{A.15})$$

Again no cancellation of the asymmetry occurs, when the following ratio is constructed,

$$\frac{M_2 - \bar{M}_2}{M_0 + \bar{M}_0} = a. \quad (\text{A.16})$$

In summary, a method of moments measures the fundamental asymmetry a without any cancellation, irrespective of the Γ_+/Γ_- ratio. The Γ_+/Γ_- ratio determines, however, the optimal moments which maximize accuracy on the fundamental asymmetry. Quite often an order of magnitude increase in statistics is available in determining this Γ_+/Γ_- ratio from isospin related channels, which by themselves do not harbour CP asymmetries. Then the ratio Γ_+/Γ_- can be determined and the optimal moments chosen. For tables of applicable modes, see Ref. 12.

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29. All the arguments below also hold when the K^{*+} , $K^+ \pi^+ \pi^-$, etc. are used instead of the K^+ .
30. Instead of the neutral charmed mode $K^- \pi^+$ used in the examples, we could as well have chosen $g = K^- \pi^+ \pi^- \pi^+$, $K^- \pi^+ \pi^0$, etc.
31. We thank J. D. Bjorken for pointing this out.
32. In addition to superb K^+ and π^+ discrimination, excellent tagging and time-resolution are required to study additional B^0 (B_s or B_d) modes that are fed from both the B^0 and $\bar{B}^0$. For instance, a time-dependent measurement is required to isolate the pure B_d and $\bar{B}_d$ amplitudes, with which the following asymmetry is formed,
- $$\frac{\Gamma(B_d \rightarrow (K^- \pi^+) D^- X^0) - \Gamma(\bar{B}_d \rightarrow (K^+ \pi^-) D^+ X^0)}{\Gamma(B_d \rightarrow (K^- \pi^+) D^- X^0) + \Gamma(\bar{B}_d \rightarrow (K^+ \pi^-) D^+ X^0)}, \text{ where } X^0 = \pi^0, \rho^0, a_1^0, \text{ etc.}$$
33. If $D^0 - \bar{D}^0$ mixing occurs at the doubly Cabibbo suppressed level, a third comparable amplitude interferes. Within the Standard Model, this amplitude has the same weak phase as that driven by the CKM favoured $\bar{b} \rightarrow \bar{c} u \bar{d}$ transition followed by a doubly Cabibbo suppressed $\bar{D}^0$ mode, $\bar{D}^0 \rightarrow g$. Large CP violating effects still occur. As for the extraction of γ , one must distinguish the two amplitudes with the same weak phase. This can be achieved with a careful time dependent study of the subsequent $\bar{D}^0$ decay, as discussed, for instance, by J. C. Anjos et al., *Phys. Rev. Lett.* **60** (1988) 1239; I. I. Bigi, in *Proceedings of the Tau-Charm Factory Workshop*, SLAC, 1989, SLAC report, SLAC-343, edited by L. V. Beers, p. 169.

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