An Improved Determination of the Ratio of $W$ and $Z$ Masses at the CERN $\bar{p}p$ Collider

The UA2 Collaboration

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Abstract

The $W$ and $Z$ bosons masses, $m_W$ and $m_Z$, are measured using samples of $W \rightarrow e \nu$ and $Z \rightarrow e^+e^-$ decays observed in $\bar{p}p$ collisions at $\sqrt{s} = 630$ GeV. The ratio is found to be $m_W/m_Z = 0.8813 \pm 0.0036 \pm 0.0019$. This gives a value $\sin^2 \theta_W = 0.2234 \pm 0.0064 \pm 0.0033$, and in combination with precise $m_Z$ measurements from LEP yields $m_W = 80.35 \pm 0.33 \pm 0.17$ GeV. This result is in good agreement with other experiments, and with the Standard Model for a top quark mass lighter than 250 GeV.

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1 Introduction

In 1990, new measurements of the $W$ mass ($m_W$) were published by the UA2 [1] and CDF [2] collaborations. The precision of these determinations far exceeded that of earlier measurements. The $W$ mass continues to be a subject of great interest in testing the Standard Model, and thus UA2 has combined the data from the 1990 run of the CERN $\bar{p}p$ collider with the data set of the previous publication to obtain an improved measurement of the $W$ mass, based on a total integrated luminosity of 13 pb$^{-1}$.

The method used to determine $m_W$ follows closely that used in the previous UA2 measurement [1]. The $W$ mass is measured from fits to transverse mass and momentum spectra in $W \rightarrow e\nu$ decays, while the $Z$ mass ($m_Z$) is determined concurrently from the $Z \rightarrow e^+e^-$ decays. The calibration scale errors largely cancel in the ratio of the two masses, so a precise value of $m_W$ is obtained by rescaling the ratio with the $m_Z$ value from LEP.

The present analysis has benefitted doubly from the increase in statistics. The statistical uncertainty on $m_W/m_Z$ has decreased, and in addition the larger sample of $Z \rightarrow e^+e^-$ events has been used to study the detector response in greater detail and has thereby made possible a reduction in the systematic errors in the $W \rightarrow e\nu$ event reconstruction.

2 Event Selection and Reconstruction

The data were collected from 1988 to 1990 at the CERN $\bar{p}p$ collider at an energy of $\sqrt{s} = 630$ GeV. After removing events where not all of the detector elements used in this analysis were functioning, the useful integrated luminosity is $13.0 \pm 0.7$ pb$^{-1}$. Requirements are imposed in order to select events where $\bar{p}p \rightarrow W + X$, $W \rightarrow e\nu$ or $\bar{p}p \rightarrow Z + X$, $Z \rightarrow e^+e^-$. The upgraded UA2 detector is described in the accompanying letter [3].

2.1 Electron Identification

A standard electron candidate must have a track reconstructed in the tracking detectors which points to an electromagnetic cluster in the calorimeter. The track must originate from a reconstructed vertex which is not displaced more than 250 mm along the beam direction from the centre of the detector. The lateral and longitudinal profile of the shower in the calorimeter is required to be consistent with that expected from an electron incident along the track trajectory as measured in test beams. Furthermore, a preshower cluster must be reconstructed which is consistent with the position of the electron candidate track. In addition, a set of looser electron cuts is defined for the region covered by the central calorimeter, in order to recover electrons for which either the track or the preshower cluster is not correctly reconstructed by the standard pattern recognition algorithms [4].

The detected energy is summed in a small number (typically two or three) of calorimeter cells ("core" cells) which are assigned to the electron. Energy corrections are applied according to the precise electron direction and impact point in the calorimeter based on data obtained from 40 GeV test beam electrons. The corrected energy is used together with the direction given by the tracking detectors to define the electron momentum, $p^e$. The program for maintaining the calibration of the calorimeters described in ref. [1] has been continued.
with periodic $^{60}$Co source measurements and test beam calibrations of representative modules preceding and following the 1990 running period. In this way, the overall scale of the energy calibration for electrons is controlled to the level of 1% for the central (non-edge) cells.

2.2 Neutrino Identification

The presence of neutrinos in $W \to e\nu$ decays is deduced by measuring the electron energy and the energies of the particles (generally hadrons) recoiling against the $W$. The missing transverse momentum ($\vec{p}_T$) is attributed to the undetected neutrino:

$$\vec{p}_T \equiv \vec{p}_T = -\vec{p}_T^e - \vec{p}_T^{had}. \quad (1)$$

Here, $\vec{p}_T^e$ is the reconstructed transverse momentum of the electron candidate and $\vec{p}_T^{had}$ is the total transverse momentum of the recoil particles, calculated as

$$\vec{p}_T^{had} = \left( \sum E_{cell} \hat{v}_{cell} \right)_T, \quad (2)$$

where $\hat{v}_{cell}$ is a unit vector from the interaction vertex to the centre of a calorimeter cell, $E_{cell}$ is the energy in that cell, and the sum extends over all cells in the calorimeter ($-3 < \eta < 3$) excluding the cells assigned to the electron.

2.3 $W$ Selection Requirements

For the $W$ mass measurement, most of the statistical information comes from events with the electron in the central calorimeter region. This arises from the fact that, due to kinematics, the $p_T^e$ and $p_T^{\nu}$ distributions are much flatter in the forward acceptance regions than in the central region where they peak at about half the $W$ mass. Therefore, this analysis uses only $W$ events in which the electron is in the central calorimeter. Additional fiducial cuts are applied so that the edge cells and the cell borders (0.5° in $\phi$ and 5 mm in $r\theta$) are excluded in order to obtain high quality energy reconstruction.

The electron must pass the standard identification cuts and have transverse momentum greater than 20 GeV. The neutrino transverse momentum reconstructed in each event must exceed 20 GeV. The transverse mass, $m_T$, is required to be between 40 GeV and 120 GeV, where $m_T = \sqrt{2p_T^e p_T^\nu (1 - \cos \phi^{\nu\nu})}$ and $\phi^{\nu\nu}$ is the azimuthal separation between the measured electron and neutrino directions. In addition, the requirement $p_T^W < 20$ GeV is imposed because the $p_T^e$ resolution is degraded in events with large amounts of hadronic energy. This leaves 2065 events, which are predominantly $W \to e\nu$ events with a 3.8% contribution from the process $W \to \tau\nu$, $\tau \to e\nu\bar{\nu}$, and a negligible QCD background ($<1\%$) [4].

2.4 $Z$ Selection Requirements

In selecting the samples for the $Z$ mass determination, it is important that the energy scale in the mass measurement comes from the same fiducial volume as defined for the $W$ events. In this way, there is maximal cancellation of the dominant calibration errors in computing the ratio $m_W/m_Z$. In a first $Z$ sample, both electron candidates are required.
to be in the fiducial volume of the central calorimeter. The mass of the electron candidate pair \( m_{ee} \) is calculated from the corrected momenta of the electrons, and it is required to be between 70 and 120 GeV. This yields a sample of 95 events.

A second independent \( Z \) sample is obtained as described in ref. [1]. One electron is required to be in the central fiducial volume while the other one must be outside, either in the forward or edge region or in the cell borders of the central calorimeter. The mass is then calculated by rescaling the momentum of the non-fiducial electron until the total event momentum balances along the \( \xi \) axis, where \( \xi \) is the outer bisector of the angle between the two electrons in the transverse plane (see inset in Fig. 3). By this procedure, the energy scale of the central calorimeter is transferred to the second electron. This “\( p_T \)-constrained” mass is required to be between 70 and 120 GeV, yielding a sample of 156 events. This sample has poorer mass resolution than the central \( Z \) sample, but with the larger number of events it makes a significant contribution to the \( Z \) mass measurement.

For both \( Z \) samples, at least one electron is required to satisfy the standard electron identification cuts, while the other may pass the looser cuts. The background from QCD two-jet events is estimated to be \(< 1\% \) [3].

3 Mass fits

3.1 \( Z \) Mass fits

The fits to \( m_Z \) are shown in Fig. 1. The function

\[
f(m_{ee}, \sigma, m_Z, \Gamma_Z) \propto \int dm' \frac{m'^2 e^{-\beta m'}}{(m'^2 - m_Z^2)^2 + m'^4 \Gamma_Z^2 / m_Z^2} e^{-(m_{ee} - m')^2 / 2\sigma^2} \tag{3}
\]

is used as the probability density function in a maximum likelihood fit. The function \( f \) combines a general relativistic Breit-Wigner resonance shape [5] with a term representing the parton luminosity \( e^{-\beta m'} \) and a convolution with the detector resolution \( \sigma \) for the event considered. The results of the \( m_Z \) fits are shown in Table 1. The fits are done both with the width \( \Gamma_Z \) left as a free parameter, and with \( \Gamma_Z \) fixed to the Standard Model value of 2.5 GeV. The fitted widths are in agreement with the Standard Model value. The mass values are rather insensitive to the width, and the final result is taken from the fits with the width fixed.

The function \( f \) does not take into account the effect of radiative decays or of the underlying event. A fraction of the sample comes from decays \( Z \to e^+e^-\gamma \) and the photon is not included in \( m_{ee} \), so the average reconstructed mass is lowered by \( \sim 190 \) MeV. Meanwhile, particles from the underlying event can contribute energy to the calorimeter cells used to determine the electron momenta, thus increasing \( m_{ee} \) by an average of \( \sim 250 \) MeV. A net correction of \(-60 \) MeV is added to the fitted mass values to compensate for these effects. The calculation of these effects and their uncertainties are discussed in the context of systematic errors (see Section 4). The results from the two samples of \( Z \) events are in good agreement and the combination gives \( m_Z = 91.74 \pm 0.28 \) GeV (statistical error only) after these corrections are applied.
Figure 1: Fits for $m_Z$ to (a) the central sample and (b) the pt-constrained sample. The curves show the fits, while the histograms show the data.
Table 1: Results of $m_Z$ fits (statistical errors only)

<table>
<thead>
<tr>
<th></th>
<th>$m_Z$(GeV)</th>
<th>$\Gamma_Z$(GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>central sample</td>
<td>91.65 ± 0.34</td>
<td>2.5 (fixed)</td>
</tr>
<tr>
<td>sample</td>
<td>91.67 ± 0.37</td>
<td>3.2 ± 0.8</td>
</tr>
<tr>
<td>$p_T$-constrained sample</td>
<td>92.10 ± 0.48</td>
<td>2.5 (fixed)</td>
</tr>
</tbody>
</table>

3.2 $W$ Mass fits

Since the longitudinal momentum of the neutrino is not measured, the $W$ mass must be obtained by fitting to a transverse kinematical variable such as $p_T^e$, $p_T^\tau$, or $m_T$. There are no simple analytical forms for these distributions, so the maximum likelihood fits are performed with sets of numerical probability density functions (pdf’s). In order to obtain these pdf’s, a simple Monte Carlo simulation is used which generates $W$ bosons, forces them to decay into electron and neutrino(s) (either $W \rightarrow e\nu$ or $W \rightarrow \tau\nu$, $\tau \rightarrow e\nu\nu$), and models the detector response. For each mass and width the events are given appropriate weights according to the general relativistic Breit-Wigner resonance shape. The rapidity distribution of the $W$ bosons is determined by the choice of structure functions, and the $p_T$ distribution is generated according to the spectrum derived from the study of $Z$ events (see below). Each electron is followed into the calorimeter and smearing is applied to the energy and direction to account for the measurement errors. The energy resolution, which depends on the impact point and cell type, is obtained from look-up tables based on test beam data (the electron response model is described in detail in ref. [1]). The hadrons in the event are modeled globally, with no individual treatment of hadrons or jets. As described in ref. [6], the model of the measurement of $p_T^{had}$ includes a resolution which is a function of the total scalar $E_T$ observed in the event excluding the decay products of the $W$ or the $Z$, and a bias on $p_T^{had}$ which varies with the transverse momentum of the $W$ or $Z$.

The $p_T$ distribution applied to the $W$ bosons is obtained largely from an empirical model based on the $Z$ data. While a theoretical calculation now exists to next-to-leading order for the $p_T$ spectra of $W$ and $Z$ bosons produced in $p\bar{p}$ interactions [7], the uncertainties at low $p_T$ are still rather large. The predictions are very similar for the $W$ and the $Z$ however, so the $p_T^Z$ spectrum is measured from a fit to the $p_T^W$ distribution (see Fig. 2(a)) and applied to the model for $W$ production, taking into account the small differences predicted by theory. The correlation between rapidity and $p_T$ is also taken from the theoretical calculation [7].

The model of the detector response to the recoiling hadrons is also studied with the aid of the $Z$ sample by examining the momentum balance in $Z$ events. For this study, the $p_T$ constraint used in defining the second $Z$ sample is not imposed. The momentum balance along the $\eta$ direction is considered ($\eta$-balance $= p_T^e + p_T^{had}$), where the $\eta$ axis is defined by the inner bisector of the two electron directions in the transverse plane (see inset Fig. 3). Along this direction, the contribution to the resolution from $p_T^Z$ is negligible, so the width of the $\eta$-balance distribution can be used to measure the resolution of the $p_T^{had}$ measurement. In
Figure 2: (a) The data for $p_T^Z$ (points) and the predictions of the central model (solid) along with the soft (dotted) and hard (dashed) variations of the $p_T$ spectrum used for systematic error studies (see text). (b) The data for $p_T^W = -p_{T^{\text{had}}}$ (points) and the prediction of the central model (curve). The predictions are modified for detector acceptance and resolution.
Figure 3: The momentum balance along the \( \eta \) axis (\( \eta \)-balance) in \( Z \to e^+e^- \) events.
The points show the data, while the histogram shows the central model.

addition, since the positive sense of the \( \eta \) axis is always in the direction of \( \vec{p}_T^\pi \), any systematic bias in the measurement of \( p_T^{had} \) manifests itself as a shift in the \( \eta \)-balance distribution. This distribution is shown in Fig. 3 along with the prediction of the model. The central model for the \( p_T^{had} \) measurement is obtained by tuning the resolution function and average response to give the correct width and offset in the predicted \( \eta \)-balance distribution.

The observed \( p_T^W \) distribution is sensitive to a combination of the true \( p_T^W \) spectrum and the measurement effects on \( p_T^{had} \). With the parameters tuned to the \( Z \) data, the prediction of the model gives very good agreement with the observed \( p_T^W \) spectrum as shown in Fig. 2(b).

The fitting is restricted to range 60-120 GeV for the \( m_T \) fits and 30-60 GeV for the \( p_T^\pi \) and \( p_T^\pi \) fits. The results are shown in Fig. 4 and Table 2. The three distributions are not independent, so they cannot be combined to give a more precise result. The \( m_T \) fit is used to obtain the final result because it gives the smallest statistical error as well as the smallest systematic error (see below). Meanwhile, the fits to \( p_T^\pi \) and \( p_T^\pi \) provide a useful cross check of the measurement systematics. When statistical fluctuations and correlations are accounted for, the expected differences between the \( m_T \) fit and the \( p_T^\pi \) and \( p_T^\pi \) fits are about 180 MeV (rms), so the three results are in good agreement. As for the \( Z \) fits, the \( W \) fits are performed with the width fixed and variable, and the fitted widths are in agreement with the Standard Model value, \( \Gamma_W = 2.1 \) GeV.
Figure 4: Fits for $m_W$ to (a) the $m_T$ spectrum, (b) the $p_T^\pi$ spectrum and (c) the $p_T^\gamma$ spectrum. The points show the data, while the curves show the fit results with the solid portions indicating the ranges over which the fits are performed.
Table 2: Results of $m_W$ fits (statistical errors only).

<table>
<thead>
<tr>
<th></th>
<th>$m_W$(GeV)</th>
<th>$\Gamma_W$(GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_T$</td>
<td>$80.84 \pm 0.22$</td>
<td>2.1 (fixed)</td>
</tr>
<tr>
<td>fit</td>
<td>$80.83 \pm 0.23$</td>
<td>2.2 $\pm$ 0.4</td>
</tr>
<tr>
<td>$p_T^e$</td>
<td>$80.86 \pm 0.29$</td>
<td>2.1 (fixed)</td>
</tr>
<tr>
<td>fit</td>
<td>$80.79 \pm 0.30$</td>
<td>2.8 $\pm$ 0.6</td>
</tr>
<tr>
<td>$p_T^\nu$</td>
<td>$80.73 \pm 0.32$</td>
<td>2.1 (fixed)</td>
</tr>
<tr>
<td>fit</td>
<td>$80.70 \pm 0.34$</td>
<td>2.3 $\pm$ 0.7</td>
</tr>
</tbody>
</table>

4 Systematic uncertainties

The systematic effects on the $m_W$ and $m_Z$ measurements are summarized in Table 3. Detailed discussions of these effects are found in ref. [8]. A short description of the individual contributions follows.

Table 3: The size (in MeV) of the systematic uncertainties in measuring $m_W$ and $m_Z$.

<table>
<thead>
<tr>
<th></th>
<th>$\delta m_W(m_T)$</th>
<th>$\delta m_W(p_T^e)$</th>
<th>$\delta m_W(p_T^\nu)$</th>
<th>$\delta m_Z$(central)</th>
<th>$\delta m_Z$(pt-con)</th>
</tr>
</thead>
<tbody>
<tr>
<td>structure fun.</td>
<td>85</td>
<td>135</td>
<td>105</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>elect. energy resolution</td>
<td>75</td>
<td>100</td>
<td>75</td>
<td>35</td>
<td>35</td>
</tr>
<tr>
<td>neutrino scale</td>
<td>70</td>
<td>-</td>
<td>140</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$p_T^W$ and $p_T^{had}$</td>
<td>60</td>
<td>120</td>
<td>90</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>underlying event</td>
<td>30</td>
<td>50</td>
<td>-</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>fitting procedure</td>
<td>30</td>
<td>40</td>
<td>40</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>radiative decays</td>
<td>30</td>
<td>50</td>
<td>20</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>elec. effic. vs. $p_T^e$</td>
<td>30</td>
<td>40</td>
<td>30</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$u_\parallel$ effect</td>
<td>25</td>
<td>95</td>
<td>350</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$p_T$ constraint</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>total syst.</td>
<td>160</td>
<td>240</td>
<td>420</td>
<td>80</td>
<td>130</td>
</tr>
</tbody>
</table>

Structure functions: For the central model, the structure function set HMRSB [9] is used. For studying systematic effects, we consider all available structure function sets which are evolved at next-to-leading order in the $\overline{MS}$ renormalization scheme (see ref. [10] and references therein). The structure functions determine a parton luminosity as a function of $\sqrt{s}$ which distorts slightly the resonance shape of the bosons. For the $Z$'s, this effect is represented by the constant $\beta$ in equation (3), where $\beta = 0.020 \pm 0.003$ GeV$^{-1}$. The resulting uncertainty on the $m_Z$ determination is less than 20 MeV and is neglected. The $m_W$ determination is much more sensitive to the structure functions because when the acceptance is taken into account, the rapidity distribution can distort
the spectra of the transverse variables. For the $m_T$ fits, the extreme variations are obtained with the structure function sets MT-El [11] (+80 MeV) and GRV [12] (-90 MeV).

If the range of the fits is extended downward, the sensitivity to structure functions increases. For example, fitting the $m_T$ spectrum over the range $40 < m_T < 120$ GeV instead of $60 < m_T < 120$ GeV results in an increase in the structure function uncertainty from 85 MeV to 115 MeV. This motivates the change in fit range with respect to ref. [1].

**Electron resolution:** The energy resolution of an individual calorimeter cell is studied with test beam data and is known with a relative error of 10% for the fiducial volume used. The cell to cell gain variations contribute a constant term of $1.5 \pm 1.0\%$ to the resolution, as determined from test beam recalibration and studies on the $W$ events themselves. An additional constant term of $1.3 \pm 0.2\%$ comes from the contribution from offset vertices. Finally, the resolution can be expressed as

$$\sigma_E/E = 17\%/\sqrt{E(\text{GeV})} \oplus 1.5\% \oplus 1.3\%,$$

leading to $\sigma_E/E = 3.3 \pm 0.5\%$ at $E = 40$ GeV.

**Neutrino scale:** To a good approximation, the scale of the $p_T^e$ measurement will match the scale of the $p_T^\nu$ measurement, but a few small effects can contribute to an imbalance between the two which will not cancel in the mass ratio $m_W/m_Z$. The electron can contribute some energy to cells outside of the core which then contributes to $p_T^{\text{had}}$. The uncertainty on the size of this effect produces an uncertainty of $\pm 60$ MeV in the neutrino scale. The effects of the underlying event inside the core cells contributes another uncertainty of $\pm 20$ MeV, and the contribution of the photon to $p_T^{\text{had}}$ in $e\nu\gamma$ decays of the $W$ adds another $\pm 30$ MeV. Taken in quadrature, the total uncertainty on $p_T^e - p_T^\nu$ is $\pm 70$ MeV.

**$p_T$ distributions and measurement of $p_T^{\text{had}}$:** The systematic effects due to the $p_T$ distribution of the $W$ are investigated by applying to the $W$ model the hard and soft spectra shown in Fig. 2(a). These spectra are obtained by varying the parameters in the fit to the $p_T^\nu$ spectrum and the significance of each variation ($-1.8\sigma$ and $+2.5\sigma$) is evaluated from the difference in likelihood of the $p_T^\nu$ prediction for the central $Z$ sample. An additional cross check is obtained by applying the theoretical model for the $p_T$ distribution [7] in place of the empirical model. The mass from the $m_T$ fit changes by less than 20 MeV.

In order to understand the $p_T^\nu$ and $m_T$ spectra, one must also consider the systematic uncertainties in the measurement of $p_T^{\text{had}}$. These errors are evaluated by varying in the model the resolution and average response for $p_T^{\text{had}}$ within the ranges allowed by the statistical errors from the $\eta$-balance distribution.

The model is further constrained by requiring that the combination of true $p_T^W$, $p_T^{\text{had}}$ resolution, and average $p_T^{\text{had}}$ response predict a mean for the observed $p_T^W$ distribution
which is consistent with the one actually measured. It is possible, for example, to
increase the true $p_T^W$ while decreasing the average response so as to maintain the
same average observed $p_T^W$. Because of the different ways these components of the
model affect the various transverse variables, constraining the $p_T^W$ prediction reduces
the systematic error on the $p_T^\gamma$ and $p_T^Z$ fits, but has almost no influence on the $m_T$ fit.
The overall sensitivity to $p_T^W$ and $p_T^{had}$ uncertainties gives a contribution of $\pm 60$ MeV
to the error on $m_W$ for the $m_T$ fit.

**Underlying event:** Frequently, particles from the underlying event can deposit a small
amount of energy in the core cells used to measure $p_T^\gamma$. The distribution of this energy
is determined by examining in real $W$ events the energy in cells not used in the electron
core but at the same azimuthal position. This effect increases $p_T^\gamma$ by an average of
$120 \pm 20$ MeV. For $m_Z$ a correction of $-250 \pm 40 (stat) \pm 50 (syst)$ MeV is included to
compensate for this increase, where the statistical error comes from the fluctuations of
the underlying event effect in the small sample. In the $m_W$ fits, the effect is already
included in the model, and the uncertainty depends on which transverse variable is
used, as shown in the table. For the $m_T$ fit, the effect is, as expected, approximately
half as large as for the $m_Z$ fit.

**Fitting procedure:** The errors arising from the use of the numerical pdf's for the $m_W$ fits
are checked by dividing the large Monte Carlo sample used in generating the pdf's into
many independent subsamples and fitting a fixed sample of 2000 Monte Carlo events.
The resulting spread of the fit values corresponds to an uncertainty of $\pm 30$ MeV for the
$m_T$ fits. Fits to large samples of Monte Carlo events are also used to confirm that the
fits return the input value to this same precision. Fits to many Monte Carlo samples
of the same size as the actual data sample verify that the statistical errors of the fits
are correct.

**Radiative Decays:** The decays $W \rightarrow e\nu\gamma$ and $Z \rightarrow e^+e^-\gamma$ are simulated with the $O(\alpha)$
Monte Carlo program of ref. [13]. The response of the calorimeter to low energy photons is modeled with a GEANT simulation [14] which includes the effect of the
preshower radiator. In order to account for the configurations where a photon lies close
to an electron, the calorimeter efficiency and energy measurement are parameterized
as a function of photon energy and angular separation according to a parameterization
obtained by superimposing two test beam electrons where one is rescaled to the photon
energy. For the systematic error evaluation, this model is compared with a naive model
where the photon is merged with the electron whenever the separation is less than 15\degree.
For the $m_Z$ determination, the effect of radiative decays is included as a separate
correction of $+190 \pm 50$ MeV. For the $m_W$ fits, the effect is already included in the
model, and it amounts to $\sim +90$ MeV for the $m_T$ fit, about half the size of the influence
on the $Z$, as expected. The uncertainties in photon response give the errors indicated
in the table.
The corrections beyond $O(\alpha)$ are estimated by using an exponentiation prescription
[15]. The mass difference $m_W - m_Z$ changes by $+45$ MeV ($-45$ MeV in the $m_W$ fit

11
Figure 5: The average value of $u_\parallel$ (see text) as a function of $p_T^W$. The points are the data and the solid curve is the prediction of the central model. The dashed curve shows the model with the $u_\parallel$ dependence of the electron efficiency removed.

and $-90$ in the $m_Z$ fit). These corrections are not included in the final result, but this serves to illustrate the size of higher order radiative effects.

**Electron efficiency vs. $p_T^Z$:** If there is a change in electron identification efficiency with $p_T^z$, it can distort the transverse variable spectra and give a slight change in the measured $m_W$. The combined efficiency of the calorimeter, tracking and preshower requirements was studied with a combination of test beam data and $W$ events. The change in efficiency between 40 GeV and 10 GeV is estimated to be $-5 \pm 5\%$. The influence of this uncertainty on $m_W$ is given in Table 3, and the effect on $m_Z$ is negligible.

**Electron efficiency vs. $u_\parallel$:** The variable $u_\parallel$ is defined as the component of $\vec{p}_T^{had}$ along the electron direction in $W$ events. When the electron lies close to the direction of $\vec{p}_T^{had}$, there is a greater chance that the electron signature will be spoiled by the hadrons. Consequently, one expects a decrease in electron efficiency for large positive $u_\parallel$. This effect is estimated for the calorimeter signature by superimposing electrons from real $W$ decays on $W$ underlying events from the PYTHIA Monte Carlo [16] and running the standard pattern recognition and electron quality cuts. For the tracking and preshower, the efficiency is measured from the $W$ data as a function of $u_\parallel$. The effect of applying the $u_\parallel$-dependent efficiency which results from these studies can be seen in Fig. 5 as a function of $p_T^W$ (where $p_T^W = -\vec{p}_T^{had}$). The tendency at large $p_T^W$ for the average value of $u_\parallel$ to become increasingly negative results from a combination of kinematic effects and the loss of events with large positive $u_\parallel$. The uncertainties on the influence of $u_\parallel$
on the electron efficiency are similar to the size of the effect itself, and the resulting errors on \(m_W\) are evaluated by removing and doubling the effect in the model. The resulting uncertainties are quite large on the fits to \(p_T\) and \(p_{T}^b\), but cancel to first order in \(m_T\), as shown in Table 3. This is one of the main reasons for the choice of \(m_T\) for quoting a final value for \(m_W\).

**\(p_T\) constraint:** The \(m_Z\) measurement from the second \(Z\) sample has an additional systematic error of 100 MeV associated with the application of the \(p_T\) constraint. The uncertainty on the \(p_{T}^b\) response contributes \(\sim 70\) MeV and the treatment of the calorimeter cells assigned to the non-fiducial electron contributes \(\sim 80\) MeV.

### 5 Results

The combined results from the two samples of \(Z\) events give \(m_Z = 91.74\pm 0.28\) (stat) \(\pm 0.12\) (syst) \(\pm 0.92\) (scale) GeV. This can be compared with the result from LEP of \(m_Z = 91.175\pm 0.021\) GeV [17]. For the \(W\) mass, the result of the \(m_T\) fit, \(m_W = 80.84\pm 0.22\) (stat) \(\pm 0.17\) (syst) \(\pm 0.81\) (scale) GeV, is taken because it has the smallest errors. The scale errors from the calorimeter calibration cancel in taking the ratio \(m_W/m_Z\) aside from a residual \(\pm 80\) MeV effect of possible nonlinearities in the calorimeter energy response. In addition, some of the systematic errors contain some correlations which are taken into account. The ratio

\[
m_W/m_Z = 0.8813 \pm 0.0036\) (stat) \(\pm 0.0019\) (syst)
\]

(5)

can be multiplied by the LEP value of \(m_Z\) to give a more precise value for the \(W\) mass:

\[
m_W = 80.35 \pm 0.33\) (stat) \(\pm 0.17\) (syst) GeV.
\]

(6)

Combining the statistical and systematic errors, one obtains \(m_W = 80.35 \pm 0.37\) GeV. Using the Sirius [18] convention \(\sin^2 \theta_W \equiv 1 - m_W^2/m_Z^2\), equation (5) implies

\[
\sin^2 \theta_W = 0.2234 \pm 0.0064 \pm 0.0033.
\]

(7)

This value is in agreement with the result derived from low energy data \(\sin^2 \theta_W = 0.2309 \pm 0.0029\) (stat) \(\pm 0.0049\) (syst) [19].

The CDF experiment has measured \(m_W = 79.91 \pm 0.39\) GeV [2], which is in good agreement with the present measurements. When the results of UA2 and CDF are combined the results are \(m_W = 80.14 \pm 0.27\) GeV and \(\sin^2 \theta_W = 0.2274 \pm 0.0052\).

Within the Standard Model, the ratio \(m_W/m_Z\) is determined at the Born level from the parameters \(\alpha\), \(G\), and \(m_Z\). Radiative corrections can modify this prediction significantly. In the minimal Standard Model, these corrections depend strongly (quadratically) on the mass of the top quark \(m_{t_{op}}\) [20] and weakly (logarithmically) on the mass of the Higgs boson \(m_H\). Consequently, the measurement of \(m_W/m_Z\) can be used to place some (model dependent) bounds on \(m_{t_{op}}\). This is illustrated in Fig. 6 [21]. From the UA2 result alone, one can conclude that \(m_{t_{op}} = 160 \pm 50\) GeV for \(m_H = 100\) GeV, and \(m_{t_{op}} < 250\) GeV at the 95% confidence level for \(m_H < 1\) TeV. The combined results of UA2 and CDF yield \(m_{t_{op}} = 130 \pm 50\) for \(m_H = 100\) GeV and \(m_{t_{op}} < 215\) GeV at the 95% confidence level for \(m_H < 1\) TeV (the
calculations [21] are no longer valid for $m_H \gtrsim 1$ TeV). Note that direct searches place a limits of $m_{\text{top}} > 89$ GeV [22] and $m_H > 48$ GeV [23].

The LEP measurements of mass, width, and asymmetries at the $Z$ pole can be combined, within the assumptions of the minimal Standard Model, to give a prediction of the $W$ mass of $m_W = 80.14 \pm 0.19$ GeV [17]. Similarly, the $\sin^2 \theta_W$ measurement from neutrino scattering [19] can be combined with $m_Z$ from LEP to give a minimal Standard Model prediction of $m_W = 79.96 \pm 0.30$ GeV. Both of these indirect determinations are in excellent agreement with the collider measurements of $m_W$.

6 Conclusions

The final data sample of the UA2 experiment has been used to make a direct determination of the ratio of $W$ and $Z$ masses. The result is

$$m_W/m_Z = 0.8813 \pm 0.0036(\text{stat}) \pm 0.0019(\text{syst}).$$

In combination with the $m_Z$ measurement from LEP, this gives

$$m_W = 80.35 \pm 0.33(\text{stat}) \pm 0.17(\text{syst}) \text{ GeV}.$$  

The result agrees well with a similar determination from CDF and with Standard Model predictions based on LEP data. Within the minimal Standard Model, this measurement of $m_W$ implies that $m_{\text{top}} < 250$ GeV at the 95% confidence level.
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