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Decay correlations in $e^+e^- \rightarrow \tau^+\tau^-$ as a test of Quantum Mechanics

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Abstract

Tau leptons which are pair produced in e^+e^- annihilation show a strong correlation in their spin directions. The process is interpreted as an Einstein-Podolski-Rosen type of experiment. The induced correlation in the τ decay products is analyzed in terms of possible tests of Quantum Mechanics versus local hidden variables theories, studying τ pair production at energies between threshold and the Z^0 mass.

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1 Introduction

In 1935 Einstein, Podolski, and Rosen (EPR) produced a famous argument [1] for the incompleteness of the Copenhagen interpretation of Quantum Mechanics. This work greatly contributed to the development of hidden variables theories, where the introduction of ‘hidden’ parameters is used to restore determinism and causality in the micro-physics world. In 1965 Bell, using Bohm’s [2] reformulation of the EPR argument in terms of spin components, proved the fundamental result [3] that a local hidden variable theory cannot reproduce all statistical predictions of Quantum Mechanics.

Several experimental tests have been performed since that time, mainly studying the polarization correlation of photons emitted in an atomic cascade [4]. Almost all the experiments agree with the quantum mechanical predictions, but they always tested Bell’s inequalities deduced from local realism and some additional assumptions [5]. The search for complementary tests has stimulated the discussion on some examples of the EPR problem in Particle Physics [6].

In this paper, the process $e^+e^- \rightarrow \tau^+\tau^-$ is proposed as a possible EPR-like experiment, and the experimental study of limits from Bell-type inequalities at different production energies is investigated. In Section 2, the EPR argument is presented, and the Bell’s inequalities are reviewed in Section 3. The spin correlation in τ pair production and decay, and its interpretation in the framework of the EPR problem is discussed in Section 4. Possible experimental tests of Quantum Mechanics versus local hidden variables theories are discussed in Section 5, studying τ decay correlation at production energies between threshold and the Z^0 mass. The conclusions are summarized in Section 6.

2 The EPR argument

‘If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity’. On the basis of this famous ‘criterion of physical reality’, Einstein, Podolski and Rosen in 1935 [1], discussing the reality of the position and momentum of two correlated particles, argued that Quantum Mechanics was not a complete theory.

A reformulation of the EPR argument, involving spin functions, was proposed by Bohm [2]. Consider two spin one-half particles in a singlet state, and let them move freely in opposite directions. The system state is described by the following function:

$$\Psi = \frac{1}{\sqrt{2}}(\psi_+(1)\psi_-(2) - \psi_-(1)\psi_+(2)), \quad (1)$$

where $\psi_{\pm}(i)$ is the wave function of the state in which particle i has a spin $\pm\hbar/2$ in the direction in which is measured. Once the particles have separated, any spin component of particle 1 can be measured. Since the total spin of the system is zero, one knows, without having interfered with particle 2, that its spin component along any axis is opposite to that of particle 1. According to the EPR criterion of physical reality, the spin component

of particle 2 represents an element of physical reality, and thus existed even before the measurement was performed. But since one could have equally chosen any other direction for measuring the spin of particle 1, all three spin components of particle 2 must have simultaneously definite values after the two particles have separated. This deduction is in clear contradiction with Quantum Mechanics which allows only one spin component at a time to have definite values, since spin operators do not commute. Thus Quantum Mechanics does not provide a *complete* description of physical reality in the EPR sense: *'Every element of physical reality attributable to a certain physical system in a given state must have a counterpart in the mathematical description provided by the theory for that physical system in the state considered'*.

The intimate connection between the EPR reasoning and the nonfactorizable form of the state function (1) was discussed for the first time by Furry [7]. The fact that the measurement of the spin component of particle 1 along a certain direction determines with certainty the result of the measurement along the same direction for particle 2, has to be interpreted as the spin components of the two particles along the chosen direction have definite values before any measurement is actually performed, if one wants the EPR *locality* principle (*'since at the time of the measurement the two systems no longer interact, no real change can take place in the second system in consequence of anything that may be done to the first system'*) to hold when the two particles are far apart. The two particles, thus, are in a definite quantum state $\psi_i(1)\psi_{-i}(2)$ ($i = +, -$). In Quantum Mechanics, the necessary and sufficient condition for two quantum systems to be in a definite quantum state is that the system state is factorizable. The comparison with Eq. (1) gives paradoxical results analogous to the EPR argument.

Bohm and Aharonov [8] interpreted Furry's conclusions providing a possible interpretation of the EPR argument. One can make the hypothesis that, after the two particles separate, the wave function for the system is no longer given by Eq. (1), and the spins of the two particles becomes definite in *some* direction. The wave function will be given by the product:

$$\Psi = \psi_{+\theta,\phi}(1)\psi_{-\theta,\phi}(2), \quad (2)$$

where $\psi_{+\theta,\phi}(1)$ is a wave function of particle 1 whose spin is positive in the direction given by θ and ϕ , while the spin of particle 2 is negative in the same direction. Assuming a uniform probability for any direction of θ and ϕ , this model predicts the experimentally observed fact of conservation of the angular momentum on the average. Since there is no precise correlation of an arbitrary component of the spin of each particle in every individual case, the EPR 'paradox' is solved. Bohm and Aharonov could show that, for the case of positronium annihilation, such a model can be experimentally tested, since it gives predictions for the polarization correlation of the two photons different from that of Quantum Mechanics.

3 The Bell inequalities

A reinterpretation of Quantum Mechanics in terms of a statistical account of an underlying hidden variables theory was proposed as a possible solution to the EPR 'paradox'.

The introduction of some supplementary parameters can *complete* the quantum mechanical state specification, and a classical-looking picture of the EPR correlations is recovered. The quantum mechanical predictions are, then, obtained when averaging over the supplementary parameters.

In this context, Bell [3] made a fundamental contribution, showing that for the *Gedankenexperiment* of Bohm, no deterministic hidden variables theory can reproduce all the statistical predictions of Quantum Mechanics. Let $A_{\hat{a}}(B_{\hat{b}})$ be the result of a measurement of the spin of particle 1(2) in the direction $\hat{a}(\hat{b})$. The product of $A_{\hat{a}}$ and $B_{\hat{b}}$ is an observable of the two particle system, and its expectation value, according to Quantum Mechanics, is:

$$E(\hat{a}, \hat{b}) = P_{++}(\hat{a}, \hat{b}) - P_{+-}(\hat{a}, \hat{b}) - P_{-+}(\hat{a}, \hat{b}) + P_{--}(\hat{a}, \hat{b}) = -\hat{a} \cdot \hat{b}, \quad (3)$$

where $P_{ij}(\hat{a}, \hat{b})$ is the joint probability of having the result i in the spin measurement of particle 1 along the direction $\hat{a}$ and the result k in the spin measurement of particle 2 along the direction $\hat{b}$, with $i, j = \pm 1$.

In a deterministic hidden variables theory, the observable $A_{\hat{a}} \cdot B_{\hat{b}}$ has a definite value $\langle A_{\hat{a}} \cdot B_{\hat{b}} \rangle_{\lambda}$ for the state λ , where the symbol λ denotes a complete specification of the system state, without any assumption on its possible complexity. Bell introduced a definition of locality:

$$\langle A_{\hat{a}} \cdot B_{\hat{b}} \rangle_{\lambda} = A_{\hat{a}}(\lambda) \cdot B_{\hat{b}}(\lambda), \quad (4)$$

which implies that once the state λ is specified and the two particles separate, the measurement $A_{\hat{a}}(B_{\hat{b}})$ depends only on λ and $\hat{a}(\hat{b})$. The expectation value of $A_{\hat{a}} \cdot B_{\hat{b}}$ for such theories is:

$$E(\hat{a}, \hat{b}) = \int_{\Lambda} A_{\hat{a}} \cdot B_{\hat{b}} d\rho, \quad (5)$$

where ρ is the distribution function for the states λ in the space Λ . Using Eq. (5) and the perfect anticorrelation in the case of parallel directions of the analyzers ($E(\hat{a}, \hat{a}) = -1$), Bell could write an inequality for all deterministic local hidden variables theories involving the expectation value $E(\hat{a}, \hat{b})$ which was violated by the quantum mechanical prediction for some particular directions of the analyzers.

Bell's proof, of fundamental heuristic value, is not, however, experimentally testable, since it requires the existence of a pair of spin analyzer orientations with perfect anticorrelation, while, in a real experiment, weak correlation in the source emissions or poor detector efficiencies strongly limit this assumption.

A more general inequality was obtained by Bell [9] without the requirement of a perfect anticorrelation:

$$-2 \leq E(\hat{a}, \hat{b}) - E(\hat{a}, \hat{b}') + E(\hat{a}', \hat{b}) + E(\hat{a}', \hat{b}') \leq 2, \quad (6)$$

which is maximally violated, by a factor $\sqrt{2}$, if the quantum mechanical prediction of Eq. (3) is used, choosing the four analyzing directions to be coplanar and the angle between $\hat{a}$ and $\hat{b}$, $\hat{b}$ and $\hat{a}'$, $\hat{a}'$ and $\hat{b}'$ equal to $\pi/4$.

Inequalities expressed in terms of coincidence counts between detectors, which are more suitable for a real experiment, have been derived by several authors [10]. Clauser and Horne proposed a general experimental set-up shown in Fig. 1, where a source of coincident

Figure 1: Experimental arrangement of Clauser and Horne proof.

two particle emissions is viewed by two analyzer-detector assemblies. Each apparatus has an adjustable parameter and let a be the value of the parameter for apparatus 1 and b that for apparatus 2. The parameters a and b could denote the angles specifying the orientation of the analyzers, like the axes of linear polarizers for photons, or the directions of the field gradients of Stern-Gerlach magnets, but the specific interpretation of these adjustable parameters is not essential for the proof. With the help of a general locality definition, Clauser and Horne found the following inequality:

$$-1 \leq p_{12}(a, b) - p_{12}(a, b') + p_{12}(a', b) + p_{12}(a', b') - p_1(a') - p_2(b) \leq 0, \quad (7)$$

where $p_{12}(a, b)$ is the probability of coincidence counts in detector 1 and 2 with the analyzers in position a and b , and $p_1(a)$, $p_2(b)$ are the probabilities of single counts respectively in detector 1 with the analyzer in position a , and in detector 2 with the analyzer in position b . Inequality (7) is also maximally violated by a factor $\sqrt{2}$ for some specific values a, b, a', b' .

4 Decay correlations in $e^+e^- \rightarrow \tau^+\tau^-$

Decay correlations in heavy leptons production through electron-positron annihilation were discussed for the first time in 1971 by Tsai [11], before the τ lepton was actually discovered [12]. The process is described in Fig. 2. In the limit $m_e/E_e \rightarrow 0$, the total angular momentum of the electron-positron system is unity, and its direction is either parallel or antiparallel to the direction of the incident electron. This specific initial state spin configuration results in a strong correlation in the spins of τ^+ and τ^- , produced in the annihilation of e^+e^- into a virtual photon (or a Z^0 boson). Near the threshold ($m_\tau/E_\tau \approx 1$) the directions of spins of τ^+ and τ^- prefer to be parallel to each other, and their total spin prefers to be either parallel or antiparallel to the direction of the incident electron, whereas far above the threshold ($m_\tau/E_\tau \rightarrow 0$), the helicities of τ^+ and τ^- prefer to be opposite to each other. A general expression for the spin dependent τ

Figure 2: Feynman diagram of the process $e^+e^- \rightarrow \tau^+\tau^-$

pair production cross section [13] can be written as

$$d\sigma = d\sigma_0(1 + a_\mu s_1^\mu + b_\mu s_2^\mu + c_{\mu\nu} s_1^\mu s_2^\nu), \quad (8)$$

where $s_1^\mu(s_2^\mu)$ is the spin of the $\tau^+(\tau^-)$ in the τ rest frame and the coefficients a , b and c are functions of the momenta of the incoming and outgoing fermions. The terms involving single powers of s_1^μ or s_2^μ result in net polarization of the τ (coming from Z^0 exchange and $\gamma - Z^0$ interference) while the terms involving two powers result in spin correlation between the two τ 's.

Since τ^+ and τ^- decay via weak interactions where parity conservation is violated maximally, the angular distribution of decay products depends strongly on the spin orientation of the τ . Since the spins of τ^+ and τ^- are strongly correlated in the production one expects the angular distributions of decay products of τ^+ to be strongly correlated to those of τ^- . A general form for τ^- decay is:

$$d\Gamma = d\Gamma_0(1 + h_\mu s^\mu), \quad (9)$$

where the polarimeter vector h_μ depends on the particular decay mode, the τ momentum and the momenta of the decay products.

The cross section for the combined production and decay process will therefore result:

$$d\sigma \propto (1 - a_\mu h_1^\mu - b_\mu h_2^\mu + c_{\mu\nu} h_1^\mu h_2^\nu) d\Gamma_1 d\Gamma_2. \quad (10)$$

Notice that the correlation between the spins of the τ 's contained in the terms $c_{\mu\nu}$ induces an analogous correlation between the polarimeter vectors of the final decay products, which can be studied experimentally looking at the final state characteristics.

In the decay $\tau \rightarrow \pi \nu_\tau$ of a polarized τ , the polarimeter vector h_μ is simply the pion momentum:

$$d\Gamma(\tau^\mp) = d\Gamma_0(1 \pm \vec{w} \cdot \hat{p}_\pi), \quad (11)$$

where $\vec{w}$ is the polarization of the τ in its rest frame and $\hat{p}_\pi$ is the unit vector in the direction of motion of the pion. Since this is a two body decay, ν_τ and π^- come out back

to back in the τ rest frame, and the component of the orbital angular momentum along the direction of ν_τ is thus zero. Now, ν_τ has a negative helicity, and hence prefers to be emitted opposite to the direction of the spin of τ^- . Therefore, π^- prefers to be emitted in the direction of the spin of the τ^- . The parity violating τ decay works thus as a sort of polarimeter, in the sense that the τ can be thought of as polarized in the direction of the pion momentum.

The other decays of the τ have a more complicated form of the polarimeter vector, which generally results in a lower sensitivity to the τ polarization. For this reason, only the decay $\tau \rightarrow \pi\nu_\tau$, which accounts for 10% of the τ branching ratio, will be considered in the following.

The *nonlocal* spin correlation described by Eq. (8) leads to an interpretation of the process $e^+e^- \rightarrow \tau^+\tau^-$ as an EPR-like experiment, since the τ^- spin direction is influenced by that of the τ^+ . The same *nonlocal* correlation appears in the τ decay products, in the sense that the τ^- decay is influenced by how τ^+ has decayed (or will decay)¹.

The highest sensitivity to the spin correlation is found in the process $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$, and since its spin dependent rate can be measured, it is interesting to study possible limits from Bell's inequalities in a field different from the usual atomic physics tests of the EPR problem.

5 Tests of Quantum Mechanics with

$$e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$$

A possible interpretation of the process $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ is shown in Fig. 3. The electron-positron annihilation is seen as a source of a $\tau^+\tau^-$ pair. The two τ 's travel in

Figure 3: $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ as a Clauser and Horne type of experiment

opposite directions, and at certain times each of them decays into a pion and a neutrino.

¹Similar considerations have been made by Törnqvist with respect to the process $e^+e^- \rightarrow \Lambda\bar{\Lambda} \rightarrow \pi^-p\pi^+\bar{p}$ [14].

The parity violating weak decay of the τ is interpreted as a spin analyzing device. That is, whenever a $\pi^+(\pi^-)$ is seen in the direction $\hat{p}_1(\hat{p}_2)$ this is understood as a count in detector 1(2) with analyzer in direction $\hat{p}_1(\hat{p}_2)$, in analogy to the Clauser-Horne experiment described in Fig. 1.

Using the spin dependent rate of the joint pions detection, which will be explicitly given in the following, one could study a possible violation of Bell's inequalities, thus testing the hypothesis of hidden variables theories. However, it is easy to verify that no *direct* violation of Bell's inequalities is achieved with this approach. The reason stands in the fact that the pion direction is only an index of the actual τ polarization, and neither τ is forced to make a dichotomic decision.

The situation described is similar to that of Kasday, Ullman and Wu [15], who used Compton scattering of the photons coming from electron-positron annihilation to measure the photons linear polarization correlation. Using their approach, that is making the additional assumptions that 1) it is possible in principle to construct an ideal τ polarization analyzer, and 2) the results obtained in an experiment with ideal analyzers and the results obtained in a τ decay experiment are correctly related by the theory², Bell's inequalities applied to the ideal analyzers can be used to calculate corresponding restrictions on the angular distribution of the pions coming from the τ decays.

Another interesting possibility is the test of the Bohm-Aharonov hypothesis discussed in Section 2. When the two τ 's are separated by some *large distance*, they get polarized in a random direction, and the spontaneous decay of one τ will not affect the decay of the other τ , which results in a sensible decrease of the two pions correlation.

These tests can be performed for $\tau^+\tau^-$ production energies in all the range from threshold to the Z^0 mass, which provide increasing spatial separation of the two τ 's ($\gamma_\tau \approx 1 \div 25$), giving, thus, additional interest to the measurement.

5.1 Threshold production of τ pair

In electron-positron annihilation near the τ threshold ($E_{\text{beam}} \approx m_\tau$), τ pairs are produced almost at rest. The two-body decay $\tau \rightarrow \pi\nu_\tau$ will, thus, provide monochromatic pions, and the τ spin correlation results in an angular correlation between π^+ and π^- directions.

The expected correlation rate [17] is given by:

$$N(\hat{p}_1, \hat{p}_2) \propto 1 - \frac{1}{3} \hat{p}_1 \cdot \hat{p}_2, \quad (12)$$

where $\hat{p}_1$ and $\hat{p}_2$ are unit vectors respectively in the direction of motion of π^+ and π^- .

The term $\hat{p}_1 \cdot \hat{p}_2 = \cos \theta_{12}$ in the correlation rate is equivalent to Bell's inequality violating expectation value of Bohm's *Gedankenexperiment* (cf. Eq. (3)). If one writes the correlation rate as $(A - B \cos \theta_{12})$, the experimental measurement of the slope B/A will be a test of Quantum Mechanics, since B/A is limited by Bell's inequality to no more than $1/\sqrt{2}$ of the value predicted by Quantum Mechanics (1/3).

²Note that assumption 2) corresponds to the correctness of Eq. (9), which has been recently tested with the measurements of the τ polarization at the Z^0 peak [16], and of Eq. (10).

Figure 4: $\pi^+\pi^-$ correlation rate versus $\cos \theta_{12}$: a) Quantum Mechanics prediction, b) Bell's inequality upper limit, c) Bohm-Aharonov hypothesis. The points are Monte Carlo data corresponding to one year of running at a τ -charm factory ($10^5 \pi^+\pi^-$).

Taking the Bohm-Aharonov hypothesis that the τ becomes polarized in a random direction $\hat{x}$, an additional factor $1/3$ is expected for the slope parameter, since one has to average over a term of the type $(1 - \hat{x} \cdot \hat{p}_1)(1 + \hat{x} \cdot \hat{p}_2)$.

A Monte Carlo [18] distribution, generated with $10^5 \pi^+\pi^-$ at $E_{beam} = 1.785$ GeV, is shown in Fig. 4, together with the upper limit of Bell's inequality, and the Bohm-Aharonov prediction.

Such an experiment, as well as other independent tests³, could be actually realized at a τ -charm factory [19], where one expects of the order of 10^7 τ pair per year. The correspondent 10^5 events with both τ 's decaying into a pion and a neutrino, would exclude Bell's limit and the Bohm-Aharonov hypothesis at 99.9% confidence level. A valuable test of Quantum Mechanics at a τ -charm factory is, thus, given by the experimental study of angular correlation in the process $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$.

5.2 High energy production of τ pair

At high energy ($m_\tau/E_{beam} \rightarrow 0$) the spin correlation in the τ production results in a correlation of the τ helicities: when the τ^+ has a negative(positive) helicity the τ^- has a positive(negative) helicity. The decay $\tau \rightarrow \pi\nu_\tau$ is, thus, sensitive to the τ helicity:

$$d\Gamma(\tau^\pm) = d\Gamma_0(1 \mp h_\pm \cos \bar{\theta}_\pi), \quad (13)$$

³A detailed analysis is discussed in: P. Privitera, Tests of Quantum Mechanics at a τ -charm factory, in preparation.

where $h_{\pm}$ is the $\tau^{\pm}$ helicity and $\bar{\theta}_{\pi}$ is the angle between the pion momentum and the τ direction.

The Lorentz boost from the τ rest frame to the Laboratory determines a simple relation between the angle $\bar{\theta}_{\pi}$ and the energy of the pion measured in the Lab:

$$\cos \bar{\theta}_{\pi} = \frac{4E_{\pi}m_{\tau}^2 - 2E_{beam}(m_{\tau}^2 + m_{\pi}^2)}{(m_{\tau}^2 - m_{\pi}^2)\sqrt{4E_{beam}^2 - 4m_{\tau}^2}} \approx 2x_{\pi} - 1, \quad x_{\pi} = \frac{E_{\pi}}{E_{beam}}, \quad (14)$$

where the approximation is valid when terms of the order $(m_{\pi}/m_{\tau})^2$ and $(m_{\tau}/E_{beam})^2$ are neglected.

The spin dependence of the rate for the process $e^{+}e^{-} \rightarrow \tau^{+}\tau^{-} \rightarrow \pi^{+}\bar{\nu}_{\tau}\pi^{-}\nu_{\tau}$ results, thus, in a strong correlation between the pions energies [13]:

$$N(x_1, x_2) \propto 1 + (2x_1 - 1)(2x_2 - 1), \quad (15)$$

where x_1 and x_2 refer respectively to positive and negative pion.

The pions energy-energy correlation reflects only the longitudinal spin correlation of the τ 's, which is in a factorized form $(\cos \bar{\theta}_1 \cdot \cos \bar{\theta}_2)$ and does not violate Bell's inequality.

The complete τ spin correlation results in an angular correlation between π^{+} and π^{-} directions in the τ rest frame. The expected correlation rate [17] is given by:

$$N(\hat{p}_1^*, \hat{p}_2^*) \propto 1 - \frac{1}{3} \hat{p}_1^* \cdot \hat{p}_2^*, \quad (16)$$

where $\hat{p}_1^*$ and $\hat{p}_2^*$ are unit vectors, defined in the τ^{-} rest frame, respectively in the direction of motion of π^{+} and π^{-} . In the following, the starred quantities are defined in the τ rest frame, with the z axis along the τ^{-} direction.

The term $\hat{p}_1^* \cdot \hat{p}_2^* = \cos \theta_{12}^*$ can be expressed by observables which are measurable in the Laboratory through the following relations [20]:

$$\cos \theta_{12}^* = \cos \theta_1^* \cos \theta_2^* + \sin \theta_1^* \sin \theta_2^* \cos(\phi_1^* - \phi_2^*), \quad (17)$$

$$\cos \theta_1^* \cos \theta_2^* = -\cos \bar{\theta}_1 \cos \bar{\theta}_2, \quad (18)$$

$$\sin \theta_1^* \sin \theta_2^* \cos(\phi_1^* - \phi_2^*) =$$

$$\frac{4m_{\tau}^2}{(m_{\tau}^2 - m_{\pi}^2)^2} \left[p_1 p_2 \cos \psi + \frac{(2E_{beam}E_1 - m_{\tau}^2 - m_{\pi}^2)(2E_{beam}E_2 - m_{\tau}^2 - m_{\pi}^2)}{4E_{beam}^2 - 4m_{\tau}^2} \right], \quad (19)$$

where $p_{1(2)}$ and $E_{1(2)}$ are $\pi^{+}(\pi^{-})$ momentum and energy and ψ is the angle between the pions directions in the Laboratory.

Using the same arguments discussed in Section 5.1, one can provide interesting tests of Quantum Mechanics measuring the correlation parameter B/A of the experimental distribution $(A - B \cos \theta_{12}^*)$.

A Monte Carlo [18] distribution, generated with 10^4 $\pi^{+}\pi^{-}$ events at $E_{beam} = 5$ GeV, is shown in Fig. 5, together with the upper limit of Bell's inequality, and the Bohm-Aharonov prediction.

Figure 5: $\pi^+\pi^-$ correlation rate versus $\cos\theta_{12}^*$: a) Quantum Mechanics prediction, b) Bell's inequality upper limit, c) Bohm-Aharonov hypothesis. The points are Monte Carlo data corresponding to the statistics collected by existing experiments ($10^4 \pi^+\pi^-$).

The high statistics collected by the ARGUS and CLEO collaborations [21] (around $10^6 \tau^+\tau^-$) could already exclude the Bohm-Aharonov hypothesis at 95% confidence level, but more statistics is needed to exclude Bell's limit. It could also provide the first evidence for spin correlation in τ pair production.

A future B factory could produce of the order of $10^7 \tau$ pair per year, with a sensitivity to the correlation parameter B/A equal to that of a τ -charm factory.

5.3 Production of τ pair at the Z^0 peak

At center of mass energies around the Z^0 mass (≈ 91.17 GeV), electron-positron annihilation is dominated by the Z^0 resonance. The process $Z^0 \rightarrow \tau^+\tau^-$ produces a polarization [22] of the τ lepton, due to the difference in the coupling of the Z^0 to right-handed and left-handed $\tau^\pm$:

$$P_{\tau^\pm} \approx \pm 2 \frac{a_\tau v_\tau}{a_\tau^2 + v_\tau^2} \approx \pm 2(1 - 4 \sin^2 \theta_W), \quad (20)$$

where a_τ and v_τ are the τ axial and vector-axial coupling constants, and θ_W is the electroweak mixing angle.

The unique possibility to determine final state helicities using the spin analyzing decay of the τ allows the measurement of the small polarization P_τ ($P_\tau \approx -0.16$ for $\sin^2 \theta_W = 0.23$).

The measurement of P_τ at the Z^0 peak has been recently performed by LEP detectors [16], providing evidence for a small negative value of the τ polarization.

The existence of a net polarization results in some additional terms, linear in the polarimeter vectors h^μ , in the differential cross section (cf. Eq. (10)). Thus, the $\pi^+\pi^-$ energy-energy correlation function contains also a term proportional to P_τ :

$$N(x_1, x_2) \propto 1 + (2x_1 - 1)(2x_2 - 1) - 2P_\tau(x_1 + x_2 - 1), \quad (21)$$

where x_1 and x_2 are the positive and negative pion normalized energies defined in Eq. (14).

The pions energy-energy correlation contains only the longitudinal spin correlation of the τ 's, whose factorized form $(\cos \bar{\theta}_1 \cdot \cos \bar{\theta}_2)$ does not violate Bell's inequality.

The $\pi^+\pi^-$ correlation rate with respect to $\cos \theta_{12}^*$ as defined in Eq. (17) recuperates the full τ spin correlation. Due to the Z^0 exchange, the transverse spin correlation term $\sin \theta_1^* \sin \theta_2^* \cos(\phi_1^* - \phi_2^*)$ contains a factor

$$\kappa = \frac{a_\tau^2 - v_\tau^2}{a_\tau^2 + v_\tau^2}. \quad (22)$$

However, since $\kappa \approx 1$ ($\kappa = 0.987$ for $\sin^2 \theta_W = 0.23$), the effect is negligible for the specific tests discussed here, and the $\pi^+\pi^-$ correlation rate is still given by Eq. (16). This is shown by the Monte Carlo [23] distribution of Fig. 6, generated with 10^5 $\pi^+\pi^-$ events at $E_{beam} = M_{Z^0}/2$.

Figure 6: $\pi^+\pi^-$ correlation rate versus $\cos \theta_{12}^*$: a) Quantum Mechanics prediction, b) Bell's inequality upper limit, c) Bohm-Aharonov hypothesis. The points represent the Monte Carlo prediction for 10^5 $\pi^+\pi^-$ generated with $\sin^2 \theta_W = 0.23$.

The spin correlation in the process $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ provides many interesting physics possibilities [20]: the measurement of P_τ and a test for maximal P, maximal C violation are amongst them.

Following the discussion of Section 5.1, interesting tests of Quantum Mechanics can be performed by extracting the correlation parameter B/A of the experimental distribution ($A - B \cos \theta_{12}^*$) and comparing the result with Bell's inequality upper limit and the Bohm-Aharonov hypothesis.

The high luminosity option [24] at the LEP collider could produce around $0.3 \cdot 10^7$ τ pair per year, giving a sensitivity comparable with a τ -charm or a B factory.

6 Conclusions

Since the original EPR argument was formulated, several tests of the incompatibility between quantum theory and local realism have been performed, almost exclusively in the domain of atomic physics. An analysis of the EPR 'paradox' in different fields and physical processes leads to a more critical understanding of the EPR problem.

In this context, τ pair production through electron-positron annihilation has been proposed as an example of EPR experiment. The strong spin correlation between the two τ 's contains, in fact, the *nonlocal* character of the EPR argument. The subsequent τ decay works as a spin analyzer, and the *nonlocal* correlation is transferred to the decay products, since the decay of one of the τ 's is influenced by how the other has decayed (or will decay).

The possibility of using these properties to make a test of Quantum Mechanics versus local hidden variable theories has been investigated. In particular, the sequential τ production and decay $e^+e^- \rightarrow \tau^+\tau^- \rightarrow \pi^+\bar{\nu}_\tau\pi^-\nu_\tau$ has been analyzed as a Clauser-Horne type of experiment. With proper assumptions, limits from Bell's inequalities are derived and can be easily tested. The Bohm-Aharonov hypothesis of a breakdown of the system state after the two τ 's have separated is shown to result in a significant decrease of the predicted pion-pion correlation.

Such studies should be performed for τ production energies between threshold and the Z^0 mass, in order to test the *nonlocal* τ spin correlation for increasing spatial separation of the two τ 's.

Future accelerators (τ -charm factory, B factory, high luminosity option for LEP) will provide high statistics samples of $\tau^+\tau^-$ pair, and precision measurements of the degree of correlation will be easily achieved. Available data could already be looked into.

Valuable tests of Quantum Mechanics can, thus, be obtained from the study of decay correlations in $e^+e^- \rightarrow \tau^+\tau^-$, giving a complementary insight of the EPR problem in the field of Particle Physics.

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