CERN-TH.6186/91
FTUAM-26/91

Chiral Lagrangians and Precision Tests of the Symmetry Breaking Sector of the Standard Model

Domènec ESPRIU* and María J. HERRERO†

CERN, Geneva
Switzerland

ABSTRACT

We consider extensions of the minimal Standard Model with various alternatives for the symmetry breaking sector. Different underlying theories are parametrized in terms of the coefficients of a Chiral Lagrangian. At present and future energies, assuming an exact custodial symmetry in the symmetry breaking sector, all relevant information is contained in just four coefficients. We analyze how to determine these coefficients in the minimal Standard Model with a large Higgs mass, where ordinary perturbation theory is not well suited, and in a class of vector-like technicolor models, but the extension to more general theories is straightforward. We study how to compute radiative corrections in an unambiguous manner and how to extract precise information even if the symmetry breaking sector is described by an effective theory. In doing that we clarify the relationship between the linear and non-linear theories. We apply our techniques to the processes $e^+e^- \rightarrow \bar{f}f$ and $e^+e^- \rightarrow W^+W^-$, introducing in the latter case an improved Born approximation with a number of phenomenological constants. We identify those where it will be possible to unambiguously probe the symmetry breaking sector and compute the full set of one-loop corrections to them in different underlying theories.

* On leave from DECM, Universitat de Barcelona, 08028 Barcelona

† On leave from Dept. de Física Teórica, Universidad Autónoma de Madrid, 28049 Madrid

CERN-TH.6186/91

FTUAM-26/91

August 1991

1. Introduction

The Standard Model has managed so far to hide any relevant information on its symmetry breaking sector with absolute perfection. We have really no experimental clue, at present, of what dynamical mechanism gives the $W^\pm$ and Z intermediate bosons a mass. In the minimal Standard Model (by that we mean the one with a doublet of scalar fields) or in extensions of it in the same spirit the answer to the above question is provided by the spontaneous breaking of the global $SU(2)_L \times SU(2)_R$ symmetry in the scalar sector to its diagonal subgroup by the scalar field acquiring a vacuum expectation value. This same mechanism makes quarks and some leptons massive.

Nature may well have decided otherwise. Quarks and leptons might get their mass from some other unknown mechanism, perhaps at very high energies. On the contrary, everything seems to indicate that the origin of the $W^\pm$ and Z masses could be experimentally probed in the coming years. If the mechanism of the minimal Standard Model is correct, there are two alternatives. If the Higgs is a light particle of, say, less than 500 GeV it will be found either at LEP II or in the next generation of pp colliders, when they are commissioned. If the Higgs is heavy, somewhere in the TeV scale, the scalar sector of the Standard Model is strongly interacting and one expects signals of a strongly interacting sector once we reach the TeV region. The third possibility is that there exists no elementary Higgs and the $SU(2)_L \times SU(2)_R$ global symmetry is broken by some dynamical mechanism. If this is the alternative chosen by Nature, the intermediate vector boson must get their mass through dynamical symmetry breaking. Composite models or some variant of technicolor theories would be required to provide the dynamics leading to the symmetry breaking. The typical scale involved must again lie somewhere in the TeV region. The origin of this scale has been discussed many times in the literature, and is very briefly reviewed in the coming section. The outcome of this discussion is often referred to as the "no-lose" corollary. Something interesting must happen before we arrive to the multi-TeV region [1].

Of course, when this "something" happens we want to be able to identify what it is. We have in mind, for instance, that one *must* be able to tell a strongly interacting Higgs from a composite model. Chiral lagrangians are ideally suited for this task since the only physical input they use is the symmetry breaking pattern, which must at least contain $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ [2,3]. In terms of the different coefficients of those phenomenologically-inspired effective theories (but which can be rigorously derived nonetheless) there is a systematic way in which we can pose and answer the question with complete generality. The weak chiral lagrangian, borrowed from pion physics, is organized as an expansion in terms of derivatives (hence good at energies which are low compared with the relevant multi-TeV scale, as we will see) and external fields. It involves only the

physical degrees of freedom; in our case the longitudinal components of the $W^\pm$ and Z fields [4,5]. We will see that different possibilities, such as a strongly interacting Higgs or a technicolor model would simply lead to different values for the parameters appearing in the chiral lagrangian just as it is known that, in pion physics, scalar or vector interchanges lead to very different values for the higher dimensional coefficients in the chiral lagrangian [3,6].

To be able to set reliable constraints on any new physics it is necessary that we have a way of estimating the effect of any unseen particles in the relevant observables that is as reliable as doing radiative electroweak corrections in the minimal Standard Model. We have to learn how to make precise and unambiguous predictions, even if our theory is a non-linear one (as unavoidably it will be), plagued with new divergences. The techniques learned from chiral perturbation theory come in handy here to save the day. We will discuss this issue in section 2.

With the amazing level of accuracy that will be attained in e^+e^- colliders, singularly at LEP, it seems appropriate to investigate at what level the differences between the various symmetry breaking alternatives would set in and whether this is within experimental reach in the near future. For instance, for the parameter Δr , the present accuracy is set to the level of just 1.7×10^{-2} and is essentially due to the uncertainty in the measurement of M_W , $M_W/M_Z = 0.8807 \pm 0.0031$ [7]. The future measurement of M_W at LEP II and CDF will no doubt reduce considerably the above uncertainty, leading to an accuracy in $\delta(\Delta r)$ at the 10^{-3} level [8]. As for the effective Weinberg angle, which currently is $\sin^2 \theta_W = 0.2317 \pm 0.0030$ [9], the error could be reduced to $\delta \sin^2 \theta_W = 4 \times 10^{-4}$ or less with polarized beams [10]. We review how different symmetry breaking sectors contribute to the relevant LEP I parameters in section 3.

The commissioning of LEP II will not only allow a more precise measurement of the W mass, but also probe the triple gauge-boson vertex and possible departures from its value in the minimal Standard Model due to the symmetry breaking sector. For a discussion on the experimental possibilities of the process $e^+e^- \rightarrow W^+W^-$ we refer the reader to section 4. To extract information from this process and compare it with the predictions of the effective theory we have been forced to introduce an improved Born approximation that contains a number of phenomenological constants. These allow us to make also contact with the very abundant existing literature on effective parametrizations of the triple-boson-vertex. However, to our knowledge, no complete calculation containing the full radiative corrections in the Standard Model exists for these phenomenological constants, let alone for modifications of the minimal Standard Model involving an effective lagrangian. We have been able to obtain results for all the effective vertices (to be used at tree level) from

the set of radiative corrections to the amplitudes that exists in form of computer code, complemented with analytic calculations when necessary. This is discussed in section 5. Our results for the $e^+e^- \rightarrow W^+W^-$ process are presented in section 6.

The information that can be obtained from present and planned e^+e^- machines complements, in a very elegant manner, that which will be learned from the scattering of longitudinal W and Z bosons. This is believed to be the most efficient way of probing a strongly interacting symmetry breaking sector in pp machines [11,12]. We will briefly comment on this in section 2. It is worth mentioning that another interesting source of constraints on the parameters of the effective chiral lagrangian is provided by atomic parity violation experiments. We will briefly review their relevance in section 7 and we will also make contact with alternative and complementary approaches there.

2. Chiral Lagrangians and the Symmetry Breaking Sector

The Standard Model with the $U(1)_Y$ coupling g' set equal to zero has a global $SU(2)_L \times SU(2)_R$ invariance in the unbroken phase. Once the electroweak symmetry breaking takes place the global group is broken to its diagonal subgroup $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$, the so-called custodial symmetry, responsible for keeping the ρ parameter equal to one at tree level (exactly equal to one if one does not switch g' on). The Goldstone theorem implies the existence in the broken phase of 3 massless bosons w^a , which will eventually become the longitudinal degrees of freedom of the $W^\pm$ and Z . If we restrict ourselves to the scalar sector, the interactions amongst the goldstones can be described, entirely on symmetry grounds, by a lagrangian of the form

$$\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \dots \quad (2.1)$$

The first term of this infinite expansion with an increasing number of derivatives of the fields is given by

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \text{tr} \partial_\mu U \partial^\mu U^\dagger \quad (2.2)$$

where v is a dimensionful constant to be determined, and the U field is a unitary matrix belonging to the quotient space $SU(2)_L \times SU(2)_R / SU(2)_V$ that can be parametrized as $U = \exp(i\sigma^a w^a/v)$ where the three w^a fields are the would be longitudinal components of the intermediate bosons. We will consider only effective theories for which the custodial symmetry $SU(2)_V$ is exactly preserved in the symmetry breaking sector.

Operators are classified according to their number of derivatives and/or external fields. At low energies only those with the smallest number of derivatives will be relevant. With the sole inclusion of the scalar fields, and assuming that the custodial symmetry is exactly

preserved, there is only one term of $\mathcal{O}(p^2)$ (the one given in eq. (2.2)), and two terms at $\mathcal{O}(p^4)$ that involve two new constants [4]

$$\mathcal{L}^{(4)} = L_1 \text{tr} \partial_\mu U \partial^\mu U^\dagger \text{tr} \partial_\nu U \partial^\nu U^\dagger + L_2 \text{tr} \partial_\mu U \partial_\nu U \partial^\mu U^\dagger \partial^\nu U^\dagger \quad (2.3)$$

The lagrangian (2.1) has exactly the same form as the well-known non-linear sigma model of pion physics [3]. Its form is equally dictated by symmetry arguments alone and is, therefore, completely general. The information about the underlying physics is contained in a number of parameters (such as, in principle, v or the coefficients of the $\mathcal{O}(p^4)$ terms L_i). If we keep terms only up to $\mathcal{O}(p^4)$ (and we will see in a moment that this is fully justified for our purposes) we have a surprisingly economical description of the symmetry breaking sector.

Although eqs. (2.2) and (2.3) are formally quite similar to the type of effective lagrangians that have been widely used in pion physics, the energy scale is very different. In strong interactions at very low energies a lagrangian of the type (2.2) governs the interactions between pions with v replaced by $f_\pi \simeq 93\text{MeV}$. In the electroweak theory at sufficiently low energies the lagrangian $\mathcal{L}^{(2)}$ is also the only relevant piece of the effective lagrangian (2.1), and, given that there is only one dimensionful scale, the Fermi constant G_F , this one must therefore set the order of magnitude of v . In fact, $v = (G_F \sqrt{2})^{-1/2} = 246 \text{ GeV}$ [1].

In the minimal Standard Model we can use the classical equations of motion to "integrate out" the physical Higgs particle. We will obtain that at long distances the interactions among the longitudinal degrees of freedom are indeed described by $\mathcal{L}^{(2)}$ with a value for v that is simply the vacuum expectation value of the scalar field, i.e. $v = 246 \text{ GeV}$. (The use of the classical equations of motion is good enough if one intends to use $\mathcal{L}^{(2)}$ only at tree level, which, in turn, is all one needs to determine v for the present discussion.) At extremely low energies that is the end of the story; higher dimensional terms such as $\mathcal{L}^{(4)}$ are suppressed by powers of the energy over some scale (M_H or v itself in the minimal Standard Model) and can be ignored. Since the form of $\mathcal{L}^{(2)}$ is quite universal and based on symmetry arguments *any* effective theory that reproduces the long-distance properties of the Standard Model, with or without elementary scalars, strongly interacting or not, must necessarily yield the same value for v . This fixes uniquely the lowest dimensional operator in the series (2.1).

In the minimal Standard Model the chiral lagrangian (2.1) provides us with a simple book-keeping device to expand observables in powers of p^2/M_H^2 . If the scalar sector turns out to be strongly interacting due to a heavy Higgs (recall that the scalar quartic coupling

$\lambda \sim M_H^2/M_W^2$, all higher orders beyond $\mathcal{O}(p^4)$ which are, by power counting, suppressed by powers of p^2/M_H^2 can be ignored. On the other hand, if nature has ruled that there is no elementary Higgs, and the Goldstone bosons are the result of some strong interaction in the scalar sector that breaks the global $SU(2)_L \times SU(2)_R$ symmetry to its diagonal subgroup, we expect, by analogy with pion physics, that the scale that normalizes the contribution of higher dimensional operators is, roughly speaking, $4\pi v$ [2] (see however the last reference in [13]). The expansion (2.1) is therefore an expansion in powers of $p^2/16\pi^2 v^2$. Since $4\pi v = 3.1 \text{ TeV}$, this expansion is a very good one and neglect of the terms beyond $\mathcal{L}^{(4)}$ is again fully justified. We need to keep, of course, the operators of $\mathcal{O}(p^4)$, because, having seen that v is universal, in practice all measurable information on the underlying theory is contained in their coefficients. In what follows we will consistently neglect all terms of $\mathcal{O}(p^6)$ and higher.

In addition to the global $SU(2)_L \times SU(2)_R$ symmetry, the Standard Model has a local $SU(2)_L \times U(1)_Y$ symmetry, which is spontaneously broken to $U(1)_{em}$. This local symmetry can be easily implemented in (2.1) by introducing the covariant derivative $D_\mu U$ defined as

$$D_\mu U = \partial_\mu U + U L_\mu - R_\mu U \quad D_\mu U^\dagger = \partial_\mu U^\dagger + U^\dagger R_\mu - L_\mu U^\dagger \quad (2.4)$$

Here, L_μ and R_μ are the left and right-handed gauge fields $L_\mu = ig\sigma^a W_\mu^a/2$ and $R_\mu = ig'\sigma^3 B_\mu/2$. The complete effective lagrangian that complies with all the symmetries of the Standard Model is thus, terms of $\mathcal{O}(p^4)$ included,

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \text{tr} D_\mu U D^\mu U^\dagger \quad (2.5)$$

$$\begin{aligned} \mathcal{L}^{(4)} = & L_1 \text{tr} D_\mu U D^\mu U^\dagger \text{tr} D_\nu U D^\nu U^\dagger + L_2 \text{tr} D_\mu U D_\nu U^\dagger \text{tr} D^\mu U D^\nu U^\dagger \\ & - i L_3 \text{tr} (F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U D_\nu U^\dagger) \\ & + L_4 \text{tr} U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} + H_1 (F_R^{\mu\nu} F_{R\mu\nu} + F_L^{\mu\nu} F_{L\mu\nu}) \end{aligned} \quad (2.6)$$

with $F_{L,R}^{\mu\nu}$ being the field strengths associated to the left and right gauge fields

$$i F_{\mu\nu}^L = \partial_\mu L_\nu - \partial_\nu L_\mu - [L_\mu, L_\nu] \quad i F_{\mu\nu}^R = \partial_\mu R_\nu - \partial_\nu R_\mu \quad (2.7)$$

Again, the lagrangian in eqs. (2.5) and (2.6) has the same form as the chiral lagrangian for pion physics [3]. For an easier comparison we have used the same normalization for the L_i parameters as in the second reference in [3]. Comparing with (2.3) we see that two new constants have appeared when the gauge fields are introduced. These new constants, L_3 and L_{10} , are, in fact, relevant constants for processes involving intermediate vector

bosons. As we will review in section 3, L_{10} enters in the oblique corrections to $e^+e^- \rightarrow f\bar{f}$ physics and has a number of interesting properties that have been studied with some detail recently. L_{10} along with L_9 also contributes to the γWW and ZWW effective vertices. We will see that in detail in sections 4, 5 and 6. Due to the level of precision of e^+e^- machines, L_9 and L_{10} offer the most promising chances of unveiling the symmetry breaking sector of the Standard Model (short of finding the Higgs, of course). L_1 and L_2 are, by virtue of the Equivalence Theorem [11], relevant in scattering of longitudinally polarized W 's and Z 's and have been amply discussed in the literature before in connection to pp machines [4,12]. The lagrangian (2.6) also contains a term —the one proportional to H_1 — that was already present in the gauge sector of the theory. It can be thought of as the contribution of the symmetry breaking sector to the coupling constant renormalization, although this remark requires further elaboration if one uses a scheme in which g itself is not a physical input. At any rate we will see that H_1 does not appear in any observable quantity. We thus see that the combined information from e^+e^- and pp machines is able to constrain all relevant coefficients. In fact, as far as probing departures from the minimal Standard Model in the scalar sector goes, these coefficients will be the *only* testable quantities in the foreseeable future; $\mathcal{O}(p^6)$ operators being very small at the planned energies.

Terms which vanish by the equations of motion can be neglected at this order. Notice that the gauge fields always appear through covariant derivatives or field strengths. This reflects the requirement that the unknown symmetry breaking sector, whose dynamics determines the $\mathcal{O}(p^4)$ coefficients, is $SU(2)_L \times U(1)_Y$ gauge invariant.

Choosing the unitary gauge, $U = 1$, $\mathcal{L}^{(2)}$ transforms into

$$\mathcal{L}^{(2)} = \frac{v^2}{4} g^2 W_\mu^+ W_\mu^- + \frac{v^2}{8} g^2 W_\mu^3 W_\mu^3 + \frac{v^2}{8} g'^2 B_\mu B_\mu - \frac{v^2}{4} g g' W_\mu^3 B_\mu \quad (2.8)$$

or equivalently

$$\mathcal{L}^{(2)} = \frac{v^2}{4} g^2 W_\mu^+ W_\mu^- + \frac{v^2 g^2}{8 \cos^2 \theta_W} Z_\mu Z_\mu \quad (2.9)$$

Here $\sin^2 \theta_W$ is defined as

$$\sin^2 \theta_W \equiv 1 - \frac{M_W^2}{M_Z^2} \quad (2.10)$$

and g and g' are defined at any order as $g = e/\sin \theta_W$ and $g' = e/\cos \theta_W$. From now on we use the notation $s = \sin \theta_W$ and $c = \cos \theta_W$.

This makes apparent that eq. (2.5) reduces to the mass term of the SM in the unitary gauge. It is also another way of showing that (2.5) is indeed universal. Any underlying theory whose long distance behaviour is that of the Standard Model admits a derivative

expansion (2.1) whose first term (of $\mathcal{O}(p^2)$) is unambiguously fixed. The terms of $\mathcal{O}(p^4)$ are the ones that contain all the relevant, and so far unknown, information about the nature of the symmetry breaking sector in the electroweak theory.

Let us begin by studying the choice of $\mathcal{O}(p^4)$ constants that corresponds to the minimal Standard Model with a heavy Higgs. For simplicity let us ignore fermion and ghost fields, which are irrelevant for the present discussion, and we retain the scalar (w, σ) and vector (A_μ) fields. σ is the Higgs field, while w are the would-be Goldstone bosons. We understand that all the functional integrals in the following discussion are properly regularized and, therefore, finite. In the minimal Standard Model the expectation value of any observable is a finite number which, among other things, will depend on the physical Higgs mass M_H .

We introduce an effective action obtained after integrating out the Higgs field σ

$$\exp i\Gamma(A, w) = \int d\sigma \exp iS_M(A, w, \sigma) \quad (2.11)$$

After performing the functional integrals over the remaining fields A_μ and w , Γ will reproduce the Green functions of the Standard Model. This can ultimately be traced back to the fact that, all integrals being properly regularized and the final answer being finite, the result must be independent of the order of the functional integrals are carried out. In diagrammatic terms, Γ consist of the sum of all the graphs of the theory that contain only σ as internal particle, as is, for instance, represented in fig. 1 for the 4-point function. The resulting effective action Γ can be expanded in inverse powers of M_H (See Appendix A for details.) On symmetry grounds

$$\Gamma(A, w) = \int d^4x \mathcal{L}(A, w) \quad (2.12)$$

with an effective lagrangian $\mathcal{L}$ of the form given by (2.1). The value of the coefficients L_i that one obtains after this calculation are at one loop (in the field σ)

$$L_1^{MSM} = \frac{v^2}{8M_H^2} + \frac{1}{16\pi^2} \left(\frac{9\pi}{16\sqrt{3}} - \frac{47}{48} \right) \quad (2.13)$$

$$L_2^{MSM} = 0$$

$$L_9^{MSM} = L_{10}^{MSM} = 0$$

While L_1^{MSM} gets corrections from self-interactions of the Higgs field, which are large when $M_H \rightarrow \infty$, the fact that $L_2^{MSM} = L_9^{MSM} = L_{10}^{MSM} = 0$ remains true at any order of perturbation theory in the Higgs quartic coupling λ . This is a unique feature of the minimal Standard Model (for the case of L_{10} it was first observed in the first reference in

[5]). This, combined with the amazing level of precision that is being reached (and that will be reached in the future) in e^+e^- machines makes observables that depend on L_9 and L_{10} ideally suited to search for possible departures from the minimal Standard Model. We would like to stress that these L_i 's are not immediately comparable with the ones of, e.g., Gasser and Leutwyler[3], which absorb part of the Goldstone boson loop contribution.

In order to compute the value of an observable in the minimal Standard Model we have now to use the lagrangian $\mathcal{L}$ with the values for the L_i just quoted. We must also include, of course, the pure gauge piece

$$-\frac{1}{2g^2} \text{tr}(F_{L\mu\nu} F_L^{\mu\nu}) - \frac{1}{2g'^2} \text{tr}(F_{R\mu\nu} F_R^{\mu\nu}) \quad (2.14)$$

the ghost piece, and the gauge fixing piece, which we take to be the one corresponding to the 't Hooft-Feynman gauge in which we chose to work. None of these pieces have been modified by removing the Higgs particle. (The fermion part of the lagrangian is modified, but this plays no important role, at leading order, neither in pp nor on e^+e^- machines, at least for light quarks.) If we include the terms of order $\mathcal{O}(p^4)$, we also have to include, according to the chiral counting rules, chiral loops, i.e. the loops of Goldstone bosons that are generated by $\mathcal{L}^{(2)}$ at the one loop level. These are dominant at high energies, but, given that we are interested in making precise predictions at LEP energies, the contribution from loops of transverse W and Z modes and from the fermionic loops can not be ignored at all. Mixed loops of scalars and gauge particles will also be generated at the one loop level. The Higgs field is, of course, not explicitly present since its effects have been *exactly* replaced by the effective lagrangian $\mathcal{L}$. This poses at once the problem of ultraviolet divergences, which are much more severe in the non-linear effective theory than in the (linear) minimal Standard Model. As is known in chiral perturbation theory, these divergences are removed by adding suitable counterterms to the original $\mathcal{L}^{(4)}$ lagrangian. However, we have to come to grips with the problem of what the L_i 's precisely mean and of what is predictable and what is not when one considers modifications of the Standard Model.

At this point if we were able to compute the contribution of each one of the infinite terms in the series $\mathcal{L}^{(2)} + \mathcal{L}^{(4)} + \dots$, to our Feynman rules, plug them in the loops, and sum them all, we would clearly recover the same result as in the Standard Model; we would just have done things in a somewhat bizarre order. This is not, of course, the point of the above exercise, among other reasons because the loop expansion is not a good one in the scalar sector if the Higgs mass is very large.

What we want to do is to cut the expansion of the effective lagrangian retaining only up to the first terms that are non-universal and estimate these non-universal terms (the

L_i 's the best we can in different underlying theories. In the case of the minimal Standard Model, keeping terms up to $\mathcal{O}(p^4)$ only in the effective expansion (2.1) is tantamount to neglecting contributions of $\mathcal{O}(1/M_H^2)$. The effective lagrangian treatment is thus really useful in the minimal case only if M_H is large enough (provided that one is able to have a good theoretical control on the coefficients L_i). In practice, a Higgs of 500 GeV is already heavy enough to justify neglect of higher orders for LEP I physics, for instance.

We have discussed with some detail how to derive the effective lagrangian (2.2) and (2.3) from the minimal Standard Model in the limit of a large Higgs mass. We would now like to turn to alternative theories, such as technicolor. The basic idea is the same. One integrates out the heavy fields that trigger the dynamical breakdown of the $SU(2)_L \times SU(2)_R$ symmetry and, in this way, obtains finite values for the $\mathcal{O}(p^4)$ coefficients. For theories with a QCD-like $SU(N)_{TC}$ symmetry breaking sector, the value of these constants can be approximately derived by a combination of analogies with strong interaction and large N_{TC} calculations. If we simply take the leading (in α_s) calculation of L_1, L_2, L_9 and L_{10} given in [3,13], we have

$$\begin{aligned} L_1 &= \frac{N_{TC}}{384\pi^2} & L_2 &= \frac{N_{TC}}{192\pi^2} \\ L_9 &= \frac{N_{TC}}{48\pi^2} & L_{10} &= -\frac{N_{TC}}{96\pi^2} \end{aligned} \quad (2.15)$$

This is close enough to the values obtained by rescaling the analogous quantities from low energy data using large N_c arguments for our purposes. In the above expression N_{TC} is the number of technicolors. We assume that there is only one doublet under $SU(2)_L$. If not the above formulae should be modified accordingly. As we see, L_9 and L_{10} are different from zero, in contrast to the minimal Standard Model. In fact, for any vector-like theory we expect non-zero values for L_9 and L_{10} . The latter is related to the spectral function $\Pi_{LR}(0) = \Pi_{VA}(0) - \Pi_{AA}(0)$ that is positive definite in theories with matter in vector-like representations (and, in fact, proportional to the number of $SU(2)_L$ representations; for N_d doublets $L_{10} \propto N_d$). In this type of theories L_{10} has therefore a definite (negative) sign and basically counts the number of heavy particles. In vector-like theories with an enlarged content one expects a variety of pseudo-Goldstone bosons to appear. Typically these theories have a symmetry breaking pattern $G_L \times G_R \rightarrow G_{L+R}$, where G must at least contain $SU(2)$. The pseudo-Goldstone bosons will give an additional contribution to L_{10}

$$\Delta L_{10} = -\frac{1}{16\pi^2} \frac{1}{36} \sum I(I+1)(2I+1) \log \frac{\Lambda}{M_I} + \dots \quad (2.16)$$

where I is the weak isospin of the multiplets of pseudo-Goldstone bosons, and M_I their mass. This type of contributions have been considered in [14]. The dots refer to constant,

non-universal pieces. Note that (2.16) is subleading in $1/N_{TC}$ with respect to (2.15). The order of magnitude of all these coefficients dictates at what order departures from the minimal Standard Model can be expected. On the whole, the contributions from $\mathcal{O}(p^4)$ at moderate energies are comparable to "normal" electroweak radiative corrections in the Standard Model. Therefore it is necessary to include consistently one loop corrections, as expected from the chiral counting rules. To these loop corrections in the non-linear sigma model approach to the Standard Model and its associated divergences we now turn.

Because the expansion in (2.1) is one in derivatives, keeping only a finite number of terms in the effective action $\mathcal{L}$ is bound to change the *high energy* behaviour of the effective theory. This is reflected in the appearance of new ultraviolet divergences in perturbation theory in addition to those that have been subtracted by requiring mass and charge renormalization. They are the usual divergences of a non-linear theory, well known to the practitioners of pion scattering. To make sense of the model one is thus forced to introduce additional counterterms.

Some of the divergences have the same structure that the original $\mathcal{O}(p^4)$ lagrangian and can be eliminated by way of redefining the L_i 's that appear in $\mathcal{L}^{(4)}$, i.e. $L_i \rightarrow L_i + \delta L_i$. These counterterms have been extensively studied in chiral perturbation theory and are well known [3]. In the MS scheme

$$\begin{aligned} \delta L_1 &= -\frac{1}{16\pi^2} \frac{1}{12\epsilon} & \delta L_2 &= -\frac{1}{16\pi^2} \frac{1}{6\epsilon} \\ \delta L_9 &= -\frac{1}{16\pi^2} \frac{1}{6\epsilon} & \delta L_{10} &= -\frac{1}{16\pi^2} \frac{1}{6\epsilon} \end{aligned} \quad (2.17)$$

Some combinations are finite, for instance $L_1 - \frac{1}{2}L_2$ and $L_9 + L_{10}$. Since $\mathcal{L}^{(2)}$ is universal, the chiral infinities are universal too (at the one loop level) and hence quite independent of the underlying theory. This is an important point. The counterterms that have to be added to make sense of the theory have the same value in any theory with the same symmetry breaking pattern as the one of the Standard Model. For instance they are the same in a composite model, provided the number of Goldstone bosons is the same. Likewise the combinations that are renormalization group invariant will be the same regardless of the precise dynamics giving the $W^\pm$ and Z a mass.

Since the $U(1)_Y$ gauge interactions explicitly break the global $SU(2)_L \times SU(2)_R$ symmetry of the chiral lagrangian (much like electromagnetism breaks $SU(3)$ in pion physics), there are also other $SU(2)_L \times U(1)_Y$ gauge invariant – but not $SU(2)_L \times SU(2)_R$ chiral invariant – counterterms that have to be included when $U(1)_Y$ gauge loops are taken into account. The origin of these counterterms can be traced back to mixed loops, in which

gauge particles and Goldstone bosons are interchanged in the same diagram. There is only one operator of dimension two of this type

$$\frac{v^2}{4}\delta\beta(trU\sigma^3U^\dagger D_\mu U U^\dagger)^2 \quad (2.18)$$

and several of dimension four [15]. It turns out that only the operator (2.18) is required to eliminate infinities. Note that from the underlying physics we never generate such a term. This counterterm has been computed by Longhitano (see the first reference in [15]) some time ago.

$$\delta\beta = \frac{g^2}{16\pi^2} \frac{s^2}{c^2} \frac{3}{4\epsilon} \quad (2.19)$$

Note that the g^2 in $\delta\beta$ betrays its gauge origin, and that it vanishes when $s \rightarrow 0$. With the subtractions implied by eqs. (2.17) and (2.19) all observables are made finite, although, in general, dependent on some unknown scale and on the renormalization prescription. Clearly if one wishes to do *precise* statements these uncertainties have to be removed.

Even if we cannot give a precise value to the cut-off, we know for sure that all these counterterms are universal at the order we are working (up to $\mathcal{O}(p^4)$) and hence completely independent of the details of the symmetry breaking sector. All the interesting information is hidden in the values of the L_i 's of the effective action (2.12), which only enter at tree level. Therefore our aim will be to identify observables which are sensitive to the *finite* parts of the L_i 's and not to the chiral infinities themselves, and compare them to their minimal Standard Model values. In short, we have to look for quantities which are renormalization group invariants with respect to divergences in the non-linear sigma model.

It is well known that the linear sigma model (which in this context is just the usual Standard Model) can be regarded from a field-theoretical point of view as a regulator of the non-linear sigma model [15]. This means that to identify observables which are free from chiral infinities we have simply to look for observables that are finite in the large M_H limit. Since by virtue of the screening theorem [16], the divergences are at worst logarithmic at the one-loop level, we have to identify observables that do not contain $\log M_H$ in the Standard Model. It is somewhat paradoxical that to investigate the symmetry breaking sector of the Standard Model in an unambiguous way one has to look for observables which are (in the large M_H limit) independent of the Higgs mass, but the reason is clear. For those observables that cancel the divergent logs in the large M_H limit, we can take the limit of a heavy Higgs mass safely. The limit is a well defined, finite number. Now, in the minimal Standard Model we can compute the above observable in two different ways: 1) working consistently at the one loop level with a propagating physical Higgs and then expand in inverse powers of M_H^2 and retain only the constant piece. 2) work with the

non-linear sigma model (the effective lagrangian $\mathcal{L}$), including the appropriate values of the L_i^{MSM} . What certainly is not correct is to work with the lowest dimensional operator of the non-linear sigma model $\mathcal{L}^{(2)}$ and ignore the terms of $\mathcal{O}(p^4)$. For observables that in the minimal Standard Model have no $\log M_H$, the divergences of the non-linear theory, that appear when perturbing $\mathcal{L}^{(2)}$ at the one loop level, cancel and we are left with a finite piece from the loops plus the contribution of the finite parts of the L_i 's. Since at this order $\mathcal{L}^{(4)}$ enters at tree level, it is quite easy now to determine the same observable in a different theory; we just have to replace L_i^{MSM} by the appropriate L_i 's. The recipe is the following

$$A^X = A^{MSM} - A^{(4)}(L_i^{MSM}) + A^{(4)}(L_i^X) \quad (2.20)$$

where A is some observable that in the MSM is *finite* when we take $M_H \rightarrow \infty$ and $A^{(4)}$ is the contribution from $\mathcal{L}^{(4)}$. The difference between the value of the observable A in the model X and in the minimal Standard Model with a large Higgs mass can be worked out by a tree level calculation only. This is quite remarkable.

The above result is particularly useful because many observables have been computed in the minimal Standard Model already. Of course it would be equally correct (always for observables which are finite in the non-linear theory, i.e. finite when $M_H \rightarrow \infty$) to compute A^X directly in the non-linear theory, according to the recipe

$$A^X = A^{(2)} + A^{(4)}(L_i^X) \quad (2.21)$$

where $A^{(2)}$ is the contribution from the effective lagrangian $\mathcal{L}^{(2)}$ (plus gauge, ghost and matter part eventually) and $A^{(4)}$ is, as above, the contribution from $\mathcal{L}^{(4)}$. As we will see in the next section in a particular case both methods give the same results.

In the rest of this section we want to illustrate the preceding discussion by applying the chiral effective lagrangian to the analysis of the scattering of longitudinal $W^\pm$ and Z . A lot of work has been done in this field [11,12], so we will be very succinct.

At very high energies the Equivalence Theorem allows us to obtain the various amplitudes for scattering of longitudinally polarized W 's and Z 's from the amplitudes for scattering of Goldstone bosons. We have

$$\begin{aligned} T(W_L^+ W_L^- \rightarrow Z_L Z_L) &= A(s, t, u) \\ T(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) &= A(s, t, u) + A(t, s, u) \\ T(Z_L Z_L \rightarrow Z_L Z_L) &= A(s, t, u) + A(t, s, u) + A(u, t, s) \end{aligned} \quad (2.22)$$

The remaining amplitudes can be obtained by crossing from the above ones. Everything can be expressed in terms of the function $A(s, t, u)$, which can be calculated in the framework of the chiral lagrangian $\mathcal{L}$. Up to order p^4 one obtains [3,4,11,12]

$$A(s, t, u) = \frac{s}{v^2} + \frac{1}{v^4}(3s^2\beta_1 + t(t-u)\beta_2 + u(u-t)\beta_2) - \frac{1}{96\pi^2 v^4}(3s^2 \log -s + t(t-u) \log -t + u(u-t) \log -u) \quad (2.23)$$

The Mandelstam variables appearing in the logarithms in (2.23) are normalized by an appropriate logarithmic dependence on the renormalization scale ν contained in β_1, β_2 . These two constants contain several contributions. First of all they contain a finite piece from the one loop calculation in the chiral lagrangian (obtained from $\mathcal{L}^{(2)}$ at one loop). These finite parts of β_1 and β_2 are regularization dependent. β_1 and β_2 will also depend on the values of L_1 and L_2 as defined in this paper through the contribution from $\mathcal{L}^{(4)}$ at tree level. β_1 and β_2 will, finally, contain, as we just said, some $\log \nu$ pieces arising from the ultraviolet divergences at one loop which are unambiguously determined by chiral perturbation theory. The divergences themselves are, as usual, eliminated by a redefinition of L_1 and L_2 (see eq. (2.17)). As is well known, the logarithmic dependence can be included into "running" $L_1(\nu)$ and $L_2(\nu)$, as is customarily done in pion physics. However, here we prefer to keep them separate for the simple reason that the chiral logs contain no information on the underlying theory and this information is hidden into L_1 and L_2 which must be expressible in terms of the fundamental quantities of the theory (such as, for instance, M_H).

Since they depend on some unknown scale (and on a renormalization prescription), β_1 and β_2 are not separately predictable. There is, however, one combination $\beta_1 - \beta_2$ in which the $\log \nu$'s do not appear. It is

$$\beta_1 - \beta_2 = \frac{8}{3}(L_1 - \frac{1}{2}L_2) - \frac{1}{16\pi^2} \frac{1}{18} \quad (2.24)$$

This particular combination appears directly in the $J = 1, I = 1$ channel. Substituting the values (2.13) for L_1 and L_2 one has

$$\beta_1 - \beta_2 = \frac{v^2}{3M_H^2} + \frac{1}{16\pi^2} \left(\frac{\pi\sqrt{3}}{2} - \frac{8}{3} \right) \quad (2.25)$$

which is the value in the minimal Standard Model. The value of $\beta_1 - \beta_2$ in other theories would be obtained by using the appropriate L_i 's.

3. Application to $e^+e^- \rightarrow \bar{f}f$

When computing radiative corrections in the Standard Model one must, first of all, select a renormalization scheme and a set of parameters to define the model. Here we will use the on-shell renormalization scheme and take as input parameters to make contact with the real world the fine structure constant, α , M_Z and M_W and the rest of the particle masses. An alternative to M_W could be the Fermi constant G_F , which is extremely well measured. In the minimal Standard Model we can pass easily from one to the other. Here we prefer to think of G_F as giving a further restriction on the radiative corrections and on the relevant $\mathcal{O}(p^4)$ coefficients, bearers of the underlying physics in the symmetry breaking sector.

All the effects in the process $e^+e^- \rightarrow \bar{f}f$ that are sensitive to the symmetry breaking sector can be summarized into three basic process-independent quantities. The first one is precisely the relation between the Fermi constant G_F (obtained from muon decay) and the W mass.

$$G_F = \frac{\pi\alpha}{\sqrt{2}M_W^2 \sin^2 \theta_W} \frac{1}{1 - \Delta r} \quad (3.1)$$

In terms of unrenormalized Green functions [17,18],

$$\Delta r = \frac{\Sigma_{WW}(0) - \Sigma_{WW}(M_W^2)}{M_W^2} + \Sigma'_{\gamma\gamma}(0) + \frac{c^2}{s^2} \left(2 \frac{s}{c} \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} - \frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Sigma_{WW}(M_W^2)}{M_W^2} \right) + \text{vertex corrections} + \text{box corrections} \quad (3.2)$$

For further use we introduce the quantity $\Delta r(q^2)$ defined as

$$\Delta r(q^2) = - \frac{\Sigma_{WW}(q^2) - \Sigma_{WW}(M_W^2)}{q^2 - M_W^2} + \Sigma'_{\gamma\gamma}(0) + \frac{c^2}{s^2} \left(2 \frac{s}{c} \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} - \frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Sigma_{WW}(M_W^2)}{M_W^2} \right) \quad (3.3)$$

If we neglect the renormalized vertex and box contribution that appear in Δr , we have $\Delta r = \Delta r(0)$. We will see that $\Delta r(M_W^2)$ will play an important role when considering radiative corrections to $e^+e^- \rightarrow W^+W^-$.

Another important quantity is ρ , the ratio between neutral and charged currents at $q^2 = 0$:

$$\rho = 1 + \Delta\rho = \frac{G_{NC}(0)}{G_{CC}(0)} \quad (3.4)$$

where

$$\Delta\rho = \frac{\Sigma_{ZZ}(0)}{M_Z^2} - \frac{\Sigma_{WW}(0)}{M_W^2} - 2\frac{s}{c}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2} \quad (3.5)$$

Finally, we have the ratio between J_m and J^3 in the one-loop corrected Born approximation. This is the effective Weinberg angle

$$\tilde{s}^2(q^2) = \kappa(q^2)s^2 = s^2(1 + \Delta\kappa(q^2)) \quad (3.6)$$

with

$$\begin{aligned} \Delta\kappa(q^2) = & -\frac{c}{sq^2}(\Sigma_{\gamma Z}(q^2) - \Sigma_{\gamma Z}(0)) \\ & -\frac{c^2}{s^2}(2\frac{s}{c}\frac{\Sigma_{\gamma Z}(0)}{M_Z^2} - \frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} + \frac{\Sigma_{WW}(M_W^2)}{M_W^2}) \end{aligned} \quad (3.7)$$

Like for $\Delta r(q^2)$, this equation includes only the self-energy ("oblique") corrections.

The effective Weinberg angle $\tilde{s}^2$ is extremely well measured at LEP, and its precision could improve substantially with improved luminosity or polarization (or both). A precise measurement of M_W as it will, hopefully, be possible in LEP II will not only severely restrict Δr to the point where the symmetry breaking sector contributions will be tested, but also, by using (3.6) constrain $\Delta\kappa(M_Z^2)$. We will see that $\Delta\kappa(q^2)$ plays a crucial role in resumming some of the radiative corrections in $e^+e^- \rightarrow W^+W^-$.

The "oblique" corrections to Δr , $\Delta\rho$ and $\Delta\kappa$ can be separated into a fermionic and a bosonic part. For instance we write $\Delta\rho = \Delta\rho^{(f)} + \Delta\rho^{(b)}$. In turn in $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$ we distinguish between the part coming from $\mathcal{L}^{(2)}$ (plus the gauge piece) at one loop, and the part coming from $\mathcal{L}^{(4)}$ that we have to use only at tree level.

$$\begin{aligned} \Delta\rho^{(b)} &= \Delta\rho_{(2)}^{(b)} + \Delta\rho_{(4)}^{(b)} \\ \Delta\kappa^{(b)} &= \Delta\kappa_{(2)}^{(b)} + \Delta\kappa_{(4)}^{(b)} \\ \Delta r^{(b)} &= \Delta r_{(2)}^{(b)} + \Delta r_{(4)}^{(b)} \end{aligned} \quad (3.8)$$

The following results are obtained for the universal parts $\Delta\rho_{(2)}^{(b)}$, $\Delta r_{(2)}^{(b)}$ and $\Delta\kappa_{(2)}^{(b)}$ by computing the unrenormalized self-energies Σ_{ZZ} , Σ_{WW} , $\Sigma_{\gamma\gamma}$ and $\Sigma_{\gamma Z}$ [5]

$$\begin{aligned} \Delta\rho_{(2)}^{(b)} &= 2\delta\beta + \left(\frac{g^2}{16\pi^2}\right)\left(-\frac{3}{4}\frac{s^2}{c^2}N_c - \frac{5}{8}\frac{s^2}{c^2}\right. \\ &\quad \left. + \frac{1}{c^2}\log c^2\left(\frac{17}{4s^2} - 7 + 2s^2\right) + \frac{17}{4} + \frac{3}{4}\tan^2\theta_W \log\left(\frac{M_W^2}{\mu^2}\right)\right) \end{aligned} \quad (3.9)$$

$$\begin{aligned} \Delta\kappa_{(2)}^{(b)}(q^2) &= 2\frac{c^2}{s^2}\delta\beta + \left(\frac{g^2}{16\pi^2}\right)\left(-\frac{5}{6}N_c + \frac{1}{18} + \log\left(\frac{M_W^2}{\mu^2}\right)(7c^2 + \frac{1}{6} - \frac{5c^2}{6s^2})\right. \\ &\quad \left. + \log\left(\frac{M_Z^2}{\mu^2}\right)\left(-7c^2 - \frac{1}{6} + \frac{5}{6s^2}\right) + \frac{1}{18}(157 - 154s^2)\right) \\ &\quad - \frac{c^2}{s^2}\log c^2\left(\frac{59}{6} - 7c^2 - \frac{2}{3c^2}\right) - \frac{c^2}{s^2}(I_1(c^2) - I_2(c^2)) \\ &\quad - (3c^2 + \frac{1}{6} + (4c^2 + \frac{4}{3})\frac{M_W^2}{q^2})F(q^2, M_W, M_W) \end{aligned} \quad (3.10)$$

$$\Delta r_{(2)}^{(b)} = \Delta\alpha + vertex + box$$

$$\begin{aligned} -2\frac{c^2}{s^2}\delta\beta + \left(\frac{g^2}{16\pi^2}\right)\left(\frac{11}{12}N_c - \frac{11}{12}\log\frac{M_W^2}{\mu^2} + \log c^2\left(\frac{5}{c^2} - 1 + 3\frac{c^2}{s^2} - \frac{17}{4s^2c^2}\right)\right. \\ \left. - s^2(3 + 4c^2)F(M_Z^2, M_W, M_W) + I_2(c^2)\left(1 - \frac{c^2}{s^2}\right) + \frac{c^2}{s^2}I_1(c^2)\right) \\ + \frac{1}{8c^2}(43s^2 - 38) + \frac{1}{18}(154s^2 - 166c^2) + \frac{1}{4c^2} + \frac{1}{6} \end{aligned} \quad (3.11)$$

where

$$N_c = \frac{2}{\epsilon} - \gamma_E + \log 4\pi$$

$$\Delta\alpha = 0.0602 + \frac{40}{9}\frac{\alpha}{\pi}\log\frac{M_Z}{92(GeV)}$$

$$vertex + box = \left(\frac{g^2}{16\pi^2}\right)\left(6 + \frac{7 - 4s^2}{2s^2}\log c^2\right) \quad (3.12)$$

and explicit expressions for the I_1, I_2 and F functions can be found in eqs (A7) and (A8) of ref.[17] and in [18]. Eq.(3.11) for $\Delta r_{(2)}^{(b)}$ corrects a printing error in the first reference in [5]. $\mathcal{L}^{(4)}$ gives, on the other hand, the following contributions to $\Delta\rho$, Δr and $\Delta\kappa$

$$\begin{aligned} \Delta\rho_{(4)}^{(b)} &= 0 \\ \Delta\kappa_{(4)}^{(b)} &= g^2(L_{10} + \delta L_{10}) \\ \Delta r_{(4)}^{(b)} &= -2g^2(L_{10} + \delta L_{10}) \end{aligned} \quad (3.13)$$

We have included in eqs (3.9-12) the δL_{10} and $\delta\beta$ counterterms given in eqs. (2.17) and (2.19) which were not explicitly written in our previous work[5]. Apart from these counterterms only L_{10} contributes to these quantities. When we put together (3.13) and (3.9-12) we notice that the divergences due to the non-linear nature of the theory cancel. Unfortunately, for each individual observable we have to trade this divergence for the knowledge of

the renormalized $L_{10}(\mu)$ and $\beta(\mu)$ at some scale μ , extracting them from the experiment. In other words, beyond the minimal Standard Model each of the quantities $\Delta\rho$, $\Delta\kappa$ and Δr are not really predictable by itself.

As we discussed in the previous section and was done in the first work in [5] the way out is to look for combinations of observables that are renormalization group invariant. In other words, combinations that in the minimal Standard Model do not depend on $\log M_H$. For instance we can form the combinations

$$\begin{aligned} O_1 &= \Delta\rho - \frac{9s^2}{10c^2}\Delta\kappa \\ O_2 &= \Delta r + \frac{11}{10}\Delta\kappa \\ O_3 &= \Delta\rho + \frac{9s^2}{11c^2}\Delta r \end{aligned} \quad (3.14)$$

In these combinations the counterterms δL_{10} and $\delta\beta$ cancel. So we would have proceeded correctly if we had left out the counterterms in the first place.

By using the above expressions for $\Delta\rho^{(b)}$, $\Delta\kappa^{(b)}$ and $\Delta r^{(b)}$ and the fermionic part (unchanged) one gets the contribution to these observables, O_1 , O_2 and O_3 , in terms of just one unknown quantity i.e. the finite part of the parameter L_{10} as defined in the previous section. In fact, all the counterterms (δL_{10} and $\delta\beta$) drop from the above observables. We also note that, as announced, H_1 does not appear in Δr , $\Delta\rho$ or $\Delta\kappa$. As we saw in section 2, it turns out that $L_{10}^{MSM} = 0$. The minimal Standard Model does not contribute to O_1 , O_2 and O_3 through $\mathcal{L}^{(4)}$. The value of these observables in any other model is obtained by just using the appropriate value of L_{10} .

Remarkably, in the observables O_1 , O_2 and O_3 (constructed to eliminate the ultraviolet divergences due to the non-linearity of the theory) there is an almost exact cancellation of the leading quadratic dependence on m_t . A simple exercise shows that an experimental uncertainty of 10 GeV in the top mass propagates, for instance in $O_3^{(b)}$, to an uncertainty of $\delta O_3^{(b)} \simeq 1 \times 10^{-4}$, which is below the foreseen experimental errors in direct measurements of $O_3^{(b)}$. Fig. 2 shows the relation between Δr and $\Delta\rho$ for a variety of underlying theories in the symmetry breaking sector. The curved solid line corresponds to the Standard Model (calculated in the linear theory for a variety of M_H values and three different m_t values). We see that for those observables that cancel the $\log M_H$ piece the result is in perfect agreement with the one loop calculation in the non-linear theory ($\mathcal{L}^{(2)}$) plus $L_{10}^{MSM} = 0$. The latter corresponds to the straight line with $N_{eff} = 0$ of fig. 2. We refer the reader to [5] for more details and experimental implications of this type of analysis.

4. $e^+e^- \rightarrow W^+W^-$ at High Energies

The study of the W and Z self interactions in general, and the precise measurement of the γWW and ZWW couplings in particular are some of the most important tasks of the next generation e^+e^- colliders that will operate in the range from 200 GeV to 1 TeV. The second phase of LEP, LEP-II, will start operating above the W^+W^- threshold in a few years from now and will reach a center-of-mass energy of $\sqrt{s} = 200$ GeV with an expected one-year-luminosity of $1.2fb^{-1}$. Higher energy e^+e^- linear colliders are also projected. Among these next linear e^+e^- linear colliders (NLC) two have been recently under consideration [19]. One with a center-of-mass energy of 500 GeV and yearly integrated luminosity of $10fb^{-1}$ and the other one with 1 TeV and $40fb^{-1}$ respectively.

In addition to e^+e^- colliders a way has also been proposed recently [20] of constructing a dedicated $\gamma\gamma$ collider in the few hundred GeV energy range by starting with a e^+e^- linear collider and converting the high energy electrons to photons by backscattering laser beams from the electron beams near the interaction point. Such a collider offers essentially the same luminosity as the original e^+e^- collider with only a 20 – 30% reduction of the center-of-mass-energy. The interesting possibility of this $\gamma\gamma$ collider relies on the advantage of being able to study separately the γWW coupling by means of the $\gamma\gamma \rightarrow W^+W^-$ process (the ZWW coupling being involved only at the one-loop level).

The conventional process that has been considered in the literature for a long time to study the W and Z interactions at e^+e^- colliders is $e^+e^- \rightarrow W^+W^-$. Due to the subtle cancellations taking place in the Standard Model at tree level among the s-channel γ exchange, s-channel Z exchange and t-channel ν exchange, one expects this process to be particularly sensitive to deviations from the Standard Model. The common lore is that the ultimate responsible for these subtle cancellations is the gauge invariance of the Standard Model which implies tree level unitarity and the Yang-Mills prescription for the three-vector-boson couplings, although one could entertain gauge-invariant effective operators in which there is no tree-level cancellation at high energies (see below). "Big" deviations of these couplings from their values in the $SU(2) \times U(1)$ gauge theory would very quickly produce observable effects like for instance an anomalous high energy behaviour of the $e^+e^- \rightarrow W^+W^-$ amplitude. However, these couplings enter in a number of radiative corrections, well tested by now, and "big" deviations can be already excluded. Any contribution to the three-vector-boson vertex beyond the minimal Standard Model must be necessarily of the size of a typical radiative correction. In other words, there is no reason to expect at moderate energies, say well below 1 TeV, sizeable deviations from the tree-level amplitudes but rather one will have to deal with deviations that are typically of the same order of magnitude as the electroweak radiative corrections.

In order to illustrate the possibility of these potentially “small” corrections with one simple example let us assume we are looking for deviations to the Standard Model γWW coupling that come from the effect of integrating out the heavy degrees of freedom of a certain assumed underlying theory that respects the $SU(2) \times U(1)$ gauge symmetry. The term in L_9 of the effective lagrangian in eq.(2.6) is a clear representative of such a deviation to the Standard Model γWW coupling respecting both the $SU(2) \times U(1)$ gauge symmetry and the $SU(2)_V$ custodial symmetry and will contribute by an amount $eg^2 L_9$ to an anomalous γWW coupling. Furthermore if the underlying theory respects the electroweak gauge symmetry the size of L_9 itself is expected to be small. For instance, in the case of technicolor theories the value of L_9 as computed in the large N_{TC} limit, eq.(2.15), is $L_9 \sim 2N_{TC} \times 10^{-3}$ and will produce a relative change in the coupling of $\delta g_{\gamma WW}/e \sim 2g^2 N_{TC} \times 10^{-3}$, typically of the same order of magnitude as any of the (custodially symmetric) pieces contributing to the Standard Model electroweak radiative corrections in the $e^+e^- \rightarrow W^+W^-$ process. Take for instance the one loop diagram with γ , W^+ and W^- as external legs and the Goldstone bosons w^+ , w^- and the Higgs particle H as internal propagators. It gives a contribution to the γWW coupling that for large values of M_H goes as $\delta g_{\gamma WW}/e \sim (g^2/16\pi^2)(-1/12)(\log M_H^2/m_W^2)$. For e.g. $M_H = 800\text{GeV}$ this gives $\delta g_{\gamma WW}/e \sim g^2(-2 \times 10^{-3})$, a value perfectly comparable to the prediction in technicolor theories.

The above example shows explicitly our point. It is inconsistent to consider the corrections to γWW and ZWW couplings coming just from the terms of order p^4 in the effective lagrangian, such as, for instance, L_9 and not to consider at the same time the corrections coming from the loops constructed with the lowest order lagrangian. The chiral lagrangian formalism provides a natural framework to make qualitative and quantitative predictions in a systematic way for the deviations to the γWW and ZWW effective couplings from both types of contributions, i.e. loops with known particles of the Standard Model and effective operators incorporating the effect of unknown ones.

The most general VWW coupling ($V = \gamma$ or Z), consistent with Lorentz invariance, that characterizes the departure from the Standard Model tree level coupling has been extensively studied in the literature. For the case in which terms proportional to $\partial_\mu W^\mu$ and $\partial_\mu V^\mu$ are excluded it is customary to express this general coupling in terms of seven form factors that reduce to four (four for γWW and four for ZWW) if we are restricting ourselves to the case of effective couplings preserving CP invariance. The terms proportional to $\partial_\mu W^\mu$ and $\partial_\mu V^\mu$ vanish if the vector bosons are on mass shell or are coupled to massless fermions which, to a good approximation, is the case in the process we are considering. The neglect of anomalous couplings that are CP odd is also a good approximation for the case studied here where neither the underlying theory governing the interactions

in the symmetry breaking sector (this is so by assumption) nor the electroweak radiative corrections to the tree level couplings can generate to the one loop level CP odd form factors. Following refs. [21-23] these four independent form factors can be derived from the following effective lagrangian:

$$\begin{aligned} \mathcal{L}_{VWW}/g_{VWW} = & i g_1^V(q^2) (W_\mu^+ W^\mu V^\nu - W_\mu^+ V_\nu W^{\mu\nu}) + i \kappa^V(q^2) W_\mu^+ W_\nu W^{\mu\nu} + \\ & i \frac{\lambda^V(q^2)}{M_W^2} W_\mu^+ W_\nu^\mu V^{\nu\lambda} + \\ & \frac{z^V(q^2)}{M_W^2} \partial^\mu \tilde{V}^{\nu\sigma} (W_\mu^+ (\partial_\sigma W_\nu) - (\partial_\sigma W_\mu^+) W_\nu + W_\mu (\partial_\sigma W_\nu^+) - (\partial_\sigma W_\mu) W_\nu^+) \end{aligned} \quad (4.1)$$

where W_μ denotes the W_μ^- field, V^μ is the Z_μ or the A_μ field,

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu, \quad V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad \tilde{V}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu\nu\rho\sigma} V^{\rho\sigma}$$

and the overall coupling constants g_{VWW} can be chosen as in the Standard Model:

$$g_{\gamma WW} = -e, \quad g_{ZWW} = e \cot \theta_W$$

In eq.(4.1) the first three effective couplings $g_1^V(q^2)$, $\kappa^V(q^2)$ and $\lambda^V(q^2)$ respect the two discrete symmetries C and P separately whereas the fourth coupling $z^V(q^2)$ is C and P violating but CP preserving. The latter is generated in the Standard Model with radiative corrections through quark loops.

The effective lagrangian in eq.(4.1) is a parametrization of all possible deviations to the VWW couplings in a particular gauge and it does not contain other fields that may mix with the $W^\pm$ and Z under a gauge transformation and, therefore, it makes little sense to require it to be manifestly gauge invariant. In fact in eq.(4.1) it is only the term in λ_V that is gauge invariant in an obvious manner. If one is interested in studying deviations to the Standard Model coming from a certain underlying theory that respects the $SU(2) \times U(1)$ gauge invariance what should be done instead is to demand this invariance at the level of the “parent” effective lagrangian; that is, before fixing the gauge. This is the case of the chiral lagrangian in eq.(2.6) that parametrizes different symmetry breaking sectors; it is (by construction) manifestly gauge invariant. Thus one can start with the invariant lagrangian in eq.(2.6) and end up with the lagrangian in eq.(4.1) in the unitary gauge $U=1$. There will be some relations among the L_9 and L_{10} of eq.(2.6) and the g_1^V and κ^V of eq.(4.1) that we shall present in section 6. Note that in the language of chiral lagrangians the terms in λ^V and in z^V will not appear up to the next order, i.e. $O(p^6)$, and therefore they will be suppressed respect to g_1^V and κ^V by one power of $p^2/4\pi v^2$.

Within the Standard Model the values of the effective couplings at tree level are

$$\begin{aligned} g_{1SM}^{\gamma}(q^2) &= g_{1SM}^Z(q^2) = 1 & \kappa_{SM}^{\gamma}(q^2) &= \kappa_{SM}^Z(q^2) = 1 \\ \lambda_{SM}^{\gamma}(q^2) &= \lambda_{SM}^Z(q^2) = 0 & z_{SM}^{\gamma}(q^2) &= z_{SM}^Z(q^2) = 0 \end{aligned} \quad (4.2)$$

Once we switch on the radiative corrections in the minimal Standard Model we will have

$$\begin{aligned} g_{1SM}^{\gamma}(q^2) &= 1 + \delta g_{1SM}^{\gamma}(q^2) & g_{1SM}^Z(q^2) &= 1 + \delta g_{1SM}^Z(q^2) \\ \kappa_{SM}^{\gamma}(q^2) &= 1 + \delta \kappa_{SM}^{\gamma}(q^2) & \kappa_{SM}^Z(q^2) &= 1 + \delta \kappa_{SM}^Z(q^2) \\ \lambda_{SM}^{\gamma}(q^2) &= \delta \lambda_{SM}^{\gamma}(q^2) & \lambda_{SM}^Z(q^2) &= \delta \lambda_{SM}^Z(q^2) \\ z_{SM}^{\gamma}(q^2) &= \delta z_{SM}^{\gamma}(q^2) & z_{SM}^Z(q^2) &= \delta z_{SM}^Z(q^2) \end{aligned} \quad (4.3)$$

where the δ -corrections are all of the order of g^2 . The sole restriction is for on-shell photons $g_1^{\gamma}(q^2 = 0) = 1$, coming from the condition of the charge of W^+ being one. The form factor $z^V(q^2)$ is sometimes written in the literature in a slightly different form. It is given in terms of an effective coupling called $g_5^V(q^2)$ that is related with $z^V(q^2)$ through [21-23]

$$g_5^V(q^2) = z^V(q^2) \left(\frac{q^2}{M_W^2} \right) \quad (4.4)$$

This expression shows explicitly the vanishing condition for on-shell photons $g_5^{\gamma}(q^2 = 0) = 0$.

A lot of studies about the sensitivities to the various anomalous 3-gauge-boson couplings at future high energy e^+e^- colliders have been done. There are also alternative studies for the Tevatron [24] and for future pp supercolliders [25] but we shall not discuss them here. Generally, these studies are based in the construction of appropriate observables starting from the full set of $e^+e^- \rightarrow W^+W^-$ helicity amplitudes as, for instance, cross-sections, angular distributions, polarization and forward-backward asymmetries, $W^{\pm}$ spin density matrix elements, etc. [19,23]. Usually, the way one proceeds is by treating the effect of $\kappa^V \neq 1$, etc. purely at the tree level, i.e. as if they were truly fundamental couplings, without including electroweak radiative corrections or the effect of the anomalous couplings into the renormalization of the parameters that define the Standard Model [26]. The rationale behind this procedure is the assumption that the deviation from the minimal Standard Model can be “large” while we have seen that we have all the right to expect the size of those deviations being comparable to the electroweak radiative corrections. It is probably a good idea, however, to try to express also the Standard Model radiative corrections in terms of the mentioned effective couplings. We will see in the next section

that this can indeed be done, but for the time being let's discuss what are the sensitivities to the various anomalous couplings regardless of whether their origin are the Standard Model radiative corrections, effective operators, or both.

The outcome is very much dependent on the assumed energy and integrated luminosity at the future experiments. For illustrative purposes we will assume here the particular values of the energy and luminosity parameters quoted in ref.[19]. By assuming that a full helicity analysis can be done from the reconstruction of the $e^+e^- \rightarrow W^+W^- \rightarrow (q\bar{q})(\nu\bar{\nu})$ events and that a degree of polarization for the e^- of 50% can be reached, the study of ref.[19] concludes that the following sensitivities to $\delta\kappa$ and $\delta\lambda$ at 95% C.L. can be reached: 0.2 at LEP II (200 GeV, 1.2 fb^{-1}), 0.01 at NLC (500 GeV, 10 fb^{-1}) and 0.0025 at NLC (1 TeV, 40 fb^{-1}). In addition the study of ref.[27] shows that a dedicated $\gamma\gamma$ collider with 350 GeV and 10 fb^{-1} will be sensitive through the $\gamma\gamma \rightarrow W^+W^-$ process to a deviation in the photon couplings of 0.03-0.05 (90% C.L.). From the above quoted values one can conclude that although LEP II will, most likely, not be sensitive enough to the small deviations we are concerned with, the next generation of linear colliders will indeed start being sensitive to them.

The whole set of electroweak radiative corrections to $e^+e^- \rightarrow W^+W^-$ have been calculated in refs.[28,30] and include the complete one-loop virtual and soft photon bremsstrahlung corrections. Numerically, for the total cross section they are found to be negative. They are moderate at LEP II energies amounting up to a maximum of $\sim 10\%$ of the Born cross-section and increase continuously with energy reaching in the TeV region a value of up to $\sim 50\%$ of the Born cross-section in the backward direction (fig.3).

One interesting part of the Standard Model electroweak radiative corrections to the $e^+e^- \rightarrow W^+W^-$ process is the one that involves the symmetry-breaking sector. In the minimal Standard Model, in particular, one can study the sensitivity of these radiative corrections to large values of the Higgs mass. By virtue of the screening theorem [16] these corrections in the large M_H limit are known to be at most logarithmically dependent on the Higgs mass and are more important for larger energies. One therefore may wonder how these logarithmic corrections would affect to the lowest order prediction for the $e^+e^- \rightarrow W^+W^-$ cross-section and in general to the whole set of relevant observables, differential cross-sections, asymmetries, etc. or, for that matter, how would they affect the effective couplings we are concerned with. As has been pointed out in the literature [14-16], [28] the interest of following the track of these logarithmic corrections is twofold. On one hand they are the relevant pieces in the minimal Standard Model coming from the effect of a heavy Higgs particle. On the other hand they provide the clue for the building up of observables that are the most sensitive ones to deviations from the Standard Model coming from a

hypothetical strongly-interacting symmetry-breaking sector as, for instance, in technicolor theories.

The leading terms for the $e^+e^- \rightarrow W^+W^-$ cross-section in the limit of large Higgs mass were worked out in detail by Lemoine and Veltman in ref.[28]. They used the dimensional regularization scheme and the on-shell renormalization prescription taking as input parameters the coupling constant g , the weak mixing angle and the mass of the W boson. We will adhere to the same regularization and renormalization schemes, but take as input parameters instead the fine structure constant α and the physical masses M_W , M_Z and M_H . Our choice of input parameters and renormalization prescriptions coincides with the ones in ref.[30] where the whole set of radiative corrections are computed. This will allow us to use their results for a latter comparison.

We would now like to summarize the $\log M_H$ contributions to the different observables. We will be in complete agreement with [28]. Let us start by defining our normalization for the differential cross-section of the process $e^+e^- \rightarrow W^+W^-$ with polarized electrons that includes radiative corrections up to $O(\alpha^3)$

$$\left(\frac{d\sigma}{d\Omega}\right)^\sigma = \left(\frac{d\sigma}{d\Omega}\right)_0^\sigma (1 + \delta_B) + 2Re(T_0^\sigma \delta T^\sigma) \frac{\alpha}{4\pi} \frac{1}{4q^2} \sqrt{1 - \frac{4M_W^2}{q^2}} \quad (4.5)$$

where, δ_B denotes the contribution from soft photon bremsstrahlung, $\left(\frac{d\sigma}{d\Omega}\right)_0^\sigma$ is the Born cross-section, T_0^σ is the lowest order amplitude and δT^σ parametrizes the total contribution from virtual corrections. Here and in the following we shall use the shorthand notation for the helicity amplitudes:

$$T^\sigma = T_0^\sigma + \delta T^\sigma = T(\sigma_+, \sigma_-, \lambda_+, \lambda_-, s, t) \quad (4.6)$$

with $\sigma = \sigma_- = -\sigma_+$ and σ_+ , σ_- , λ_+ and λ_- are the helicities of the e^+ , e^- , W^+ and W^- respectively; $s = q^2$ and $t = (p_+ - p_-)^2 = (k_+ - k_-)^2$ are the usual Mandelstam variables. We use p_- and p_+ for the incoming electron and positron and k_+ and k_- for the outgoing W^+ , W^- , respectively. In the neutrino diagram $t = p_\nu^2$.

The explicit expressions for the lowest order amplitudes corresponding to left-handed electrons, T_0^- , and right-handed electrons, T_0^+ , are the following

$$\begin{aligned} T_0^- &= T_0^{-\nu} + T_0^{-\gamma} + T_0^{-Z} \\ T_0^+ &= T_0^{+\gamma} + T_0^{+Z} \\ T_0^{-\nu} &= \mathcal{M}_1^- e^2 (g_1^\nu)^2 \frac{1}{p^2} \end{aligned} \quad (4.7)$$

$$T_0^{-\gamma} = -(\mathcal{M}_2^- - \mathcal{M}_3^-) e^2 \frac{2}{q^2}, \quad T_0^{-Z} = (\mathcal{M}_2^- - \mathcal{M}_3^-) (e^2 \frac{c}{s} g_e^-) \frac{2}{q^2 - M_Z^2}$$

$$T_0^{+\gamma} = -(\mathcal{M}_2^+ - \mathcal{M}_3^+) e^2 \frac{2}{q^2}, \quad T_0^{+Z} = (\mathcal{M}_2^+ - \mathcal{M}_3^+) (e^2 \frac{c}{s} g_e^+) \frac{2}{q^2 - M_Z^2}$$

where the Z boson couplings to left and right handed electrons and the $e\nu W$ coupling are given by

$$g_e^- = \frac{1}{sc} \left(-\frac{1}{2} + s^2\right), \quad g_e^+ = \frac{s}{c}, \quad g_1^\nu = \frac{1}{\sqrt{2}s} \quad (4.8)$$

Explicit expressions for the Dirac matrix elements $\mathcal{M}_i^\pm(\lambda_+, \lambda_-)$, $i = 1, 2, 3$ can be found in ref.[30].

The total virtual corrections include self-energy, vertex and box corrections that we shall denote as follows

$$\delta T^\sigma = \delta T_S^\sigma + \delta T_V^\sigma + \delta T_B^\sigma \quad (4.9)$$

The $\log M_H$ dependence of the differential cross-section in the large M_H limit comes entirely from the virtual corrections and more precisely from the self-energy, δT_S^σ , and vertex corrections, δT_V^σ . A compact and complete expressions for δT_S^σ and δT_V^σ in terms of unrenormalized Green functions will be presented in the following section. We have used those expressions in order to extract the large M_H limit. As far as the $\log M_H$ terms goes they come exclusively from the following pieces: W -wave function renormalization, W and Z mass renormalization, g coupling constant renormalization and from the virtual corrections to the W and Z self-energies and to the γWW and ZWW vertices. By putting all the pieces together we get the following final expression for the one loop correction to the amplitudes in the large M_H limit

$$\begin{aligned} \delta T^{-\nu} &\sim T_0^{-\nu} \left(\frac{g^2}{16\pi^2} \left(\frac{11}{12} \right) \log M_H^2 \right) \\ \delta T^{-\gamma} &\sim T_0^{-\gamma} \left(\frac{g^2}{16\pi^2} (0) \log M_H^2 \right) \\ \delta T^{-Z} &\sim T_0^{-Z} \left(\frac{g^2}{16\pi^2} \left(\frac{1}{24c^2} + \frac{1}{6} \frac{1}{1-2s^2} \right) \log M_H^2 \right) \\ \delta T^{+\gamma} &\sim T_0^{+\gamma} \left(\frac{g^2}{16\pi^2} (0) \log M_H^2 \right) \\ \delta T^{+Z} &\sim T_0^{+Z} \left(\frac{g^2}{16\pi^2} \left(\frac{1}{24c^2} \right) \log M_H^2 \right) \end{aligned} \quad (4.10)$$

in agreement with the results of ref.[28].

5. Radiative Corrections to $e^+e^- \rightarrow W^+W^-$: Improved Born Approximation

Given the complexity of radiative corrections, in order to make comparison between experiment and theory easier it seems convenient to express the amplitude for the process $e^+e^- \rightarrow W^+W^-$ in such a way that only tree-level expressions are involved. By analogy with what has been done in LEP I physics (see section 3) we call this procedure an improved Born approximation. The name is, however, a bit misleading. In fact, we will be dealing with *exact* formulae that summarize the effect of radiative corrections in a small number of parameters.

The natural counterparts of $\Delta\kappa$, $\Delta\rho$ and Δr , used in $e^+e^- \rightarrow \bar{f}f$ physics will be the effective coupling constants κ^V , g_1^V , λ^V and g_s^V that have been introduced in the previous section to parametrize the most general VWW vertex. We will also have to introduce two effective coupling constants for the t -channel diagram, which we call g_1^{tV} and g_2^{tV} . It turns out that g_1^{tV} is related to $\Delta r(M_W^2)$, defined in section 3, while g_2^{tV} does not depend at the order we are working on the symmetry breaking sector. The effective Weinberg angle $\bar{s}(q^2)$, or, equivalently, $\Delta\kappa(q^2)$ play an important role in summing up some of the oblique corrections to $e^+e^- \rightarrow W^+W^-$, just as they do for $e^+e^- \rightarrow \bar{f}f$. (We apologize for the proliferation of κ 's, but we have preferred to stick to conventional notations as much as possible.) Of all these constants we will be mostly interested in $\bar{s}$, κ^V , g_1^V and g_s^V , since we expect that these will be the ones that contain information on the symmetry breaking sector (as is, for instance, evidenced by (4.11) in the minimal Standard Model). However, we will keep our analysis general enough in this section so that our expressions can be of use to somebody that is not just interested in the scalar sector of the Standard Model. In the next section we will discuss what is the contribution of different symmetry breaking sectors to all these constants.

As discussed in the previous section, the one loop correction to the amplitude contains, in addition to the vertex and self-energy corrections, box diagrams and bremsstrahlung corrections due to the emission of soft photons. We will not attempt to give Born expressions for the box and bremsstrahlung contributions. We prefer to deal with the box diagrams explicitly for aesthetic and practical reasons, and, also, because boxes contain neither a significant contribution from the symmetry breaking sector nor information about radiative corrections to the triple gauge vertex. We do not try either to give improved Born formulae for the corrections due to soft photons and instead we treat them explicitly. The correction due to photon radiation is anyway incomplete since the contribution from hard photons is best included by Monte Carlo methods taking properly into account the detector cuts. As a result of not including the soft photon bremsstrahlung, the effective constants (κ^V and g_1^V in particular) will depend on an infrared cut-off that is traded by the experimental resolution ΔE . Following Bohm et al. [30], we take $\Delta E = 0.1E$.

From eq.(4.10) one can immediately conclude that in the large M_H limit all the relevant radiative corrections coming from the Higgs sector, namely the $\log M_H$ pieces, are proportional to the Born amplitude and therefore one can systematically take them into account by redefining a new set of effective coupling constants that contain these logarithmic pieces. More specifically for the $e^+e^- \rightarrow W^+W^-$ process we find the following $\log M_H$ contributions to the effective couplings in the large M_H limit

$$\begin{aligned} \frac{\delta g_1^{tV}(q^2)}{g_1^{tV}} &\sim \frac{g^2}{16\pi^2} \frac{11}{24} \log M_H^2 \\ \delta g_1^V(q^2) &\sim 0 \\ \delta \kappa^V(q^2) &\sim 0 \\ \frac{\delta g_1^Z(q^2)}{g_1^Z} &\sim \frac{g^2}{16\pi^2} \frac{1}{24c^2} \log M_H^2 \\ \frac{\delta \kappa^Z(q^2)}{\kappa^Z} &\sim \frac{g^2}{16\pi^2} \frac{1}{24c^2} \log M_H^2 \end{aligned} \quad (4.11)$$

It is interesting to note the total effect of the various contributing pieces in the renormalization procedure for the photon channel that leads finally to the absence of any $\log M_H$ dependence in the effective γWW couplings. It is also remarkable that these logarithmic pieces are the same for the two effective ZWW couplings $g_1^Z(q^2)$ and $\kappa^Z(q^2)$. Besides, there are no $\log M_H$ contributions to the $\lambda^V(q^2)$ or $g_s^V(q^2)$ couplings.

Finally, it should be mentioned that apart from the $\log M_H$ pieces there will also be additional pieces in the large M_H limit that are not explicitly dependent on M_H and that are generated from the scalar loops. These additional "constant" pieces will not have in general the same structure as the tree-level amplitudes. For instance they could contribute in principle to $\lambda^V(q^2)$. We will discuss about these additional contributions together with the whole set of remaining SM electroweak radiative corrections in the next sections.

We will thus be concerned with those radiative corrections that can be naturally expressed as a s -channel or a t -channel contribution. These correspond to the self-energy corrections and vertex corrections depicted in fig. 4, as well to the contribution of the counterterms. In the on-shell scheme, the renormalization counterterms are

$$\frac{\delta\alpha}{\alpha} = \Sigma'_{\tau\tau}(0) - 2\frac{s}{c} \frac{\Sigma_{\tau Z}(0)}{M_Z^2} \quad (5.1)$$

$$\delta M_W^2 = \text{Re}\Sigma_{WW}(M_W^2) \quad \delta M_Z^2 = \text{Re}\Sigma_{ZZ}(M_Z^2) \quad (5.2)$$

At the one loop order we can ignore the imaginary parts of the self-energies. From $s^2 = 1 - (M_W^2/M_Z^2)$, one derives the relations

$$\frac{\delta s^2}{s^2} = \frac{c^2}{s^2} \left(\frac{\Sigma_{ZZ}(M_Z^2)}{M_Z^2} - \frac{\Sigma_{WW}(M_W^2)}{M_W^2} \right) \quad (5.3)$$

and

$$\frac{\delta g^2}{g^2} = \frac{\delta\alpha}{\alpha} - \frac{\delta s^2}{s^2} \quad (5.4)$$

We also need perform wave function renormalization of the external $W^\pm$ legs. This is accomplished by multiplying each leg by a factor $(Z_2^W)^{-1/2}$ where

$$\tilde{Z}_2^W = 1 - \frac{\partial \Sigma_{WW}(p^2)}{\partial p^2} \Big|_{p^2=M_W^2} \quad (5.5)$$

which differs from the usual Z_2^W

$$Z_2^W = 1 - \frac{\delta g^2}{g^2} - \frac{2}{sc} \frac{\Sigma_{\tau Z}(0)}{M_Z^2} \quad (5.6)$$

by a finite amount that corrects for the fact that with Z_2^W the renormalized $W^\pm$ two point function, although is finite, does not have a residue equal to one.

The contribution from all the counterterms and the self-energy insertions is always proportional to the different tree level structures T_0^ν , $T_0^{\pm\gamma}$, and $T_0^{\pm Z}$, while vertex corrections generate pieces proportional to the different T_0 's and also to new structures. On account of renormalizability only the former can be divergent in the minimal Standard Model. (As we will see in the next section this statement remains true even in the type of non-linear effective theories that describe the possible symmetry breaking sectors.) We take as a basis for the different spinorial structures appearing in the amplitudes

$$M_1^\pm, M_2^\pm - M_3^\pm, M_3^\pm$$

$$M_4^\pm, M_5^\pm, M_6^\pm \quad (5.7)$$

following the notation and conventions of [30]. The tree level contribution only involves M_1 and $M_2 - M_3$ (see eq. 4.7).

The contribution to the one loop corrected amplitude δT coming from the self-energy insertions, from the counterterms (eqs. (5.1) and (5.2)), and from those vertex corrections that are proportional to the tree-level structure are, for left handed electrons

$$\begin{aligned} \delta T^{-\nu} = & T_0^{-\nu} \left(-\Sigma_\nu(p_\nu^2) - \frac{\Sigma_{WW}(k_+^2) - \Sigma_{WW}(M_W^2)}{k_+^2 - M_W^2} - \frac{\Sigma_{WW}(k_-^2) - \Sigma_{WW}(M_W^2)}{k_-^2 - M_W^2} \right. \\ & + \frac{\delta g^2}{g^2} - \delta \tilde{Z}_2^W \\ & \left. + V_1^{\nu\nu}(k_-, p_\nu) + V_1^{\nu\nu}(k_+, p_\nu) + \delta Z_L \right) \end{aligned} \quad (5.8)$$

where $V_1^{\nu\nu}$ is the bare $e - \nu$ vertex correction proportional to the tree level matrix element in the t -channel M_1^- . Σ_ν is the unrenormalized invariant neutrino self energy, and δZ_L is the wave function renormalization for left-handed electrons. We have used the fact that in the on-shell scheme, for left-handed electrons $\delta Z_L = -\Sigma_L(0)$ to express self-energy corrections in terms of δZ_L , Σ_L being the invariant self-energy for massless left-handed electrons.

In the s -channel for left-handed electrons we have the following one loop corrections that respect the tree-level structure

$$\begin{aligned} \delta T^{-\gamma} = & T_0^{-\gamma} \left(-\frac{\Sigma_{\tau\tau}(q^2)}{q^2} - \frac{\Sigma_{WW}(k_+^2) - \Sigma_{WW}(M_W^2)}{k_+^2 - M_W^2} - \frac{\Sigma_{WW}(k_-^2) - \Sigma_{WW}(M_W^2)}{k_-^2 - M_W^2} \right. \\ & + \left(g_e^- - \frac{c}{s} \frac{\Sigma_{\tau Z}(0)}{M_Z^2} \right) + \frac{\delta\alpha}{\alpha} - \delta \tilde{Z}_2^W \\ & \left. + V_1^{\gamma WW}(q^2) + V_-^{\gamma\gamma}(q^2) + \delta Z_L \right) \end{aligned} \quad (5.9)$$

$$\begin{aligned} \delta T^{-Z} = & T_0^{-Z} \left(-\frac{\Sigma_{ZZ}(q^2) - \Sigma_{ZZ}(M_Z^2)}{q^2 - M_Z^2} \right. \\ & - \frac{\Sigma_{WW}(k_+^2) - \Sigma_{WW}(M_W^2)}{k_+^2 - M_W^2} - \frac{\Sigma_{WW}(k_-^2) - \Sigma_{WW}(M_W^2)}{k_-^2 - M_W^2} \\ & + \left(\frac{s}{c} - \frac{1}{g_e} \right) \frac{\tilde{\Sigma}_{\tau Z}(q^2)}{q^2} \\ & \left. + \frac{\delta\alpha}{\alpha} + \frac{\delta(\frac{s}{s} g_e^-)}{\frac{s}{s} g_e^-} - \delta \tilde{Z}_2^W + V_1^{ZWW}(q^2) + V_-^{eZ}(q^2) + \delta Z_L \right) \end{aligned} \quad (5.10)$$

where

$$\tilde{\Sigma}_{\gamma Z}(q^2) = \Sigma_{\gamma Z}(q^2) + \frac{(q^2 - M_Z^2)}{M_Z^2} \Sigma_{\gamma Z}(0)$$

In the above expressions, $V_1^{\gamma WW}(q^2)$ and V_1^{ZWW} are the pieces of the unrenormalized γWW and ZWW vertices that yield a contribution proportional to the tree level tensorial structure $\mathcal{M}_2^- - \mathcal{M}_3^-$. To be precise, we decompose the γWW and ZWW vertices into four independent form factors for each, taking into account that the W external lines are on-shell and that the γ or Z is attached to a line of massless fermions

$$\begin{aligned} \Gamma_{VVWW}^{\alpha\beta\mu} = & V_1^{\gamma WW}(k_- - k_+)^{\mu} g^{\alpha\beta} - \frac{V_1^{\gamma WW}}{M_W^2} (k_- - k_+)^{\mu} q^{\alpha} q^{\beta} \\ & + V_3^{\gamma WW}(q^{\alpha} g^{\mu\beta} - q^{\beta} g^{\mu\alpha}) + i V_5^{\gamma WW} \epsilon^{\mu\alpha\beta\nu} (k_- - k_+)_{\nu} \end{aligned} \quad (5.11)$$

In the basis given in (5.7) and (5.11) the value of $V_1^{\gamma WW}$ determines the coefficient of the vertex contribution to the tree level structure $\mathcal{M}_2 - \mathcal{M}_3$. The $V_1^{\gamma WW}, \dots, V_5^{\gamma WW}$ form factors would be linear combinations of the effective parameters $\kappa^V, \dots$ introduced in the previous section if we neglected the contributions from the self-energies and counterterms. However κ^V and g_1^V do get contributions from self-energies and counterterms.

In eqs. (5.9) and (5.10) $V_-^{\gamma\gamma}$ and V_-^{eZ} are the unrenormalized γee and Zee vertices for left-handed electrons. The actual expressions for all these vertices can be deduced from the renormalized vertices presented in [31] and the knowledge of the renormalization constants. The unrenormalized self-energies entering in these formulae can be found, for instance in [18], [31]. Note that $\tilde{\Sigma}_{\gamma Z}(q^2) \propto q^2$, so eqs. (5.9) and (5.10) indeed have a pole at $q^2 = 0$ and $q^2 = M_Z^2$, respectively.

The reason why we choose to express everything in terms of unrenormalized self-energies is the following. We have in mind using the formulae for the effective vertices we are about to derive in a more general setting than just in the minimal Standard Model. Indeed, we want to compute deviations from the minimal Standard Model in a variety of different symmetry breaking sectors that we will parametrize with the help of the effective chiral lagrangian introduced in section 2. Some observables that are finite and certainly predictable in the minimal Standard Model are not so in the non-linear theory. It is always possible to make all the Green functions finite by an appropriate tuning of the $\mathcal{O}(p^4)$ coefficients, as discussed in section 2, but it is not possible to make all Green functions of the theory unambiguous when one is dealing with an effective theory — only some combinations which are independent of the cut-off can be unambiguously predicted. The “renormalized” Green functions that can be introduced by tuning the $\mathcal{O}(p^4)$ are not

directly related to “renormalized” Green functions of the minimal Standard Model, rather they differ from them by an arbitrary subtraction constant absorbed in the renormalized L_i 's. For this reason we prefer to work with unrenormalized Green functions when these can get contributions from an effective theory.

Now we turn to the amplitudes for right-handed electrons. The contribution to the amplitudes coming from self-energies, counterterms and vertex parts proportional to tree level are

$$\delta T^{+\nu} = 0 \quad (5.12)$$

$$\begin{aligned} \delta T^{+\gamma} = & T_0^{+\gamma} \left(-\frac{\Sigma_{\gamma\gamma}(q^2)}{q^2} - \frac{\Sigma_{WW}(k_+^2) - \Sigma_{WW}(M_W^2)}{k_+^2 - M_W^2} - \frac{\Sigma_{WW}(k_-^2) - \Sigma_{WW}(M_W^2)}{k_-^2 - M_W^2} \right. \\ & + (g_e^+ - \frac{c}{s}) \frac{\Sigma_{\gamma Z}(0)}{M_Z^2} \\ & \left. + \frac{\delta\alpha}{\alpha} - \delta\hat{Z}_2^W + V_1^{\gamma WW}(q^2) + V_+^{ee\gamma}(q^2) + \delta Z_R \right) \end{aligned} \quad (5.13)$$

and, finally,

$$\begin{aligned} \delta T^{+Z} = & T_0^{+Z} \left(-\frac{\Sigma_{ZZ}(q^2) - \Sigma_{ZZ}(M_Z^2)}{q^2 - M_Z^2} \right. \\ & - \frac{\Sigma_{WW}(k_+^2) - \Sigma_{WW}(M_W^2)}{k_+^2 - M_W^2} - \frac{\Sigma_{WW}(k_-^2) - \Sigma_{WW}(M_W^2)}{k_-^2 - M_W^2} \\ & + (\frac{s}{c} - \frac{1}{g_e^+}) \frac{\tilde{\Sigma}_{\gamma Z}(q^2)}{q^2} + \frac{\delta\alpha}{\alpha} - \delta\hat{Z}_2^W \\ & \left. + V_1^{ZWW}(q^2) + V_+^{eeZ}(q^2) + \delta Z_R \right) \end{aligned} \quad (5.14)$$

$V_+^{ee\gamma}$ and V_+^{eeZ} are the unrenormalized vertices for right-handed electrons. (Only one tensorial structure contributes finally to the cross section, both for left and right handed massless electrons, even if the most general eeV vertex contains twelve independent structures.) It can be checked that the divergent parts of all these Green functions cancel among themselves in the minimal Standard Model.

Now we want to modify the tree level couplings so that all radiative corrections can be expressed as “tree level” contributions. For the γ and Z channels we would like to factorize the radiative corrections that have been listed above into “final” effective vertices κ^V and g_1^V and “initial” effective vertices $g_{\pm}^{ee\gamma} = 1 + \delta g_{\pm}^{ee\gamma}$ and $g_{\pm}^{eeZ} = g_{\pm}^Z(1 + \delta g_{\pm}^{eeZ})$. The effective vertices κ^V and g_1^V should not, logically, depend on the helicity of the fermions

to which the Z or photon is attached. We would also like the initial effective vertices $g_{\epsilon^*V}^{\epsilon^*V}$ to contain only the renormalized fermion vertex correction (which is independent, at leading order, of the symmetry breaking sector). So we have to include all oblique corrections (that contain the information on the symmetry breaking sector we are after) into the effective vertices κ^V and g_1^V . By looking at eqs. (5.9) to (5.14) this looks difficult, since the oblique corrections are manifestly different for the right and left channels. This problem is solved by two observations. First of all, $\Sigma_{\gamma Z}(0)$ is, in the minimal Standard Model, a pure divergence. It is there simply to cancel some of the $1/\epsilon$ that appear in these expressions. Since the whole set is finite, we can ignore $\Sigma_{\gamma Z}(0)$ if we just retain the finite parts of eqs (5.9), (5.10), (5.13) and (5.14). It turns out that in the extensions of the symmetry breaking sector of the Standard Model that we will consider $\Sigma_{\gamma Z}(0)$ never receives any contribution beyond its value in the minimal Standard Model. (This can be inferred from the fact that differences between different symmetry breaking sectors lie in operators of $\mathcal{O}(p^4)$, which vanish at zero momentum. (There is an $\mathcal{O}(p^2)$ operator (2.18), but it does not contribute to $\Sigma_{\gamma Z}$, and, furthermore, is a pure counterterm.) Secondly, we still have some liberty in redefining the Weinberg angle that appears in $T_0^{\pm Z}$. We know from the analysis of radiative corrections to $e^+e^- \rightarrow W^+W^-$ that changing s to $\bar{s}$ absorbs a number of oblique corrections. Let's therefore change

$$T_0^{\pm Z}(s) \rightarrow T_0^{\pm Z}(\bar{s}) \quad (5.15)$$

The relation between s and $\bar{s}$ is given in eq. (3.6) and it involves $\Delta\kappa(q^2)$. After neglecting $\Sigma_{\gamma Z}(0)$ we arrive at the following expressions

$$\begin{aligned} \delta T^{\pm\gamma} = T_0^{\pm\gamma} & \left(-\frac{\Sigma_{\gamma\gamma}(q^2)}{q^2} - \frac{\partial\Sigma_{WW}(q^2)}{\partial q^2} \right) \Big|_{q^2=M_W^2} \\ & + \Sigma'_{\gamma\gamma}(0) + V_1^{\gamma WW}(q^2) + \hat{V}_{\pm}^{\epsilon\epsilon\gamma}(q^2) \end{aligned} \quad (5.16)$$

and

$$\begin{aligned} \delta T^{\pm Z} = T_0^{\pm Z}(\bar{s}) & \left(-\frac{\Sigma_{ZZ}(q^2) - \Sigma_{ZZ}(M_Z^2)}{q^2 - M_Z^2} - \frac{\partial\Sigma_{WW}(q^2)}{\partial q^2} \right) \Big|_{q^2=M_W^2} \\ & + \frac{s^2 - c^2}{sc} \frac{\Sigma_{\gamma Z}(q^2)}{q^2} + \Sigma'_{\gamma\gamma}(0) + V_1^{Z WW}(q^2) + \hat{V}_{\pm}^{\epsilon\epsilon\gamma}(q^2) \end{aligned} \quad (5.17)$$

Where we have not written terms proportional to $\Sigma_{\gamma Z}(0)$ and we have particularized to the case in which the external $W^\pm$ particles are on shell. $\hat{V}_{\pm}^{\epsilon\epsilon\gamma}$ and $\hat{V}_{\pm}^{\epsilon\epsilon Z}$ are the renormalized $\epsilon\epsilon\gamma$ and $\epsilon\epsilon Z$ vertex functions respectively. At $\mathcal{O}(g^2)$ they contain no information on the symmetry breaking sector. We absorb these minimal Standard Model corrections into the effective constants

$$g_{\pm}^{\epsilon\epsilon\gamma} = 1 + \hat{V}_{\pm}^{\epsilon\epsilon\gamma} \quad g_{\pm}^{\epsilon\epsilon Z} = g_{\pm}^{\pm}(1 + \hat{V}_{\pm}^{\epsilon\epsilon Z}) \quad (5.18)$$

And everything that is left can be included into effective couplings of the γWW and ZWW vertices. For the photon couplings

$$\begin{aligned} g_1^{\gamma} = 1 - \frac{\Sigma_{\gamma\gamma}(q^2)}{q^2} - \frac{\partial\Sigma_{WW}(q^2)}{\partial q^2} \Big|_{q^2=M_W^2} & + \Sigma'_{\gamma\gamma}(0) \\ & + V_1^{\gamma WW}(q^2) - \frac{q^2}{2M_W^2} V_2^{\gamma WW}(q^2) \end{aligned} \quad (5.19)$$

$$\begin{aligned} \kappa^{\gamma} = 1 - \frac{\Sigma_{\gamma\gamma}(q^2)}{q^2} - \frac{\partial\Sigma_{WW}(q^2)}{\partial q^2} \Big|_{q^2=M_W^2} & + \Sigma'_{\gamma\gamma}(0) \\ & + V_3^{\gamma WW}(q^2) - V_1^{\gamma WW}(q^2) - \left(1 - \frac{q^2}{2M_W^2}\right) V_2^{\gamma WW}(q^2) \end{aligned} \quad (5.20)$$

and for the Z couplings

$$\begin{aligned} g_1^Z = 1 + \frac{s^2 - c^2}{sc} \frac{\Sigma_{\gamma Z}(q^2)}{q^2} - \frac{\Sigma_{ZZ}(q^2) - \Sigma_{ZZ}(M_Z^2)}{q^2 - M_Z^2} \\ - \frac{\partial\Sigma_{WW}(q^2)}{\partial q^2} \Big|_{q^2=M_W^2} & + \Sigma'_{\gamma\gamma}(0) \\ & + V_1^{Z WW}(q^2) - \frac{q^2}{2M_W^2} V_2^{Z WW}(q^2) \end{aligned} \quad (5.21)$$

$$\begin{aligned} \kappa^Z = 1 + \frac{s^2 - c^2}{sc} \frac{\Sigma_{\gamma Z}(q^2)}{q^2} - \frac{\Sigma_{ZZ}(q^2) - \Sigma_{ZZ}(M_Z^2)}{q^2 - M_Z^2} \\ - \frac{\partial\Sigma_{WW}(q^2)}{\partial q^2} \Big|_{q^2=M_W^2} & + \Sigma'_{\gamma\gamma}(0) \\ & + V_3^{Z WW}(q^2) - V_1^{Z WW}(q^2) - \left(1 - \frac{q^2}{2M_W^2}\right) V_2^{Z WW}(q^2) \end{aligned} \quad (5.22)$$

The effective couplings λ^V and g_s^V do not get contributions from self energies or renormalization counterterms, although they are certainly non-zero due to radiative vertex corrections.

For the t -channel contributions we proceed in a similar fashion. The only complication is due to the fact that the neutrino is not on-shell, and this brings in a new spinorial structure $\mathcal{M}_t^-$ and a new form factor $g_2^{\nu\nu}$, which is zero at tree level. For the structure that is already present at tree level, we introduce the effective coupling $g_1^{\nu\nu}$, defined as $g_1^{\nu\nu} \rightarrow g_1^{\nu\nu} + \delta g_1^{\nu\nu}$

$$\frac{\delta g_1^{\nu\nu}}{g_1^{\nu\nu}} = -\frac{1}{2} \frac{\hat{\Sigma}_\nu(p_\nu^2)}{\hat{\Sigma}_\nu(p_\nu^2)} + \frac{1}{2} \Delta r(M_W^2) + \hat{V}_1^{\nu\nu}(k_-, p_\nu) + \hat{V}_1^{\nu\nu}(k_+, p_\nu) \quad (5.23)$$

Finally, we list the full one-loop correction to the s and t -channel amplitudes in terms of the effective couplings introduced. For the t -channel

$$\delta T^{-\nu} = e^2 \left(\frac{1}{s\sqrt{2}} \right) 2\delta g_1^{\nu} \frac{1}{p_\nu^2} \mathcal{M}_1^- + e^2 \left(-\frac{1}{s\sqrt{2}} \right) \delta g_2^{\nu} \frac{1}{p_\nu^2} \mathcal{M}_4^- \quad (5.24)$$

$$\delta T^{+\nu} = 0 \quad (5.25)$$

For the photon and Z s -channels

$$\delta T^{\pm\gamma} = -\frac{2e^2}{q^2} \left((\delta g_1^\gamma + \frac{q^2}{2M_W^2} \delta\lambda^\gamma) \mathcal{M}_2^\pm - \frac{1}{2} (\delta g_1^\gamma + \delta\kappa^\gamma + \delta\lambda^\gamma) \mathcal{M}_3^\pm \right. \\ \left. - \frac{\delta\lambda^\gamma}{M_W^2} \mathcal{M}_5^\pm \pm \frac{\delta g_5^\gamma}{2} \mathcal{M}_6^\pm \right) \quad (5.26)$$

$$-\frac{2e^2}{q^2} \delta g_\pm^{\gamma\gamma} (\mathcal{M}_2^\pm - \mathcal{M}_3^\pm) \quad (5.27)$$

$$\delta T^{\pm Z} = \frac{2e^2}{q^2 - M_Z^2} \left(\frac{\bar{c}}{s} \right) g_e^\pm \left(\bar{s} \right) \left((\delta g_1^Z + \frac{q^2}{2M_W^2} \delta\lambda^Z) \mathcal{M}_2^\pm - \frac{1}{2} (\delta g_1^Z + \delta\kappa^Z + \delta\lambda^Z) \mathcal{M}_3^\pm \right. \\ \left. - \frac{\delta\lambda^Z}{M_W^2} \mathcal{M}_5^\pm \pm \frac{\delta g_5^Z}{2} \mathcal{M}_6^\pm \right) \\ + \frac{2e^2}{q^2 - M_Z^2} \left(\frac{\bar{c}}{s} \right) \delta g_\pm^{eZ} (\mathcal{M}_2^\pm - \mathcal{M}_3^\pm) \quad (5.27)$$

6. Chiral Predictions for $e^+e^- \rightarrow W^+W^-$

Having defined the set of effective constants that implement an improved Born approximation for $e^+e^- \rightarrow W^+W^-$ physics, and having identified those constants where we expect to find interesting information on the symmetry breaking sector, we would now like to estimate the value of these phenomenological parameters in different theoretical settings.

The way to estimate the values of κ^V , g_1^V , g_2^V and the rest of parameters within the framework of an effective theory has been explained in detail in sections 2 and 3. We take advantage of the fact that different symmetry breaking alternatives differ from the result in the minimal Standard Model by terms of $\mathcal{O}(p^4)$ that, to the chiral order to which we are working, enter only at tree level (recall that the contribution from the L_i 's are nominally $\mathcal{O}(g^2)$ already). Knowing the predictions of the minimal Standard Model in the limit of large M_H and the appropriate values of the L_i is enough, using the procedure summarized in eq. (2.20), to have a *precise* prediction in any other theory. The procedure has three steps. 1) Pick your favourite theory and determine the values of the corresponding L_i 's. 2) Find observables that are finite in the non-linear theory. These coincide with those observables that in the minimal Standard Model are free of $\log M_H$. 3) Determine the value of the selected observables in the minimal Standard Model. Subtract the contribution from the L_i^{MSM} and add the contribution from the values of L_i appropriate to the underlying theory.

We will begin by listing the contribution of the $\mathcal{O}(p^4)$ coefficients to the different unrenormalized Green functions that enter in the definition of the effective parameters, eqs. (5.18-23). Since we work in the 't Hooft-Feynman gauge we give only the terms proportional to $g^{\mu\nu}$. We read the $\mathcal{O}(p^4)$ from (2.6) and (2.18)

$$\begin{aligned} \Sigma_{\gamma\gamma}(q^2) &= (-4g^2 s^2 H_1 - 2g^2 s^2 (L_{10} + \delta L_{10})) q^2 \\ \Sigma_{\gamma Z}(q^2) &= (2g^2 \frac{s}{c} (c^2 - s^2) H_1 + g^2 \frac{s}{c} (c^2 - s^2) (L_{10} + \delta L_{10})) q^2 \\ \Sigma_{WW}(q^2) &= -2g^2 H_1 q^2 \\ \Sigma_{ZZ}(q^2) &= (-2g^2 \frac{c^4 + s^4}{c^2} H_1 + 2g^2 s^2 (L_{10} + \delta L_{10})) q^2 + 2\delta\beta M_Z^2 \end{aligned} \quad (6.1)$$

We have not bothered to add the counterterms to H_1 because it can be checked that H_1 drops from any observable. For L_{10} we distinguish between the term obtained from integrating out the Higgs field in the minimal Standard Model, i.e. (2.13), or their counterpart in another theory (see e.g. (2.15) for some technicolor theories) and the divergent counterterm δL_{10} which is universal. We will do the same for L_9 that contributes through the unrenormalized vertices. From (2.6) we read

$$V_1^{ZWW} = \frac{g^2}{2c^2} (L_9 + \delta L_9) - 2g^2 H_1$$

$$V_1^{\gamma WW} = -2g^2 H_1$$

$$V_3^{ZWW} = -g^2 \frac{s^2}{c^2} (L_{10} + \delta L_{10}) + g^2 (L_9 + \delta L_9) - 4g^2 H_1$$

$$V_3^{\gamma WW} = g^2 (L_{10} + \delta L_{10} + L_9 + \delta L_9) - 4g^2 H_1$$

We have listed only the unrenormalized Green functions that get any contribution from the $\mathcal{O}(p^4)$ coefficients. Using these expressions it is easy to keep track of the $\log M_H$ terms. We know already that a $\log M_H$ corresponds to a $1/\epsilon$ in the non-linear theory. These $1/\epsilon$ are cancelled by the δL_{10} , δL_9 and $\delta\beta$ listed in eqs. (2.17) and (2.19). Therefore it is enough to keep track of the counterterms δL_{10} , δL_9 and $\delta\beta$ to identify $\log M_H$'s. Using eqs. (5.9), (5.10), (5.13) and (5.14) we reproduce in this way the $\log M_H$ behaviour quoted in (4.10), but with a much smaller calculational effort than the one required to trace the $\log M_H$'s in the Green functions of the minimal Standard Model.

The above expressions for the Green functions translate into the effective parameters in terms of which we parametrize the improved Born approximation. The contributions they receive can be derived from eqs. (5.19) to (5.23). Note that these are not the complete results for g_1^{γ} , etc., but only the contribution from the $\mathcal{O}(p^4)$ lagrangian. We will have to add the contribution from $\mathcal{L}^{(2)}$ later.

$$g_1^{\gamma} = 1$$

$$\kappa^{\gamma} = 1 + g^2 L_{10} + g^2 L_9$$

$$g_1^Z = 1 + g^2 \frac{1}{2c^2} (L_9 + \delta L_9) - g^2 \frac{1}{c^2} (L_{10} + \delta L_{10}) \quad (6.3)$$

$$\kappa^Z = 1 - g^2 \frac{s^2 - c^2}{2c^2} (L_9 + \delta L_9) - g^2 \frac{1 + s^2}{c^2} (L_{10} + \delta L_{10})$$

$$g_1^{e\nu} = \frac{1}{\sqrt{2}s} \left(1 - g^2 (L_{10} + \delta L_{10}) - \frac{c^2}{s^2} \delta\beta \right)$$

The rest of the effective couplings are independent of the $\mathcal{O}(p^4)$ coefficients and of $\delta\beta$. Accordingly, they must be finite in the large M_H limit (and they are). By the same token they must also be universal in the limit of large M_H . Notice that in g_1^{γ} and κ^{γ} the divergences cancel and, as a consequence, the effective couplings for the photon vertex are fully predictable on their own even in the effective theory. This is not quite the case for the Z couplings or $g_1^{e\nu}$, but it is easy to see that $\kappa^Z - g_1^Z$ is predictable

$$\kappa^Z - g_1^Z = -g^2 \frac{s^2}{c^2} (L_9 + L_{10}) \quad (6.4)$$

Recall that $L_9 + L_{10}$ is independent of chiral logs. $g_1^{e\nu}$ is, in turn, expressible in terms of Δr as we have seen, eq.(5.23).

The above are the tree level contribution from the effective lagrangian to the observables. Before being able to make any predictions we need to know the value of these quantities in the minimal Standard Model. To compute the full radiative corrections in the Standard Model to the process $e^+e^- \rightarrow W^+W^-$ is a formidable task. We have taken advantage of the complete one loop correction carried out by Bohm et al.[30], which is in agreement with the previous work of Lemoine and Veltman [28]. The full radiative corrections have been implemented in a computer code by the first set of authors. We have used this code in a way that the values of the phenomenological parameters κ^{γ} , g_1^{γ} , λ^{γ} , g_5^{γ} , $g_{\pm}^{e\nu}$, $g_1^{e\nu}$ and $g_2^{e\nu}$ can be extracted with ease. A number of cross checks have been done in order to convince ourselves that the modifications have been correctly implemented. The radiative corrections are very much dominated by the, so far unknown, top mass. In our numerical analysis we have used the value $m_t = 130$ GeV, not very different from the central value of the range allowed from an analysis of radiative corrections to $e^+e^- \rightarrow f\bar{f}$ and a combination of the four LEP experiments [9]. We include in our results an error bar of $m_t = \pm 10$ GeV. We do not believe that it is really feasible to rigorously analyze the contribution of the symmetry breaking sector before the uncertainties due to the top quark mass are removed. Since we take as input the M_W mass in our analysis and this is not very well measured, we have to decide on a value for $\sin^2 \theta_W$. We have taken $\sin^2 \theta_W = 0.230$, which, with the value of the Z mass $M_Z = 91.185 \pm 0.020$ GeV gives $M_W = 80.015$ GeV. It is hoped that M_W will be measured with a precision of up to 60 MeV in LEP II [8]. Since the numbers we will present depend on these two uncertainties, we state once more that they have to be taken only as indicative of the possibilities to explore the symmetry breaking sector. The experimental resolution on m_t and M_W has to be improved before definite conclusions can be drawn.

The numerical results for the photon effective couplings are shown in fig.5.a for various choices of the center-of-mass energy. In this case the prediction of the effective theory is a point in the $\kappa^{\gamma} - g_1^{\gamma}$ plane. The numerical values for κ^{γ} and g_1^{γ} written on the top of fig.5.a correspond to the announced universal contribution coming from $\mathcal{L}^{(2)}$ to one-loop level (plus gauge, ghost and matter part). We show four different points. One is the prediction of the minimal Standard Model (labelled SM in the plot) for a very large Higgs mass, which, as pointed out is a finite limit for the photon couplings. We also show, for this point, a line showing the variations due to effects $\mathcal{O}(1/M_H^2)$ for the range $M_H = 200 - 3000$ GeV (the line in the plot with crossed points). As expected the variations are tiny. (An error in the top quark mass of ± 10 GeV around the assumed value of $m_t = 130$ GeV is also represented for the particular choice of $M_H = 1000$ GeV. See the black squared points in the plot). The modifications induced by technicolor theories can be included by adding the appropriate L_i 's. We show the results for vector-like theories. We use the values given in (2.15). Fig.5.a shows the results for $N_{TC} = 3$, $N_{TC} = 8$ and $N_{TC} = 12$. We take $N_d=1$, but the analysis

can be easily extended. Although unfortunately below the experimental possibilities of LEP II, they are not negligible. In fact, the total contribution to κ^V and g_1^V from the whole set of radiative corrections is of the order of a few per cent and therefore they will be within the sensitivity range of the next linear collider generation with center-of-mass energy of 500 GeV and integrated luminosity of 10 fb^{-1} . On the other hand, the numerical differences among the predictions for the various considered possibilities for the symmetry breaking sector are of the order of a few per mil and, therefore, it will be necessary to wait till the next linear collider generation with 1 TeV of center-of-mass energy and integrated luminosity of 40 fb^{-1} start operating. The predictions for the photon effective coupling are to a large extent free of uncertainties due to the scale of the new physics.

For the Z couplings the situation is a little less clear. If we set $L_9 = L_{10} = 0$ we have the minimal Standard Model result. A variation of the Higgs mass in the range $M_H = 200 - 3000 \text{ GeV}$ combined with a top mass in the range $m_t = 130 \pm 10 \text{ GeV}$ leads to the allowed regions in fig.5.b. In this case, the same type of technicolor theories as considered above lead only to relations between κ^Z and g_1^Z , which correspond to the straight lines of fig.5.b.

The predictions for κ^V , g_1^V , κ^Z and g_1^Z in the minimal Standard Model are also shown separately in figs.6 and 7. They are presented for different energies and as a function of the Higgs mass, with bars indicating the influence of the top quark mass in the mentioned range.

Also shown in fig.8 are the predictions for λ^V and g_3^V in the Standard Model for different values of q^2 and top mass. These constants are (for heavy Higgses) independent of the Higgs mass. (We have also checked this fact numerically). They receive no contribution at all from the $\mathcal{O}(p^4)$ coefficients and so are to a large extent independent of the symmetry breaking sector. One could think of effective operators that might contribute to λ^V but, at the relatively small values of q^2 that we are considering, naive power counting suggests that its influence will be minute (recall that the normalizing scale is of the order of 3.1 TeV). We expect its contribution to be roughly 10% of that of the L_i and that in the most favourable case of a e^+e^- collider at 1 TeV. The effective coupling g_3^V is C and P violating and it is quite an interesting exercise to construct operators that may give rise to it in the effective theory. Certainly it cannot be generated in theories of the type we are considering. At any rate, the contribution in the minimal Standard Model, which we just argued it is very approximately all that can contribute to these two constants, is very small. They will be hard to measure, although they are surely relevant to explore sectors of the Standard Model other than the symmetry breaking one.

What helicity amplitudes are the most interesting ones to explore κ^V and g_1^V ? The answer to this question requires a detailed analysis of the Dirac matrix elements $\mathcal{M}_i$. Assume for a moment that one can clearly separate the contribution from s and t channels and that the latter can be subtracted from the results. Assume also that the γ and Z contributions can be studied separately. The cross section for longitudinally polarized W 's will then depend on a combination of g_1^V and κ^V [21]. On the other hand, the cross section for transversally polarized W 's will depend on a combination of g_1^V and λ^V . If we assume that λ^V has its minimal Standard Model value (which is not unreasonable, as we have seen) and search for possible departures from orthodoxy in the couplings κ^V and g_1^V one can make do with that. Otherwise the transversal-longitudinal cross section must be measured as well. To disentangle the photon and Z couplings it is also necessary to be able to separate between left and right electrons. One must not forget, however, that $g_1^{\nu\nu}$ also depends on the $\mathcal{O}(p^4)$ coefficients, and hence on variations in the symmetry breaking sector.

7. Conclusions

The coming years will witness a variety of ongoing and planned experiments reaching the level of precision where, for the first time, it will be possible to tell between alternative explanations for the origin of the W and Z masses. At that point, in order to be able to discriminate between these alternatives, we must be able to determine, without any ambiguities, the theoretical predictions for a number of experimental observables in different competing possibilities (technicolor, minimal Standard Model, supersymmetry,...). The most efficient and elegant way to discriminate amongst these different alternatives is provided by a Chiral Lagrangian.

The choice of which are the "right" observables to distinguish amongst various possibilities is dictated by the Chiral Lagrangian itself. The privileged observables are those where chiral logs drop out and are, accordingly, free from cut-off ambiguities. From these observables, the value of the higher-order coefficients in the Chiral Lagrangian can be extracted from experiment and compared with theoretical predictions. In the linear theory, these are observables where the logarithmic dependence on the Higgs mass cancels. The higher order coefficients (as have been defined in this paper) contain information *only* on the heavy degrees of freedom which, of course, is just what one wants. We have emphasized in the text that the relevant coefficients are those of $\mathcal{O}(p^4)$. These coefficients accompany operators of dimension four and, therefore, they are *not* suppressed by any power of some large scale. This is why they are so important.

We have argued in some detail how the higher-order coefficients are essential to relate the linear and non-linear theories in the case of a heavy Higgs. We have also pointed out

the distinction between universal contributions, that are the same for any theory whose low-energy realization is the Standard Model, and non-universal pieces coming from the higher dimensional coefficients, depending on the precise way that the symmetry breaking is realized. When computing Green's functions up to $\mathcal{O}(p^4)$ the latter only enter at tree level.

We have concentrated mostly in two types of symmetry breaking possibilities: a heavy elementary Higgs and vector-like composite theories. We have explored in detail what are the experimental consequences for LEP I and LEP II physics. For LEP I the results are very promising. We do not believe that it is possible to conclude much yet, but it is certainly true that by the end of the run of LEP (assuming, of course, that the Higgs or something else is not found before) it will be possible to extract very interesting conclusions on the symmetry breaking sector. This is particularly so if the polarization or high luminosity options are implemented. The main source of uncertainty here is, however, the lack of definite knowledge on the top quark mass. One can only hope that our ignorance about it will soon disappear thanks to our colleagues from CDF.

All the quantities relevant for LEP I physics can be expressed in terms of the Chiral Lagrangian $\mathcal{O}(p^4)$ coefficient L_{10} . Let us here make contact with the notation employed in the work of the other authors in ref. [5]

$$\begin{aligned} S &= -16\pi L_{10} \\ \epsilon_3 &= \frac{\alpha}{4g^2} S = -g^2 L_{10} \end{aligned} \quad (7.1)$$

We also note that

$$\begin{aligned} \alpha T &= \Delta\rho \\ \epsilon_1 &= \alpha T = \Delta\rho \end{aligned} \quad (7.2)$$

There is no equivalent term (at the order we are working in the custodially symmetric Lagrangian we use) to ϵ_2 (or U) [5,32].

It would not be appropriate to conclude this discussion without a reference to the atomic parity violation experiments [33]. Indeed the precision that can be obtained with these simple, table-top experiments is similar at the current experimental uncertainties coming from LEP I as of this date. From the measurement of the weak charge of cesium one gets, after subtracting the radiative corrections (with values $m_t = 140$ GeV and $M_H = 100$ GeV)

$$S = -2.7 \pm 2.0 \pm 1.1 \pm 0.16 \quad (7.3)$$

where the first error is statistical, the second reflects an atomic theory uncertainty and the third the error in estimating the hadronic corrections. This implies

$$S < 0.6 \quad (7.4)$$

This should be compared with the technicolor prediction for vector-like theories with N_f doublets

$$S \geq 0.1 N_{TC} N_f \quad (7.5)$$

As for LEP I, the experimental resolution will no doubt improve in the future. There is of course some ambiguity in the conclusions that can be drawn from these numbers as the radiative corrections contain a contribution from the physical Higgs, which cannot be removed unambiguously.

Returning to accelerator experiments, LEP II will yield for the first time some information (even if, alas, too imprecise) on the triple gauge boson vertex. This information will considerably improve with the generation of Next Linear Colliders currently under discussion. This is genuinely new information beyond what will be learned from LEP I and it will complement very nicely the information gained from LEP I and pp colliders. Everything taken into account, the coming years will provide an interesting test on the $\mathcal{O}(p^4)$ coefficients of the Chiral Lagrangian. As explained in detail, assuming custodial symmetry to be an exact one, these four coefficients, L_1 , L_2 , L_9 and L_{10} , are all one can hope to know on the symmetry breaking sector in the near future. From one point of view it is nice that everything is so economically described by a few numbers. On the other hand, it may be difficult to open a window on the new physics beyond the Standard Model with only the knowledge of these four quantities, more or less as if one would have hoped to "guess" QCD just from knowing some low-energy parameters.

A large part of the computational effort of this paper has been devoted to implement a kind of Born approximation, with as few effective coupling constants as possible and to estimate the value of the radiative corrections to these in the Standard Model. We have also estimated all these effective couplings in different symmetry breaking sectors. Amongst all the effective couplings and parameters we have identified those where *precise* predictions can be made. The photon effective couplings are particularly interesting. Experience from LEP I shows that it is very useful for experimentalists and theorists alike to try to systematize and absorb the theoretical predictions into such effective parameters. We do not pretend to have said the last word on this point, but we do hope that our work will be useful to interpret the LEP II results when they come.

The chiral counting rules teach yet one more lesson. The order of magnitude of the $\mathcal{O}(p^4)$ coefficients is expected to be of the order of a "normal" electroweak radiative

correction. This implies that it is really not consistent to include one and not the other. We have estimated the full radiative corrections to all the effective couplings that have been introduced. This is, we believe, interesting *per se*, quite independent of the light it may shed on the symmetry breaking sector. Our task has been greatly facilitated by the existence of a computer code[30] calculating the one loop correction to the $e^+e^- \rightarrow W^+W^-$ amplitude. We have consistently worked in the on-shell renormalization scheme. This should be borne in mind when comparing our results with other work as the dependence on the higher order coefficients of the Chiral Lagrangian is scheme-dependent.

In conclusion, we are confident that we will have the long awaited clue on the symmetry breaking sector soon.

Acknowledgements

We are specially grateful to A.Dobado who collaborated in the early stages of this work and thank him for helping us. We are also grateful to many of our colleagues at CERN and elsewhere for many interesting discussions, particularly to F. del Aguila, J. Bernabeu, J. Bijnens, B. Gavela, M. Martínez, E. de Rafael and A. de Rújula. We thank W. Beenakker and T. Sack for providing us with their code computing the electroweak radiative corrections to the amplitude for the process $e^+e^- \rightarrow W^+W^-$. D.E. acknowledges the financial support of the EEC Science Twinning Grant no. SCI-000337.

Appendix

In this appendix we estimate the value of the coefficients of the $\mathcal{O}(p^4)$ lagrangian in the minimal Standard Model (referred in the text as L_{SM}^4) obtained from the linear theory when one integrates out the physical Higgs field.

We take advantage of the gauge invariance of the Standard Model and select the unitary gauge to remove the Goldstone bosons. In the non-linear theory this corresponds to setting $U = 1$. The relevant pieces of the lagrangian that will be relevant to us to determine the effective action $\Gamma(A, w)$ introduced in (2.11) are

$$\begin{aligned} & \frac{1}{4}g^2v^2W_\mu^+W_\mu^- + \frac{1}{8}(g^2 + g'^2)v^2Z_\mu Z_\mu \\ & + \frac{1}{4}g^2W_\mu^+W_\mu^-(2v\sigma + \sigma^2) + \frac{1}{8}(g^2 + g'^2)Z_\mu Z_\mu(2v\sigma + \sigma^2) \\ & + \frac{1}{2}(\partial\sigma)^2 + v(\mu^2 - \lambda v^2)\sigma + \frac{1}{2}(\mu^2 - 3\lambda v^2)\sigma^2 \\ & - v\lambda\sigma^3 - \frac{1}{4}\lambda\sigma^4 \end{aligned} \quad (A.1)$$

The reader will recognize the terms in the first line of (A.1) as those obtained from $\mathcal{L}^{(2)}$ when going to the unitary gauge (equation (2.9)). When we integrate out the σ field at the one loop level v gets a radiative correction from the terms in the second line of (A.1). Since v is ultimately related to the W and Z mass, and we wish to work in the on-shell scheme, we set v equal to its physical value by appropriately adjusting a counterterm, i.e. by replacing v^2 by $v^2 + \delta v^2$. In dimensional regularization at one loop

$$\delta v^2 = M_H^2 \frac{1}{16\pi^2} \left(\frac{2}{\epsilon} - \gamma_E + \log 4\pi + 1 - \log \frac{M_H^2}{v^2} \right) \quad (A.2)$$

Due to the cubic term in (A.1) tadpoles are generated in perturbation theory. They can be eliminated thanks to the linear term in the penultimate line of (A.1) by demanding that, once the appropriate counterterms are included in v , λ and μ , the difference $\mu^2 - \lambda v^2$, which vanishes at tree level, satisfies

$$\mu^2 - \lambda v^2 = -\frac{1}{v}\delta T \quad (A.3)$$

δT being precisely the tadpole contribution. This fixes the renormalization of μ^2 as a function of the other two parameters in the lagrangian λ and v .

Next we turn to the L_i 's. We first observe that a two-point function (other than the mass term we just referred to) or a three-point function for the W and Z fields will never

be generated from integrating out the σ field. The only functions which can appear are four-point functions and beyond. This statement remains true at any order of perturbation theory in λ (i.e. even for very large Higgs masses). Since in the unitary gauge the chiral lagrangian has two- and three-point functions involving the gauge fields with coefficients L_{10} and L_9 , respectively, this automatically implies that in the minimal Standard Model $L_9 = L_{10} = 0$.

To determine L_1 and L_2 we reduce the appropriate operators in (2.6) by going to the unitary gauge. Among many other terms one obtains

$$\dots + g^4 L_1 W_\mu^3 W_\nu^+ W_\nu^- + g^4 L_2 W_\mu^+ W_\nu^3 W_\nu^- + \dots \quad (A.4)$$

Given the tensorial structure of the indices in (A.1) is again obvious that no term corresponding to L_2 is generated by integrating out the physical Higgs (and only the Higgs). This implies that in the minimal Standard Model $L_2 = 0$. On the other hand, L_1 is non-zero. To estimate L_1 we must evaluate the diagrams shown in fig. 1 in the limit of zero momentum transfer, where the first pair of wavy lines correspond to $W^+ W^-$ and the second one to $W^3 W^3$ (or the other way around). The result of the calculation is

$$L_1 = \frac{v^2}{8M_H^2} + \frac{1}{16\pi^2} \left(\frac{9\pi}{16\sqrt{3}} - \frac{13}{16} + \gamma \right) \quad (A.5)$$

The result takes into account the appropriate counterterms arising from (A.1), including a renormalization of the Higgs mass (related to that of λ , v ; see (A.1)) and an additional counterterm related to the tadpole. Due to the fact that L_1 contains both a tree level and a one-loop contribution from the physical Higgs, the way the Higgs mass is renormalized does matter. Since it is hard to match the renormalization conditions in the process of integrating out the σ field it is easier to pick a simple scheme in the underlying theory (such as an on-shell renormalization) and parametrize the renormalization ambiguity with the help of the finite quantity γ . Alternatively, we can view γ as an ambiguity arising from the renormalization of λ (since the renormalization of v has already been determined). The ambiguity can be resolved from a comparison between amplitudes in the $J = 1, I = 1$ channel computed in the linear and the non-linear theories. This channel is free from chiral logs as mentioned in the text and allows for an unambiguous determination of $L_1 - L_2/2$. This gives

$$\gamma = -\frac{1}{6} \quad (A.6)$$

We insist once more that the values of the constants L_1 , L_2 , L_9 and L_{10} that have been obtained above will be renormalized by chiral loops and must not be confused with

the running $L_1(\nu)$, $L_2(\nu)$, $L_9(\nu)$ and $L_{10}(\nu)$, all of which are, for a generic ν , non-zero. The reasons for this distinction have been explained in the text.

Due to the different choice of renormalization procedure these results cannot be directly compared with the ones presented in the appendix B of ref. [3].

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Figure Captions

Fig.1 Feynman diagrams contributing to the Standard Model amplitude $W^+W^- \rightarrow ZZ$ to one-loop and containing only the Higgs field as internal particle.

Fig.2 Prediction for the parameters Δr and $\Delta \rho$ in the various scenarios for the symmetry breaking sector. The curved lines are the predictions in the Standard Model (linear σ model) for various values of the Higgs mass from 50 GeV to 3000 GeV (the crossed points over the line) and for three choices of the top quark mass. The straight lines give the relation between Δr and $\Delta \rho$ as predicted with the chiral lagrangian in eqs.(2.5), (2.6) (non-linear σ model), with the parameters in eq.(2.15). and for the values $N_{eff} = 0, 8, 12$.

Fig.3.a Standard Model electroweak radiative corrections to $e^+e^- \rightarrow W^+W^-$ for various values of the center-of-mass energy. The definition of δ is: $(d\sigma/d\Omega)_0(1+\delta)$. In this plot the Higgs mass and the top mass where fixed to the values 200 GeV and 130 GeV respectively. A soft photon energy cut-off of $\Delta E = 0.1E$ was chosen.

Fig.3.b Angular dependence of the differential Born cross section for the same choices of the parameters and the same center-of-mass energy values as in fig.3.a.

Fig.4 Feynman diagrams for self-energy and vertex corrections to $e^+e^- \rightarrow W^+W^-$. The white blob of the last four diagrams represents any of the possible topologies, i.e. the neutrino, the photon, and the Z channels.

Fig.5.a Prediction with the chiral lagrangian of eqs. (2.5), (2.6) for the effective parameters entering in the triple gauge boson vertex γW^+W^- for various center-of-mass energy values and for various choices of N_{eff} . The prediction in the Standard Model with radiative corrections is also shown for comparison (the values chosen for M_H of 50 GeV through 3000 GeV and for m_t of 120, 130, 140 GeV give the crossed line shown in the plot).

Fig.5.b Relation between the effective parameters κ^Z and g_1^Z entering in the triple gauge boson vertex ZW^+W^- as predicted with the chiral lagrangian of eqs.(2.5),(2.6) for various values of the center-of-mass energy and for various choices of N_{eff} . The closed line shown represents the region covered by the Standard Model predictions for the same choices of M_H and m_t as in fig.5.a.

Fig.6.a The effective parameter g_1^Z in the Standard Model as a function of M_H and m_t

Fig.6.b The effective parameter g_1^Z in the Standard Model as a function of M_H and m_t

Fig.7.a The effective parameter κ^Z in the Standard Model as a function of M_H and m_t

Fig.7.b The effective parameter κ^Z in the Standard Model as a function of M_H and m_t

Fig.8 The effective parameters λ^γ , λ^Z , g_3^γ and g_3^Z in the Standard Model as a function of m_t . M_H was fixed to 1000 GeV

Fig. 1

Fig.2

Fig.3.a

Fig.3.b

Fig.4

Fig.5.a

Fig.5.b

Fig.6.a

Fig.6.b

Fig.7.a

Fig.7.b

Fig.8