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Higher Order Bose-Einstein Correlations in $p\bar{p}$ Collisions at $\sqrt{s} = 630$ and 900 GeV

UA1-MINIMUM BIAS-Collaboration

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Abstract

Higher order Bose-Einstein correlations up to 5th order, of particles produced in proton-antiproton collisions, are presented using UA1 data at $\sqrt{s} = 630$ and 900 GeV. The results are compared with theoretical calculations to investigate the primary assumptions for the parametrization of the correlation functions.

(submitted to Physics Letters B)

1 Introduction

Hanbury-Brown and Twiss [1] were the first who applied the Bose-Einstein correlations (BEC) to measure the size of distant stars. The method was first applied to particle physics by Goldhaber et al. [2,3] in 1959. Since then the size of the interaction region has been measured by numerous experiments in high energy collisions between many different types of particles [4-8].

The study of BEC is based on the idea of intensity interferometry [9]. The analogue to “intensity” in optics is “probability density” in quantum mechanics. The wave functions of identical bosons are symmetric with respect to particle exchange and this symmetrization requirement results in an interference term, which contains information about the space-time structure of the boson emitting source. So the probability of finding two identical bosons in the same phase-space element is enhanced, compared to the probability of finding two non-identical particles, and the size of the boson emitting source can be measured.

In this paper we present higher order BEC up to the 5th order¹ using data from $p\bar{p}$ collisions at center of mass energies of 630 and 900 GeV. The data for this analysis were collected in 1985 by the UA1 experiment at the SPS proton-antiproton collider. The theoretical background for higher orders is reviewed in section 2. In section 3 we report on the data taking and discuss the selection criteria used in the analysis. In section 4 the methods of our analysis are described. The results obtained are discussed in section 5.

2 Higher Order Correlations

To study BEC one compares the number of like-charged pion pairs $N^{(2)}$ with the number of uncorrelated pairs N_2^{BG} , as a function of the relative pion momentum squared. The ratio often has been parametrized with a Gaussian function of the four momentum difference squared:

$$N^{(2)}/N_2^{BG} = 1 + \lambda e^{-r^2 Q_{2\pi}^2} \quad \text{with} \quad Q_{2\pi}^2 = -(p_1 - p_2)^2, \quad (1)$$

where p_1 and p_2 are the four momenta of the two pions. The two parameters λ and r are to be determined by fitting the data. In the limit of a completely incoherent source and identical momenta the ratio is expected to produce $N^{(2)}/N^{BG}(Q_{2\pi} \rightarrow 0) = 2$. The parameter λ expresses the degree of incoherence of the source. For a fully coherent source λ is zero. One can interpret the parameter r as the extent in space of the pion producing region.

The analysis of BEC for more than two particles follows the procedure for pairs [5]. The three particle correlation function depends on the three variables q_{12} , q_{13} and q_{23} . If one uses the pair parametrization (1), and

$$Q_{3\pi}^2 = q_{12}^2 + q_{13}^2 + q_{23}^2 \quad \text{with} \quad q_{ij}^2 = -(p_i - p_j)^2 \quad (2)$$

¹We use here the term “ n^{th} order” BEC for n -body correlations, in agreement with the notation of [10].

the three particle function can be parametrized [11] in analogy to the pair analysis:

$$N^{(3)}/N_3^{BG} = 1 + \lambda_3 e^{-r_3^2 Q_{3\pi}^2}. \quad (3)$$

In the limit of small $q_{12}^2 \approx q_{23}^2 \approx q_{31}^2 \equiv q_{ij}^2$, $Q_{3\pi}^2 \approx 3q_{ij}^2$, the parameters λ_3 and r_3 are restricted by the parameters λ and r of the two particle correlation function with the inequality relations [5]:

$$\lambda_3 \leq 3\lambda + 2 \leq 5 \quad \text{and} \quad \frac{r^2}{3} < r_3^2 < \frac{r^2}{2}. \quad (4)$$

The calculation for 4th and 5th order is straightforward but using this technique it becomes too complicated.

In [10] a new set of formulas to describe the higher order correlation functions is given. These formulas were obtained using methods from quantum optics assuming the coherent part of the source to be pointlike and the incoherent one to be of a Gaussian form.

In the symmetric configuration for n particles

$$q_{12}^2 = q_{13}^2 = q_{23}^2 = \dots = q_{(n-1)n}^2 \quad \text{with} \quad q_{ij}^2 = -(p_i - p_j)^2 \quad (5)$$

there are $\frac{n(n-1)}{2}$ combinations of q_{ij}^2 for $Q_{n\pi}^2$:

$$Q_{n\pi}^2 = q_{12}^2 + \dots + q_{(n-1)n}^2 \quad (6)$$

Now the formulas for second to fifth order are [10]:

$$N^{(2)}/N_2^{BG} = 1 + 2\lambda(1 - \lambda)e^{-r^2 Q_{2\pi}^2} + \lambda^2 e^{-2r^2 Q_{2\pi}^2} \quad (7)$$

$$N^{(3)}/N_3^{BG} = 1 + 6\lambda(1 - \lambda)e^{-\frac{1}{3}r^2 Q_{3\pi}^2} + 3\lambda^2(3 - 2\lambda)e^{-\frac{2}{3}r^2 Q_{3\pi}^2} + 2\lambda^3 e^{-r^2 Q_{3\pi}^2} \quad (8)$$

$$N^{(4)}/N_4^{BG} = 1 + 12\lambda(1 - \lambda)e^{-\frac{1}{6}r^2 Q_{4\pi}^2} + 6\lambda^2(7 - 8\lambda + 2\lambda^2)e^{-\frac{1}{3}r^2 Q_{4\pi}^2} + 4\lambda^3(11 - 9\lambda)e^{-\frac{1}{2}r^2 Q_{4\pi}^2} + 9\lambda^4 e^{-\frac{2}{3}r^2 Q_{4\pi}^2} \quad (9)$$

$$N^{(5)}/N_5^{BG} = 1 + 20\lambda(1 - \lambda)e^{-\frac{1}{10}r^2 Q_{5\pi}^2} + 10\lambda^2(13 - 18\lambda + 6\lambda^2)e^{-\frac{1}{5}r^2 Q_{5\pi}^2} + 20\lambda^3(16 - 21\lambda + 6\lambda^2)e^{-\frac{3}{10}r^2 Q_{5\pi}^2} + 5\lambda^4(53 - 44\lambda)e^{-\frac{2}{5}r^2 Q_{5\pi}^2} + 44\lambda^5 e^{-\frac{1}{2}r^2 Q_{5\pi}^2} \quad (10)$$

In (7) to (10) there are only two free parameters r and λ . In [10] it has been pointed out, that these parameters should be the same in all orders. Higher order BEC are a powerful tool to check the assumptions for two-particle correlations and to clarify further corrections (Coulomb interactions, etc.).

3 Data

The data for this analysis were obtained during two periods of data taking, one at a fixed center of mass energy of $\sqrt{s} = 630$ GeV and the other at $\sqrt{s} = 900$ GeV. All

data were taken using a 'minimum bias' trigger [12], requiring at least one charged particle in the pseudorapidity range of $1.5 < |\eta| < 5.6$ in each of the downstream arms of the detector. The first data-set with $\sqrt{s} = 630$ GeV includes about 147,000 events and the second with $\sqrt{s} = 900$ GeV about 66,000 events. All information used for this analysis was obtained from reconstructed trajectories measured by the UA1 Central Detector [13] in a region of pseudorapidity $|\eta| \leq 3$. The azimuthal regions with $|\Phi| \leq 15^\circ$ and $|\Phi| \geq 165^\circ$ were excluded because of the high uncertainty of momentum measurement in those regions. Only those tracks with projected length, in the plane normal to the magnetic field, larger than 40 cm and a $p_T \geq 0.15$ GeV/c are considered for this analysis. Also a good fit to the vertex and a good track quality are required for all tracks. We assumed that all particles retained from these conditions are charged pions, which is true for about 85 % of all tracks.

We have observed that a substantial number of low momentum tracks (about 2%) are misidentified as two tracks by the pattern recognition program. This happens mainly when the particles are crossing the walls of the different parts of the central detector. To avoid this problem and the problem with the two track resolution of the drift chambers a special algorithm was applied: This algorithm searches for track-pairs which are close together in phase-space (i.e. $\Delta\eta \leq 0.35$, $\Delta\Phi \leq 7.5^\circ$, $\Delta p \leq 0.1$ GeV/c) and checks, if they are really different tracks. If the spurious two tracks belong to only one split track, then our algorithm removes the one that has more distance to the vertex. Our results are very sensitive to these misidentifications, because two tracks which are nearby in phasespace produce a very low Q^2 . Our algorithm removes only 98% of all misidentified pions. After a visual examination of accepted track-pairs we additionally require each track-pair to have an invariant momentum-difference-squared $Q^2 = -(p_1 - p_2)^2 \geq 0.0001(\text{GeV}/c)^2$.

Because we want to compare the results of higher orders (up to 5) of the BEC, it is necessary that each event consists of at least 5 like-charged tracks.

4 Analysis

4.1 Reference Sample

In order to analyse our data to obtain the radius r and the incoherence parameter λ of the pion source we had to divide the Q^2 -distribution $N^{(n)}$ of like-charged n -tuples of tracks from the same event by a reference- Q^2 -distribution N_n^{BG} . An ideal reference sample must represent the kinematical phase space of the studied sample, but should be free from any correlations (BEC, resonances, etc.). We used the method of event mixing: for the n^{th} order reference sample distribution we took n like-charged tracks, each of a different event, and calculated $Q_{n\pi}^2 = \sum_{i < j}^n q_{ij}^2$. If there are no correlations present and if the total number of track-tuples is the same for both distributions, the ratio $N^{(n)}/N_n^{BG}$ should be equal to one. However the method of event mixing produces many more track-pairs than one gets from correlated pairs. Therefore we normalized the ratio to unity in the region $0.4 \leq Q^2 \leq 1.0 (\text{GeV}/c)^2$ where correlations are absent.

To fit our data for r and λ , we applied the normalizing correction for the reference

sample distribution as outlined in [14], i.e. dividing all our functions used for fitting by a term $(1 + \delta Q^2)$. The parameter δ is introduced to describe the slight Q^2 -dependence of the reference sample.

4.2 Corrections

To measure the undisturbed BEC we have to subtract all other correlation effects from our distributions. For the correlated distributions of charged particles one has to correct for Coulomb-interaction by weighting each like-charged pair with the inverse Gamow-factor [9]:

$$G_{ij}^{-1} = \frac{\exp(2\pi y) - 1}{2\pi y} \quad \text{with} \quad y = \frac{\alpha m_\pi}{\sqrt{q_{ij}^2}}, \quad (11)$$

where $\alpha = \frac{1}{137}$ is the fine structure constant and m_π the pion mass. This method is only valid for a two particle correlation, however in [5] an approximation for three particles is given:

$$W_3 = G_{12}^{-1} \cdot G_{13}^{-1} \cdot G_{23}^{-1}. \quad (12)$$

Since now there exist no exact calculations for the Coulomb interaction for more than three particles, but we generalized this approximation for n^{th} order in a straightforward extension:

$$W_n = \prod_{i < j}^n G_{ij}^{-1}. \quad (13)$$

The correction is done by weighting each like-charged n -tuple with W_n .

One has to correct for the limited detector acceptance in some phase-space regions. The central detector (CD) acceptance has been evaluated as a function of the charge, η , Φ and p_T [15]. All tracks are corrected for the CD acceptance. For particle-multiplets the correction factors are multiplied. It is not really straightforward to use the acceptance for many particle distributions, because the acceptance is only defined for single particle distributions. But the uncertainty in the acceptance is on average small: $\pm 4\%$ [16].

In addition, the contribution to the systematic error due to our procedure of selecting our data sample described in section 3 was studied varying the data selection parameters and found to be of the order of 10% in the lowest bins of the Q^2 distributions. We have then estimated the systematic uncertainty inherent in our fitting for the Bose-Einstein parameters and conclude that the overall systematic error on the parameter r is of the order of 13%.

The calculation of the correction for the influence on BEC of the $\pi\pi$ -final state interaction has been given in [17], where the effect on λ is estimated to be roughly a 15% reduction. Because of the difficulties involved, we did not correct for this effect [5,18].

With the UA1-detector there is no possibility to correct for pion-misidentification. Only 85% of all tracks are pions. Arguing that all particles have the same region of production, the radius r is independent of pion misidentification, whereas the incoherence parameter λ will be lowered by impurity of the detected particles [5].

For the following analysis we take into account the Coulomb and the acceptance corrections (see tables 1 and 2). The corrections for $\pi\pi$ -final state interactions and particle impurities are ignored. We believe that the corresponding uncertainties in r and λ are small enough not to spoil our argumentation in the subsequent sections.

4.3 Fits

The resulting Bose-Einstein ratio is then fitted to various functions whose parameters r and λ describe the particle source in the case of a good fit-quality. We fitted our data samples with the following functions:

- (i) simple Gaussian: $(1 + \lambda e^{-r^2 Q^2})/(1 + \delta Q^2)$
- (ii) exponential: $(1 + \lambda e^{-rQ})/(1 + \delta Q^2)$
- (iii) (7) to (10) divided by the normalizing factor $(1 + \delta Q^2)$
- (iv) (7) to (10) divided by the normalizing factor $(1 + \delta Q^2)$, but the Gaussian form for the two particle correlation function is replaced in all orders by an exponential assumption:

$$N^{(2)}/N_2^{BG} = 1 + 2\lambda(1 - \lambda)e^{-rQ_{2\pi}} + \lambda^2 e^{-2rQ_{2\pi}} \quad (14)$$

$$N^{(3)}/N_3^{BG} = 1 + 6\lambda(1 - \lambda)e^{-\frac{1}{3}rQ_{3\pi}} + 3\lambda^2(3 - 2\lambda)e^{-\frac{2}{3}rQ_{3\pi}} + 2\lambda^3 e^{-rQ_{3\pi}} \quad (15)$$

$$N^{(4)}/N_4^{BG} = 1 + 12\lambda(1 - \lambda)e^{-\frac{1}{6}rQ_{4\pi}} + 6\lambda^2(7 - 8\lambda + 2\lambda^2)e^{-\frac{1}{3}rQ_{4\pi}} + 4\lambda^3(11 - 9\lambda)e^{-\frac{1}{2}rQ_{4\pi}} + 9\lambda^4 e^{-\frac{2}{3}rQ_{4\pi}} \quad (16)$$

$$N^{(5)}/N_5^{BG} = 1 + 20\lambda(1 - \lambda)e^{-\frac{1}{10}rQ_{5\pi}} + 10\lambda^2(13 - 18\lambda + 6\lambda^2)e^{-\frac{1}{5}rQ_{5\pi}} + 20\lambda^3(16 - 21\lambda + 6\lambda^2)e^{-\frac{3}{10}rQ_{5\pi}} + 5\lambda^4(53 - 44\lambda)e^{-\frac{2}{5}rQ_{5\pi}} + 44\lambda^5 e^{-\frac{1}{2}rQ_{5\pi}} \quad (17)$$

For comparing our data sample with the results of [4-8], we fitted with (i) and (ii). Expressions (i) and (iii) fail to describe the data in low $Q_{n\pi}^2$ regions, but using an exponential expression for the incoherent source (ii) and (iv) we obtain a much better agreement with the data, and therefore the interpretation of the parameters is more meaningful. The parameters of (7) to (10) and (14) to (17) are listed in table 1 and table 2.

5 Results and Discussion

We studied BEC from second to fifth order. Typical plots to show the differences between the orders are given in Fig. 1. The values of our fitted parameters for second and third order are in agreement with [4,5]. The normalized data were fitted with (4.3.i) to (4.3.iv). Fitting our data with the functions (7) to (10) (i.e. Gaussian assumption) we found, that with this set of functions it is not possible to describe all orders of

the BEC with the same parameters as claimed in [10]. As shown in table 1 the radius r rises with the order, only the incoherence parameter λ is almost constant. We modified the functions given in [10] using an exponential expression to describe the incoherent part (14) - (17). We found, that functions with an exponential assumption² for the shape of the incoherent source reproduce the data much better than functions with a Gaussian assumption. The differences between the two sets of functions are shown in figure 2 a,b and the fitted parameters are given in table 1 and table 2. Also in the exponential fits (14) to (17) the incoherence parameter is nearly constant and the radius r is rising with the order. We introduce a purely phenomenological factor $\sqrt{\frac{2}{n(n-1)}}$, so that the modified radius $R = r \cdot \sqrt{\frac{2}{n(n-1)}}$ is only slightly increasing with the order (almost constant, compare Fig. 3).

The fact, that we cannot describe the effect in all orders with the same parameters r and λ does not necessarily contradict the quantum statistical description of [10] since the formulas (7)-(10) have been derived under several assumptions which need not be fulfilled: pointlike coherent source, symmetric configuration for n particles, stationarity of the source [19,20,21], etc.

It is interesting that the radius of the source is not sensitive to the corrections (Coulomb, detector acceptance). The incoherence parameter is more sensitive on all our corrections. Although our method for correcting the Coulomb interaction for more than three particles is just an empirical one, it is a straightforward extension.

We used two data samples with different center of mass energies, one with 630 GeV and the other with 900 GeV. So it is possible to look for a dependence of r and λ on the center of mass energy. Fig. 3 shows, that there is no significant dependence on $\sqrt{s}$ between 630 and 900 GeV. The small increase of the radius r with energy in the second and third order may be partly due to the higher multiplicity at $\sqrt{s} = 900$ GeV in agreement with [4]. It should be noted however that the observed rise is comparable with the systematic errors of 13% which arise because of the different run conditions.

6 Conclusions

For the first time second to fifth order BEC are measured and analysed with the same method. The most remarkable result of our analysis is, that an exponential assumption for the two particle correlation function describes the data much better than functions, which have a Gaussian parametrization. This is proved by a nearly perfect fit of the functions (14) to (17) for all investigated orders of the BEC. To obtain this result it was essential to study higher order BEC, whereas the usual study of second order will not yield such a clear distinction.

In our modified functions (14) to (17) the radius r is rising and only λ is constant with the order. However we have found a modified radius R , which is almost independent of the order in the exponential parametrization. λ is sensitive to our corrections,

²Using an exponential assumption it is not straightforward to find an interpretation for the parameter r . Also the simple exponential (4.3.ii) fits our data very well, but there is just one function for all orders, and the variation of λ and r is considerably bigger than in (7) to (10) and (14) to (17).

whereas r , and therefore also R , shows no significant dependence on it (Fig. 3).

In agreement with [4] we do not find a significant energy dependence of the parameters r and λ .

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Table 1

The results of fitting the various corrected data samples to the functions (7) to (10) upon gaussian parametrisation for the incoherent part, corresponding to the order of the correlations. *a)* stands for uncorrected data, *b)* represents the data corrected for detector acceptance, *c)* represents the data corrected for Coulomb interactions and *d)* stands for the fully corrected data. The errors are only statistical.

r in fm		2 nd order	3 rd order	4 th order	5 th order
a)	630 GeV	0.719 ± 0.011	0.794 ± 0.008	0.954 ± 0.014	1.228 ± 0.043
	900 GeV	0.795 ± 0.017	0.836 ± 0.010	0.951 ± 0.018	1.146 ± 0.077
b)	630 GeV	0.728 ± 0.010	0.821 ± 0.006	0.937 ± 0.013	1.245 ± 0.034
	900 GeV	0.801 ± 0.016	0.832 ± 0.008	0.922 ± 0.016	1.124 ± 0.067
c)	630 GeV	0.733 ± 0.010	0.796 ± 0.007	0.962 ± 0.015	1.230 ± 0.042
	900 GeV	0.811 ± 0.017	0.833 ± 0.009	0.959 ± 0.017	1.163 ± 0.057
d)	630 GeV	0.742 ± 0.009	0.821 ± 0.005	0.944 ± 0.010	1.208 ± 0.027
	900 GeV	0.814 ± 0.016	0.831 ± 0.007	0.933 ± 0.013	1.146 ± 0.048
λ					
a)	630 GeV	0.147 ± 0.004	0.124 ± 0.003	0.120 ± 0.004	0.151 ± 0.017
	900 GeV	0.158 ± 0.006	0.131 ± 0.004	0.119 ± 0.006	0.107 ± 0.020
b)	630 GeV	0.143 ± 0.003	0.130 ± 0.002	0.113 ± 0.004	0.150 ± 0.013
	900 GeV	0.158 ± 0.005	0.132 ± 0.003	0.110 ± 0.005	0.082 ± 0.013
c)	630 GeV	0.172 ± 0.004	0.144 ± 0.003	0.140 ± 0.006	0.175 ± 0.020
	900 GeV	0.186 ± 0.007	0.151 ± 0.004	0.140 ± 0.007	0.131 ± 0.019
d)	630 GeV	0.168 ± 0.004	0.150 ± 0.003	0.133 ± 0.004	0.156 ± 0.011
	900 GeV	0.186 ± 0.006	0.152 ± 0.004	0.130 ± 0.005	0.104 ± 0.012
δ in $(\text{GeV}/c)^{-2}$					
a)	630 GeV	-0.002 ± 0.001	0.035 ± 0.002	0.164 ± 0.006	0.353 ± 0.025
	900 GeV	-0.003 ± 0.001	0.026 ± 0.002	0.163 ± 0.007	0.370 ± 0.040
b)	630 GeV	-0.002 ± 0.001	0.029 ± 0.001	0.165 ± 0.005	0.346 ± 0.018
	900 GeV	-0.003 ± 0.001	0.028 ± 0.002	0.172 ± 0.006	0.329 ± 0.030
c)	630 GeV	-0.003 ± 0.001	0.040 ± 0.002	0.183 ± 0.006	0.404 ± 0.027
	900 GeV	-0.003 ± 0.002	0.030 ± 0.002	0.182 ± 0.007	0.421 ± 0.036
d)	630 GeV	-0.003 ± 0.001	0.033 ± 0.001	0.185 ± 0.005	0.416 ± 0.018
	900 GeV	-0.003 ± 0.002	0.032 ± 0.002	0.191 ± 0.006	0.379 ± 0.023

Table 2

The results of fitting the various corrected data samples to the functions (14) to (17) upon exponential parametrisation for the incoherent part, corresponding to the order of the correlations. *a)* stands for uncorrected data, *b)* represents the data corrected for detector acceptance, *c)* represents the data corrected for Coulomb interactions and *d)* stands for the fully corrected data. The errors are only statistical.

<i>r</i> in <i>fm</i>		2 nd order	3 rd order	4 th order	5 th order
a)	630 GeV	1.234 ± 0.021	2.566 ± 0.034	3.808 ± 0.150	6.247 ± 0.210
	900 GeV	1.386 ± 0.033	2.741 ± 0.040	3.755 ± 0.134	5.752 ± 0.707
b)	630 GeV	1.251 ± 0.021	2.671 ± 0.027	3.712 ± 0.117	6.452 ± 0.139
	900 GeV	1.390 ± 0.030	2.728 ± 0.032	3.434 ± 0.127	5.348 ± 0.848
c)	630 GeV	1.247 ± 0.018	2.536 ± 0.025	3.812 ± 0.087	5.917 ± 0.122
	900 GeV	1.374 ± 0.020	2.653 ± 0.021	3.765 ± 0.115	5.821 ± 0.439
d)	630 GeV	1.264 ± 0.017	2.613 ± 0.015	3.726 ± 0.086	5.820 ± 0.245
	900 GeV	1.374 ± 0.018	2.643 ± 0.016	3.510 ± 0.114	5.583 ± 0.671
λ					
a)	630 GeV	0.335 ± 0.013	0.365 ± 0.016	0.339 ± 0.032	0.510 ± 0.074
	900 GeV	0.396 ± 0.025	0.417 ± 0.027	0.328 ± 0.030	0.319 ± 0.088
b)	630 GeV	0.322 ± 0.011	0.397 ± 0.014	0.313 ± 0.022	0.553 ± 0.096
	900 GeV	0.395 ± 0.022	0.422 ± 0.022	0.271 ± 0.019	0.211 ± 0.053
c)	630 GeV	0.427 ± 0.016	0.463 ± 0.022	0.429 ± 0.033	0.618 ± 0.130
	900 GeV	0.538 ± 0.043	0.527 ± 0.031	0.415 ± 0.041	0.425 ± 0.099
d)	630 GeV	0.410 ± 0.015	0.505 ± 0.019	0.393 ± 0.024	0.470 ± 0.067
	900 GeV	0.531 ± 0.038	0.538 ± 0.027	0.343 ± 0.024	0.288 ± 0.067
δ in (GeV/c) ⁻²					
a)	630 GeV	0.004 ± 0.002	0.061 ± 0.002	0.268 ± 0.014	0.598 ± 0.049
	900 GeV	0.002 ± 0.002	0.049 ± 0.002	0.271 ± 0.012	0.622 ± 0.087
b)	630 GeV	0.004 ± 0.001	0.054 ± 0.002	0.271 ± 0.011	0.564 ± 0.035
	900 GeV	0.002 ± 0.002	0.051 ± 0.002	0.299 ± 0.014	0.560 ± 0.085
c)	630 GeV	0.004 ± 0.002	0.069 ± 0.002	0.301 ± 0.010	0.706 ± 0.061
	900 GeV	0.002 ± 0.002	0.058 ± 0.002	0.304 ± 0.014	0.716 ± 0.075
d)	630 GeV	0.004 ± 0.001	0.062 ± 0.002	0.306 ± 0.011	0.745 ± 0.053
	900 GeV	0.002 ± 0.002	0.060 ± 0.002	0.332 ± 0.014	0.646 ± 0.088

Figure Captions

Fig.1: Plots of the Bose-Einstein ratio for second: a), b), third: c), d), fourth: e), f) and fifth: g), h) order at 630 GeV with the best fitting functions (14) to (17). a), c), e), and g) are uncorrected data samples. b), d), f) and h) show data corrected for Coulomb interactions. Q^2 is equal to $Q_{n\pi}^2 = q_{12}^2 + \dots + q_{(n-1)n}^2$ depending on the order n .

Fig.2a: Bose-Einstein ratio of two and three particles. The dashed line represents the fit with (7) and (8) and the solid line represents the fit with (14) and (15). All data are corrected for Coulomb interactions.

Fig.2b: Bose-Einstein ratio of four and five particles. The dashed line represents the fit with (9) and (10) and the solid line represents the fit with (16) and (17). All data are corrected for Coulomb interactions.

Fig.3: Comparison of the modified radii $R = r \cdot \sqrt{\frac{2}{n(n-1)}}$ and of the incoherence parameters λ , obtained by fitting the functions (14) to (17) to the data samples, at 630 and 900 GeV for second to fifth order. In a) and c) the parameters r and λ are obtained by fitting the uncorrected data. In b) and d) the data sample is fully corrected. The errors indicated are only statistical.

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3