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STOP NEAR THRESHOLD

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Abstract:

The observation that perturbative QCD suffices to describe threshold production of heavy particles if their decay width is sufficiently large is extended to SUSY particles: the production of squarks - and there in particular of stops - and of gluinos is analyzed, both for e^+e^- annihilation and hadronic collisions. Some qualitatively new features - like intrinsic limitations to a non-relativistic treatment of P wave production - are pointed out and discussed.

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I. Introduction

Supersymmetry (hereafter referred as SUSY), in particular in its minimal version, constitutes the "standard extension" of the Standard Model. Whenever the physics is analyzed that a new accelerator might open up its potential for discovering SUSY is always addressed as an obvious point of study. To this already existing vast literature we want to add here a detailed discussion of one very specific aspect, namely the *threshold* production of squarks – and in particular of stops – and of gluinos. The sizeable decay widths of these objects provides an infrared cut-off that allows us to calculate their production rate also in the *threshold* region *reliably* within *perturbative* QCD as a function of their mass and the strong coupling α_s . A similar situation is encountered in the production of top quarks and has been analyzed by two of us (V.F. and V.K.)^[1]. Subsequent refinements and extensions can be found in the papers of ref.2,3. Yet there are several quantitative and even qualitative differences in the impact QCD has on, say,

$$e^+ e^- \rightarrow t \bar{t} \quad (1)$$

on one hand and

$$e^+ e^- \rightarrow \tilde{t} \tilde{t}^* [\tilde{q} \tilde{q}^*], \tilde{g} \tilde{g} \quad (2)$$

on the other. It is the purpose of this note to describe these differences in detail in order to prepare *precision* tools for SUSY studies in very high energy e^+e^- or hadronic collisions. Furthermore many important aspects of reaction (1) can be elucidated by the differences and similarities encountered in the processes (2).

The paper will be organized as follows. In Section II we sketch those aspects of SUSY that are relevant for our analysis. In Section III we calculate the threshold production rate for stops (and gluinos) in e^+e^- annihilation and in Section IV we treat stop production in hadronic collisions. Section V finally contains our conclusions.

II. Squarks and Gluinos

As already stated our analysis rests on the following observations:

- Perturbative QCD is sufficient to treat threshold production of heavy particles as long as the decay width of these particles is large enough.
- These strong final state interactions are insensitive to the flavour structure of the produced particles.
- They depend however very much on the colour charges carried by these objects and on their relative orbital angular momenta.

Accordingly we are interested here only in rather generic features of SUSY, namely

- the overall mass scale for SUSY particles;

- the pattern of their mass differences;
- the rough order of magnitude for their decay rates.

These three aspects will be addressed now:

If SUSY is to protect us against fine tuning the gauge hierarchy in an unacceptable way, a host of sparticles has to exist with masses below 1 TeV, and gluinos and squarks are certainly among them. The squarks actually enter in many varieties: left- and right-handed fermions each have scalar partners in SUSY, $\tilde{f}_L$ and $\tilde{f}_R$.

From the so far unsuccessful SUSY searches at colliders one infers the following 90 % C.L. lower bounds on various masses^[4]:

$$m(\tilde{g}) \geq 135 - 150 \text{ GeV} \quad (3)$$

$$m(\tilde{q}) \geq \begin{cases} 170 \text{ GeV} & \text{for } m(\tilde{g}) < m(\tilde{q}) \\ 130 \text{ GeV} & \text{for } m(\tilde{q}) < m(\tilde{g}) \leq 400 \text{ GeV} \end{cases} \quad (4)$$

In inferring (4) from collider data it was assumed that all squarks are practically mass degenerate. This is a sound assumption for the u, d, s, c and even b squarks as seen in the following way^[5]: The 2×2 mass matrices for these scalar fields in general contain off-diagonal elements which are proportional to the corresponding fermion mass. Since the overall SUSY breaking scale M_0 that sets also the scale for the sfermion masses (and which is usually identified with the gravitino mass) is of order M_W the mixing between "left-" and "right-handed" sfermions $\tilde{f}_L$ and $\tilde{f}_R$ is rather insignificant for all known fermions thus making these sfermions practically degenerate in mass.

For the partners of top quarks on the other hand such mixing could become quite significant and lead to a large splitting in mass for the two stop mass eigenstates $\tilde{t}_+$ and $\tilde{t}_-$ which in the simplest version reads^[6]:

$$m(\tilde{t}_{\pm}) \sim |m_t \pm M_0| \quad (5)$$

Since as already stated

$$M_0 \sim \mathcal{O}(M_W) \quad (6)$$

one concludes that the lightest squark will be one of the stop fields (in its three color copies)

$$m(\tilde{t}_-) < m(\tilde{f}) < m(\tilde{t}_+) \quad (7)$$

$f = u, d, s, c, b, \text{ leptons.}$

The relation (5) should be seen as a semi-quantitative guideline only. Even in minimal SUSY the two stop masses depend on five parameters^[5]; this will not change the general hierarchy stated in (7), yet allows for a wide range in stop masses. For our subsequent discussion we only invoke the expectation that one of the squark mass

eigenstates, presumably one of the stop states, will be substantially lighter than the others. Even a "low" mass like $m(\tilde{t}) \leq 100 \text{ GeV}$ is still allowed.

It is intriguing to note that invoking SUSY to provide a solution to the dark matter problem in astrophysics - clearly a bold assumption - leads to a further constraint [7]:

$$100 \text{ GeV} \leq m(\tilde{L}) \leq 300 \text{ GeV} \quad (8)$$

This is certainly an interesting range since it could be studied at the LHC/SSC and/or a high energy e^+e^- collider like the NLC.

We had already mentioned that the size of the decay width is a crucially important ingredient in our analysis, since it provides an *infrared cut-off* for our perturbative computations; more specifically *if the decay width exceeds 1 GeV or so, the perturbative result can be trusted.*

(a) The non-top sfermions will presumably decay predominantly by emitting the lightest neutralino, denoted by $\tilde{\chi}_1^0$

$$\tilde{f} \rightarrow f + \tilde{\chi}_1^0 \quad (9)$$

yielding

$$\Gamma(\tilde{f}) \sim \alpha m(\tilde{f}) \quad (10)$$

thus meeting the requirement of a sufficiently large width. (This would be even more the case if the strong decay $\tilde{q} \rightarrow q + \tilde{g}$ were kinematically allowed) The same general situation would hold if these sfermions decayed instead into charginos: $\tilde{U} \rightarrow D \tilde{\chi}_1^+$, $\tilde{D} \rightarrow U \tilde{\chi}_1^-$.

(b) As far as the decay of the lightest stop is concerned no such unequivocal statement can be made. For a crucial distinction is provided by the relative size of $m(\tilde{t})$ vs. $m(t)$.

(i) If

$$\tilde{t} \rightarrow t \tilde{\chi}_1^0 \quad (\text{or} \quad \tilde{t} \rightarrow b \tilde{\chi}_1^+) \quad (11)$$

are allowed kinematically then again we have $\Gamma(\tilde{t}) \sim \alpha m(\tilde{t})$, i.e. a sufficiently large width.

(ii) Otherwise stop would be rather long lived since it could decay only via

$$\tilde{t} \rightarrow \begin{cases} b \tilde{W} \rightarrow b f \bar{f}' \tilde{\chi}_1^0 \\ \tilde{b} W \rightarrow \end{cases} \quad (12)$$

with its 4-body phase space suppression or via the one-loop process

$$\tilde{t} \rightarrow c \tilde{\chi}_1^0 \quad (13)$$

yielding a typical decay width of $\Gamma(\tilde{t}) \sim 10^{-10} m(\tilde{t})$ [8]!

The analysis to be presented below applies to the threshold production of all squarks (and gluinos) with a decay width in excess of 1 GeV or so. We phrase our discussion more

specifically in terms of stops because, as sketched above, one of the stop states can be expected to be the lightest squark. This lightest squark might actually have a very small width, as explained above under (ii); we will briefly comment on such a situation.

III. Threshold Production of Stop in e^+e^- Annihilation

The production near threshold even of heavy particles is drastically affected if those particles can strongly interact.^[16] Among other things the two coloured particles can form bound states like the quarkonium resonances that show up so spectacularly especially in e^+e^- annihilation. To describe these phenomena interquark potentials have been employed that – though certainly *inspired* by QCD – are not (yet) *derived* from it and thus are *phenomenological* in nature. However two of us (V.F. and V.K.) obtained the following result:

- The production of heavy coloured particles near threshold can be described in terms of the *Schrödinger Green's function* for the relative motion of the produced particles.
- The *decay width* of these coloured particles generates an *infrared cut-off* for the production process. Therefore if the width exceeds one GeV or so then the threshold cross section is controlled by the *short distance* behavior of this Green's function and thus can be *computed reliably within perturbative QCD*.

Top quarks represent one realization of this scenario, stop quarks another. To be more specific let us concentrate at first on

$$e^+e^- \rightarrow \tilde{t} \tilde{t}^* \quad (17)$$

and compare it with

$$e^+e^- \rightarrow t \bar{t} \quad (18)$$

The process (18) proceeds in an S wave whereas (17) in a P wave. The production in (17) is thus considerably more suppressed than in (18): in the Born approximation, i.e. when the strong final state interactions are ignored this suppression is (in the zero width approximation) given by $\beta^3(\tilde{t})$ vs. $\beta(t)$ where $\beta(t[\tilde{t}])$ denotes the velocity of the top [stop] relative to the speed of light; the strong final state interactions affect the reactions (17) and (18) differently with some of those features being of a rather subtle nature, as we will see.

The original analysis in ref. 1 has been extended and refined in two respects:

- The more complex role played by strong final state interactions in hadronic top production has been analyzed.^[3]
- The fact that the strong coupling is not fixed, but scale dependant was incorporated in a systematic way by solving the appropriate Schrödinger equation *numerically*.^[2]

In this note we will address the question how stop production in hadronic and in two-photon collisions is affected by these phenomena. However we will not tackle point (ii).

(a) Zero width approximation

The drastic modifications enforced by strong final state interactions on the Born cross section near threshold can – as discussed in detail in ref.1 and 2 – be expressed via Green's functions for the relative motion of coloured particles. Extending this treatment to a P wave final state one obtains in the zero width approximation^[9]

$$\sigma(e^+e^- \rightarrow \tilde{t} \tilde{t}^*; s) = \sigma^{(B)}(e^+e^- \rightarrow \tilde{t} \tilde{t}^*; s) \frac{K_P^{(C=1)}(E)}{K_P^{(B)}(E)} \quad (19)$$

where $\sigma^{(B)}(e^+e^- \rightarrow \tilde{t} \tilde{t}^*, s)$ denotes the Born cross section at C.M. energy $\sqrt{s}$; the second factor in (19) contains two functions of E, the amount of energy that *exceeds* the threshold

$$E = \sqrt{s} - 2m(\tilde{t}) \quad (20)$$

These functions are

$$K_P^{(B)}(E) = \frac{m(\tilde{t})}{4\pi} (m(\tilde{t}) E)^{3/2} \quad (21)$$

and

$$K_P^{(C=1)}(E) = \text{Im} \left\langle \frac{\partial}{\partial \vec{r}} \frac{\partial}{\partial \vec{r}'} G_E^{(1)}(\vec{r}, \vec{r}') \right\rangle_{\vec{r} = \vec{r}' = 0} \quad (22)$$

where $G_E^{(1)}$ is the Schrödinger Green's function for the relative motion of the produced particles in a colour singlet state:

$$G_E^{(1)}(\vec{r}, \vec{r}') = \langle \vec{r} | (H^{(1)} - E - i\epsilon)^{-1} | \vec{r}' \rangle \quad (23)$$

The derivatives in (22) project out the $l=1$ component in the Green's function and the averaging indicated there is performed over the angles between $\vec{r}$ and $\vec{r}'$; this avoids uncertainties that are encountered in evaluating first $\partial^2 G_E(\vec{r}, \vec{r}') / \partial \vec{r} \partial \vec{r}'$ at $\vec{r} = \vec{r}' = 0$. Note at this point that (21) represents the Born approximation to (22).

The non-relativistic Hamilton operator $H^{(1)}$ contains the potential for colour singlet configurations of the $(\tilde{t} \tilde{t}^*)$ system

$$H^{(1)} = -\frac{\nabla^2}{m(\tilde{t})} + V^{(1)}(\vec{r}) \quad (24)$$

Next we anticipate a result that will be described in subsect.(b), namely that a *finite decay width* for the squarks that are being studied here provides an infrared cut-off; the Green's functions are then dominated by their short distance behavior and can thus be calculated *perturbatively*. The leading contribution is actually provided by a series of ladder diagrams

containing multiple gluon exchanges between the stop fields. Summing them up is equivalent to inserting an *attractive* color Coulomb potential

$$V^{(1)}(\vec{r}) = -\frac{4}{3} \frac{\alpha_S}{r} \quad (25)$$

for colour triplet particles as it is the case for (17, 18). [There exist non-leading non-Coulombic corrections to this potential ansatz; we will however ignore them in this paper.]

Thus one can express $G_E^{(1)}(\vec{r}, \vec{r}')$ in terms of the familiar eigenfunctions $\psi_{nlm}(\vec{r})$ of the Hamiltonian operator $H^{(1)}$

$$G_E^{(1)}(\vec{r}, \vec{r}') = \sum_{l,m} \left\{ \sum_{n=l+1}^{\infty} \frac{\psi_{nlm}(\vec{r}) \psi_{nlm}^*(\vec{r}')}{E_{nl} - E - i\epsilon} + \int_0^{\infty} \frac{dk}{2\pi} \frac{\psi_{klm}(\vec{r}) \psi_{klm}^*(\vec{r}')}{\frac{k^2}{m} - E - i\epsilon} \right\} \quad (26)$$

Performing the operations stated in (22) we then obtain

$$K_P^{(1)}(E) = \frac{9}{4\pi} \left\{ \theta(E) \frac{1}{4\beta_{\tilde{t}}} \left| \left(\frac{\partial}{\partial r} R_{k1}^{(1),\text{cont.}} \right) (0) \right|^2 + \pi \sum_{n=2}^{\infty} \delta(E - E_n) \left| \left(\frac{\partial}{\partial r} R_{n1}^{(1),\text{bound}} \right) (0) \right|^2 \right\} \quad (27)$$

where a rather compact notation has to be explained:

$$\beta_{\tilde{t}} = \sqrt{1 - 4 m^2(\tilde{t})/s} = \sqrt{E/m(\tilde{t})} \quad (28)$$

denotes the velocity of the produced stops and

$$E_n = -\frac{p_s^2}{mn^2} \quad (29)$$

the binding energy for the n -th $(\tilde{t}\tilde{t})$ bound state; furthermore

$$\left(\frac{\partial}{\partial r} R_{n1}^{(1),\text{bound}} \right) (0) = \frac{2}{3n^3} \sqrt{\frac{(n+1)!}{(n-2)!}} |p_s|^{5/2} \quad (30)$$

with $R_{n1}^{(1),\text{bound}}$ being the radial wave function of the n -th *bound* state with $l=1$ [thus the derivative in (27, 30)] and

$$p_s = \frac{2}{3} \alpha_S m(\tilde{t}) \quad (31)$$

Finally

$$\left(\frac{\partial}{\partial r} R_{k1}^{(1),\text{cont.}} \right) (0) = \frac{2}{3} \frac{k p_s}{k^2} |\psi_{(1)}(0)| \left(1 + \frac{k^2}{p_s^2} \right)^{1/2} \quad (32)$$

here $R_{k1}^{(1),\text{cont.}}$ is the radial wavefunction of a *continuum* state with $k = \sqrt{mE}$ and $\psi_{(1)}^{(0)}$ the wave-function at the origin for a colour singlet S wave state, see ref.1,3,9:

$$|\Psi_{(1)}(0)|^2 = \frac{X_{(1)}}{1 - \exp(-X_{(1)})}, \quad X_{(1)} = \frac{2\pi p_s}{k} \quad (33)$$

It is easily seen that the production of P wave bound states is indeed suppressed by a factor $(p_s/m)^2$ relative to the S wave case. At or just above production threshold, namely for

$$E \ll \frac{p_s^2}{m} \quad (34)$$

there is a *finite* cross section since the colour Coulomb exchanges in the final state compensate for the kinematical threshold factor β^3 contained in the Born cross section of eq. (19):

$$\beta_t^3 \frac{K_P^{(1)}(E)}{K_P^{(B)}(E)} \rightarrow (2\pi) \left(\frac{p_s}{m}\right)^3 = \frac{16\pi}{27} \alpha_s^3 \quad (35)$$

The corresponding phenomenon in QED, namely how the production of charged leptons is affected by Coulomb attraction was first discussed by A.Sakharov, ref.17.

Eq.(27) can be rewritten as follows:

$$K_P^{(1)}(E) = (p_s^2 + mE) \frac{m^2}{4\pi} \left\{ \theta(E) \left[\beta_t + \frac{\pi p_s}{m} + \frac{2p_s^2}{m^2} \sum_{n=1}^{\infty} \frac{\sqrt{mE}}{n^2 \left(E + \frac{p_s^2}{mn^2}\right)} \right] + \frac{4\pi p_s^3}{m^2} \sum_{n=1}^{\infty} \delta\left(E + \frac{p_s^2}{mn^2}\right) \frac{1}{n^3} \right\} \quad (36)$$

Both expressions (36) and (27) clearly exhibit two contributions:

- (i) The first one which is proportional to the step function $\theta(E)$ represents *open stop* production;
- (ii) the second one containing a sum over δ functions reflects *stoponium* formation.

It should be pointed out that there already exists a considerable literature on stoponium formation in e^+e^- annihilation where "QCD inspired" phenomenological potentials are introduced. A detailed recent discussion can be found in ref.10. Due to the slow opening of the P wave production channel one finds that the effects of squarkonia formation are much less spectacular than for quarkonia production in e^+e^- annihilation. The rate for the production of a stoponium resonance with principal quantum number n due to γ exchange is given by (in the *narrow width approximation*)

$$R_{\text{peak}}^{(\gamma)}([\tilde{t}\tilde{t}]_n) = \frac{\sigma_{\text{peak}}(e^+e^- \rightarrow \gamma^* \rightarrow [\tilde{t}\tilde{t}]_n)}{\sigma_{\text{QED}}} \approx \frac{4}{3} \frac{p_s^5}{m^4(\tilde{t}) \Gamma(\tilde{t})} \left(1 - \frac{1}{n^2}\right) \frac{1}{n^3} \eta_R = 3 \left(\frac{2}{3}\right)^7 \alpha_s^5 \frac{m(\tilde{t})}{\frac{1}{2} \Gamma([\tilde{t}\tilde{t}]_n)} \eta_R \left(1 - \frac{1}{n^2}\right) \frac{1}{n^3} \quad (37a)$$

where the usual QED corrections have been lumped into the factor η_R (see ref.1 for details),

$$\sigma_{\text{QED}} = \frac{4\pi\alpha^2}{3s}$$

denotes the QED cross section for $e^+e^- \rightarrow \mu^+\mu^-$ and $\Gamma([\tilde{t}\tilde{t}]) \sim 2\Gamma(\tilde{t})$ was used This is to be compared with the corresponding expression for the (S wave) production of toponium

$$R_{\text{peak}}^{(\gamma)}([\tilde{t}\tilde{t}]_{n=1}) \approx \alpha_S^3 \frac{m(\tilde{t})}{\Gamma(\tilde{t})} (2.3\eta_R) \quad (38a)$$

Similarly one finds for the γ exchange contribution to the continuum production right at threshold, i.e. for $\beta \ll \pi \alpha_S$

$$R^{(\gamma)}(\tilde{t}\tilde{t}) \approx \frac{16\pi}{81} \alpha_S^3 \eta_R \quad (37b)$$

versus

$$R^{(\gamma)}(\tilde{t}\tilde{t}) \approx \frac{8}{3} \alpha_S \pi \eta_R \quad (38b)$$

Our treatment can be generalized in a straightforward way to describe other colour configurations denoted by T: one inserts the appropriate colour Coulomb potential into (24)

$$H^{(T)} = -\frac{\nabla^2}{m(\tilde{t})} + V^{(T)}(\tilde{r}), \quad V^{(T)}(\tilde{r}) = c^{(T)} \frac{\alpha_S}{r} \quad (39)$$

with $c^{(T)}$ being the corresponding colour Casimir coefficient; e.g. for colour singlet and octet configurations formed from colour triplet [octet] stop [gluino] fields one has

$$c^{(1)}(\tilde{t}\tilde{t}) = -\frac{4}{3} \quad (40a)$$

$$c^{(8)}(\tilde{t}\tilde{t}) = \frac{1}{6} \quad (40b)$$

$$c^{(1)}(\tilde{g}\tilde{g}) = -3 \quad (40c)$$

Following the analogous procedure one arrives at

$$K_P^{(T)}(E) = (p_T^2 + mE) K_S^{(T)}(E) = (p_T^2 + mE) \text{Im } G_E^{(T)}(0, 0) \quad (41)$$

where

$$p^{(T)} = -\frac{1}{2} c^{(T)} \alpha_S m(\tilde{t} \text{ or } \tilde{g}) \quad (42)$$

and the function $K_S^{(T)}$ refers to the case of S wave production. The explicit expressions for $G_E^{(T)}$ can be found in ref.1. One should note that the potential is *repulsive* for colour octet configurations, see (40b); *no* resonances can then form.

We have thus obtained a result that is reminiscent from positronium production in QED^[12] and can be generalized as follows:

- The cross section for the production of l wave states made up of colored particles is suppressed by a factor $\sim (p_s/m)^{2l+1}$
- yet even at threshold the cross sections are finite!

(b) Finite width effects

As already stressed the exclusive reliance on perturbation theory (even when summing over an infinite set of diagrams) requires the presence of a finite decay width to intervene as infrared cut-off. To be more specific only if $\Gamma(\tilde{t})$ is large enough does the calculation of the Green's function in eqs. (19) involve short distances only. It was shown in refs. 1-3 that there exists a rather simple prescription for taking proper account of finite width effects in S wave production of color objects, namely to replace E by $E+i\Gamma$ in the Green's function. The analogous prescription is adopted for P wave production:

$$K_P^{(1)}(E) = (p_s^2 + mE) K_S^{(1)}(E) + \left(\frac{m^2 \Gamma}{4\pi} \right) \left\{ -p_1 + 2 p_s \log \left| \frac{p_s}{p_1 - ip_2} \right| - \frac{2 p_s^2}{m} \sum_{n=1}^{\infty} \frac{1}{n^3} \frac{(n p_1 + p_s)(E + p_s^2/mn^2) - p_2 \Gamma}{(E + p_s^2/mn^2) + \Gamma^2} \right\} \quad (43)$$

with

$$p_{1,2} = \left[\frac{m}{2} \left(\sqrt{E^2 + \Gamma^2} \mp E \right) \right]^{1/2} \quad (44)$$

This expression was derived from perturbative QCD by relying on the decay width Γ as an infrared cut-off. Yet once it has been obtained one can consider its behaviour in the narrow width approximation: not surprisingly one recovers the result given in eq.(37a).

There arises however one *qualitatively new feature* here: *the non-relativistic approximation is, strictly speaking, no longer valid for the production of heavy particles in an $l \geq 1$ configuration if those particles possess a finite width.* The basic reason for this phenomenon is easy to see: the momentum for a stable particle near threshold is fixed at $|\vec{p}| = \sqrt{mE}$; on the other hand the width of unstable particles generates a distribution for its invariant mass and thus allows for large momenta; the weight of this region is then *enhanced by the kinematical factor $|\vec{p}|^{2l}$ in the production cross section!*

We will discuss this potential problem now in more detail to see how it can affect our general conclusions: in the phase space integral for *unstable* particles $\delta(p^2 - m^2) \theta(p^0)$ is replaced by a factor

$$\Delta(p^2) = \frac{m\Gamma(p^2)\theta(p^0)}{\pi[(p^2 - m^2)^2 + m^2\Gamma^2(p^2)]} \quad (45)$$

with $\Gamma(p^2)$ defined via the imaginary part of the particle self energy and with the normalization $\Gamma(m^2) = \Gamma$ for the physical width. Taking $\Gamma(p^2)$ actually to be a constant one can approximate $\Delta(p^2)$ in the non-relativistic limit by

$$\Delta_{NR}(p^2) = \frac{\Gamma}{2\pi \left[\left(\epsilon - \frac{|\vec{p}|^2}{2m} \right)^2 + \frac{\Gamma^2}{4} \right] m} \quad (46)$$

with $\epsilon = p^0 - m$. The two-particle phase space element

$$d\rho = \frac{d^4 p_+}{(2\pi)^3} \frac{d^4 p_-}{(2\pi)^3} \Delta(p_+^2) \Delta(p_-^2) (2\pi)^4 \delta(q - p_- - p_+) m^2 \quad (47)$$

is then given by a simple expression in the C.M. frame

$$d\rho_{NR} = \frac{d^3 p}{(2\pi)^3} \frac{2\Gamma}{\left| \frac{|\vec{p}|^2}{m} - E - i\Gamma \right|^2} \quad (48)$$

The integral $\int d\rho |\vec{p}|^{2l}$ yields the Born threshold function $K_l^{(B)}(E)$

$$K_l^{(B)}(E) = \frac{m}{4\pi} (mE)^{l+1/2} \quad (49)$$

which was already introduced in (21) for the P wave case; however there arises a *new* contribution $\tilde{K}_1^{(B)}$ that is *proportional to Γ* , comes from the region of *large momenta*

$$\tilde{K}_1^{(B)} \sim \frac{\Gamma m^2}{\pi^2} \int d|\vec{p}| |\vec{p}|^{2(1-l)} \quad (50)$$

and accordingly is controlled by $p_{\max}$, the upper limit of integration in (50). Eq. (50) thus represents a statement of the problem.

In a fully relativistic treatment one has to include the p^2 dependance of $\Gamma(p^2)$ which will in turn determine $p_{\max}$. The explicit expression for $\Gamma(p^2)$ depends on which decay channels are relevant – a topic that is beyond the scope of this paper.

Instead we will estimate the weight of these contributions in a more global fashion for $l = 1$. For P wave production *only the Born* function $\tilde{K}_p^{(B)}$ contains a term proportional to $p_{\max}$, see (50). The first Coulomb correction introduces merely a logarithmic dependance on $p_{\max}$ and the higher order Coulomb corrections are insensitive to large momenta.⁽⁹⁾ The main deviation of $K_p^{(1)}(E)$ from the non-relativistic result of (43) is thus given by the *Born* function $\tilde{K}_p^{(B)}$ and largely insensitive to the strong final state interactions.

Kinematics relates $p_{\max}$ to the minimal invariant mass of the decay products, referred to as μ , i.e. $p^2 > \mu^2$:

$$p_{\max}^2 \equiv \frac{(9m^2 - \mu^2)(m^2 - \mu^2 + 2mE)}{16m^2} \quad (51)$$

This quantity can be large compared to the characteristic scale $\sim |mE|$ – and only then would it be of concern to us – if

$$(m - \mu) \gg |E| \quad (52)$$

This leads to a considerable simplification in the relativistic expression for the phase space element dp :

$$dp = \frac{d^3 p}{(2\pi)^3} \frac{2m\Gamma(p^2)}{\epsilon_p [(2m + E - 2\epsilon_p)^2 + \Gamma^2(p^2)]} \quad (53)$$

with $E \ll M$, $\epsilon_p = \sqrt{m^2 + |\vec{p}|^2}$ and

$$p^2 = m^2 - 2m(2\epsilon_p - 2m - E) \quad (54)$$

For because of (52) one can ignore the dependance on E in (51, 53, 54) when computing the large momentum contribution to $\tilde{K}_p^{(B)}$:

$$|\vec{p}|^2 dp \equiv \frac{(9m^2 - p^2)^{3/2}}{(2\pi)^2} \frac{\Gamma(p^2) dp^2}{4m(m^2 - p^2)^{1/2}} \quad (55)$$

with $\mu^2 < p^2 < m^2$. As already stated using (55) and an explicit expression for $\Gamma(p^2)$ one can compute the large momentum contribution for any concrete decay channel. Lacking such knowledge on $\Gamma(p^2)$ we assume Γ can be expanded around μ^2 with $\Gamma(\mu^2) = 0$

$$\Gamma(p^2) \sim \left(\frac{p^2 - \mu^2}{m^2} \right)^\kappa \Gamma_0 \quad (56)$$

Inserting this ansatz into (55) we finally arrive at

$$\tilde{K}_p^{(1)}(E) = \tilde{K}_p^{(B)}(1 + \mathcal{O}(\alpha_s)) \sim \frac{m^3 \Gamma}{\pi^2} \left(1 - \frac{\mu^2}{m^2} \right)^{\kappa + 1/2} \quad (57)$$

Comparing this result with (43) we conclude that the non-relativistic expression of (43) provides a good description of threshold phenomena

- at all energies E if

$$\frac{m^2 \Gamma_0}{\pi} \left(1 - \frac{\mu^2}{m^2} \right)^{\kappa + 1/2} \ll (p_s)^3 \quad (58a)$$

holds or

- at energies sufficiently above threshold s.t.

$$\frac{m^2 \Gamma_o}{\pi} \left(1 - \frac{\mu^2}{m^2}\right)^{\kappa + 1/2} \ll (mE)^{3/2} \quad (58b)$$

is satisfied.

One should also note the following and this might actually be the most important point here: $\tilde{K}_p^{(1)}$, the contribution from large momenta, is *independent of E*, as discussed above eq. (55). Therefore even if $\tilde{K}_p^{(1)}$ were not small, one can nevertheless rely on (43) to describe *correctly the energy dependance* of the production cross section near threshold!

(c) A dramatic scenario: $e^+e^- \rightarrow \tilde{g}\tilde{g}$

In principle the reaction

$$e^+e^- \rightarrow Z^0 \rightarrow \tilde{g}\tilde{g}$$

could proceed via a one-loop process involving the axial vector part of the Z^0 coupling (charge conjugation symmetry forces the vector coupling to $\tilde{g}\tilde{g}$ to vanish identically).^[11] Gluon exchanges yield an attractive (colour) Coulomb potential that is quite strong due to the large colour charge of gluinos (and gluons); namely $c^{(1)}(\tilde{g}\tilde{g}) = -3$

$$V_{\tilde{g}\tilde{g}}^{(1)}(r) = -\frac{3}{r} \alpha_s \quad (59a)$$

$$p_{\tilde{g}\tilde{g}}^{(1)} = \frac{3}{2} \alpha_s m(\tilde{g}) \quad (59b)$$

$$E_n = -\frac{9}{4} \alpha_s^2 \frac{m(\tilde{g})}{n^2} \quad (59c)$$

Strong final state interactions then substantially enhance the production rate at or near threshold as can be seen from the analog of eq. (35) [in the zero width approximation]:

$$\beta_{\tilde{g}}^3 \frac{K_p^{(1)}(\tilde{g}\tilde{g}; E)}{K_p^{(B)}(\tilde{g}\tilde{g}; E)} \rightarrow \frac{27\pi}{4} \alpha_s^3 \quad (60)$$

This would be a dramatic effect indeed relative to lowest order expectations – alas, presumably quite an academic one, too, in view of the lower bound $m(\tilde{g}) \geq 135$ GeV that is inferred from negative searches for gluinos in hadronic collisions.

IV. Hadronic Production of Stops (and Gluinos)

Hadronic collisions provide two different mechanisms for producing squarks, namely gg and $q\bar{q}$ fusion. Their contributions to the invariant mass distributions of the $t\bar{t}$ pair take the usual form^[13]

$$\frac{d\sigma_{gg}}{d\tau} = \sigma(gg \rightarrow \tilde{t}\tilde{t}^* ; \hat{s}) \frac{d\mathcal{L}_{gg}}{d\tau} \quad (61)$$

$$\frac{d\sigma_{q\bar{q}}}{d\tau} = \sigma(q\bar{q} \rightarrow \tilde{t}\tilde{t}^* ; \hat{s}) \frac{d\mathcal{L}_{q\bar{q}}}{d\tau} \quad (62)$$

where $d\mathcal{L}_{gg, q\bar{q}}/d\tau$ denote the corresponding parton luminosities with

$$\tau = \frac{\hat{s}}{s} ; \hat{s} = (2m + E)^2 \quad (63)$$

$\sqrt{s}$ is the total CM energy and $\sqrt{\hat{s}}$ the invariant mass of a $\tilde{t}\tilde{t}^*$ pair.

The Born cross sections (averaged over the initial parton polarizations) are for the two subprocesses near threshold^[13]

$$\sigma_{thr}^{(B)}(q\bar{q} \rightarrow \tilde{t}\tilde{t}^* ; \hat{s}) = \frac{1}{54} \beta_{\tilde{t}}^3 \frac{\pi \alpha_s^2}{m^2} \quad (64)$$

$$\sigma_{thr}^{(B)}(gg \rightarrow \tilde{t}\tilde{t}^* ; \hat{s}) = \frac{7}{384} \beta_{\tilde{t}} \frac{\pi \alpha_s^2}{m^2} \quad (65)$$

with $\beta_{\tilde{t}} = \sqrt{1 - 4m_{\tilde{t}}^2/\hat{s}} = \sqrt{E/m_{\tilde{t}}}$.

Near threshold the $q\bar{q} \rightarrow \tilde{t}\tilde{t}^*$ mechanism is obviously greatly suppressed since the final pair is necessarily produced in a P wave; the $gg \rightarrow \tilde{t}\tilde{t}^*$ process on the other hand yields a pair with spin-parity 0^- , i.e. S Wave, for incoming gluons with total spin $S_{gg} = 0$ and relative orbital angular momentum $L_{gg} = 0$.

Strong final state interactions further strengthen this trend.

(1) $q\bar{q}$ fusion necessarily yields $\tilde{t}\tilde{t}^*$ in a colour octet state (as was the case for $t\bar{t}$ production). Coulomb exchanges then generate a *repulsive* force since

$$c^{(8)}(\tilde{t}\tilde{t}^*) = \frac{1}{6} \quad (66)$$

in eq. (37) leading to an additional suppression in the production cross section.

(2) Again in analogy to the $t\bar{t}$ case gg collisions generate $\tilde{t}\tilde{t}^*$ in a *mixture* of colour octet and singlet configurations. The ratio of the Born cross sections for octet over singlet production is given by properly normalized factors

$$\frac{\sigma^{(B)}(gg \rightarrow [\tilde{t}\tilde{t}^*]_8)}{\sigma^{(B)}(gg \rightarrow [\tilde{t}\tilde{t}^*]_1)} = \frac{(d^{abc}/\sqrt{2})^2}{(\delta^{ab}/\sqrt{3})^2} = \frac{5}{2} \quad (67)$$

[The agreement of this ratio with the corresponding one in $gg \rightarrow t\bar{t}$ is due to the identical colour group structure.]

From this discussion it follows in a straightforward way:

- Since the reaction

$$gg \rightarrow \tilde{t}\tilde{t}^* \quad (68)$$

can be treated in complete analogy to

$$gg \rightarrow t \bar{t} \quad (69)$$

the threshold factors are the same in both cases and can be found in ref.3.

- Yet

$$q\bar{q} \rightarrow \tilde{t} \tilde{t}^* \quad (70)$$

is *strongly reduced* due to P wave suppression and a *repulsive* colour Coulomb force. Nevertheless for completeness we give the expression for threshold production

$$\sigma^{(8)}(q\bar{q} \rightarrow \tilde{t} \tilde{t}^*) = \sigma^{(B)}(q\bar{q} \rightarrow t\bar{t}) \frac{K_P^{(8)}(E + i\Gamma)}{K_P^{(B)}(E)} \quad (71)$$

in analogy to (19). The explicit expression for $K_P^{(8)}$ is obtained from (43) by replacing

$$p_s \rightarrow p^{(8)} = -\frac{1}{12} m_t^2 \alpha_s \quad (72)$$

see (42). In close analogy to $q\bar{q} \rightarrow t\bar{t}$ one can rewrite $K_P^{(8)}$ in a form where the spurious

singularities at $E_n = -\frac{p^{(8)2}}{mn^2} \pm i\Gamma$ are removed

$$K_P^{(8)}(E + i\Gamma) = (p_{(8)}^2 + mE) K_S^{(8)}(E + i\Gamma) + \frac{m^2\Gamma}{4\pi} \left\{ -p_1 + 2p_{(8)} \log \left| \frac{p_{(8)}}{p_1 - ip_2} \right| - \frac{2p_{(8)}^2}{m} \sum_{n=1}^{\infty} \frac{p_{(8)} - np_1}{n[(np_1 - p_{(8)})^2 + p_2^2 n^2]} \right\} \quad (73)$$

where

$$K_S^{(8)}(E + i\Gamma) = \text{Im } G_{E+i\Gamma}^{(8)}(0, 0) \quad (74)$$

Possible deviations from this *non-relativistic* result can be treated as discussed above in Sect. III(b).

Gluinos will most likely be observed first in hadronic collisions; their threshold production can be treated in close analogy to the discussion given above.

V. Summary

The *threshold* production of strongly interacting heavy particles can be predicted *reliably by perturbative QCD* (although one has to go beyond the lowest order) if the decay width of these particles is sufficiently large, namely $\Gamma \geq 1 \text{ GeV}$ or so. For Γ acts as an infrared cut-off. This general feature which was first pointed out for top production is valid also for stop and gluino production near threshold.

On the quantitative level, there are however several significant differences in particular between

$$e^+e^- \rightarrow t\bar{t} \quad (75)$$

and

$$e^+e^- \rightarrow \tilde{t}\bar{\tilde{t}} \quad (76)$$

The most obvious one is the absence of a boundstate with $n = 1$ and a much weaker manifestation of the other bound states in (76) due to the P wave suppression which introduces an additional factor of $(p_s^2 + mE)/m^2$, see eqs.(37a) vs. (38a). Yet there is also a more subtle new feature appearing in (76) relative to (75): the finite decay width which is essential for justifying the reliance on perturbative QCD leads to the emergence of contributions in the threshold production of P wave (or in general $l \geq 1$) states that are *outside a strictly nonrelativistic ansatz*. To predict the actual size of these new contributions more dynamical information on specific decay channels is required than presently available. We have however presented criteria under which these new terms would be numerically insignificant. Secondly we have shown that the energy *dependence* of the production cross section – in contrast to the *absolute* scale – can still be computed reliably within the non-relativistic Coulomb ansatz.

This already shows that we have outlined here a general procedure rather than given the full numerical answers. There are three other areas where improvements can and will be made in due course:

- We have used a fixed value for α_s rather than a "running" value. It appears quite natural to employ α_s as it is probed at the Coulomb scale. However a more systematic approach that would also test the significance of the next-to-leading contributions would be quite desirable. Extending the numerical analysis that was presented in ref.2 to P wave production - while not straightforward - might be feasible for implementing the running of α_s here.
- A priori further significant forces can be generated by the exchange of Higgs fields. We have however refrained from including these effects for "practical" reasons: too little definitive is known about the Higgs sector even in minimal SUSY beyond the statement that there have to exist at least two Higgs doublets. In particular the masses of the various Higgs states are quite undetermined and it is not known whether they exceed $m(\tilde{t})$ or not. This has important technical consequences: if

$$m(\text{Higgs}) < m(\tilde{t})$$

Higgs exchanges can effectively be described by a Yukawa potential; yet for

$$m(\text{Higgs}) > m(\tilde{t})$$

this ceases to be true since relativistic effects can become significant thus invalidating a pure potential ansatz^[14].

- For the final analysis one has to include corrections due to initial state radiation. This can be done using the standard procedures^[1,2].

Finally while squark and gluino production will presumably - if ever - be first discovered in hadronic collisions, two-photon collisions

$$e^+ e^- \rightarrow e^+ e^- \tilde{t} \tilde{t}^*$$

might provide the best and richest laboratory to study the threshold production of squarks^[15].

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References

- [1] V.Fadin and V.Khoze, JETP Lett.46 (1987) 525; Sov.J.Nucl.Phys.48 (1988) 309; Proceed.of the 24th LNPI Winter School, Leningrad, Vol. 1, p. 3.
- [2] M.Strassler and M.Peskin, Phys.Rev.D43 (1991) 1500, with references to other related work.
- [3] V.Fadin, V.Khoze and T.Sjostrand, Z.Phys.C48 (1990) 613.
- [4] see for example: W.Majerotto, talk given at LPHEP-91, Geneva, July 1991.
- [5] see for example: H.Haber and G.Kane, Phys.Rep.117 (1985) 75.
- [6] J.Ellis and S.Rudaz, Phys.Lett.B 128 (1983) 248.
- [7] J.Ellis and F.Zwirner, Nucl.Phys.B 338 (1990) 317.
- [8] H.Hikasa and M.Kobayashi, Phys.Rev.D 36 (1987) 724; I.Bigi and S.Rudaz, Phys.Lett.B 153 (1985) 335.
- [9] The discussion here parallels that for P wave production of top quarks, see: V.Fadin and V.Khoze, Yad.Fiz. 54 (1991) 1118.
- [10] K.Hagiwara et al., Nucl.Phys.B344 (1990) 1.
- [11] P.Nelson and P.Osland, Phys.Lett.115B (1982) 407; B.Campbell et al., Phys.Lett.126B (1983) 376; G.Kane and W.Rolnick, Nucl.Phys.B217 (1983) 117.
- [12] J.Schwinger: "Particles, Sources and Fields", Addison-Wesley, Reading, 1970, Vol.II.
- [13] S.Dawson, E.Eichten and C.Quigg, Phys.Rev.D31 (1985) 1581.
- [14] V.Fadin and O.Yakolev, preprint NPI-90-138, to be published in Yad.Fiz.; see also: W.Beenakker and W.Hollik, preprint CERN-TH-6185/91, 1991; R.J.Guth and J.H.Kuehn, Karlsruhe preprint TKP 91-3, 1991.
- [15] I.Bigi and F.Gabbiani, preprint UND-HEP-91-BIG05.
- [16] For an authoritative review see: J.H.Kuehn and P.M.Zerwas, Phys.Rep.167(1988) 321.
- [17] A.Sakharov, JETP 18 (1948) 631.