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IS YOUR WEIGHT GAUGE DEPENDENT ? *

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ABSTRACT

In the presence of a continuous degeneracy of stationary points of the effective action the standard proofs of the gauge independence of the S -matrix become inapplicable. This explains the apparent lack of gauge independence of physical quantities in (perturbative) quantum gravity found in explicit calculations. We show that the S -matrix is still gauge independent when expressed in terms of physically relevant parameters.

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Clearly, as long as all connected diagrams are well defined, at least in the context of some suitable regularization, the gauge independence of the (perturbative) S -matrix is beyond doubt. However, in certain cases there are diagrams that are not well defined, irrespective of the regularization scheme. What we have in mind here are so-called tadpole diagrams associated with massless fields: diagrams with a single external propagator line, which tend to diverge due to the singular behaviour of the massless zero-momentum propagator. Such diagrams are not only encountered in the calculation of scattering amplitudes, but also in various diagrammatic identities. For instance, in the Ward identity that governs the change of the S -matrix, massless ghost fields can give rise to such tadpole diagrams. Another example is quantum gravity, where tadpole diagrams can occur with a single external graviton line. Of course, we are not concerned here about fields that are massless just because of a special choice of the parameters or of the value of the fields about which one has chosen to expand the effective action. In those cases one can usually choose a different set-up, which would avoid the ambiguous terms. What concerns us here is the situation where the quantum effective action vanishes for certain fields taking arbitrary constant values. In that case these fields are obviously massless and the one-particle irreducible diagrams with zero-momentum external lines (corresponding to these fields) will vanish as well. Hence we are dealing with a continuous degeneracy in the stationary points of the effective action. For convenience we will remain in the context of a flat-space background, which is not always the situation one encounters in quantum gravity. However, we should stress that, in principle, our arguments do not depend on the precise nature of the degeneracy.

When the quantum effective action is degenerate for constant fields, then the vacuum-expectation values of these fields are arbitrary parameters, which are not constrained by the dynamics. Obviously the mass associated with these fields must be zero and there are no non-zero diagrams with a single line disappearing into the vacuum. Therefore ill-defined tadpole diagrams associated with these massless fields can be consistently suppressed. On the other hand there are extra free parameters corresponding to the vacuum-expectation values of the massless fields, that characterize the field configurations associated with the possible degenerate ground states. It is easy to write classical Lagrangians that have this property (i.e., Lagrangians with only kinetic terms or "velocity-dependent" potentials, such as non-linear sigma models), although usually this feature will not be preserved at the quantum level.

Whether or not different vacuum-expectation values will correspond to an inequivalent realization of the theory is another matter, which has to be studied separately. For gauge theories the ground states that are related by the action of the gauge group are strictly gauge equivalent and one should view the ground state as being non-degenerate. The existence of a continuous degeneracy of the effective action for field configurations associated with this ground state may be a feature that depends on the choice of the gauge condition. The classical action for gauge theories has an obvious degeneracy associated with the possibility of performing gauge transformations. In the quantum theory, most of this degeneracy will disappear. By its very nature

As students in Utrecht in the late Sixties, when we started working on gauge theories, it was unavoidable that we had to acquaint ourselves with Tini Veltman's work on gauge theories. I still remember being struck by two remarkable features. First of all, Tini was not afraid of ghosts. It seemed that he, more than anybody else in the physics community, had accepted the idea that it was perfectly all right to live in an extended version of the physical Hilbert space, just because life was more convenient there. Secondly, he had developed a very personal style in dealing with Feynman diagrams and combinatorics. He freely introduced all sorts of new lines, vertices and even fish into the diagrams¹, was very experienced in cutting graphs, and was a renowned expert in extracting practical results from them. Occasionally, he would confess to us that he thought that diagrams were really much more fundamental than the underlying Lagrangian or Hamiltonian. In those days Tini belonged to a small minority of theorists who still took field theory seriously, and he was practically the only one working on gauge field theory with the ultimate goal of setting up a consistent quantum field theory for the weak interactions².

Looking back it is clear that Tini's methods for dealing with ghosts and diagrams were very instrumental in making the remarkable progress in gauge field theory that was about to take place in subsequent years. Ghosts and diagrams will also play a role in this lecture, which deals with the old problem of how to unambiguously extract physics results from the extended Hilbert space, populated by ghosts and phantoms and what have you. As is well known, this problem is solved by establishing the gauge independence of the S -matrix: one must set up a consistent procedure in order to restrict oneself to the physical subspace and extract the physically relevant information in an unambiguous way, irrespective of the gauge condition that is used.

The gauge independence of S -matrix elements in gauge theories has been proven by many authors (see, for instance, refs. 3-5). The proof makes use of the Ward identities for connected Green's functions and the kinematical structure of these functions. After a truncation of the propagator poles associated with the external lines corresponding to physical particles and an appropriate normalization of each of these lines (wave function renormalization), the result becomes independent of the gauge condition employed in the calculations. Of course, one should be careful and choose an admissible gauge condition, i.e., a gauge condition that breaks the gauge invariance at the perturbative level without inducing a spontaneously broken realization of the BRST invariance of the gauge-fixed action⁶. In principle the proofs apply to generic gauge theories.

the gauge condition breaks the gauge invariance, although it is by no means excluded that some harmless residual invariance remains. An example of such a residual invariance is the subgroup of rigid (i.e., space-time independent) gauge transformations, which is still manifest in many cases. Another example, which is more relevant for the purpose of this paper, is the following. Consider an abelian gauge field, A_μ , subject to standard gauge transformations $A_\mu \rightarrow A_\mu + \partial_\mu \Lambda$. Obviously, the Lorentz gauge, based on the gauge fixing term $F = \xi \partial_\mu A^\mu$, is not affected by gauge transformations satisfying $\partial^2 \Lambda = 0$. In particular, the theory must still be invariant under those transformations that shift the gauge potentials by a constant vector a_μ , i.e.,

$$A_\mu(x) \rightarrow A_\mu(x) + a_\mu. \quad (1)$$

These are the transformations corresponding to $\Lambda(x) = a_\mu x^\mu$. From this residual invariance it then follows that the part of the quantum effective action that depends exclusively on the gauge field and not on the matter fields must depend on the derivatives of the gauge field and not on the gauge field itself. Therefore the effective action vanishes for arbitrary constant gauge fields.*

In fact it is not so essential that (1) corresponds to a residual symmetry. As long as the gauge condition vanishes for constant potentials one may still expect a degeneracy of the effective action for these potentials. To prove this at the quantum level is not entirely straightforward. For instance, take the gauge condition $F(A) = f(A) \partial^\mu A_\mu$, where $f(A)$ is some arbitrary function of the gauge field A_μ . Although this gauge-fixing term still vanishes on the space of constant potentials, it is not, in general, invariant under (1). Therefore one can no longer invoke an invariance argument to show that the effective action is degenerate for constant gauge potentials. Indeed, an elementary one-loop calculation shows that the gauge-field part of the effective action no longer depends exclusively on derivatives of the gauge field. Nevertheless, it remains proportional to such derivatives, so that the effective action still vanishes for constant gauge potentials in accord with more intuitive expectation.

We were motivated to study the question of gauge independence in the presence of a continuous degeneracy of stationary points of the effective action by certain explicit calculations performed some time ago⁷ in quantum gravity, which exhibited an apparent lack of gauge independence for physical quantities such as the mass. These one-loop calculations were performed in the context of dimensional regularization.⁸ No explanation for this unusual phenomenon was offered, but the authors of ref. 7 claimed that it was still possible to regain gauge independence by exploiting certain symmetries of the gauge-fixed action, which were assumed to be typical for gravity. Later on the phenomenon was analyzed in ref. 9, where the analogy with spontaneously broken gauge theories was emphasized in an approach based on the effective action and one-particle irreducible graphs. Just recently Hari Dass and I argued that the degeneracy of the effective action was responsible for this apparent lack of gauge

* The effective action is determined modulo an additive constant, which we choose such that the action vanishes for zero fields.

independence. We showed that in the degenerate case there is indeed a qualitative difference with regard to the way in which the gauge independence of the S -matrix is realized as compared to the standard situation¹⁰. We then established that the phenomenon is not restricted to quantum gravity, but is generic to all theories with a continuous degeneracy of stationary points of the effective action.

In this lecture I shall discuss the work of ref. 10 and derive a Ward identity, in a diagrammatic way just as in ref. 4, that governs the change of the S -matrix under a change of gauge, paying proper attention to the troublesome tadpole-like diagrams that may be present when the effective action exhibits the degeneracy discussed above. Our starting point is the standard quantized action in the presence of external sources J^i ,

$$\mathcal{L}_{\text{qu}} = \mathcal{L}_{\text{inv}} - \frac{1}{2}(F^\alpha)^2 + i\partial^\alpha \frac{\partial \delta F^\alpha}{\partial \xi^\beta} \xi^\beta + J^i \phi^i, \quad (2)$$

where δF^α is the variation of the gauge-fixing term under infinitesimal gauge transformations with parameters ξ^α . Furthermore ξ^α and ϕ^α are the (real) ghost and anti-ghost fields. We use a standard notation where the indices i and α refer not only to the type of field or gauge group generator, but also incorporate their dependence on space-time. The statistics of the ghost and anti-ghost fields is always opposite to the statistics of the corresponding gauge fields; for convenience, we restrict ourselves to bosonic symmetries, so that the (anti-)ghost fields are anticommuting. The infinitesimal gauge transformations of the fields ϕ^i are

$$\delta \phi^i = \{t_\alpha^i + s_\alpha^i(\phi)\} \xi^\alpha, \quad (3)$$

where the quantities t_α^i and $s_\alpha^i(\phi)$ denote the field-independent and field-dependent term. It is straightforward to generalize this notation and include composite operators $Q(\phi)$, which then transform under infinitesimal gauge transformations according to

$$\delta Q(\phi) = \{t_\alpha^Q + s_\alpha^Q(\phi)\} \xi^\alpha. \quad (4)$$

The quantities t_α^Q and $s_\alpha^Q(\phi)$ appear as extra operators in the Ward identity that couple to the ghost fields. The diagrammatic form of this identity, which applies to connected diagrams, is⁴

$$\begin{aligned}
& \left[\text{Diagram 1} \right] + \sum_J \left[\text{Diagram 2} \right] + \left[\text{Diagram 3} \right] = 0, \quad (5)
\end{aligned}$$

where the summation over J indicates that we have to sum over all possible attachments of the ghost line to the external sources. This identity can be directly generalized to more complicated operators coupling to external sources. When the external sources are taken on the mass shell, the last two sets of diagrams cancel. Hence we have

$$\begin{aligned}
& \left[\text{Diagram 4} \right] = 0. \quad (6)
\end{aligned}$$

J on the mass shell

To establish the gauge independence of S -matrix elements one considers the Ward identity (5) in the presence of an extra source coupled to some operator $(\Delta F)^\beta$. The corresponding identity takes the form

$$\begin{aligned}
& \left[\text{Diagram 5} \right] + \left[\text{Diagram 6} \right] + \left[\text{Diagram 7} \right] = 0. \quad (9)
\end{aligned}$$

The procedure of joining the vertices leads to new operators. For the first three diagrams (9) these operators coincide with the terms generated from the quantum action (2) by a change of the gauge-fixing term equal to

$$F^\alpha \longrightarrow F^\alpha + (\Delta F)^\alpha. \quad (10)$$

$$\begin{aligned}
& \left[\text{Diagram 8} \right] + \sum_J \left[\text{Diagram 9} \right] + \left[\text{Diagram 10} \right] = 0. \quad (7)
\end{aligned}$$

The vertices in the second and third diagram originate from the infinitesimal gauge transformation of ΔF , which we defined by

$$\delta(\Delta F)^\alpha = \{t_\beta^\alpha + s_\beta^\alpha(\phi)\}\xi^\beta. \quad (8)$$

Subsequently, one joins the two upper vertices in the diagrams (7), sums over α and β (which implies both a summation over the gauge group generators and an integration over the momentum associated with the newly formed loop, such that the net momentum flowing into the combined vertex is equal to zero). The result reads

$$\begin{aligned}
& \left[\text{Diagram 11} \right] + \left[\text{Diagram 12} \right] + \left[\text{Diagram 13} \right] = 0. \quad (9)
\end{aligned}$$

$$\begin{aligned}
& \left[\text{Diagram 14} \right] + \sum_J \left[\text{Diagram 15} \right] + \left[\text{Diagram 16} \right] = 0. \quad (9)
\end{aligned}$$

The procedure of joining the vertices leads to new operators. For the first three diagrams (9) these operators coincide with the terms generated from the quantum action (2) by a change of the gauge-fixing term equal to

$$F^\alpha \longrightarrow F^\alpha + (\Delta F)^\alpha. \quad (10)$$

Taking into account the proper minus sign for a closed ghost loop, the sum of the operators corresponding to the first three graphs is equal to minus the change of the quantum action (2) under a gauge variation (10). Therefore (9) expresses the change of the connected diagrams under (10) in terms of the last two types of graphs containing the ghost line attachments to the external sources. However, certain contributions are still missing in (9). In the first graph of (9), before the actual joining of the two vertices, the operators F^α and $(\Delta F)^\alpha$ are part of the same connected diagram. There is an alternative situation, which also leads to diagrams that contribute to the gauge variation of the connected diagrams, where a diagram is formed by joining two separate connected diagrams, one containing F^α and the other one $(\Delta F)^\alpha$. Hence one has the following diagrams,

In principle, those diagrams should be added to the diagrams (9), in order to describe completely the change of the connected diagrams. However, because the diagrams (11) vanish on the mass shell by virtue of (6), they will be ignored throughout this paper.

Hence we conclude that, modulo diagrams of type (11) that vanish when the external sources are on the mass shell, $\Delta\Gamma$, the change of the connected diagrams induced by a change of gauge (10) is expressed in terms of the last two sets of diagrams (9), where the ghost lines are attached to the sources J . On the mass shell each of these diagrams leads to a factor ΔJ , which depends on the type of source, multiplying the full amplitude, i.e.,

$$\Delta\Gamma = \sum_J \Delta J \Gamma, \quad (12)$$

where Γ denotes the sum of all connected diagrams with a given configuration of external lines. In the proper normalization of the S -matrix the factors ΔJ are precisely cancelled by the change of the wave-function renormalization constants under the gauge change (10). As these features are standard and do not play a special role in our discussion, we refrain from giving further details and refer to refs. 3-5. We should stress, however, that it is assumed throughout this analysis that the propagators exhibit at most single poles.

This completes the standard proof of the gauge-independence of the S -matrix, where hitherto we have assumed that no ambiguities arise in the definition of the various amplitudes, at least in a regularization scheme such as dimensional regularization⁸.

However, as already pointed out, such ambiguities may arise when dealing with tadpole diagrams corresponding to massless fields. Such tadpole diagrams may be formed in (9) upon joining the two upper vertices. If the top of the diagram is connected to the main body that contains all external source attachments via a single propagator line, then the momentum flowing through this line is forced to zero when the two upper vertices are joined. This is so because joining the two operators implies that we adjust the momenta flowing to the each of these operators, such that the total momentum flowing into the combined vertex vanishes. Subsequently we integrate over the momentum variable associated with the new closed loop, so that the result corresponds to the insertion of a space-time integral of a local operator. The diagrams of this type that contribute to (9) are

Whenever (some of) the ghosts and some of the ordinary fields corresponding to the propagator lines that connect the upper and the lower part of the graphs (13) are massless, we have diagrams that are ill-defined. This happens in the situation sketched previously, where the effective action has a continuous degeneracy of stationary points. Of course, we did not discuss the ghost-dependent terms in the effective action, but the ghost mass is always related to the mass of some of the ordinary fields. This follows, for instance, from a special case of the Ward identity (5),

$$F^\alpha \quad + \quad b^\alpha \quad + \quad i s_\beta^\dagger c^\beta = 0. \quad (14)$$

As explained previously, in the case where the effective action is degenerate for certain fields taking arbitrary constant values, it is not meaningful to include the ambiguous tadpole graphs corresponding to these fields. Instead their vacuum-expectation values play the role of arbitrary parameters that characterize the field configuration corresponding to one of the possible (degenerate) ground states. Obviously, when we suppress the ambiguous tadpole diagrams from the connected diagrams, we should also suppress their variation under (10), which coincides with the first three diagrams (13). Let us first rewrite these graphs by applying the Ward identity (9) to the top part of the diagrams (13),

$$\begin{aligned}
 & F^\alpha (\Delta F)^\alpha + i b^\alpha t_\beta^\alpha c^\beta + i b^\alpha s_\beta^\alpha c^\beta \\
 & = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} \\
 & = \text{[diagram 4]} + \text{[diagram 5]} + \text{[diagram 6]} \\
 & = \text{[diagram 7]} + \text{[diagram 8]} + \text{[diagram 9]}
 \end{aligned} \tag{15}$$

For those diagrams of the last line of (15) that can be divided into two diagrams by cutting only one internal (ghost) line, we can make use of the identity (14). Diagrams that cannot be divided in this way will be called irreducible; such diagrams do not contain a massless (ghost) propagator pole associated with the external line. It is convenient to introduce the following notation for the irreducible graphs,

$$\begin{aligned}
 (\Delta T)_\alpha & \equiv \text{[diagram 10]} \\
 (\Delta S)^i & \equiv \text{[diagram 11]} \\
 i s_\alpha^i c^\alpha & \equiv \text{[diagram 12]}
 \end{aligned} \tag{16}$$

Observe that we have truncated the ghost propagator from the diagrams contributing to $(\Delta T)_\alpha$ and that the diagrams contributing to $(\Delta S)^i$ are irreducible. Hence both diagrams are well defined in the limit of vanishing external momentum. The irreducible character of the diagrams (16) is indicated by using a double boundary circle.

Combining these steps leads to the following result for the sum of the first three diagrams (13),

$$\begin{aligned}
 & F^\alpha (\Delta F)^\alpha + i b^\alpha t_\beta^\alpha c^\beta + i b^\alpha s_\beta^\alpha c^\beta \\
 & = \text{[diagram 13a]} + \text{[diagram 13b]} + \text{[diagram 13c]} \\
 & = (\Delta T)_\alpha - (\Delta S)^i
 \end{aligned} \tag{17}$$

where the index i is kept fixed for the moment. Note that the second diagram on the right-hand side of (17) no longer contains a (full) propagator associated with the field ϕ^* .

Now we return to the identity (9). The first three graphs describe the effect of a change of gauge (10) on the connected Green's functions which include the attachments of ambiguous tadpole diagrams associated with the massless fields. As we intend to suppress the tadpole diagrams associated with massless fields from the connected Green's functions, we should exclude the diagrams corresponding to their variation, i.e., the diagrams (13a-c) with a massless propagator. Hence we treat these diagrams separately and rewrite them with the aid of (17), where the index i will now be replaced by $\bar{i}$, to indicate that we restrict ourselves to the massless fields. Then there are the tadpole diagrams (13d-e), which contribute to the last two diagrams of (9). These diagrams can be rewritten by using the Ward identity (5) for the lower part of the diagram; the top part of the diagram corresponds precisely to $(\Delta T)_\alpha$, as defined in (16). Putting these elements together then leads to the following form of (9),

labelled by $\bar{i}$ must still cancel. We would like to show that this first diagram is in fact proportional to the third diagram,

$$\frac{\partial \hat{\Gamma}}{\partial v^i} \equiv \text{diagram} \quad (19)$$

Here $\hat{\Gamma}$ denotes the sum of the connected diagrams with a given configuration of external lines without any ambiguous tadpole attachments associated with the massless fields ϕ^i ; v^i are the vacuum-expectation values of these fields, which, as explained before, play the role of extra parameters characterizing the degenerate ground states. The connected diagrams are evaluated for a given value of the parameters v^i .

Although the massless propagator poles cancel in the diagrams of the third line of (18), their presence is important when evaluating the non-singular remainder. In order to see this, one must reconsider the joining of the vertices, which causes the momentum of the intermediate propagator line to vanish.* The result is that the contribution of the third line in (18) will involve both terms that are proportional to (19), where the momentum flowing through the top line of the diagram is zero, and terms proportional to the derivative of (19) with respect to the momentum of the top line. However, we will show that the derivative term is suppressed if one makes the additional assumption that the nature of the degeneracy of the effective action is not affected by the change of gauge (10). This is not a severe restriction; if the degeneracy were (partly) lifted by a change of gauge, then it would be best to proceed from a class of gauge conditions with a lower degree of degeneracy as a limiting case. Of course, a change in the nature of the degeneracy is a subtle matter, which often signals a phase transition. However, in the present context the phenomenon is only relevant to the gauge-dependent sector of the theory.

Assuming that the degeneracy is not affected by the change of gauge, the singular behaviour of the massless external propagator in the first three diagrams in (15) is cancelled, because the truncated version of these graphs (i.e., the graphs without the (full) external propagator) represents the change under (10) of the first derivative

* In ref. 10 we postponed joining the vertices until the end by introducing a limiting procedure that replaces the direct joining of two operators X^α and Y^β by

$$X^\alpha Y^\beta \rightarrow [X^\alpha Y^\beta]_\epsilon \equiv \mathcal{F}_{\alpha\beta} X^\alpha Y^\beta,$$

where the test function $\mathcal{F}_{\alpha\beta}$ is such that a finite amount of momentum flows into the $X^\alpha Y^\beta$ vertex. In this way the singular behaviour of troublesome tadpole-like diagrams is avoided and the calculations can be carried out without ever encountering ill-defined expressions. At the end one then takes the limit $\mathcal{F}_{\alpha\beta} \rightarrow \delta_{\alpha\beta}$, which corresponds to the zero-momentum limit.

$$\begin{aligned} & \left[\text{diagram with } F^\alpha(\Delta F)^\alpha \right] + \left[\text{diagram with } i b^\alpha t_\beta^\alpha c^\beta \right] + \left[\text{diagram with } i b^\alpha s_\gamma^\beta c^\gamma \right] \\ & + \sum_j \left[\text{diagram with } b^\alpha(\Delta F)^\alpha \right] + \left[\text{diagram with } b^\alpha(\Delta F)^\alpha \right] + \left[\text{diagram with } i j^i s_\alpha^i c^\alpha \right] \\ & + \left[\text{diagram with } F^\alpha \right] + \left[\text{diagram with } F^\alpha \right] - (\Delta T)_\alpha - (\Delta S)^\alpha = 0. \end{aligned} \quad (18)$$

where we sum over the indices $\bar{i}$ associated with the massless fields. The double boundary circles in the first line of (18) indicate that we suppressed the troublesome tadpole diagrams (13a-c). Therefore the first line coincides with minus the change of the connected Green's function without ambiguous tadpole diagrams attached (again modulo diagrams of type (11), which vanish on the mass shell). The double boundary circles in the second line indicate that we suppressed the diagrams (13d-e). Just as before, on the mass shell, these diagrams yield a multiplicative factor Δ_I , one for each external line (cf. (12)). Finally, the diagrams in the third line are such that the troublesome tadpole diagrams with a massless propagator are absent, so that equation (18) is well defined.

We now proceed and take the external sources on the mass shell. In that case the second diagram in the third line of (18) vanishes by virtue of (6), while we know that the singularity in the first diagram associated with the massless propagator

of the effective action with respect to a massless constant field. As this derivative vanishes for constant values of these fields (because of the degeneracy) and the nature of the degeneracy is assumed to remain the same after a change of the gauge condition, it must remain zero. It then follows from (15) that the divergence caused by massless ghost propagators in the fourth diagram of (15) is also suppressed. This suppression suffices to prove that the on-shell contribution from the third line in (18) will be proportional to the diagram (19) and not to its derivative with respect to the external momentum of the top line. Hence we find that the standard equation (12) for the change of the connected on-shell Green's functions takes a modified form,

$$\Delta \hat{\Gamma} = \sum_j \Delta_j \hat{\Gamma} + (\Delta \phi)^i \frac{\partial \hat{\Gamma}}{\partial v^i}, \quad (20)$$

where the expression for $(\Delta \phi)^i$ follows from explicitly evaluating the third line of (18). In simple cases one finds that $(\Delta T)_\alpha$ is equal to zero, which shows that the first three diagrams of (15) are indeed non-singular, so that the nature of the degeneracy remains the same after a choice of gauge. In that case we have $(\Delta \phi)^i = (\Delta S)^i$.

It is straightforward to derive the corresponding condition for the S -matrix. Introducing parameters ξ^a to parametrize a variety of gauge-fixing terms, the S -matrix satisfies the condition

$$\left\{ \frac{\partial}{\partial \xi^a} - \beta^a v(\lambda, \xi^a, v^i, \epsilon) \frac{\partial}{\partial v^i} \right\} S(\lambda, \xi^a, v^i, \epsilon) = 0. \quad (21)$$

Here λ generically denotes the (bare) parameters of the gauge-invariant Lagrangian and ϵ the cut-off; the functions β^a follow from $(\Delta \phi)^i$. In the standard case without a continuous degeneracy of stationary points of the effective action, the second term is absent and one finds that the regularized S -matrix elements are independent of the gauge parameters ξ^a . In the degenerate case, the second term is present and seems to give rise to a lack of gauge independence. This is the phenomenon noted in ref. 7 for quantum gravity (see also, ref. 9), whose origin was explained in terms of the degeneracy in ref. 10. However, the correct interpretation of this phenomenon is not that gauge independence is violated. According to (21) the S -matrix elements still depend on fewer parameters than explicitly indicated and, in fact, this equation suffices to prove that the S -matrix elements are gauge independent, provided they are expressed in terms of an independent set of physical parameters. Those are the parameters that can be extracted from S -matrix elements, such as masses, coupling constants, scattering lengths, etc.. This amounts to replacing the original parameters λ and v^i by physically relevant parameters G which are themselves S -matrix elements. Each of these parameters depends on λ , ξ^a , v^i and ϵ , and satisfies (21). According to (21), the S -matrix elements expressed in terms of G , ξ^a and ϵ no longer depend on the gauge-fixing parameters ξ^a .

It is clear that in the presence of a continuous degeneracy of stationary points of the effective action, the gauge independence is realized in a qualitatively different way.

In the usual situation S -matrix elements are independent of the gauge condition when we keep the (bare) parameters of the Lagrangian constant. Hence, the expression for a physical mass or a coupling constant is a function of the parameters of the Lagrangian, but does not depend on the gauge parameters. On the other hand, the degenerate case gives rise to S -matrix elements that do depend on the gauge parameters as well as on the parameters of the Lagrangian. Moreover they depend on parameters v^i that characterize the degenerate field configurations corresponding to the ground state. In actual calculations (like in quantum gravity) these parameters are often implicitly fixed. In spite of this complication, one finds that gauge independence is regained after expressing the results of the calculation in terms of physical parameters. Although this form of gauge independence is thus weaker than in the usual situation, it is sufficient for all practical purposes.

In the presence of a continuous degeneracy of stationary points of the effective action the number of parameters occurring in the S -matrix is larger than the number of parameters occurring in the Lagrangian. It should be emphasized that this enlargement of the parameter space is a phenomenon in its own right and has nothing to do with the issue of the gauge-independence of the S -matrix. In practice some of these parameters could be physically irrelevant, because the space of degenerate ground states may have certain invariances resulting in a physical equivalence of the corresponding theories. Even when symmetry arguments are lacking to relate the different ground states, it may still be possible to absorb these extra parameters into a smaller number of physically relevant parameters.

The latter phenomenon takes place in quantum gravity. In that case (for simplicity we assume that the ground state corresponds to flat space-time) the metric associated with the ground state can be given modulo a multiplicative factor ρ ,

$$g_{\mu\nu} = \rho \eta_{\mu\nu}. \quad (22)$$

This parameter ρ may be viewed as the vacuum-expectation value of the trace of $g_{\mu\nu}$, which is a massless field. Already at the classical level, physical quantities will depend on the parameters in the Lagrangian and on the parameter ρ . For instance, consider gravity coupled to a scalar field described by the Lagrangian

$$\mathcal{L} = -\kappa^{-2} \sqrt{g} R - \frac{1}{2} \sqrt{g} \{ g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + m^2 \phi^2 \}. \quad (23)$$

Expanding about the background (22), one finds that the physical graviton-graviton coupling constant, the (static) graviton-scalar coupling and the scalar mass all depend on ρ in the following way (for arbitrary space-time dimension d),

$$\begin{aligned} K_{ggg}^{\text{phys}} &\propto \rho^{\frac{1}{2}(2-d)} \kappa, \\ K_{g\phi\phi}^{\text{phys}} &\propto \rho^{\frac{1}{2}(6-d)} \kappa m^2, \\ M^{\text{phys}} &\propto \rho^{\frac{1}{2}} m, \end{aligned} \quad (24)$$

Consider now two different scalar fields, ϕ_1 and ϕ_2 with (Lagrangian) masses m_1 and m_2 , respectively, and calculate the one-graviton exchange amplitude in the static limit. The invariant amplitude again depends on ρ and is proportional to

$$\mathcal{M}(1+2 \rightarrow 1+2) \propto \rho^{\frac{1}{2}(6-d)} \kappa^2 m_1^2 m_2^2. \quad (25)$$

However, the explicit ρ -dependence disappears when expressing this result in terms of physical parameters, Newton's constant K_{ggg}^{phys} and the masses M_1^{phys} and M_2^{phys} . After correctly normalizing the incoming and outgoing particle states, one finds Newton's law with the $1/r$ potential and the proper coefficient. The latter is defined in terms of the physical parameters and not the parameters that appear in the Lagrangian (23). Although all these quantities depend on ρ , there is no explicit ρ -dependence left.

This example illustrates how the extra parameters associated with a degeneracy of the effective action can still be physically irrelevant. For gravity this phenomenon is not unexpected. One can always change the length scale by performing a special coordinate transformation $x^\mu \rightarrow \alpha x^\mu$. This transformation will change the value of ρ in (22). In order to write down expressions for dimensionful quantities, the length scale, and thus the value of ρ , has to be fixed. In quantum gravity the issue is more subtle, because the full effective action is not invariant under general coordinate transformations. We refer to ref. 10 for a further discussion of this point.

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