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THEORETICAL IMPLICATIONS OF PRECISION ELECTROWEAK DATA

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1. Introduction

The main goal of LEP 1 is to perform precision tests of the standard electroweak theory^[1] at the Z peak. Theoretical predictions in the Standard Model for all relevant observables have been developed in detail^[2]. I refer the reader to my talks^[3] at some recent conferences for concise summaries and for many relevant discussions that I will not repeat here. One starts from the Standard Model Lagrangian and a conveniently chosen set of input parameters. The interesting quantities are computed in perturbation theory. The lowest-order formulae plus one-loop radiative corrections^[4], often improved by important renormalization group resummations, provide a sufficiently accurate approximation to match the precision of realistic experiments and to allow quite significant tests of the theory. For LEP physics, a self-imposing set of input parameters is given by α_s , α , G_F , m_Z , m_t and m_H . Clearly the Fermi coupling $G_F = 1.16637(22) \times 10^{-5} \text{GeV}^{-2}$ is conceptually more complicated than $\alpha_{\text{weak}} = \frac{g^2}{4\pi}$ (which would more naturally accompany $\alpha = 1/137.036$ and α_s) or $\sin^2 \theta_W$ or m_W , but is preferred for practical reasons because it is known with all the desirable accuracy. Similarly, m_Z has now been measured at LEP with remarkable precision. This preliminary task of LEP in view of precision tests of the Standard Model has already been accomplished to a nearly final degree of accuracy.

The LEP results on m_Z , as summarized at the Geneva Conference^[5], are reported in Table 1[†]. The resulting relative precision is impressive: $\delta m_Z/m_Z = 2.3 \times 10^{-4}$.

TABLE 1

Experiment	m_Z (GeV)
ALEPH	91.182 ± 0.009
DELPHI	91.177 ± 0.010
L3	91.181 ± 0.010
OPAL	91.161 ± 0.009
AVERAGE	91.175 ± 0.005 (Stat.) ± 0.020 (LEP) $\simeq 91.175 \pm 0.021$

Among the quark and lepton masses, m_t , the main unknown is the top quark mass. Our ignorance of m_t is at present a serious limitation for precise

[†] In writing this Proceedings I decided to update the experimental data by also including results not yet available when the talk was given.

tests of the electroweak theory because the radiative corrections are relatively large for large m_t and depend quadratically on $m_t^{[3],[4]}$. This fact can be used to put stringent constraints on m_t from the existing electroweak measurements, in particular an upper bound on m_t , to be discussed in detail later. As for lower bounds on m_t the best results arise from the failure to observe the t quark at e^+e^- and hadron colliders. LEP and SLC lead to a model-independent bound $m_t \gtrsim 45$ GeV. From CDF one learns that $m_t \gtrsim 89$ GeV, provided that the t quark semi-leptonic branching ratio is as predicted by the Standard Model.

The Higgs mass m_H is largely unknown. One of the most impressive performances of LEP up to now has been the dwarfing^[6] of all previous lower bounds on m_H . For the mass of the minimal Standard Model Higgs boson, ALEPH was able to establish the lower limit $m_H \gtrsim 51$ GeV. Less stringent but comparable limits were also obtained by the other LEP experiments (OPAL $m_H \gtrsim 47.3$ GeV, L3: $m_H \gtrsim 47.6$ GeV, DELPHI: $m_H \gtrsim 42$ GeV). For the two-doublet Higgs sector of the minimal supersymmetric extension of the Standard Model^{[7],[8]}, the corresponding limit is: $m_h \gtrsim 43$ GeV (ALEPH).

The upper limit on m_H is mainly from theoretical arguments of consistency and is not equally sharp. It is well known that for $m_H \gtrsim 0.6$ -1 TeV the Standard Model becomes affected by serious problems^[9] (e.g., Landau singularities moving down to energies of order 1 TeV), certainly the perturbative framework is no more reliable and weak interactions become strong. For this reason, most computations of radiative corrections are given for $m_H < 1$ TeV. The sensitivity of the radiative corrections to variations of m_H in the range $50 \text{ GeV} < m_H < 1 \text{ TeV}$ is not large. In a sense, this level of accuracy fixes the goal for precision tests of the Standard Model because the clarification of the symmetry breaking sector of the theory is the main target of present-day experiments.

Finally, for electroweak calculations involving hadrons, the value of the QCD coupling α_s , must also be specified. The best value of α_s at the Z mass, obtained from experiments at energies lower than m_Z (in particular from deep inelastic scattering) is given by^[10] $\alpha_s(m_Z) = 0.11 \pm 0.01$. At LEP one finds^[10] something like $\alpha_s(m_Z) \simeq 0.12 \pm 0.01$. (Note that I am more conservative than usual on errors which are dominated by theoretical uncertainties.) The QCD corrections to processes involving quarks are typically of order $\frac{\alpha_s}{\pi}$. As a consequence the stated error on α_s , leads to a few per mille relative uncertainty on the corresponding predictions.

2. Precision Tests of the Standard Electroweak Theory

From the above discussion it is clear that the set of input parameters can be separated into two parts. On the one hand, α , G_F , m_Z , m_{flight} are well known and the ambiguities associated with these quantities on the radiative corrections are quite small. We can add α_s to this class, in that, if it is true that the experimental error on α_s is relatively large, it only enters as a small correction to electroweak processes involving hadrons and is practically irrelevant for purely leptonic processes. Also, by fitting the electroweak data one cannot obtain a better value for $\alpha_s(m_Z)$ than that derived from QCD tests. On the other hand m_t and m_H are largely unknown. Thus, for each relevant observable, one can only express the prediction of the Standard Model as a function of m_t and m_H , obtained by using the best available calculations of radiative corrections, with α , α_s , G_F , m_Z and m_{flight} fixed at their experimental values with the corresponding errors. By comparing these predictions with experiment one can check their mutual consistency and derive constraints on m_t and m_H .

Actually the sensitivity on m_H is so small that for all the measured quantities the ambiguity due to varying m_H in the range $50 \text{ GeV} < m_H < 1 \text{ TeV}$ is far below the present experimental error, so that for practical purposes, at the present stage of accuracy, the relevant predictions can be plotted as functions of m_t in the form of a band of values determined by δm_H , δm_Z , $(\delta \alpha_s)$ (see figs. 1,7).

Note that from this point of view $\sin^2 \theta_W$ is not a primary quantity. It is not part of the set of input parameters. It is a derived quantity that one could even decide not to introduce at all. I stress this point in order to make it clear that all disputes over which is the better definition of $\sin^2 \theta_W$ beyond the tree level are completely secondary. First of all it is always true that physical results are independent of definitions. Differences in physical results obtained from a different definition of input parameters (scheme dependence) can at most occur by terms of higher order, due to the truncation of the perturbative series at a given order. But for $\sin^2 \theta_W$ its precise definition is only necessary to compute it from the input parameters, but cannot matter for the prediction of observables because, with the choice specified above, $\sin^2 \theta_W$ is not taken as an input parameter of the theory.

The widespread use of expressing the experimental result to each given observable in terms of the corresponding value of $\sin^2 \theta_W$ (within a specified definition for it) is no longer adequate at the present level of sophistication. For

comparing the constraining power of different experiments it would be more appropriate to quote the range of m_t implied by each of them^[11] (see Table 4). In fact, $\sin^2\theta_W$ is just one particular observable of the theory. Outside the domain of precision tests, with appropriate definitions of $\sin^2\theta_W$, one can write simple improved Born approximations that include the main contributions of radiative corrections (e.g., large logarithms and terms of order $G_F m_t^2$). While for precision tests the use of as complete as possible radiative corrections is mandatory, these approximate formulae are very useful for our understanding of the pattern of radiative corrections and for everyday-life estimates of rates and experimental sensitivities.

One common definition^[12] of $\sin^2\theta_W$ is:

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \equiv s_W^2 \quad (1)$$

to all orders of perturbation theory. Clearly in this case the observables s_W^2 and m_W are directly equivalent given that m_Z is among the input parameters. In the Standard Model, s_W^2 can be computed from the input parameters by the relation:

$$s_W^2 c_W^2 \equiv \left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W^2}{m_Z^2} = \frac{\pi\alpha}{\sqrt{2}G_F} \frac{1}{m_Z^2} \frac{1}{1 - \Delta r} \quad (2)$$

where $c_W^2 = 1 - s_W^2$ and $\Delta r \equiv \Delta r(\alpha_s, G_F, m_Z, m_f, m_H)$ is the effect of radiative corrections. The quantity Δr as a function of the input parameters has been studied in great detail^[13]. The result for s_W^2 , obtained starting from the average LEP value for m_Z (see Table 1), as a function of m_t , is plotted in fig. 1, where the uncertainties for $50 \text{ GeV} < m_H < 1 \text{ TeV}$ and $\delta m_Z = \pm 21 \text{ MeV}$ are also visible. We see that m_t is the main unknown in the calculation of $\frac{m_W}{m_Z}$ from m_Z , followed in importance by the ambiguity from varying the Higgs mass in the above range, while the remaining uncertainty from the experimental error on m_Z is very small.

When the available direct experimental information on $\frac{m_W}{m_Z}$ is added, the sensitivity of s_W^2 to m_t provides a very strong constraint on m_t . $\frac{m_W}{m_Z}$ is directly measured at hadron colliders and can also be obtained (assuming the validity of the Standard Model) from the ratio $R_\nu = \sigma^{\text{NC}}/\sigma^{\text{CC}}$ of neutral current (NC) to charged current (CC) cross-sections in neutrino-nucleus deep inelastic scattering. The value of $\frac{m_W}{m_Z}$ has been measured at hadron colliders^[14]. From CDF and UA2 we have the results reported in Table 2.

TABLE 2

Experiment	$\frac{m_W}{m_Z}$	$s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$
CDF	0.8768 ± 0.0046	0.231 ± 0.008
UA2	0.8841 ± 0.0043	0.2184 ± 0.0077
AVERAGE	0.8807 ± 0.0031	0.2245 ± 0.006

Fig. 1. Labelled by m_Z is the theoretical prediction for s_W^2 obtained from m_Z as a function of m_t for $m_H = 50 - 1000$ GeV (the uncertainty due to $\delta m_Z = 21$ MeV leads to the double boundaries). The error bands implied by the CDF and UA2 measurements of m_W/m_Z and by the data on R_ν are also shown.

By combining $\frac{m_W}{m_Z}$ with the LEP value for m_Z one obtains $m_W = 80.30 \pm 0.28$ GeV. The corresponding average value of s_W^2 is also shown in fig. 1 as a horizontal band, obviously independent of m_t , in the $s_W^2 - m_t$ plane.

As is well known, the value of s_W^2 extracted from R_ν is also nearly independent of m_t in the interesting range of values for the top mass. This fact arises from a largely accidental cancellation^[15], specific to this process and to the

Standard Model, between two different sources of m_t dependence, as discussed in the following.

In general, at tree level, the four-fermion interaction from Z exchange is given by:

$$M_{if} = \frac{\sqrt{2}G_F m_Z^2}{D(s)} \rho_{\text{tree}} (J_3^i - 2\sin^2\theta_W J_{\text{em}}^i) \cdot (J_3^f - 2\sin^2\theta_W J_{\text{em}}^f) \quad (3)$$

where $D(s)$ is the Z propagator and $J_3^{i,f}, J_{\text{em}}^{i,f}$ are the weak isospin and electromagnetic currents for the fermion i or f . Excluding pure QED corrections, electroweak radiative corrections^[4] modify M_{if} according to:

$$M_{if} = \frac{\sqrt{2}G_F m_Z^2}{D(s)} \rho_{i,f} (J_3^i - 2k_i \sin^2\theta_W J_{\text{em}}^i) \cdot (J_3^f - 2k_f \sin^2\theta_W J_{\text{em}}^f) + \dots \quad (4)$$

where $\rho_{i,f} = \rho_{\text{tree}}(1 + \delta\rho_{i,f})$, $k_a = 1 + \delta k_a$ ($a = i, f$) are different for different fermions and depend on the scheme adopted (for example δk_i depend on the definition of $\sin^2\theta_W$). The ellipses indicate possible additional non-factorizable terms (for example from box diagrams). Let us call “large” radiative corrections those terms containing large logarithms, i.e. $\frac{\alpha}{\pi} \ell n \frac{m_Z^2}{m_{\text{light}}^2}$ or quadratic dependences on m_t , i.e., $\sim G_F m_t^2$. For large enough m_t , the bulk of the contribution of electroweak radiative corrections arises from these terms^{[3],[4]}. The “large” contributions to $\delta\rho_{i,f}$ and δk_f in eq. (4) are universal, i.e., they are the same at fixed q^2 for all i and f (except for b quarks). If for $\sin^2\theta_W$ one adopts the definition $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$ one obtains^[4]:

$$1 - \Delta r = (1 - \Delta\alpha) \left(1 + \frac{c_W^2}{s_W^2} \delta\rho + \text{“small”} \right) \quad (5)$$

$$\rho \cong 1 + \delta\rho = 1 + \frac{3G_F m_t^2}{8\pi^2 \sqrt{2}} + \text{“small”} \quad (6)$$

$$k \cong 1 + \delta k = 1 + \frac{c_W^2}{s_W^2} \delta\rho + \text{“small”} \quad (7)$$

(for b quarks there are additional large terms) where Δr is defined in eq. 2 and $\Delta\alpha$ arises from the running of the QED coupling:

$$\alpha(m_Z) = \frac{\alpha}{1 - \Delta\alpha} \quad (8)$$

$\Delta\alpha$ is dominated by large logs and its value is given by⁴⁾:

$$\Delta\alpha \simeq 0.0601 \pm 0.0009 \quad (9)$$

(or $\alpha(m_Z)^{-1} \simeq 128.8 \pm 0.1$). Note that both δk and Δr contain the large term $\delta\rho$ enhanced by the factor $\frac{c_W^2}{s_W^2}$. Logarithmic scale violations of order $\frac{\alpha}{\pi} \ln q^2/m_Z^2$ are included in the “small” terms (which is only appropriate for $q^2 \gg m_{\text{light}}^2$). The ratio $R_\nu = \frac{\sigma_{\text{NC}}}{\sigma_{\text{CC}}}$ for ν -N scattering is given in terms of s_W^2 by:

$$R_\nu = \rho_{\nu N}^2 \left(\frac{1}{2} - k_{\nu N} s_W^2 + \frac{5}{9} (k_{\nu N} s_W^2)^2 (1 + r) \right) + \dots \quad (10)$$

where $r = (\sigma^{\bar{\nu}}/\sigma^\nu)_{\text{CC}} \simeq 0.4$ is also measured. The tree approximation (with $\rho_{\text{tree}} = 1$) is recovered for $\rho_{\nu N} = k_{\nu N} = 1$. Some large logarithms from the radiative corrections to σ^{CC} are also included in $\rho_{\nu N}$. But for the sake of this argument we are only considering the $G_F m_t^2$ terms. For fixed R_ν = (the experimental value) and $s_W^2 \sim 0.23$ there is a strong cancellation in the Standard Model between the m_t dependence of $\rho_{\nu N} \simeq 1 + \delta\rho$ and of $k_{\nu N} \simeq 1 + \frac{c_W^2}{s_W^2} \delta\rho$, so that as a result $\delta s_W^2 \simeq 0.2\delta\rho$, where $\delta\rho$ is given in eq. (6). For realistic values of m_t the resulting contribution of the quadratic m_t terms is no more dominant.

The most precise experimental results on R_ν were obtained by the CHARM^[16] and CDHS^[17] collaborations at CERN. The original results on $\sin^2_W$ were given for fixed m_t and m_H . CHARM obtained $s_W^2 = 0.236 \pm 0.005$ (exp) ± 0.005 (th) for $m_t = 45$ GeV and $m_H = 100$ GeV, while the CDHS result was $s_W^2 = 0.2275 \pm 0.005$ (exp) ± 0.005 (th) for $m_t = 60$ GeV and $m_H = 100$ GeV. The theoretical error arises from hadronic uncertainties and the effect of the charm threshold. An average at $m_t = 60$ GeV and $m_H = 100$ GeV gives $s_W^2 = 0.232 \pm 0.006$ (where the error 6×10^{-3} is obtained as $6 \times 10^{-3} = \sqrt{\left(\frac{5}{\sqrt{2}}\right)^2 + 5^2} \times 10^{-3}$). The corresponding combined result at different values of m_t and m_H can also be obtained from the known form of the radiative corrections. The result is shown^[18] in fig. 1.

There are many more less precise experimental results on s_W^2 from low energy neutral current data, most of them being well known^{[19],[20]}. These additional data are all consistent among them and with the results in fig. 1.

We now consider the implications for the standard electroweak theory of the LEP results on the Z partial widths and asymmetries. In figs. 2-4, we compare the data on the Z widths (collected in Table 3) with the predictions of the Standard Model, obtained by the programme ZSHAPE^[21] which includes a state of the art set of electroweak radiative corrections.

TABLE 3

	ALEPH	DELPHI	L3	OPAL	Average
Γ_Z (MeV)	2484 ± 17	2465 ± 20	2501 ± 17	2492 ± 16	2487 ± 10
Γ_h (MeV)	1744 ± 15	1726 ± 19	1747 ± 16	1739 ± 17	1740 ± 9
Γ_ℓ (MeV)	83.1 ± 0.7	83.4 ± 0.8	83.5 ± 0.7	83.0 ± 0.7	83.2 ± 0.4
$R = \frac{\Gamma_h}{\Gamma_\ell}$	21.00 ± 0.20	20.70 ± 0.29	20.93 ± 0.22	20.95 ± 0.22	20.92 ± 0.11
Γ_{inv} (MeV)	491 ± 13	488 ± 17	501 ± 14	504 ± 15	496.2 ± 8.8
Γ_e (MeV)	83.8 ± 0.9	82.4 ± 1.2	82.5 ± 0.9	82.9 ± 1.0	83.0 ± 0.5
Γ_ν (MeV)	81.4 ± 1.4	86.9 ± 2.1	86.0 ± 1.6	83.2 ± 1.5	83.8 ± 0.8
Γ_τ (MeV)	82.4 ± 1.6	82.7 ± 2.4	85.6 ± 1.9	82.7 ± 1.9	83.3 ± 1.0

Results from LEP. The average also includes systematic errors as given by J. Carter^[5] at the Geneva Conference. The average value of Γ_{inv} corresponds to $N_\nu = 2.99 \pm 0.05$ which is the best determination of the number of light neutrinos from LEP.

Totally equivalent predictions are obtained by other complete calculations of the line shape^{[22]–[26]}. The predicted widths are plotted as a function of m_t and compared to the data. The ambiguities corresponding to $m_H = 50$ –1000 GeV and $\alpha_s(m_Z) = 0.11$ –0.13 are also indicated. Note that the leptonic width (obtained by assuming $e - \mu - \tau$ universality) gives a particularly strong constraint on m_t , because of its insensitivity to $\alpha_s(m_Z)$, the rather low central value and the small experimental error.

Additional important information is provided by the measurement of a number of asymmetries. In particular we refer to the forward-backward asymmetries for charged leptons (A_{FB}^ℓ) and for the b-quark (A_{FB}^b) and the τ polarization asymmetry (A_{pol}^τ). The value of those asymmetries, combined over the LEP experiments, are given in the following^[5]. For A_{FB}^ℓ one has:

$$A_{\text{FB}}^\ell(\sqrt{s} = m_Z) = 0.0163 \pm 0.0036 \quad (11)$$

Fig. 2. Γ_Z vs m_t as predicted by the Standard Model for $m_H = 50 - 1000$ GeV and $\alpha_s(m_Z) = 0.11-0.13$ compared with the LEP result.

Fig. 3. Γ_h vs m_t as predicted by the Standard Model for $m_H = 50 - 1000$ GeV and $\alpha_s(m_Z) = 0.11-0.13$ compared with the LEP result.

obtained from the average value $y = \bar{v}^2/\bar{a}^2 = 0.0048 \pm 0.0012$ quoted by V.

Fig. 4. Γ_ℓ vs m_t as predicted by the Standard Model for $m_H = 50 - 1000$ GeV compared with the LEP result.

Nikolaenko^[27] and J. Carter^[5], by using the relation:

$$A_{\text{FB}}^\ell = \frac{3y}{(1+y)^2} + 0.002 \quad (12)$$

which corresponds to the definitions of ZFITTER^[24]. For A_{FB}^b the result, after correction for the $B - \bar{B}$ mixing effect, is given by:

$$A_{\text{FB}}^b(\sqrt{s} = m_Z) = 0.126 \pm 0.022 \quad (13)$$

Actually the measured value includes the effect from QCD corrections. We prefer to consider the value of A_{FB}^b with the QCD correction^[28] removed:

$$\begin{aligned} (A_{\text{FB}}^b)_{\text{measured}} &\cong (A_{\text{FB}}^b)_{\text{E-W}} \left(1 - \frac{0.79 \alpha_s(m_Z)}{\pi} \right) \\ &\cong 0.97 (A_{\text{FB}}^b)_{\text{E-W}} \end{aligned} \quad (14)$$

Dropping the E-W subscript, in the following we shall use:

$$A_{\text{FB}}^b(\sqrt{s} = m_Z) = 0.130 \pm 0.022 \quad (15)$$

and the corresponding theoretical prediction. Finally, for A_{pol}^τ one has:

$$A_{\text{pol}}^\tau(\sqrt{s} = m_Z) = 0.134 \pm 0.035 \quad (16)$$

The experimental results on the asymmetries are compared with the Standard Model predictions in figs. 5-7.

When all the data are combined I find the following range for the top quark mass:

$$m_t = 140 \pm 35 \text{ GeV} \quad (17)$$

The central value of m_t is smaller, $\simeq 120 \text{ GeV}$, for light Higgs ($m_H = 50 - 100 \text{ GeV}$), while, it is larger ($\simeq 160 \text{ GeV}$) for heavy Higgs ($m_H = 0.5 - 1 \text{ TeV}$). The above result on m_t is in agreement with other recent analyses of the data^[29]. The exact range of m_t depends on all sorts of details, e.g. the set of data which are included, the weight given to the CDF bound on m_t , the range assumed for $\alpha_s(m_Z)$ (here $\alpha_s(m_Z) = 0.12 \pm 0.01$) and so on. The ranges of m_t implied by each experiment are listed in Table 4. We repeat that m_t is a more adequate figure of merit for experiments than $\sin^2\theta_W$ (defined in one or another way) because the constraints on m_t are the obvious goal of precision tests of the electroweak theory at present.

TABLE 4

	$m_H = 50 - 1000 \text{ GeV}$	$m_H = 0.5 - 1 \text{ TeV}$
$\frac{m_W}{m_Z}$	$150 \pm 50 \text{ GeV}$	$180 \pm 50 \text{ GeV}$
R_ν	100 ± 60	140 ± 50
Γ_Z	90 ± 60	150 ± 50
Γ_h	100 ± 75	170 ± 50
Γ_ℓ	80 ± 50	90 ± 60
A_{FB}^ℓ	140 ± 60	180 ± 60
A_{pol}^τ	130 ± 120	170 ± 120
A_{FB}^b	195 ± 100	330 ± 100
Average	121 ± 22	162 ± 21

Fig. 5. $A_{FB}^{\ell} \times 10^2$ vs m_t as predicted by the Standard Model for $m_H = 50-1000$ GeV compared with the LEP result.

Values of m_t implied by different measurements [$\alpha_s(m_Z) = 0.12 \pm 0.01$].

3. Toward a Model Independent Analysis of the Data

In the present section we propose a different way^[30] of analysing the data which does not necessarily assume the validity of the Standard Model from the start and takes into account recent theoretical studies on the parametrisation of possible effects of new physics on precision experiments^{[31]–[36]}. Given the set of input parameters as specified above, we start from the basic observables m_W/m_Z , Γ_{ℓ} and A_{FB}^{ℓ} . We assume charged lepton universality, which is supported by the data at the present level of accuracy, so that Γ_{ℓ} and A_{FB}^{ℓ} refer to the corresponding average data. From these three quantities we can isolate the corresponding dynamically significant corrections Δr_W , $\Delta \rho$ and $\Delta k'$, which contain the small effects one is trying to disentangle, and are defined in the following. First we introduce Δr_W as obtained from m_W/m_Z by the relation (see eq. (2)):

$$\left(1 - \frac{m_W^2}{m_Z^2}\right) \frac{m_W^2}{m_Z^2} = \frac{\pi \alpha(m_Z)}{\sqrt{2} G_F m_Z^2 (1 - \Delta r_W)} \quad (18)$$

Fig. 6. A_{FB}^b vs m_t as predicted by the Standard Model for $m_H = 50 - 1000$ GeV compared with the LEP result.

Fig. 7. A_{pol}^τ vs m_t as predicted by the Standard Model for $m_H = 50 - 1000$ GeV compared with the LEP result.

Here $\alpha(m_Z) = \alpha/(1 - \Delta\alpha)$ is fixed to the conventional value $1/128.8$ (recall eq. 9)) so that the effect of the running of α due to known physics is extracted from $(1 - \Delta r) = (1 - \Delta\alpha)(1 - \Delta r_W)$. We take $G_F = 1.16637 \times 10^{-5} \text{ GeV}^{-2}$. A possible departure from these values would then be included in Δr_W . In order to define $\Delta\rho$ and $\Delta k'$ we consider effective vector and axial-vector couplings g_V and g_A for the on-shell Z to charged leptons, defined by the formulae:

$$\Gamma_\ell = \frac{G_F m_Z^3 (g_V^2 + g_A^2)}{6\pi\sqrt{2}} \quad (19)$$

$$A_{\text{FB}}^\ell(\sqrt{s} = m_Z) = \frac{3 g_V^2 g_A^2}{(g_V^2 + g_A^2)^2} \quad (20)$$

Note that Γ_ℓ stands for the inclusive partial width ($\Gamma(Z \rightarrow \ell\bar{\ell} + \text{photons})$). We could extract from $(g_V^2 + g_A^2)$ the factor $(1 + 3\alpha/4\pi + \dots)$ which is induced in Γ_ℓ from final state radiation, but we prefer the simpler definition of eq. (19). The asymmetry in eq. (20) is obtained from the data after deconvolution of initial state radiation. Contributions from box diagrams and imaginary parts of vertex functions and propagators are absorbed in the definition of g_A and g_V (of course, this can only be possible for a single channel, which we chose to be the charged lepton channel). In terms of g_V and g_A , $\Delta\rho$ and $\Delta k'$ are given by^[33]:

$$g_A = -\frac{\sqrt{\rho}}{2} = -\frac{1}{2} \left(1 + \frac{\Delta\rho}{2} \right) \quad (21)$$

$$\frac{g_V}{g_A} = 1 - 4 \bar{s}_W^2 = 1 - 4(1 + \Delta k') s_0^2$$

In eq. (21) $\bar{s}_W^2$ is a particular effective $\sin^2\theta_W$ for on-shell Z ^{[4],[37]–[39]}, while s_0^2 is the corresponding quantity before non pure-QED corrections, given by:

$$s_0^2 c_0^2 = \frac{\pi\alpha(m_Z)}{\sqrt{2}G_F m_Z^2} \quad (22)$$

with $c_0^2 = 1 - s_0^2$ ($s_0^2 = 0.23145$ for $m_Z = 91.175 \text{ GeV}$).

In the Standard Model Δr_W , $\Delta\rho$ and $\Delta k'$, for sufficiently large m_t , are all dominated by quadratic terms in m_t of order $G_F m_t^2$ ^[40] and in this limit one has $\Delta r_W \sim -c_0^2/s_0^2 \Delta\rho \sim (c_0^2 - s_0^2)s_0^2 \Delta k'$ ⁴⁾. As new physics can more easily be disentangled if not masked by large conventional m_t effects, it is convenient to

keep $\Delta\rho$ while trading Δr_W and $\Delta k'$ for two quantities with no contributions of order $G_F m_t^2$. We thus introduce the following linear combinations^[39]:

$$\begin{aligned}\epsilon_1 &= \Delta\rho \\ \epsilon_2 &= c_0^2 \Delta\rho + \frac{s_0^2 \Delta r_W}{(c_0^2 - s_0^2)} - 2s_0^2 \Delta k' \\ \epsilon_3 &= c_0^2 \Delta\rho + (c_0^2 - s_0^2) \Delta k'\end{aligned}\tag{23}$$

Clearly ϵ_2 and ϵ_3 no longer contain terms of order $G_F m_t^2$ but only logarithmic terms in m_t . The leading terms for large Higgs mass, which are logarithmic, are mainly contained in ϵ_1 but are also present in ϵ_3 . In the Standard Model one has the following “large” asymptotic contributions^{[4],[40]–[41]}:

$$\begin{aligned}\epsilon_1 &= \frac{3G_F m_t^2}{8\pi^2 \sqrt{2}} - \frac{3G_F m_W^2}{4\pi^2 \sqrt{2}} \operatorname{tg}^2 \theta_W \ln\left(\frac{m_H}{m_Z}\right) + \dots \\ \epsilon_2 &= -\frac{G_F m_W^2}{2\pi^2 \sqrt{2}} \ln\left(\frac{m_t}{m_Z}\right) + \dots \\ \epsilon_3 &= \frac{G_F m_W^2}{12\pi^2 \sqrt{2}} \ln\left(\frac{m_H}{m_Z}\right) - \frac{G_F m_W^2}{6\pi^2 \sqrt{2}} \ln\left(\frac{m_t}{m_Z}\right) + \dots\end{aligned}\tag{24}$$

In passing, we note the following interesting alternative expression for the leptonic width in terms of $\bar{s}_W^2$ defined in eqs. (19-22) and ϵ_3 (valid in linear approximation in ϵ_1 and ϵ_3):

$$\Gamma_\ell = \frac{\alpha(m_Z)m_Z}{48\bar{s}_W^2 \bar{c}_W^2} \left(1 + \frac{\epsilon_3}{\bar{c}_W^2}\right) [1 + (1 - 4\bar{s}_W^2)^2]\tag{25}$$

The definitions in eqs. (18-23) are quite general and do not commit us to any particular model. The epsilons are useful in that they provide a very efficient parametrization of the most important input data with respect to the sensitivity to new physics so that they represent a convenient starting point for a model-independent analysis of the data. One can then formulate a hierarchy of simple and rather general assumptions valid in large classes of models which are needed in order to relate the epsilons to a progressively larger set of observables.

Starting from the hadron collider result on m_W/m_Z given in Table 2, by combining it with the LEP value for the Z mass; $m_Z = 91.175 \pm 0.021$ GeV, and using eq. (1) one obtains:

$$\Delta r_W = (-2.2 \pm 1.7)10^{-2} \quad (26)$$

similarly from the LEP results on the charged lepton partial width and the forward-backward asymmetry given in Table 3 and eq. (11) one finds:

$$g_A^2 = 0.2495 \pm 0.0012 \quad (27)$$

$$g_V/g_A = 0.074 \pm 0.008 \text{ or } \bar{s}_W^2 = 0.2315 \pm 0.0020 \quad (28)$$

One can now use eq. (21) to derive the results for $\Delta\rho$ and $\Delta k'$:

$$\epsilon_1 = \Delta\rho = (-0.19 \pm 0.49)10^{-2} \quad (29)$$

$$\Delta k' = (0.02 \pm 0.87)10^{-2} \quad (30)$$

Finally the corresponding results for ϵ_2 and ϵ_3 are obtained from eq. (23):

$$\epsilon_2 = (-1.10 \pm 0.88)10^{-2} \quad (31)$$

$$\epsilon_3 = (-0.14 \pm 0.67)10^{-2} \quad (32)$$

We can make contact with the notation of Refs. [31-32] by setting $\Delta\rho = \alpha T$, $\epsilon_2 = -\alpha U/(4s_0^2)$ and $\epsilon_3 = \alpha S/(4s_0^2)$. We then obtain:

$$S = -0.18 \pm 0.85, \quad T = -0.26 \pm 0.67, \quad U = 1.4 \pm 1.1 \quad (33)$$

Note, however, that in Refs. [31,32] S and T are defined in the specific context of dominance of vacuum polarisation corrections from new physics, while the present definitions are general. Also, in Refs. [31],[32], S and T are defined as deviations from the theoretical predictions of the Standard Model for given values of m_t and m_H , while the values in eq. (33) are unsubtracted.

The predictions of the Standard Model for ϵ_1, ϵ_2 and ϵ_3 have been studied in detail in Ref. [30]. We refer the reader to this paper for a description of the calculations, a comparison of results from several available programmes of radiative corrections and a discussion of the theoretical errors on the epsilons. Here we only quote the main results. In fig. 8 the predicted values of the epsilons are shown together on the same scale. Note that indeed ϵ_2 and ϵ_3 are

much flatter than ϵ_1 in m_t (this is especially true for ϵ_3 , while some dependence is still visible in ϵ_2 due mainly to logarithmic terms in m_t which are three times larger for ϵ_2 than for ϵ_3 , as is seen from eq. (24)). In figs. 9-11 the results are displayed and compared with the present experimental values. Only the comparison of $\Delta\rho$ from theory and experiment implies a strong constraint on m_t , while for ϵ_2 and ϵ_3 one observes consistency with the Standard Model with no important constraints on m_t and m_H . However, ϵ_2 and ϵ_3 impose interesting bounds on new physics, as we shall see.

Fig. 8. Theoretical predictions for ϵ_1, ϵ_2 and ϵ_3 in the Standard Model for $m_H = 50 - 1000$ GeV.

In the Standard Model, the knowledge of ϵ_1 and ϵ_3 allows one to determine all other observables measured at the Z pole and related to charged leptons, such as A_{pol}^τ and A_{LR} . As mentioned in the introduction, this is also true in a very large class of models where new physics only contributes at $q^2 = m_Z^2$ through either vacuum polarisation amplitudes and/or vertex corrections of the form $\Delta V_\mu(Z \rightarrow \ell + \bar{\ell}) = \bar{u}(\Delta g_A \gamma_5 + \Delta g_V) \gamma_\mu u$. It is therefore worthwhile to establish this connection in as general terms as possible.

Together with eq. (19), but in place of eq. (20), one could have used A_{pol}^τ to define the effective on-shell vector and axial vector Z couplings to charged

Fig. 9. $\epsilon_1 \times 10^2$ vs m_t as predicted by the Standard Model for $m_H = 50 - 1000$ GeV compared with the LEP result.

Fig. 10. $\epsilon_2 \times 10^2$ vs m_Z as predicted by the Standard Model for $m_H = 50 - 1000$ GeV compared with the LEP result.

Fig. 11. $\epsilon_3 \times 10^2$ vs m_t as predicted by the Standard Model for $m_H = 50 - 1000$ GeV compared with the LEP result.

leptons, via the relation:

$$A_{\text{pol}}^\tau = \frac{2g'_A g'_V}{g_V'^2 + g_A'^2} \quad (34)$$

We have adopted different symbols for the effective couplings because g_V and g_A , defined from A_{FB} , certainly cannot be confused with g'_V and g'_A obtained from A_{pol}^τ , to the level of accuracy at which the LEP experiments are aiming. On the contrary A_{pol}^τ and A_{LR} can be considered as equivalent in this respect. In fact we have checked that the relative differences between the two asymmetries are smaller than 10^{-4} in the Standard Model.

The set of ϵ'_i , related to g'_A and g'_V in the same way as the set of ϵ_i to g_A and g_V as in eq. (21), are also sizeably different. One can easily understand the corresponding pattern of corrections. The same effective $\sin^2 \theta_W$ would describe A_{FB} and A_{pol}^τ when only quadratic and logarithmic terms in m_t and m_H are included, because such terms only arise from vacuum polarisation diagrams, which affect the asymmetries in the same way. Thus the effective values of $\sin^2 \theta_W$ defined from one or the other asymmetry can only differ by a constant term in m_t and m_H . In fact numerically we find:

$$(\bar{s}_W^2)_{\text{pol}} = (\bar{s}_W^2)_{\text{FB}} + \delta : \delta = (1.3 \pm 0.2)10^{-3} \quad (35)$$

The main contribution to δ is from the constant $+ 0.002$ that appears in eq. (12).

Clearly, by the same argument that leads to the independence of δ from m_t and m_H , one obtains that the same shifts also apply to all cases of new physics in oblique or universal vertex corrections. Starting from the experimental value of A_{pol}^r obtained by LEP (eq. (16)) (see also fig. 7) which corresponds to (from eq. (34) using $g_V'/g_A' = 1 - 4(\bar{s}_W^2)_{\text{pol}}$):

$$(\bar{s}_W^2)_{\text{pol}} = 0.2332 \pm 0.0045 \quad (36)$$

we can apply the correction in eq. (35) and obtain $(\bar{s}_W^2)_{\text{FB}} = 0.2319 \pm 0.0045$ and hence a new value for $\Delta k'$:

$$\Delta k' = (0.2 \pm 1.9)10^{-2} \quad (37)$$

We now proceed to consider quantities that also involve quarks and are still measured at the Z pole. In the Standard Model, to obtain a prediction for these quantities a range of values for $\alpha_s(m_Z)$ has to be assumed. Also, the large quadratic terms in m_t from the $Z \rightarrow b\bar{b}$ vertex are to be taken into account. Beyond the Standard Model, it is obvious that a stronger form of universality must be assumed in order to directly transfer into the quark sector the information embodied by the ϵ_i 's measured in the charged lepton sector. Quark-lepton universality is automatic in the case of oblique corrections, while it is in general violated if vertex corrections are important.

Particularly important quantities, (nearly) independent of $\alpha_s(m_Z)$, are Γ_b/Γ_h and A_{FB}^b . Similar quantities from charm or light quarks are not equally interesting both in terms of experimental precision and because of the large m_t term [42] in the $Z \rightarrow b\bar{b}$ vertex corrections. For example, the data on Γ_b/Γ_h in principle allow us to set a bound on m_t which is independent of assumptions on the absence of exotic contributions to ϵ_1 . However, such a limit is not very interesting at the moment because a comparatively large error is introduced by ambiguities associated with the semileptonic branching ratio or by other uncertainties in the case of DELPHI that uses a purely hadronic b-selection criterium. On the contrary, A_{FB}^b is almost unaffected by the presence of large m_t -dependent vertex corrections. Schematically the reason is that $A_{\text{FB}}^b = 3\eta_e\eta_b$ with $\eta = gv g_A/(g_V^2 + g_A^2)$, so that $\delta A_{\text{FB}}^b = 3(\eta_e\delta\eta_b + \eta_b\delta\eta_e)$. We see that the sensitivity on η_b , which contains the $Z \rightarrow b\bar{b}$ vertex correction, is strongly

suppressed by the small factor η_e . This is confirmed by an accurate numerical calculation of A_{FB}^b . We define a new quantity $(\bar{s}_W^2)_b$ by the identity:

$$A_{\text{FB}}^b = 3 \frac{1 - 4 (\bar{s}_W^2)_b}{1 + \left(1 - 4 (\bar{s}_W^2)_b\right)^2} \frac{\beta \left(1 - \frac{4}{3} (\bar{s}_W^2)_b\right)}{\beta^2 + \frac{3 - \beta^2}{2} \left(1 - \frac{4}{3} (\bar{s}_W^2)_b\right)^2} \quad (38)$$

where $\beta = \sqrt{1 - 4m_b^2/m_Z^2}$. We can explicitly evaluate the relation, as a function of m_t and m_H , between $(\bar{s}_W^2)_b$ and the similar quantity $(\bar{s}_W^2)_{\text{FB}}$ previously defined from the charged lepton asymmetry A_{FB}^ℓ . We obtain:

$$(\bar{s}_W^2)_b = (\bar{s}_W^2)_{\text{FB}} + \delta_b, \quad \delta_b = (0.9 \pm 0.15)10^{-3} \quad (39)$$

(valid for $m_t = 90\text{-}300$ GeV, $m_H = 50\text{-}1000$ GeV).

As a consequence, if quark-lepton universality is assumed, one can use the present combined LEP result on A_{FB}^b given in eq. (15) which is equivalent, by eq. (38) to:

$$(\bar{s}_W^2)_b = 0.2268 \pm 0.0040 \quad (40)$$

and obtain an independent input on $\Delta k'$. In fact we derive from eqs. (39) and (40) that $(\bar{s}_W^2)_{\text{FB}} = 0.2259 \pm 0.0040$, and then, from eq. (21), the result:

$$\Delta k' = (-2.4 \pm 1.7)10^{-2} \quad (41)$$

While flavour universality of new physics is the crucial assumption that is needed to relate different measured quantities at the Z pole, some hypotheses on the absence of new sources of substantial q^2 dependence have to be formulated in order to add low energy measurements to the picture. Actually, in most of the relevant cases both flavour universality and q^2 independence have to be combined in order to make contact with important experiments, such as ν (or $\bar{\nu}$) - N deep inelastic scattering and atomic parity violation. In models where oblique corrections, which directly possess flavour universality, are dominant, sizeable q^2 -dependent effects from new physics are absent if terms involving second and higher derivatives with respect to q^2 can be neglected in vacuum polarization form factors. This is a good approximation in models with no decoupling, where first-order derivatives lead to effects of order 1 in the limit when the scale Λ of new physics becomes very large. On the other hand, in models with decoupling, like those considered in Ref. [36], first order derivatives

are of order v^2/Λ^2 , where v is a parameter of the order of the electroweak scale (typically the Higgs vacuum expectation value). In this case the effect of second order derivatives, of order m_Z^2/Λ^2 , is not relatively negligible. Note, however, that in these models both effects are quite small unless the scale Λ of the new physics is very close to the domain of energies of present experiments.

If one assumes flavour and lepton-quark universality and no additional q^2 -dependence, the available data on neutrino-nucleus deep inelastic scattering and on parity violation in Cs atoms lead to further constraints on ϵ_1 and ϵ_3 , while they have no direct effect on ϵ_2 . The present data^{[16]–[17]} on R_ν and $R_{\bar{\nu}}$, the ratios of neutral to charged current processes in deep inelastic neutrinos scattering on nuclei, imply the following constraints^{[20],[34]}:

$$\begin{aligned} R_\nu : \epsilon_1 - 0.34\epsilon_3 &= (-0.07 \pm 0.45)10^{-2} \\ R_{\bar{\nu}} : \epsilon_1 - 0.02\epsilon_3 &= (-1.34 \pm 0.95)10^{-2} \end{aligned} \quad (42)$$

with our definition of epsilons (for comparison, we also report the result from the ratio of neutrino to antineutrino scattering on electrons^[43], which gives^{[20],[34]} $\epsilon_3 - 0.74 \epsilon_1 = (0.13 \pm 2.12)10^{-2}$).

Similarly, the results on parity violation in Cs^{[44],[45]} lead to a value of ϵ_3 , while the sensitivity to ϵ_1 is accidentally almost exactly cancelled due to the particular ratio of protons to neutrons in Cs^[34]. Neglecting the ϵ_1 contribution, one finds in this case the general result:

$$Q_W = -72.84 \pm 0.13 - 102\epsilon_3 \quad (43)$$

The present experimental value^{[44],[45]}:

$$(Q_W)_{\text{exp}} = -71.04 \pm 1.81 \quad (44)$$

implies the result:

$$\epsilon_3 = (-1.8 \pm 1.8)10^{-2} \quad (45)$$

Note that in eqs. (42),(45) the quoted values for ϵ_1 and ϵ_3 are inclusive of all standard and possibly non-standard effects.

In a large variety of different models the epsilon parameters are suitable for a discussion of the possible effects of new physics on the various observables. There is an extensive and still rapidly growing literature on the subject. A summary is given in Ref. [30].

We now start from the model-independent determination of the epsilons given in eqs. (29),(31),(32) and progressively make various stages of assumptions that allow us to combine an increasing large set of data. We first assume that there are no new physics effects that can invalidate the connection with A_{pol}^τ (e.g. peculiar four-fermion interactions which could affect A_{FB}^ℓ but not A_{pol}^τ). As a second step we assume flavour and lepton-quark universality and we include hadronic quantities measured at the Z-peak. In particular we consider A_{FB}^b which is precisely measured and relatively unaffected by $\alpha_s(m_Z)$. Then, in addition, we assume the absence of additional sources of q^2 dependence beyond the Standard Model and include neutrino-nucleus deep inelastic scattering and parity violation in Cs atoms.

The inclusion of the data on A_{pol}^τ allows us to combine the values of $\Delta k'$ given in eqs. (30) and (37), thus obtaining:

$$\Delta k' = (0.04 \pm 0.81)10^{-2} \quad (46)$$

We can then evaluate ϵ_1, ϵ_2 and ϵ_3 by using the known values of Δr_W and Γ_ℓ and the above value of $\Delta k'$. This gives:

$$\begin{aligned} \epsilon_1 &= \Delta\rho = (-0.19 \pm 0.49)10^{-2} \\ \epsilon_2 &= (-1.11 \pm 0.87)10^{-2} \\ \epsilon_3 &= (-0.12 \pm 0.63)10^{-2} \end{aligned} \quad (47)$$

This stage is permissible in all models, including the case of a new Z' from an extra U(1) provided the universality of charged leptons is maintained.

We now add hadronic quantities measured at the Z pole. The b-asymmetry A_{FB}^b leads to the determination of $\Delta k'$ given in eq. (41) which can be combined with those leading to eq. (46). The resulting value is:

$$\Delta k' = (-0.40 \pm 0.70)10^{-2} \quad (48)$$

At this stage one could also include the hadronic width Γ_h and/or the total width Γ_Z , for $\alpha_s(m_Z)$ varying in a specified interval, for example in the range measured at LEP, as given in Section 1. The presence of large vertex corrections in the $Z \rightarrow b\bar{b}$ vertex makes the relation with the epsilons strongly m_t dependent

(as is also the case for Γ_b/Γ_h). For these reasons we restrict our attention to A_{FB}^b at this stage. For the epsilons we then obtain:

$$\begin{aligned}\epsilon_1 &= \Delta\rho = (-0.25 \pm 0.49)10^{-2} \\ \epsilon_2 &= (-0.96 \pm 0.86)10^{-2} \\ \epsilon_3 &= (-0.41 \pm 0.58)10^{-2}\end{aligned}\tag{49}$$

The next step, valid if the effects of a large q^2 difference can be described as in the Standard Model, is to include the low energy data, in particular neutrino-nucleus deep inelastic scattering and parity violation in Cs atoms. By combining the low energy results given in eqs. (42) to (45) with the rest of the data one finds:

$$\begin{aligned}\epsilon_1 &= \Delta\rho = (-0.07 \pm 0.36)10^{-2} \\ \epsilon_2 &= (-0.80 \pm 0.81)10^{-2} \\ \epsilon_3 &= (-0.28 \pm 0.51)10^{-2}\end{aligned}\tag{50}$$

At each stage, in the $\epsilon_1 - \epsilon_3$ plane, the experimental result is compared in figs. 12a to 12d with the predictions of the Standard Model for different values of m_t and m_H . It is well known^[34] that the data on atomic parity violation in Cs push the value of ϵ_3 on the negative side [see eq. (45)]. We see that this tendency toward negative values of ϵ_3 is also supported by the very recent data on A_{FB}^b (see eq. (49)). The effect of A_{FB}^b can be appreciated from fig. 12e which is a fit to the same data as fig. 12d but with A_{FB}^b removed.

As seen from the overall summary in fig. 12d, the central value of ϵ_3 is negative and about 1σ away from the Standard Model prediction. Clearly, models where an additional positive contribution to ϵ_3 is predicted are a fortiori discouraged, as in the case for the class of technicolour models^{[30],[31]–[35],[46]} leading to the band of values (corresponding to $N_{TC} = 4$ and one technifamily) also shown in fig. 12d. The values of ϵ_2 are consistent with the Standard Model, with a still rather large error (fig. 13).

Fig. 12a.

Fig. 12b.

Fig. 12c.

Fig. 12d.

Fig. 12e. Data on ϵ_1 and ϵ_3 . The 1σ ellipses and their projections on the ϵ_1, ϵ_3 axes are shown. The Standard Model prediction is also displayed for reference purposes (the four solid lines are for different values of $m_H, m_H = 50, 100, 500, 1000$ GeV, and the dots mark values of m_t in the range $m_t = 50-270$ GeV). Cases a) to d) correspond to different input data. a) is obtained from Γ_ℓ and A_{FB}^ℓ . In b) the data on A_{pol}^τ have been also taken into account. The result for A_{FB}^b is added in c). All data, including low energy experiments (eq. 50), are included in d). The predictions of QCD-like versions of technicolour (with $N_{TC} = 4$ and one technifamily) are also shown for comparison. Case e) corresponds to the all data but for A_{FB}^b .

Summarising, we have shown that it is possible and indeed useful to introduce the parameters ϵ_1, ϵ_2 and ϵ_3 (or equivalently S, T and U) following a general definition independent on assumptions like the dominance of oblique corrections as were made in previous discussions. The presence of three parameters is related to the fact that $m_W/m_Z, \Gamma_\ell$ and A_{FB}^ℓ carry qualitatively different information and are the most precisely measured observables defined at the W/Z mass scale (beyond the input parameter m_Z). ϵ_2 and ϵ_3 are good indicators for the presence of new physics effects, because the uncertainties due to our ignorance of m_t are concentrated in ϵ_1 . The epsilons have been studied

in the Standard Model as functions of m_t and m_H and the associated theoretical errors were estimated. ϵ_3 turns out to be particularly interesting, being independent of m_t with very good accuracy, sensitive to Higgs sector and likely to collect large contributions in models with no decoupling. Present data on A_{FB}^b and on parity violation in atomic Cesium favour values of ϵ_3 smaller than in the Standard Model (although compatible with it).

Fig. 13. ϵ_2 vs m_t in the Standard Model for different values of m_H ($m_H = 50, 100, 500, 1000$ GeV) with the experimental result in eq. (50), obtained from all the data which are available at present.

4. Heavy Flavours at LEP

We have already discussed the importance of A_{pol}^τ and A_{FB}^b for precision tests of the electroweak theory. In this section a number of additional important results of LEP on heavy flavour phenomenology will be discussed. These results include Γ_b , the b semileptonic branching ratio $B_{SL} \equiv B(b \rightarrow \ell \nu X)$ ($\ell = e, \mu$) and the τ -lifetime.

A quantity which is directly measured at LEP is the product $B_{SL} \Gamma_b / \Gamma_h$. The present experimental results are given in Table 5^{[47],[48]}.

TABLE 5

$B_{SL} \frac{\Gamma_b}{\Gamma_h}$	$= 0.0224 \pm 0.0009 \pm 0.0007$ ALEPH ($e + \mu$) $= 0.0230 \pm 0.0012 \pm 0.0011$ DELPHI (μ) $= 0.0259 \pm 0.0005 \pm 0.0007$ L3 ($e + \mu$) $= 0.0226 \pm 0.0007 \pm 0.0013$ OPAL (μ)
	$= 0.0239 \pm 0.0010$ Average

Two main applications of these results are possible: either one takes Γ_b/Γ_h from the Standard Model and obtains B_{SL} , or one can input B_{SL} from other measurements and derive a value for Γ_b/Γ_h . While both ways are interesting, to me the determination of B_{SL} is more important, because the LEP number is very precise and the situation with B_{SL} is a little mysterious. Inserting the Standard Model value $\Gamma_b/\Gamma_h \simeq 0.217$ one obtains:

$$\begin{aligned}
 B_{SL} &= (10.3 \pm 0.4 \pm 0.3)\% \text{ ALEPH} \\
 &= (10.6 \pm 0.6 \pm 0.5)\% \text{ DELPHI} \\
 &= (11.9 \pm 0.2 \pm 0.3)\% \text{ L3} \\
 &= (10.4 \pm 0.3 \pm 0.6)\% \text{ OPAL}
 \end{aligned} \tag{51}$$

Note that the L3 result is somewhat different from the other ones. The L3 collaboration has also given an independent determination of B_{SL} from the ratio $N_{\ell\ell}/N_\ell$ of dilepton vs single lepton events, which leads to $B_{SL} = (11.3 \pm 1.2)\%$.

These results can be compared with those obtained at PEP and PETRA:^{[47],[48]}

$$B_{SL} = 12.0 \pm 0.7\% \tag{52}$$

At the $\Upsilon(4S)$ there is a collection of recent, precise results, which, however, in principle do not refer to the same average as measured at higher energies. The $\Upsilon(4S)$ results are^[47]:

$$\begin{aligned}
 B_{SL} &= 10.2 \pm 0.54\% \text{ ARGUS} \\
 &= 10.5 \pm 0.45\% \text{ CLEO} \\
 &= 10.0 \pm 0.5\% \text{ CUSB} \\
 &= 12.2 \pm 0.9\% \text{ X-BALL} \\
 &\hline
 &10.4 \pm 0.3\%
 \end{aligned} \tag{53}$$

We see that L3 and PEP/PETRA suggest a value B_{SL} at high energies larger than at the $\Upsilon(4S)$, while the other LEP experiments tend to confirm the low energy value.

From a purely theoretical point of view a value of B_{SL} below 12% is difficult to accommodate in a spectator picture of b decays^[49]. This is at least true in the Standard Model. For example, large $b \rightarrow sg$ or $b \rightarrow c\tau\nu$ rates which are absolutely out of scale in the Standard Model but can arise in models with charged Higgs^[50], could fix the problem. More conventionally a large W -exchange contribution (with gluon emission) could conceivably be evoked^[49]. But this explanation would require $\tau^-/\tau^0 \simeq 1.2$, which at present is rather marginal, but not excluded. Other people may simply find that a 20% violation of the parton model picture with inert spectators for the b -quark system is not surprising^[51].

Going back to $B_{SL}\Gamma_b/\Gamma_h$ the other possible path is to take B_{SL} from PEP/PETRA plus the measurement by L3 of B_{SL} from $N_{\ell\ell}/N_\ell$ and extract Γ_b/Γ_h from the results of Table 4. With $B_{SL} = 11.8 \pm 0.6\%$ one obtains^[48]:

$$\frac{\Gamma_b}{\Gamma_h} = 0.240 \pm 0.012 \text{ (LEP AVERAGE)} \quad (54)$$

This ratio is in principle interesting because it is independent of $\alpha_s(m_Z)$ and sensitive to the large $Z \rightarrow b\bar{b}$ vertex corrections of order $G_F m_t^2$ ^[42]. (The b quark is special because $V_{tb} \simeq 1$). With a precise enough measurement one could determine m_t independent of $\Delta\rho$. In fact $\Delta\rho$ depends on m_t through vacuum polarization diagrams where other new physics effects could mask the m_t dependence. In practice there is not enough precision (and there will not be in the near future) for such a test.

I conclude by recalling that there is a persistent small discrepancy between the Standard Model and the measured values of the τ -lifetime and the τ leptonic branching ratio. One can compute the τ -lifetime by:

$$(\tau_\tau)_{th} = \tau_\mu \left(\frac{m_\mu}{m_\tau} \right)^5 B(\tau \rightarrow \nu_\tau \ell \nu) \quad (55)$$

where $B(\tau \rightarrow \nu_\tau \ell \nu)$ is taken from experiment. The effects of differences in phase space and in radiative corrections are insignificant, and are easily taken

into account. The theoretical number $(\tau_\tau)_{\text{th}}$ can be compared with the experimental value. The result is a deficit in the τ width that can be described by an equivalent ratio of Fermi couplings given by^[47]:

$$\frac{G_\tau}{G_\mu} = 0.975 \pm 0.010 \quad (56)$$

The fact that perhaps $\Gamma_{\text{exp}} < \Gamma_{\text{TH}}$ suggests that possibly $\Gamma_{\text{exp}} = \cos^2 \theta \Gamma_{\text{TH}}$. In other words there could be a mixing of ν_τ with some heavy neutrino. One possibility is a fourth lepton generation with ν_4 so heavy that it does not affect the Z invisible width, or $\nu_\tau = \cos \theta \nu_1 + \sin \theta \nu_2$ with the orthogonal combination $-\sin \theta \nu_1 + \cos \theta \nu_2$ being a singlet under SU(2). ν_1 and ν_2 are mass eigenstates. If ν_2 is very heavy, then the Z invisible width is suppressed by $\cos^4 \theta$ (because there are two τ -neutrinos in the final state of $Z \rightarrow \nu_\tau \nu_\tau$). Thus a 5% (3%) suppression of Γ_τ would imply $N_\nu = 2.90$ (2.94), i.e. a 2σ stretching of the present results would be needed. If $m_{\nu_2} > m_\tau$ but $m_{\nu_2} \ll m_Z$ it is difficult to keep ν_2 invisible and to escape present bounds on a heavy neutral lepton. A more trivial way out is the possibility that the precise measurement of m_τ (which was done only by DELCO) is slightly incorrect.

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