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Topical Review

Unified Scaling Law for flux pinning in practical superconductors: II. Parameter testing, scaling constants, and the Extrapolative Scaling Expression

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Abstract

A scaling study of several thousand Nb₃Sn critical-current (I_c) measurements is used to derive the Extrapolative Scaling Expression (ESE), a relation that can quickly and accurately extrapolate limited datasets to obtain full three-dimensional dependences of I_c on magnetic field (B), temperature (T), and mechanical strain (ε). The relation has the advantage of being easy to implement, and offers significant savings in sample characterization time and a useful tool for magnet design.

Thorough data-based analysis of the general parameterization of the Unified Scaling Law (USL) shows the existence of three universal *scaling constants* for practical Nb₃Sn conductors. The study also identifies the scaling parameters that are conductor specific and need to be fitted to each conductor. This investigation includes two new, rare, and very large $I_c(B, T, \varepsilon)$ datasets (each with nearly a thousand I_c measurements spanning magnetic fields from 1 to 16 T, temperatures from ~ 2.26 to 14 K, and intrinsic strains from -1.1% to $+0.3\%$). The results are summarized in terms of the general USL parameters given in table 3 of Part 1 (Ekin J W 2010 *Supercond. Sci. Technol.* **23** 083001) of this series of articles. The scaling constants determined for practical Nb₃Sn conductors are: the upper-critical-field temperature parameter $\nu = 1.50 \pm 0.04$; the cross-link parameter $w = 3.0 \pm 0.3$; and the strain curvature parameter $u = 1.7 \pm 0.1$ (from equation (29) for $b_{c2}(\varepsilon)$ in Part 1). These constants and required fitting parameters result in the ESE relation, given by

$$I_c(B, T, \varepsilon)B = C[b_{c2}(\varepsilon)]^f (1 - t^{1.5})^{\eta-\mu} (1 - t^2)^\mu b^p (1 - b)^q$$

with reduced magnetic field $b \equiv B/B_{c2}^*(T, \varepsilon)$ and reduced temperature $t \equiv T/T_c^*(\varepsilon)$, where:

$$B_{c2}^*(T, \varepsilon) = B_{c2}^*(0, 0)(1 - t^{1.5})b_{c2}(\varepsilon)$$

$$T_c^*(\varepsilon) = T_c^*(0)[b_{c2}(\varepsilon)]^{1/3}$$

and fitting parameters: C , $B_{c2}^*(0,0)$, $T_c^*(0)$, s , either η or μ (but not both), plus the parameters in the strain function $b_{c2}(\varepsilon)$. The pinning-force shape parameters p and q are also preferably fitted (simultaneously with the other parameters), but default values $p = 0.5$ and $q = 2.0$ also give high fitting accuracy when the range of relative magnetic fields is not extensive. Default values are also essential when the magnetic field data range is insufficient to determine p and q .

The scaling constants are remarkably stable (changes less than $\sim 1\%$) with respect to different values of p and q , Nb₃Sn conductor configurations, magnetic self-field corrections, and pinning-force trim values. The results demonstrate that the scaling of transport critical current holds down to the lowest temperatures measured ~ 2.2 K, for both magnetic self-field corrected and uncorrected data. An initial comparison is also made between transport and magnetization scaling data in matched Nb₃Sn samples and significant differences are found, especially for the upper critical field $B_{c2}^*(T, \varepsilon)$, which may be a result of inhomogeneous shielding currents.

In Part 3 of this topical review series (Ekin J W 2017 *Supercond. Sci. Technol.* at press), the smallest practical *minimum dataset* for extrapolating full $I_c(B, T, \varepsilon)$ datasets is derived. Application of the ESE relation is illustrated in several new areas, including full characterization of Nb₃Sn conductors from as little as a single $I_c(B)$ curve when a few core parameters have been determined for similar conductors.

Keywords: Nb₃Sn, scaling law, flux pinning, critical current, strain, superconductor measurements, critical current extrapolation

(Some figures may appear in colour only in the online journal)

1. Introduction

The Unified Scaling Law (USL), postulated more than three decades ago (Ekin 1980), has been substantiated only recently with the advent of large datasets of critical current $I_c(B, T, \varepsilon)$ measured as a function of magnetic field (B), temperature (T), and strain (ε). It is now being extensively applied in practical magnet design through the use of various multi-parameter fitting equations, which are interpolative in nature. However, this approach considerably compromises the ability to *extrapolate* limited data to obtain complete $I_c(B, T, \varepsilon)$ datasets by use of master scaling curves (which form the basis of the USL). A principal purpose of this series of review articles is to reintroduce the extrapolation capability of unified scaling, *in an easy way*. The savings in measurement time and expense is considerable, reducing data acquisition for complete conductor characterization to about one fifth the number of measurements. In special situations where similar conductors have already been characterized, the procedure can be abbreviated to the measurement of a single $I_c(B)$ curve. The intent of this article is to facilitate engineering applications of the USL by providing a useful tool for wire characterization and magnet modeling that provides extrapolation capability from limited conductor data.

This topical review series consists of 3 parts:

Part 1 (previously published, Ekin 2010): *A review of the history of the USL and its parameterization*. Emphasis is given to the use of master scaling curves. The recently introduced alternative method of interpolative fitting equations is also reviewed, and the many parameters in these equations are organized by splitting the general parameterization of the USL into *separable parts*, each dependent on reduced temperature or strain only. The end result of Part 1 is a ‘Rosetta Stone’ table (table 3) for translating among the various parameterizations.

This table and the separable parts of the USL form the starting point of the present article. An easily implemented fitting equation is also determined in Part 1 for accurate interpolations and extrapolations at *moderate* strains, where many magnets are designed.

Part 2 (present article): *Parameter testing and derivation of the Extrapolative Scaling Expression (ESE)*. The derivation is carried out by analyzing values of the parameters in the USL, either individually or in small groups, with extensive *raw scaling data* from a number of large Nb₃Sn conductor datasets. Raw scaling data are the values of the effective upper critical field $B_{c2}^*(T, \varepsilon)$ and maximum pinning force $F_{Pmax}(T, \varepsilon)$ that register all the pinning-force curves into a master scaling curve. (This will be illustrated in figure 1 of the next section, and an example of raw scaling data for the extensive OST-RRP® dataset is shown in appendix table A2.) The results determine which USL parameters are scaling constants, and which are conductor specific and need to be fitted to each conductor. The analysis gives the ESE relation – a fitting equation that can be easily applied, but retains the extrapolation capability of fundamental scaling. The analysis is carried out with I_c data both corrected and uncorrected for magnetic self field.

Part 3 (to be published sequentially with the present article): *Extrapolations from minimum datasets, and practical application of the ESE relation*. Raw scaling data are used to also derive the effective minimum dataset needed to extrapolate a full, three-dimensional $I_c(B, T, \varepsilon)$ dataset. Extrapolation testing from such minimum datasets is then carried out with the ESE relation and compared with present fitting equations. Applications of the extrapolation capability of ESE are illustrated for several common measurement needs. In special situations, such as the characterization of large quantities of production wire where several core scaling parameters have been determined for similar

Two approaches in use to analyze $I_c(B, T, \varepsilon)$

Figure 1. Two methods currently used to analyze $I_c(B, T, \varepsilon)$ datasets: (a) *Data*: pinning force curves ($F_p \equiv I_c(B, T, \varepsilon) B$ in units of kiloampere tesla, kAT, equivalent to kN m^{-1}) plotted at different temperatures and axial strains ($I_c(B, T, \varepsilon)$ data from Taylor and Hampshire 2005; The flux pinning curves are color coordinated with those in figures (b) and (c): red $\rightarrow$ 4.2 K, green $\rightarrow$ 8.0 K, blue $\rightarrow$ 12.0 K. (b) *Method 1: Fundamental scaling*—curve registration of the data in graph (a) utilizing each curve's measured pinning-force peak value $F_{pmax}(T, \varepsilon)$ and extrapolated magnetic-field axis intercept $B_{c2}^*(T, \varepsilon)$. The curves coalesce into a single master scaling curve. (c) *Method 2: Global-fitting equation*—the data in graph (a) are simultaneously fit with a multi-parameter equation. If values of $B_{c2}^*(T, \varepsilon)$ and $F_{pmax}(T, \varepsilon)$ are calculated from the resulting equation and used to normalize the pinning force curves, a single master scaling curve does not result. Global fitting is more expedient than fundamental scaling, but it has greatly diminished extrapolation capability.

conductors, full $I_c(B, T, \varepsilon)$ datasets can be obtained from very limited data, including extrapolations from a fit of ESE to a single $I_c(B)$ curve.

Thus, the ESE parameterization of the USL is derived from a thorough one-time analysis of raw scaling data from pinning-force curves measured in many different Nb_3Sn conductors (Part 2). In practice, however, ESE can be simply applied as a fitting equation (without raw scaling data) to accurately extrapolate full $I_c(B, T, \varepsilon)$ datasets from limited data in a quick, straightforward manner (Part 3).

1.1. Methods for determining USL parameter values

Two distinct methods are presently used to determine parameter values for the USL (Ekin 2010). We briefly summarize them here, and then introduce a third method that combines the best of the first two.

Method 1: Fundamental scaling: The USL exists independent of specific modeling equations. Fundamental, true scaling is the principle that pinning-force curves (F_p versus B , where $F_p \equiv I_c B$), measured at different temperatures and strains, are *shape* invariant and thus can be registered into a single *master unified-scaling curve*. This is done by normalizing them with their maximum pinning-force values F_{pmax} and magnetic-field axis intercepts B_{c2}^* (illustrated in the left side of figure 1 in section 2.1). Raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $F_{pmax}(T, \varepsilon)$ determined from the registration process are the fundament of scaling. The great practical advantage of this scaling method is just that—scaling. *From such a master scaling curve, one pinning-force curve can be scaled to give any of the other pinning-force curves comprising the master, and thus this method offers the capability of extrapolation.*

This is a remarkable natural phenomenon that could also pay significant dividends for other superconducting materials where scaling holds (e.g., MgB_2 , Bi-2223, Nb_3Al , and YBCO). Note, however, that master scaling curves can be formed only when the basic pinning mechanisms in the superconductor remains unchanged over the magnetic field, temperature, and strain range under consideration. This may limit applicability of scaling in some high- T_c superconductors, and restricts it in low- T_c superconductors to regimes away from where the basic pinning mechanism significantly changes (for example, away from where flux creep starts to dominate near the critical temperature and upper critical magnetic field, which normally is not a practical limitation).

Method 2: Global fitting equations: About a decade after the original postulation of the USL, a shortcut was introduced which consists of fitting I_c data with an empirical or semi-empirical interpolative equation. Such relations have been proposed, starting with Summers *et al* (1991), and more recently Keys and Hampshire (2003), as well as most of the recent references cited in Part 1. The fitting of all the parameters in such an equation is carried out *simultaneously* to available data (i.e. ‘global’ fitting), skipping the step of calculating raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $F_{pmax}(T, \varepsilon)$ to build a master scaling curve. This global-fitting method is quicker than fundamental scaling, but, because so many parameters are fit simultaneously, a single master scaling curve does not result. Extrapolations are consequently severely compromised. As a result, such global-fitting equations are useful for *interpolation*, but not for extrapolation of limited data to more complete datasets. With method 2, extensive parameter-space measurements are required to fully characterize a conductor.

Method 3: ESE: Here, in Part 2, we develop a third method, an expression that combines the extrapolation capability of fundamental scaling with the application ease of a global-fitting equation. ESE (‘easy’) is a fitting equation determined from analysis of hundreds of raw scaling data points obtained from thousands of I_c measurements for many different Nb_3Sn conductors. This analysis shows the existence of several scaling constants, and also identifies the parameters that require fitting to each conductor. When combined with the general parameterization of the USL, these constants and fitting parameters determine ESE. Two new, very large datasets measured at the National Institute of Standards and Technology (NIST) for both high- J_c and moderate- J_c conductors are utilized along with other datasets to test the extrapolation accuracy of the ‘easy’ fit compared with the fitting equations in current use. Part 3 shows the remarkable improvement in extrapolation (and interpolation) capability of ESE in test cases covering most fitting-equation and fitting-method possibilities.

Method 3 is a recognition that the global-fitting method is simpler to apply than fundamental scaling, which requires the determination of raw scaling data and master scaling curves. Fortunately in practice, ESE is, well, easy. It is the simplest way to obtain extrapolation capability. No calculations of raw scaling data are needed.

- **Derivation** of ESE is a single study (the purpose of Part 2), wherein many scaled curves for the pinning force as a function of temperature and strain are used to evaluate each of the parameters in the general parameterization of the USL, either individually or in small groups of 2–4.
- **Application** of ESE, however, is just a single, simultaneous fit of its parameters to available data; and even data from different measurement apparatuses can be combined. In practice, this is relatively quick to implement. For large amounts of data it helps to use a nonlinear regression program such as SAS® or MATLAB®, but for limited data and special situations where some of the core scaling parameters are known, spreadsheet programs such as the Solver subroutine in Excel™ work effectively.

The shortcut of an extrapolative fitting equation similar to the ESE relation may also exist for superconducting materials other than Nb_3Sn , as noted previously. This is because unified scaling is a ubiquitous phenomenon with master scaling curves demonstrated for Nb–Hf/Cu–Sn–Ga, V_3Ga , and Nb_3Ge (Ekin 1981 and the references in Ekin 2006); $\text{V}_2(\text{Hf}, \text{Zr})$ (Wada *et al* 1982), V_3Si (Ekin 1984); Nb_3Al (Ekin 1984, Keys *et al* 2002, Banno *et al* 2005), PbMo_6S_8 (Ekin *et al* 1985), MgB_2 (Dhalle *et al* 2005, Cheggour *et al* 2009), Bi-2223 (van der Laan *et al* 2006), and Y–Ba–Cu–O (YBCO) (Cheggour *et al* 2003, 2005a, 2005b, van der Laan *et al* 2007, 2012). Of course, angular dependences need to be considered for the high- T_c superconductors, but the essential point is that when flux pinning curves can be scaled into a master curve, one flux pinning curve can in principal predict them all. However, the development of such an *extrapolative* fitting equation for other practical superconducting materials requires extensive analysis of pinning force curves and raw scaling data, applied with the logical sequence summarized at the beginning of section 4.

1.2. Extras

A computer algorithm to simplify the determination of raw scaling data (B_{c2}^* and F_{pmax}) from $I_c(B, T, \varepsilon)$ measurements is published online in a supplemental website accompanying this article, www.ResearchMeasurements.com. Examples of the unified master scaling curves that result from this registration process are illustrated in appendix A. Tabulations of raw scaling data calculated from a number of larger Nb_3Sn datasets are also published as Excel™ files in the supplemental website.

Particular emphasis is given to two new, rare, very large datasets, each comprising nearly a thousand critical measurements that span a wide range of magnetic field, temperature, and strain ($1 \text{ T} < B < 16 \text{ T}$, temperatures from $2.45 < T < 14 \text{ K}$, and intrinsic strains from -1.1% to $+0.3\%$):

- A high- J_c RRP® ternary Nb_3Sn multifilamentary conductor manufactured by Oxford Superconducting Technology (OST) for the High Luminosity Large Hadron

Collider (HL-LHC) magnets and nuclear magnetic resonance (NMR) magnets, billet #8781, 0.7 mm diameter, sub-element count 54/61, distributed Nb diffusion barriers, Ta doping (7.5 wt%, 4at%, relative to the original Nb filament), no Cr plating, and a final heat treatment at 640 °C for 48 h. This conductor has a very low irreversibility strain limit near 0% intrinsic strain. Appendix A.1 gives further information for this conductor, including a cross-section micrograph and a tabulation of raw scaling data, both with and without magnetic self-field corrections.

- A moderate- J_c internal-tin ternary Nb₃Sn multifilamentary conductor manufactured by WST for the International Thermonuclear Experimental Reactor (ITER) toroidal-field (TF) magnets, billet #01CW0014A01, 0.82 mm diameter, a single Ta diffusion barrier, no Cr plating, reacted with heat treatment cycle B and a final heat treatment at 650 °C for 100 h. Further conductor information, including a cross-section micrograph and self-field corrected and uncorrected master scaling curves, is given for this conductor in appendix A.2.

These NIST datasets were obtained by use of a 1 kA class Walters spring apparatus (Walters *et al* 1986, Cheggour and Hampshire 2000, 2002, and Goodrich *et al* 2013), and are published in their entirety in the supplemental website cited above.

Less extensive datasets for other conductors are also included in the analysis, as listed in appendix A. Note that the focus of this study is on practical conductors, reacted with heat treatments meant to produce high currents and strong pinning.

1.3. Organization of these articles

A synthesis of the key results of Part 2 is included in section 6. A combined executive summary of both Parts 2 and 3 is given at the end of Part 3. Parts 2 and 3 build on the framework provided in Part 1 (Ekin 2010).

Organization of Part 2. Derivation of the ESE

- (1) Introduction.
- (2) Overview of method for deriving the ESE relation.
- (3) Mathematical description of the USL—a brief review.
- (4) Determination of scaling parameter values with raw scaling data for Nb₃Sn: which are constant; which are not.
- (5) Magnetic self-field correction for transport data; comparison with magnetization data.
- (6) Summary and derivation of ESE; two types of extrapolations.

2. Overview of method for derivation of the ESE relation

Brief definition of nomenclature:

- In these articles, magnetic induction B is assumed to be perpendicular to the current direction, and denoted simply as magnetic field, defined as $B \equiv \mu_0 H$ to give the practical unit of tesla.

- The *intrinsic longitudinal strain* is defined as $\varepsilon_0 \equiv \varepsilon - \varepsilon_m$, where ε is the longitudinal strain applied along the conductor axis, and ε_m is the applied longitudinal strain at the $I_c(\varepsilon)$ maximum. This serves to compare different conductors with varying amounts of compressive prestrain, which results from differential thermal contraction between the superconducting material and the matrix of the composite conductor, plus strain introduced by differential thermal contraction from the sample holder material.
- The pinning force is characterized by the critical pinning force per unit conductor length, $F_p \equiv I_c B$, for current perpendicular to magnetic field ($\mathbf{F}_p = -\mathbf{F}_L \equiv \mathbf{B} \times \mathbf{I}_c$, where $\mathbf{F}_L$ is the Lorentz force). Although F_p has units of force per unit length, this quantity is used because it gives *practical units of ampere-tesla* (AT, equivalent to N m^{-1}). Here, F_p is denoted simply as the *pinning force*.

2.1. Illustration of the two methods currently used to analyze $I_c(B, T, \varepsilon)$ datasets

The fundamental process of scaling individual pinning force curves into a master pinning-force curve, and the raw scaling data this registration procedure yields, are demonstrated on the left side of figure 1. The other method with global fitting equations is illustrated on the right side.

The top center graph 1(a) shows pinning-force curves measured for a typical practical conductor. Each data curve in figure 1(a) corresponds to a different pair of applied strain and temperature values, with temperatures represented by different colors, and strains by different symbols. (These data have not been corrected for magnetic self field; however the results are the same within experimental error for self-field corrected data, as analyzed in section 5.)

- (1) *Fundamental scaling*—The left side, figure 1(b), shows the main points of fundamental scaling:

- *Master scaling curve*: If the individual F_p versus B pinning-force curves in figure 1(a) are scaled by their maxima $F_{p\max}(T, \varepsilon)$ and magnetic-field axis intercepts $B_{c2}^*(T, \varepsilon)$, they coalesce into a single master scaled curve, as shown in figure 1(b). This *curve registration* and the resulting master scaling curve are the physical basis of the USL.
- All we get from fundamental scaling are the shape of the master scaling curve, and the raw scaling data for each curve: that is, the x -axis intercept $B_{c2}^*(T, \varepsilon)$ and the maximum pinning force $F_{p\max}(T, \varepsilon)$ (or, equivalently, the prefactor $K(T, \varepsilon)$ of the USL)⁴.

⁴ $K(T, \varepsilon)$ is proportional to $F_{p\max}(T, \varepsilon)$. That is, from the general form of the USL $F_p = K(T, \varepsilon) b^p (1 - b)^q$ where $b \equiv B/B_{c2}^*(T, \varepsilon)$, we see that the maximum $F_{p\max}(T, \varepsilon)$ occurs at $b_{\max} = p/(p + q)$, and thus $F_{p\max}(T, \varepsilon) = K(T, \varepsilon) b_{\max}^p (1 - b_{\max})^q$. Here, the term $b_{\max}^p (1 - b_{\max})^q$ has a constant value for all temperatures and strains for any particular conductor. For example, if a conductor's master scaling curve has pinning-force shape parameters $p = 0.5$ and $q = 2.0$, then $b_{\max} = 0.2$, and thus $F_{p\max}(T, \varepsilon) = 0.286 K(T, \varepsilon)$ for this conductor.

Derivation of the Extrapolative Scaling Expression (ESE)

Figure 2. Derivation of the extrapolative scaling relation. *Left side:* scaling analysis is used to determine raw scaling data for $B_{c2}^*(T, \epsilon)$ and $K(T, \epsilon)$ in many different conductors. *Middle:* these data are then used to determine which of the general scaling parameters are constants, and which parameters are conductor specific. *Right side:* the results of this analysis determine ESE, a unique parameterization of the USL with three scaling constants and five fitted parameters, plus the parameters in the strain function $b_{c2}(\epsilon)$. This expression has the extrapolation capability of fundamental scaling when used as a global-fitting equation (without the need to scale data into a master scaling curve).

- *Extrapolation capability:* Raw scaling data provide a very powerful tool for extrapolation. A measurement of the magnetic field dependence of I_c in a conductor at a *single* temperature and strain pair (i.e., a single pinning-force curve in figure 1) gives the I_c - B dependence at any other temperature and strain, *if we know the functions $B_{c2}^*(T, \epsilon)$ and $K(T, \epsilon)$* . What makes this extrapolation possible is that *the shape of the pinning-force curves is unaltered by strain or temperature—the essence of unified temperature and strain scaling*.
- The original parameterization of the USL was built up step-by-step in small parameter groups by this process.

(2) *Global-fitting equations*—The *right side*, figure 1(c), shows the global-fitting shortcut.

This later approach consists of a global fit of data to an equation empirically formulated to parameterize the USL. Currently, this procedure has become the norm for I_c dataset analysis.

With this second method:

- The curve registration step is skipped: no scaling is performed and, instead, a many-parameter fit is made directly to the entire $I_c(B, T, \epsilon)$ dataset of figure 1(a), with all the fitting parameters determined simultaneously.
- Minimal analysis time: this method saves the analysis time of determining raw scaling values of $B_{c2}^*(T, \epsilon)$ and $K(T, \epsilon)$ for each curve.
- However, global-fitting is a low leverage technique for determining parameter values. The difficulty arises because of the interdependence of fitting so many parameters simultaneously.
- Figure 1(c) shows the results of normalizing the pinning-force curves of figure 1(a) with values of $F_{pmax}(T, \epsilon)$ and

$B_{c2}^*(T, \epsilon)$ calculated from published global-fitting results. The curves do not coalesce into a single master scaling curve. Why? Again, too many interdependent parameters are being fit at the same time when the global-fitting method is used.

Thus, the global-fitting equation method is quick, but useful for interpolation only. The step of calculating raw scaling data for $B_{c2}^*(T, \epsilon)$ and $K(T, \epsilon)$ to build a master scaling curve cannot be skipped and still retain scaling and extrapolation capability. The third method, the ESE relation, addresses this.

2.2. ESE: providing extrapolative capability for the global-fitting approach

Figure 2 gives an overview of the specific steps to derive the ESE relation—essentially a summary of this article in one figure:

Left side of figure: Large $I_c(B, T, \epsilon)$ datasets are scaled to give a database of raw scaling data for $B_{c2}^*(T, \epsilon)$ and $K(T, \epsilon)$.

Middle: Analysis of the individual scaling parameters with these raw scaling data shows that some of the parameters are effectively constant⁵ for all the conductors analyzed, while others are conductor specific and need to be determined

⁵ The term ‘scaling constant’ is used in these articles to describe a parameter in a global fitting equation that has a consistent value within a family of superconductors. In the application of the ESE fitting equation in Part 3, the scaling constants are not fit along with other parameters in a nonlinear regression, but are fixed based on the analysis of raw scaling data described here. These constants will have some uncertainty associated with them that is typically a type B statistical evaluation (i.e., based on these scaling results and engineering judgment).

for each wire rather than assigned universal constant values, as in some scaling models.

Right side: The scaling constants and fitting parameters determined in the middle step are then combined with the general fitting equation of figure 1, and the result is the ESE fitting equation.

The process illustrated in figure 2 is carried out for several thousand I_c measurements covering a wide range of different Nb₃Sn conductors. This is done in section 4 for data not corrected for magnet self-field effects, and in section 5 for corrected data.

The essential point is that raw scaling data determine which scaling parameters in ESE have a constant value for all the practical Nb₃Sn conductors tested, and which need to be fitted to each conductor. The result is the ESE relation, which results in the formation of master scaling curves and therefore extrapolation capability.

3. Mathematical description of the USL

We first give a brief review of the general parameterization of the USL and its separable form. The *separable* form and its parameters are the foundation of the analysis in sections 4 and 5 to sequentially determine the scaling parameters in small groups.

3.1. Unified Scaling Law

Mathematically, the USL states that neither strain nor temperature change the *shape* of a superconductor's pinning-force curve when expressed in terms of certain dimensionless scaling variables:

b is the reduced effective upper critical magnetic field, $b \equiv B/B_{c2}^*(t, \varepsilon)$, and $B_{c2}^*(t, \varepsilon)$ is the effective temperature-dependent and strain-dependent upper critical field determined by registering F_P - B curves into a master scaling curve as shown in figure 1; and

t is the reduced temperature $t \equiv T/T_c^*(\varepsilon)$, where $T_c^*(\varepsilon)$ is the effective strain-dependent critical temperature determined by the temperature where $B_{c2}^*(T, \varepsilon)$ versus T curves at constant strain extrapolate to zero, as shown in figure 3 of section 4.1.

Unified strain and temperature scaling is expressed by the following set of equations (Ekin 1980):

Unified strain-and-temperature scaling law (USL)

$$F_P(B, T, \varepsilon) = I_c(B, T, \varepsilon)B = K(t, \varepsilon)f(b) \quad (1a)$$

with dimensionless measurement variables:

$$b \equiv B/B_{c2}^*(t, \varepsilon) \quad \text{Reduced upper critical magnetic field} \quad (1b)$$

$$t \equiv T/T_c^*(\varepsilon) \quad \text{Reduced temperature.} \quad (1c)$$

Here F_P is the critical pinning force per unit conductor length. The two raw scaling data terms needed to coalesce all the F_P versus B curves into a master scaling curve [figure 1(b)], are the prefactor $K(t, \varepsilon)$ [or equivalently $F_{Pmax}(t, \varepsilon)$] and the

effective upper critical field $B_{c2}^*(t, \varepsilon)$, both dependent on temperature and strain. *Scaling of pinning force curves only gives data for these two terms.* They are determined by fitting each F_P - B curve with equation (1a) with a two parameter fit for individual values of $K(t, \varepsilon)$ and $B_{c2}^*(t, \varepsilon)$, while keeping the magnetic-field shape function $f(b)$ the same for all the curves. This is most conveniently done for all the curves simultaneously with an automated algorithm, as illustrated in the online information accompanying this article, but they can also be calculated individually for each curve.

3.2. Separable form of the USL: five single-variable, dimensionless scaling functions

Parameter testing is greatly simplified by parameterizing the two raw scaling datasets, $B_{c2}^*(t, \varepsilon)$ and $K(t, \varepsilon)$, with separate *single-variable* functions, each involving t or ε alone. As will be shown in section 4, this separability postulate for the parameterization is substantiated by raw-scaling-data analysis of many recent large datasets for Nb₃Sn. It has also been adopted in most proposed parameterizations of the USL.

Specifically, the separable form of the USL, is (Ekin 1980, 2006):

Unified strain-and-temperature scaling law (USL)—separable form

$$F_P(B, T, \varepsilon) = C g(\varepsilon) h(t) f(b) \quad (2a)$$

Separable form of the pinning force per unit length

$$B_{c2}^*(T, \varepsilon) = B_{c2}^*(0, 0) b_{c2}(t) b_{c2}(\varepsilon), \quad (2b)$$

Separable form of the effective upper critical field

with dimensionless measurement variables:

$$b \equiv B/B_{c2}^*(T, \varepsilon) \quad \text{Reduced magnetic field} \quad (2c)$$

$$t \equiv T/T_c^*(\varepsilon) \quad \text{Reduced temperature.} \quad (2d)$$

On the right side of equation (2a), C is a proportionality constant with units of pinning force, and the separable functions $g(\varepsilon)$, $h(t)$, and $f(b)$ are dimensionless. In equation (2b), and in the remainder of this article, a dimensionless notation is also introduced for the separable parts of the effective upper critical field $b_{c2}(t)$ and $b_{c2}(\varepsilon)$, which greatly simplifies many of the expressions:

Dimensionless notation for the separable parts of the effective upper critical field in equation (2b):

$$b_{c2}(t) \equiv B_{c2}^*(T, 0)/B_{c2}^*(0, 0). \quad (3a)$$

Temperature part of upper critical field

$$b_{c2}(\varepsilon) \equiv B_{c2}^*(0, \varepsilon)/B_{c2}^*(0, 0) \quad (3b)$$

Strain part of upper critical field,

also denoted $S(\varepsilon)$, i.e., $b_{c2}(\varepsilon) \equiv S(\varepsilon)$.

We use the notation $b_{c2}(\varepsilon)$, rather than $S(\varepsilon)$, to emphasize it is not an arbitrary strain fitting function, but the physically measurable strain dependent part of the upper critical field.

Thus, in its separable form (i.e., equation-set (2)), the USL is separated into five *single-variable, dimensionless*

scaling functions with values ranging from 0 and 1: $b_{c2}(t)$, $b_{c2}(\varepsilon)$, $g(\varepsilon)$, $h(t)$, and $f(b)$. Each of these separate functions depends alone on strain (usually expressed as a percent), reduced temperature, or reduced magnetic field. These five functions can be determined individually from measurements of raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $F_{pmax}(T, \varepsilon)$ with relatively high precision because it takes only a few (1–4) parameters to model each separable function.

3.3. Parameterization of the five separable scaling functions

In section 4 we proceed with testing the commonly used parameterizations for each of these five separable functions through the analysis of raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$. In general form, these parameterizations are:

$$b_{c2}(t) = (1 - t^\nu), \quad \text{temperature scaling parameter } \nu \quad (\text{section 4.1}), \quad (4)$$

$$b_{c2}(\varepsilon) [\text{aka } S(\varepsilon)] \quad \text{several parameterizations postulated} \quad (\text{section 4.5}), \quad (5)$$

$$g(\varepsilon) = [b_{c2}(\varepsilon)]^s \quad \text{strain scaling parameters} \quad (\text{section 4.4}), \quad (6)$$

$$h(t) = (1 - t^\nu)^{\eta-\mu} (1 - t^2)^\mu \quad \text{temperature parameters } \eta \text{ and } \mu \quad (\text{section 4.2}), \quad (7)$$

$$f(b) = b^p (1 - b)^q. \quad \text{pinning-force curve shape parameters } p \text{ and } q \quad (\text{section 5.1}). \quad (8)$$

Thus, we have three *temperature* scaling parameters (ν , η , and μ), several *strain* scaling parameters (the exponent s and those used to model $b_{c2}(\varepsilon)$), and two *magnetic field* scaling parameter (p and q).

In addition, the strain dependence of the effective upper critical temperature $T^*(\varepsilon)$ in equation (2d) is usually parameterized as

$$T_c^*(\varepsilon) = T_c^*(0) [b_{c2}(\varepsilon)]^{1/w} \quad \text{cross-link parameter } w \quad (\text{section 4.3}). \quad (9)$$

This is one of the more important parameterizations, because the cross-link parameter w in equation (9) is the only parameter in the basic USL that is fundamentally *non-separable*, because it requires large datasets of combined temperature and strain measurements for its determination. Hence we refer to it here as the *cross-link* parameter.

3.4. General parameterization of the USL equation

Thus, substituting the above parameterizations into equations (2a) and (2b), we have the most general parameterization of the separable USL (Ekin 2010). The separable parts are lightly boxed in equation-set (10):

General separable parameterization of the USL

$$F_p(B, T, \varepsilon) = I_c(B, T, \varepsilon) B = K(T, \varepsilon) f(b) = C \boxed{g(\varepsilon)} \boxed{h(t)} \boxed{f(b)}, \quad (10a)$$

$$= C \boxed{[b_{c2}(\varepsilon)]^s} \boxed{(1 - t^\nu)^{\eta-\mu} (1 - t^2)^\mu} \boxed{b^p (1 - b)^q} \quad (10b)$$

with reduced magnetic field $b \equiv B/B_{c2}^*(T, \varepsilon)$ and reduced temperature $t \equiv T/T_c^*(\varepsilon)$, where:

$$B_{c2}^*(T, \varepsilon) = B_{c2}^*(0, 0) \boxed{b_{c2}(t)} \boxed{b_{c2}(\varepsilon)} = B_{c2}^*(0, 0) \boxed{(1 - t^\nu)} \boxed{b_{c2}(\varepsilon)}, \quad (10c)$$

$$T_c^*(\varepsilon) = T_c^*(0) [b_{c2}(\varepsilon)]^{1/w} \quad (10d)$$

and scaling parameters: C , $B_{c2}^*(0, 0)$, $T_c^*(0)$, s , ν , η , μ , w , p , q , and the various strain parameters modeling $b_{c2}(\varepsilon)$.

Testing of these ten scaling parameters plus $b_{c2}(\varepsilon)$ proceeds in two steps:

- (1) First, in sections 4 and 5, we use raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$ to separately test the *individual* parameters in each of equations (10b), (10c), and (10d). This raw-scaling-data analysis yields three parameters that are scaling constants across many sets of data, and several parameters that need to be fitted (even though commonly fixed in present global-fitting equations). These results are combined in section 6 to give the ESE, shown on the right hand side of figure 2.
- (2) Then, in Part 3, we extrapolation test ESE and compare its extrapolation accuracy with the most current global-fitting equations: those of Durham (Taylor and Hampshire 2005, Lu *et al* 2008), Godeke (Godeke *et al* 2006, 2009, 2013, Mentink 2008), ITER (Bottura and Bordini 2009), and NIST (Ekin 1980, 2006).

4. Determination of scaling parameter values with raw scaling data for Nb₃Sn: which are constant, which are not

This section is the core of Part 2, where each of the general scaling parameters is determined from raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$ by breaking the task down into separable parts (as described in equation-set 10). The analysis is systematic and thorough. It is the only way to determine a USL parameterization based on the registration of scaling curves into a master curve, which preserves extrapolation capability. Although comprehensive and detailed, this one-time analysis results in the ESE fitting equation (section 6), which is easily applied.

No assumptions are made a priori about any possible constant parameter values. Thus, the results also serve to assess the underlying semi-theoretical basis for the general parameterization of the USL.

The sequence for determining scaling parameter values from raw scaling data is important.

- First, in sections 4.1 and 4.2 we consider the parameterization of the *temperature* parts of $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$,

Figure 3. Effective upper critical field $B_{c2}^*(T, \epsilon_0)$ as a function of temperature T at different intrinsic strains ($\epsilon_0 \equiv \epsilon - 0.3\%$). Solid curves plot equation (4), $B_{c2}^*(T, \epsilon_0) = B_{c2}^*(0, \epsilon_0) [1 - (T/T_c^*(\epsilon_0))^{1.5}]$, showing that the constant value $\nu = 1.5$ holds for Nb_3Sn , independent of applied strain.

because their determination from raw scaling data is straightforward (that is, it does not require extrapolation of raw scaling data to zero temperature, as do the *strain* parts).

- Then, from the temperature parameterization of $B_{c2}^*(T, \epsilon)$ in section 4.1, we determine the *cross-link* parameter w , which connects the strain dependence of T_c^* and B_{c2}^* in equation (10d). This is done in section 4.3 for a number of different Nb_3Sn conductors. The determination of w requires *complete, full matrix datasets* covering sufficiently wide ranges of temperature and strain.
- Finally, in sections 4.4 and 4.5 we carry out raw data analysis of the *strain* parts of $K(T, \epsilon)$ and $B_{c2}^*(T, \epsilon)$, respectively. This requires extrapolation of raw strain scaling data to zero kelvin by use of the temperature parameterizations determined in sections 4.1 and 4.2. The extrapolation to zero kelvin is needed to eliminate the complication of strain entering the analysis through the reduced temperature $t \equiv T/T_c^*(\epsilon)$.

4.1. Temperature dependence of the upper critical field $b_{c2}(t)$: ν is a scaling constant with value $\nu = 1.50 \pm 0.04$ (equation (4))

4.1.1. Strain independence. The constancy of the parameter ν was originally observed by Cheggour and Hampshire (2002) in raw scaling data for B_{c2}^* obtained for a bronze-processed Nb_3Sn multifilamentary wire. They found that ν has a constant value $\nu = 1.5$ over a wide range of intrinsic strain ($-1.03\% < \epsilon_0 < +0.37\%$). They also noted that this constant value of ν is approximately that calculated in WHH theory for a dirty superconductor with no Pauli paramagnetic limiting ($\nu = 1.44$) (Werthamer *et al* 1966).

4.1.2. Stoichiometry and measurement-method independence.

Godeke *et al* (2006) then observed effectively the same

Table 1. Best-fit values of the scaling constant ν at three different voltage taps along the length of the WST sample. The difference between the values $\nu = 1.52$ and 1.5 is less than the experimental error. Also, the average over many data is slightly less than 1.5 , not greater.

Tap #	Value of the scaling constant ν
1	1.48
2	1.50
3	1.49

value of ν in many samples, independent of sample stoichiometry or the method used to determine the upper critical field (i.e., resistive measurements at low transport currents, or Kramer-plot extrapolations to zero F_P). They also found this value fits a wide range of literature data for different superconductor forms, including Nb_3Sn wires, thin films, single crystals, and polycrystals (Charlesworth *et al* 1970, Foner and McNiff 1976, Orlando *et al* 1979, and Jewell *et al* 2004). More specifically, equation (4) with $\nu = 1.52$ was used by Godeke *et al* as a good approximation to the microscopic Maki-DeGennes expression (deGennes 1964, 1966, and Maki 1964, 1969).

However, the difference between Maki-DeGennes theory and equation (4) with $\nu = 1.52$ is only about $\pm 2\%$ (Godeke 2008), which is effectively negligible compared with the experimental error for $b_{c2}(t)$ data. Also, raw scaling data analysis for $b_{c2}(t)$ shows that the average over many measurements is typically slightly lower than 1.5 (instead of higher than 1.5), as shown in table 1. We conclude, based on raw scaling data, that *the added complexity of evaluating the Maki-deGennes relation or Werthamer theory is not needed for fitting accuracy, in lieu of equation (4) the simpler parameterization of equation (4) with $\nu = 1.5$.*

4.1.3. Parameterization of the temperature dependence of the upper critical field $b_{c2}(t)$. A detailed example of the quality of the fit of equation (4) with constant $\nu = 1.5$ is shown in figure 3 for extensive $B_{c2}^*(T, \epsilon)$ raw scaling data calculated from the OST-RRP[®] dataset. This dataset is an extensive test and covers a magnetic field range from 1 to 16 T, temperature range $2.5 \text{ K} < T < 15 \text{ K}$, and a compressive intrinsic strain range from $-1.07\% < \epsilon_0 < +0.0\%$. Figure 3 also gives validation of the constancy of ν , because $B_{c2}^*(T, \epsilon)$ at fixed strain is directly proportional to $b_{c2}(t)$, as seen from equation (10c). Figure 3 shows that equation (4), with a constant parameter value $\nu = 1.5 \pm 0.04$, fits these data at all strains with a very low root-mean-square (RMS) error of 0.09 T. Similar results were obtained for the WST wire, as well as for magnetic self-field corrected data for both conductors (shown in section 5), where even smaller RMSE values of 0.04 to 0.08 T were observed.

There has been a general adoption of equation (4) with $\nu = 1.50$ or 1.52 to represent the temperature-dependent part of the effective upper critical field. Because the best fit to raw scaling data for $B_{c2}^*(T, \epsilon)$ is usually for values of ν slightly less than 1.5 , (table 1) we adopt the former value

(again, the difference in these values is less than the experimental error in $B_{c2}^*(T, \varepsilon)$; for example, we have run variational tests and find that changing ν by 0.05 changes the rms fitting error in figure 3 by less than ~ 0.02 T). Thus, widespread experimental data, as well as microscopic theory, show that ν is a scaling constant

$$\nu = 1.50 \pm 0.04. \quad \text{Temperature scaling parameter for } B_{c2}^*(T, \varepsilon), \text{ equation (4).} \quad (11)$$

The quality of the fit in figure 3 is also an important confirmation of the *separability* postulate of equation (10c), $B_{c2}^*(T, \varepsilon) = B_{c2}^*(0, 0) b_{c2}(t) b_{c2}(\varepsilon)$. That is, for a wide range of strain, the temperature parameterization $b_{c2}(t) = (1 - t^{1.5})$ fits all the $B_{c2}^*(T, \varepsilon)$ curves with the same curve-shape parameter $\nu = 1.5$ (provided the reduced temperature $t \equiv T/T_c^*(\varepsilon)$ is used as the scaling variable, where $T_c^*(\varepsilon)$ is determined by the x -axis intercepts for each curve). Thus, $B_{c2}^*(T, \varepsilon)$ is indeed separable into the single-variable functions $b_{c2}(t)$ and $b_{c2}(\varepsilon)$.

4.2. Temperature dependence of the USL prefactor $h(t)$: η and μ are not scaling constants (equation (7))

In this section, we test the individual parameterizations that have been proposed for the temperature part $h(t)$ of the prefactor in equation (10a):

$$K(T, \varepsilon) = C g(\varepsilon) h(t). \quad (12)$$

The general form of $h(t)$ is given by Hampshire *et al* (1985), Keys and Hampshire (2003)

$$h(t) = [b_{c2}(t)]^\eta [\kappa_1(t)]^{-\mu}, \quad (13)$$

where $b_{c2}(t)$ is the temperature contribution to $h(t)$ from the upper critical field, and $\kappa_1(t)$ is from the Ginzburg–Landau parameter. Utilizing the temperature parameterizations for $b_{c2}(t)$ and $\kappa_1(t)$ from Keys and Hampshire (2003), the general form becomes

$$h(t) = (1 - t^\nu)^{\eta-\mu} (1 - t^2)^\mu. \quad (14)$$

The various models currently used for $h(t)$ differ simply in how they treat the temperature exponents η and μ in equation (14):

- (1) *Fietz and Webb* model (1969): an empirical relation with the kappa exponent fixed at $\mu = 0$, and the upper-critical field exponent η left a fitted parameter. This was the original $h(t)$ model used by Ekin (1980, 2006), and by Cheggour and Hampshire (1999, 2002)

$$h(t) = (1 - t^{1.5})^\eta. \quad (15)$$

- (2) *Godeke/Mentink/ITER* model (Godeke *et al* 2006, 2009, 2013; Mentink 2008, Bottura and Bordini 2009): an empirical relation with *both* exponents fixed: $\mu = 1$ and

$$\eta = 2 \text{ (Godeke } et al \text{ 2006)}$$

$$h(t) = (1 - t^{1.52})(1 - t^2). \quad (\text{no adjustable parameters}). \quad (16)$$

- (3) *Durham* model (Keys and Hampshire 2003, Taylor and Hampshire 2005, and Lu *et al* 2008): a semi-empirical relation with the kappa exponent fixed at $\mu = 2$, and the upper-critical field exponent η fitted

$$h(t) = (1 - t^{1.5})^{\eta-2} (1 - t^2)^2. \quad (17)$$

- (4) *Modified Mentink (MM)* model for $h(t)$ (Mentink 2014): an empirical relation with the kappa exponent μ fitted, and the critical-field exponent η fixed by the relation $\eta = \mu + 1$

$$h(t) = (1 - t^{1.52})(1 - t^2)^\mu. \quad (18)$$

- (5) *Hybrid1* model: an empirical relation with the kappa exponent fixed at $\mu = 1$ and the upper-critical field exponent η fitted

$$h(t) = (1 - t^{1.5})^{\eta-1} (1 - t^2). \quad (19)$$

- (6) *Hybrid2* model: a variant of the Hybrid1 model with nearly the same parameter values, wherein the kappa exponent is fixed at $\mu = \eta/2$ and the upper-critical field exponent η fitted

$$h(t) = (1 - t^{1.5})^{\eta/2} (1 - t^2)^{\eta/2}. \quad (20)$$

- (7) *Two-parameter fit*: an empirical relation with both η and μ fitted independently

$$h(t) = (1 - t^{1.5})^{\eta-\mu} (1 - t^2)^\mu. \quad (21)$$

A *Hybrid3* model is also possible, in which the kappa exponent μ is fitted, with the critical-field exponent η fixed at $\eta = 2$. This was not tested because of the considerably increased variability when using μ as the main fitting parameter, rather than η (parameter variability is evaluated in tables 3 and 7, and described in the discussion below.)

Table 2 summarizes the various combinations of η and μ proposed for $h(t)$, along with their properties.

As indicated by the last column in table 2, all these parameterizations are *empirically* determined fitting equations, except the Durham model, which is semi-empirical. Also, only the Fietz and Webb, Hybrid1, Hybrid2, and 2-parameter fit models were based on raw scaling data for more than one sample.

- The parameterization of *Fietz and Webb* was obtained from measurements of κ_1 from complete hysteretic magnetization curves in many samples of NbTi and NbTa.
- The parameterization of *Godeke/Mentink/ITER* (G/M/ITER) was an empirical fit to raw scaling data for one bronze-processed moderate- J_c Nb₃Sn sample, and

Table 2. Summary of parameterizations for $h(t)$.

Model	μ	η	Quality of fit	Comments
Fietz and Webb (F&W)	0	Fitted	Good	Empirical—based on raw scaling data for many NbTi and NbTa samples
Godeke/Mentink/ITER (G/M/ITER)	1	2	Not generally applicable	Empirical—based on raw scaling data for one Nb ₃ Sn sample
Durham (DUR)	2	Fitted, but sometimes fixed at $\eta \approx 2.5$	Good	Semi-empirical—based in part on theory, and raw scaling data for one Nb ₃ Sn sample
Hybrid1 (HYB)	1	Fitted	Good	Empirical—based on raw scaling data for many Nb ₃ Sn samples
Hybrid2, (HYB)	$\mu = \eta/2$	Fitted	Good	Empirical—based on raw scaling data for many Nb ₃ Sn samples
Modified Mentink (MM)	Fitted	$\eta = \mu + 1$	Good	Empirical—no raw scaling data used in its formulation
2 Par. Fit (2PAR)	Fitted	Fitted	Good, but no significant reduction in errors	Empirical—based on raw scaling data for many Nb ₃ Sn samples

Table 3. Values of the temperature curvature parameter η obtained from fitting different $h(t)$ models to raw scaling data for $K(T, \varepsilon)$ for a number of larger datasets. Conductor datasets are listed in the first column, and the temperature range of each dataset is given in the second column. In compiling this table, raw scaling data for $K(T, \varepsilon)$ were usually available only for zero applied strain, except for the Oxford-RRP[®] and WST-ITER conductors.

Conductor	T range (K)	F&W ($\mu = 0$)	G/M/ITER ($\mu = 1, \eta = 2$)	Durham ($\mu = 2$)	Hybrid1 ($\mu = 1$)	Hybrid2 ($\mu = \eta/2$)	MM ($\mu = \eta - 1$)
Oxford RRP [®] (HL-LHC cond.)	2.5–12	2.07	2	2.56	2.31	2.35	2.41
EM-LMI (ITER conductor)	4.2–13	2.15	2	2.52	2.34	2.37	2.41
Oxford (ITER conductor)	4.03–16	1.93	2	2.36	2.13	2.15	2.18
Luvata (ITER conductor)	2.26–14	1.78	2	2.25	2.01	2.01	2.03
WST (ITER conductor)	4.03–14	1.84	2	2.24	2.04	2.04	2.05
Variability		21%	0%	14%	16%	18%	19%

adopted as the parameterization for characterizing ITER conductors.

- The *Durham* model with $\mu = 2$ was also based on raw scaling data from one Nb₃Sn sample. This parameterization has theoretical underpinnings and corresponds to the general form of the pinning models of Kramer (1970) and Dew-Hughes (1974). More recently, the parameter η has sometimes been fixed at $\eta \approx 2.5$ (Lu *et al* 2008). However, it needs to be left a free parameter for fitting accuracy (as shown below). Consequently, we analyze here the more general variable- η form.
- The *Modified Mentink* parameterization (MM model) was recently proposed (Mentink 2014, thesis), in which μ is made a *fitted* temperature parameter to correct the relatively large $h(t)$ temperature errors of the G/M/ITER relation (as described by Ekin (2010) and Ekin *et al* (2013), and treated in greater detail here). The MM parameterization is also empirical; no raw scaling data were used in its formulation. (Note that this model was not the main focus of the thesis, however, which is a

comprehensive analysis of the origin of strain effects in Nb₃Sn conductors.)

- The Hybrid2 parameterization is a variant of the Hybrid1 model with nearly the same parameter values (table 3). It is now being implemented for EuroCirCol and the Future Circular Collider design studies, and at CERN for calculating the margin of the HL-LHC magnets.

Table 3 shows fitting results for these seven parameterizations of $h(t)$ when tested with extensive $K(T, \varepsilon)$ raw scaling data for a wide range of Nb₃Sn datasets (conductor information is given in appendix table A1 and online). Raw-scaling-data testing of $h(t)$ is straightforward because $h(t)$ is directly proportional to $K(T, \varepsilon)$ at constant strain, as seen from equation (12).

The errors in fitting these data were comparably small, except for one model. The fitting accuracy of the G/M/ITER temperature model was reasonable for many (but not all) of the moderate- J_c ITER conductors, but its applicability is not general. RMS fitting errors were up to four times those of the other models for some of the conductors; e.g., the OST restacked-rod-process (RRP[®]) and EM-LMI internal-tin ITER

Table 4. Ratio of the Ginsberg-Landau kappa $\kappa_1(T)/\kappa_1(0)$ for various $h(t)$ models, calculated from raw scaling data for several conductors. Data are tabulated for the highest available temperature for each conductor, showing unphysical kappa values for the G/M/ITER $h(t)$ model as temperature approaches T_c^* in the Oxford RRP[®] conductor and EM-LMI internal-tin ITER conductor.

$h(t)$ model	Oxford RRP [®] , $T = 14$ K	EM-LMI, $T = 13$ K	Luvata CS, $T = 14$ K
G/M/ITER	<i>1.17</i>	<i>1.14</i>	0.77
Durham	0.72	0.69	0.71
Hybrid1	0.77	0.69	0.76
Hybrid2	0.76	0.69	0.76
2 Par. Fit	0.74	0.71	0.78

conductors (figures 4(b) and 5). The two-parameter temperature model, equation (21), provided no significant reduction in the RMS error compared with the other models, which are formulated with a single parameter (η or μ). It served only to introduce an extra parameter that could not be determined precisely from any of the datasets. Consequently, it was dropped from further consideration.

Note that the temperature curvature parameter η is significantly higher for the conductors in the top 2 rows of table 3 compared with the bottom 3 rows. This correlates with the use of Ta doping and distributed diffusion barriers for the first two conductors, versus Ti doping and a single diffusion barrier for the last three. With recent micrographic information regarding increased compositional inhomogeneities for Ta versus Ti doping (Tarantini *et al* 2016), this increase in the temperature curvature parameter could also correlate with compositional inhomogeneities (see figure 6 and the discussion on compositional inhomogeneities below). However, further compositional studies and scaling parameter measurements are needed.

The last row of table 3 gives the *variability* in the fitted values of η needed to accommodate the different datasets tested. In this article, we have defined variability as the difference between the maximum and minimum parameter values, divided by the minimum value. A change in η of about 14% to 21% was required to fit the range of conductors studied. The lowest variability was obtained for the Durham and Hybrid models.⁶ Differences in variability among the models were greater for the magnetic self-field corrected data, as described in section 5.

We also conducted tests with *simultaneously* varied temperatures and strain data (the above tests were mostly for data at one strain). Examples of several of these results, shown in figures 4(a)–(c), provide a comprehensive test with temperature ranging from 2.5 to 12 K, and intrinsic strains extending to -1.05% compression. All the models had small errors (RMS = 300–400 AT), except the G/M/ITER model where the error was three to four times greater (RMS = 1189 AT). Also, fractional deviation errors at individual data points for this model were up to 10 times greater than for the other

models across the entire temperature range as shown by the blue arrows in figure 4(b).

4.2.1. Conclusion: parameterization of $h(t)$. Any of the fitted η or μ models work quite well for the general range of practical Nb₃Sn conductors, if one of the temperature parameters (either η or μ) is fitted. The fitting accuracy of the G/M/ITER parameterization (where both parameters are fixed, $\mu = 1$ and $\eta = 2$) was reasonable for many of the moderate- J_c conductors, but the relative RMS errors in many cases were almost four times greater than those of the other parameterizations. The error cannot be accounted for by the $\pm 2\%$ difference between the $(1 - t^{1.52})$ term in equation (16) and the Maki-deGennes expression for the upper critical field (Godeke 2008). This is too small to account for the large errors observed in figures 4(b) and 5 (below).

Unphysical values of the Ginsberg Landau kappa also result with the G/M/ITER parameterization. The ratio of $\kappa_1(T)/\kappa_1(0)$ is expected to decrease from 1 to a minimum of ~ 0.83 in the weak-coupling, and ~ 0.67 in the strong-coupling limits (Ranier and Bergman 1974), as temperature approaches T_c . Table 4 shows values of the ratio $\kappa_1(T)/\kappa_1(0)$ at temperatures approaching T_c^* for several of the conductors tested. The fixed-parameter G/M/ITER model gave ratio values that were greater than 1 (shown in red italic type) for the Oxford RRP[®] and EM-LMI conductors.

The results in table 4 also correlate with the difference in the conductor's Ta and Ti doping, and diffusion barriers as noted in the discussion of table 3.

Figure 5 is a *comparative* plot of all the models, evaluated at one strain ($\varepsilon = 0.035\%$, corresponding to about $\varepsilon_0 = -0.27\%$). The plot shows that the fitting error is comparable to the measurement error for all the models (except G/M/ITER). However, extrapolation tests from 4.2 K down to 2.26 K (presented later in figure 16 of section 5.3), also show significant *extrapolation* errors below ~ 4 K for the F&W and Durham models.

The interpolation and extrapolation errors of Hybrid1, Hybrid2 and MM models are significantly lower than those of the other models. These models all have fitted parameter values in the vicinity of $\eta = 2$ and $\mu = 1$. However, a practical difference between these models is that values of the fitting parameter in the Hybrid1 and Hybrid2 models have less variability than for the MM model. For the magnetic self-field corrected data presented in section 5.3, the differences are even more pronounced, with parameter variability as much as 40% less (table 7).

⁶ Mathematically, the differences stem from the relatively slow temperature dependence of the μ kappa contribution to $h(t)$, that is the *ratio* $(1 - t^{1.5})/(1 - t^2)^{-1}$, compared with the η upper-critical-field contribution $(1 - t^{1.5})$ (from equations (13) and (14)). This necessitates larger changes in models that focus on varying the temperature contribution from μ , such as the MM and Hybrid3 models, rather than η to accommodate the range of temperature dependences in different Nb₃Sn conductors.

Figure 4. (a) Durham $h(t)$ model tested with a large set of raw-scaling data for $K(T, \varepsilon)$ measured for an Oxford RRP[®] Ta-doped conductor (Oxford Superconducting Technology, OST). The root-mean-square error was relatively small, RMSE = 337 AT. (b) G/M/ITER $h(t)$ model tested with the same large dataset as in figure 4(a). The root-mean-square error (RMSE = 1189 AT) was three to four times larger than that of the other models, and the percentage errors at individual data points were up to 10 times greater across the entire temperature range, marked by arrows. (c) Hybrid1 $h(t)$ model tested with the same OST-RRP[®] dataset as in figures 4(a) and (b). This model has the same fixed kappa exponent ($\mu = 1$) as that of the G/M/ITER model (figure 4(b)), but with a fitted η . The root-mean-square error was relatively small, RMSE = 337 AT.

4.2.2. Preferences. In summary, the *extrapolation* tests of section 5.3, show the two Hybrid models are able to predict the 2.26 K data from data above 4 K to within 1%, which is comparable to the measurement error. The fits for these two parameterizations are also well behaved over the entire temperature range from 14 to 2.26 K, and thus it is expected that the relatively small one-third kelvin extrapolation from 2.26 to 1.9 K (the operating temperature of the High Luminosity LHC) should have commensurately small extrapolation error. Additionally, the Hybrid models have significantly smaller parameter variability. *Thus, the Hybrid1 and Hybrid2 $h(t)$ models would be preferred from the standpoint of parameter consistency (tables 3 and 7) and extrapolation accuracy (figures 4(c), 5, and 16 in section 5.3).*

4.2.3. Discussion: compositional inhomogeneities. The need for a fitted temperature parameter may result from

compositional inhomogeneities in practical superconductors. We illustrate this with a simple two-component model using the Hybrid1 and MM $h(t)$ models. We follow the illustration given by Mentink (2014), except we consider the complete temperature dependence of $h(t)$, in which we include the temperature dependence for the MM model that is embedded in the b^{-1} term of the MAG magnetic-field function. [That is, when fitting raw scaling data for $h(t)$, what scales is the pinning force, $b^p(1 - b^q)$, not the magnetic field function of the MAG relation, $b^{p-1}(1 - b^q)$.]

In this illustration, we consider a conductor consisting of two parallel current paths, each with the same microscopic properties and each assumed to have the same $h(t)$ dependence. For this example, we take the $h(t)$ of each component to be $(1 - t^{1.52})(1 - t^2)$ (i.e., the fixed temperature dependence of the G/M/ITER model). Only the critical temperatures of the two paths are assumed to be

Figure 5. Comparative plot of the $h(t)$ model fits to $K(T, \varepsilon)$ raw scaling data measured for the OST-RRP[®] conductor at an intrinsic strain of about -0.27% . At fixed strain, the ordinate $K(T, \varepsilon)$ is directly proportional to $h(t)$, as seen from equation (12).

different, for example $T_c^* = 15$ and 18 K (to match the earlier illustration of Mentink). [Note that T_c is the effective scaling critical temperature, determined by the temperature where the $B_{c2}^*(T, \varepsilon)$ versus T curves at constant strain extrapolate to zero, as shown in figure 3 of section 4.1]. Although simple, this two-component model could have practical application to the radial compositional variations recently observed across the A15 layer in RRP[®] high- J_c conductors (Tarantini *et al* 2016).

The temperature dependence of each current path (A and B) is plotted in the lower part of figure 6. The temperature dependence of the *total* current carried by both components is given by the sum of these two curves, shown by the upper (red) curve in figure 6 (labeled total A + B).

The fits of the Hybrid1 and MM $h(t)$ models to the total curve are also plotted in red in figure 6. The fitting errors are so small ($\sim 0.2\%$) that both fitted curves appear coincident with the total curve. The results show that the Hybrid1 parameter η for the fit to the total curve increases by about 7% compared with the starting value for each components (i.e., η increased from $\eta = 2$ to $\eta = 2.14$), whereas the MM parameter μ increases by about 37% (from $\mu = 1$ to $\mu = 1.37$). (Note that the fitted value for μ obtained in this illustration is higher than that obtained by Mentink (2014) ($\mu = 1.26$), mainly because the earlier result did not include the temperature dependence embedded in the b^{-1} term of the magnetic field function of the MAG relation.)

In figure 6, we have trimmed the curves at 15 K and also stopped the fit at that temperature to illustrate the effect for the range where temperature data are usually fit. At higher temperatures, the total curve has an elongated tail because of the difference in T_c^* values. *Trimming the fit at 15 K shows that the increase in the temperature scaling parameters is not a result of this tail, but occurs over the whole practical temperature span.*

Figure 6. Illustration of the effect of compositional inhomogeneities to increase the values of the temperature scaling parameters, η or μ . The simple model depicted consists of two parallel current paths, component A with $T_c^* = 18$ K (shown by the long dashed curve) and component B with $T_c^* = 15$ K (short dashed curve). Otherwise, the temperature dependence of the two paths have the same functional form $(1 - t^{1.52})(1 - t^2)$. The total of the two components is shown by the solid red curve (labeled total A + B). The Hybrid1 and Modified Mentink (MM) models fit the total curve to within $\sim 0.2\%$ (the fitted curves, also shown in red, appear coincident with the total curve). The fitted results have temperature parameters η or μ that are increased compared with the starting parameter value of the components. The increase for the Hybrid1 parameter η ($\sim 7\%$) was much less than for the MM parameter μ ($\sim 37\%$). The fit is trimmed at 15 K to illustrate that the increase in the temperature parameter occurs over the entire technological temperature range, independent of the high temperature ‘tail’ region.

Although simple, this two-component model serves to illustrate the possible role of compositional inhomogeneities and the need for a *fitted* temperature parameter because of compositional variations in different conductors. It may also explain, at least in part, the greater variability observed for the MM model than the Hybrid models.

4.3. Cross-link parameter w relating the strain dependencies of T_c^* and B_{c2}^* : w is a scaling constant with a value $w = 3.0 \pm 0.3$ (equation (9))

Perhaps the most significant scaling constant, from the standpoint of saving measurement time, is the *cross-link* parameter w that links the strain dependencies of B_{c2}^* and T_c^* , that is, $T_c^*(\varepsilon) = T_c^*(0) [b_{c2}(\varepsilon)]^{1/w}$ (equation (9)). This is because the determination of w from raw scaling data requires extensive B - T - ε datasets. Originally, this parameter was determined from raw scaling data obtained for Nb₃Sn over a moderate strain range, $-0.5\% < \varepsilon_0 < \varepsilon_{0irr}$ (Ekin 1980) (Here, ε_{0irr} is the irreversible tensile-strain limit where the superconductor is irreversibly damaged Ekin 1979, Goodrich

et al 2011.) Fortunately, however, the constancy of this parameter value was recently found to also extend to *high compressive* strains, and to a wide range of different types of practical Nb₃Sn conductors (Ekin 2006, 2010).

We can see the constancy of w in figure 7, which summarizes the analysis of raw scaling data for $B_{c2}^*(T, \varepsilon)$ for a number of different Nb₃Sn conductors. These data for $B_{c2}^*(0, \varepsilon)$ and $T_c^*(\varepsilon)$ were obtained from graphs of raw scaling data of $B_{c2}^*(T, \varepsilon)$ versus temperature at different strains, such as shown in figure 3. The accuracy of the temperature parameterization of $b_{c2}(t)$ given by equation (4) and the constancy of the parameter v make it possible to extrapolate the curves in figure 3 to the y and x axis intercepts, giving values of $B_{c2}^*(0, \varepsilon)$ and $T_c^*(\varepsilon)$, respectively. In figure 7, these values of $B_{c2}^*(0, \varepsilon)$ and $T_c^*(\varepsilon)$ are normalized by their maximum values at $\varepsilon_0 = 0\%$ to be able to combine different conductor results into the same plot.

The slope of the data in the log-log graph of figure 7 directly gives the value of the cross-link parameter w . We see from the graph that the slope is

$$w = 3.0 \pm 0.3, \text{ Cross-link parameter between } B_{c2}^*(0, \varepsilon) \text{ and } T_c^*(\varepsilon), \text{ equation (9).} \quad (22)$$

This value describes the wide range of Nb₃Sn conductor types shown in the figure, including: binary Nb₃Sn, binary bronze-process Nb₃Sn, (Nb-1wt%Ti)₃Sn internal-tin modified-jelly-roll (MJR) conductor, (Nb-7.5 wt%Ta)₃Sn bronze-process conductor, and (Nb-Ta)₃Sn RRP[®] conductor. The relation holds over a very wide intrinsic strain range spanning high-compressive strains and tensile strains, $-1.8\% < \varepsilon_0 < +0.39\%$.

In contrast to this *raw-scaling* value of about 3.0, the light dashed line in figure 7 shows a slope of 2.2 obtained from averaging *simultaneous globally fit* values of w (Taylor and Hampshire 2005). Such empirical fits give a wide range of w values (from 1.9 to over 3). The average, $w = 2.2$, deviates significantly from the raw scaling-data value. Erroneous values of this parameter, more than any other parameter, greatly diminish the predictive capability of the global fitting method.

The value of w cannot be determined precisely from simple global fitting because of the convolution of many parameters. It can be determined only from raw scaling data for $B_{c2}^*(T, \varepsilon)$. We are fortunate that w is, in fact, nearly constant for practical Nb₃Sn conductors because, otherwise, its value would have to be determined on a conductor-to-conductor basis from the extensive combined B - T - ε data needed to measure $B_{c2}^*(T, \varepsilon)$. Then the whole concept of an extrapolative *fitting function*, without raw scaling data, would have been precluded.

In analysis for the WST conductor, w was at the low end of this error range, about 2.7 or slightly lower, depending on the strain range being fit. This was the only conductor analyzed with w near the range limit. If a large dataset is available, as for the WST conductor, the value of w determined from raw scaling data would, of course, be preferable to use for that particular conductor. But, lacking such an extensive dataset, the value 3 gave high fitting accuracy even for the

WST conductor, as shown by the very small error for the many extrapolations carried out with $w = 3.0$ for the WST conductor in Part 3. (Making it a free global-fitting parameter is much worse, with extrapolation errors at high strains more than 10 times greater.)

From a fundamental standpoint, a coupling between the strain-induced changes in $B_{c2}^*(0, \varepsilon)$ and $T_c^*(\varepsilon)$ stems from their common dependence on the average phonon frequency and the electronic density of states at the Fermi energy, as shown by Taylor and Hampshire (2005). Also, by numerically solving the nonlinear Eliashberg equations (Eliashberg 1960), Oh and Kim (2006) have shown that the power-law dependence of $T_c^*(\varepsilon)$ on strain applies to the strain dependence of the upper critical field as well. Their calculation, with strain entering mainly through the low-frequency strong-coupling parameter, gives the power law relation of equation (9) and with an exponent w of about 3, in good agreement with the value given by raw scaling data.

4.4. Strain dependence of the USL prefactor $g(\varepsilon)$: s is not a scaling constant (equation (6))

The underlying model for the strain part $g(\varepsilon)$ of the prefactor $K(T, \varepsilon) = C g(\varepsilon) h(t)$ in equation (10a) is given in general form by (Keys and Hampshire 2003):

$$g(\varepsilon) = A^*(\varepsilon)[b_{c2}(\varepsilon)]^\eta [\kappa_1(0, \varepsilon)]^{-\mu} \quad (23)$$

$$= A^*(\varepsilon)[b_{c2}(\varepsilon)]^{\eta-\mu} [\gamma(\varepsilon)^{1/2} T_c^*(\varepsilon)]^\mu. \quad (24)$$

Here, the parameter $A^*(\varepsilon)$ depends only on microstructure (Dew-Hughes 1974), $\kappa_1(T, \varepsilon)$ is the temperature and strain dependent Ginzburg-Landau parameter, and γ is the Sommerfeld constant. The original empirical parameterization of $g(\varepsilon)$, equation (6), combined all the strain dependences in equation (24) and simply represented them by the strain part of the effective upper critical field alone (Ekin 1980)

$$g(\varepsilon) = [b_{c2}(\varepsilon)]^s. \quad (25)$$

This was based on *moderate*-strain measurements of early binary Nb₃Sn conductors, for which the exponent s was measured to vary over a fairly wide range, approximately $s \approx 1 \pm 0.3$ (Ekin 1980). However, soon afterward, in measurements on optimally doped Nb₃Sn (Ekin 1985) and other superconductor (Ekin 1981), the value of the exponent s was measured to be consistently *greater than 1*, even at moderate strains. Representative values of s for these superconductors are given at moderate strains in table 6 of the next section 4.5.

In more recent measurements at higher compressive strains, s is also consistently observed to have raw-scaling-data values greater than 1. For example, the collection of six ITER internal-tin and bronze-route conductors measured by Cheggour *et al* (2014), all had raw-scaling-data values of $s = 1.4 \pm 0.1$. In this series of measurements the value of s was constant over the entire strain range, including both

Table 5. Comparison of advantages and disadvantages of parameterizations for the strain part $b_{c2}(\varepsilon)$ of the effective upper critical field. Each parameterization is defined in appendix B.

	Longitudinal strain models			Three-dimensional strain models		
	Power Law	Extended Power Law	Polynomial	Deviatoric	Invariant Function	Exponential
Number of adjustable parameters ^a	1 (mod. ε only)	3	3	3 (2)	3	1
High compressive strain fitting?		•	•	•	•	•
Single, consistent strain-sensitivity index?	•	•				•
Utilization of curvature scaling constant, u ?	•	•			•	•
Extrapolation capability?	• (mod. ε only)	• (mod. ε only)				•

Note: A bullet denotes the $b_{c2}(\varepsilon)$ parameterization has the indicated feature.

^a In counting the number of adjustable parameters, the strain peak ε_m has been omitted for all the fitting equations, because this parameter is fixed by the location of the peak in the $b_{c2}(\varepsilon)$ data curve.

moderate and high compressive intrinsic strains, as well as tensile intrinsic strains.

Figure 8 shows a graph similar to figure 10 in Part 1, but now extended to high compressive strains. Here we have plotted $\ln[g(\varepsilon_0)]$ versus $\ln[b_{c2}(\varepsilon_0)]$ at zero kelvin obtained by taking raw scaling data for $K(4.02 \text{ K}, \varepsilon_0)/K(4.02 \text{ K}, 0)$ and $B_{c2}^*(4.02 \text{ K}, \varepsilon_0)/B_{c2}^*(4.02 \text{ K}, 0)$ at $\sim 4 \text{ K}$ (purple squares in figure 8) and extrapolating these data down to 0 K (other symbols in figure 8). [The extrapolations to 0 K are needed to extract the strain parts $g(\varepsilon)$ and $b_{c2}(\varepsilon)$ from $K(T, \varepsilon)$ and $B_{c2}^*(T, \varepsilon)$, because of the strain dependence inherent in the reduced temperature $t \equiv T/T_c^*(\varepsilon)$.] The data in figure 8 are for the wide-range temperature and strain dataset (Oxford RRP[®]) shown in figures 4(a)–(c). The relatively short extrapolations to 0 K were carried out for $b_{c2}(\varepsilon)$ with equation (10c), and for $g(\varepsilon)$ with equation (10a) using several different $h(t)$ model equations (16), (17), and (19).

A comparison of the two types of data in figure 8 shows that values of the exponent s can be obtained with minimal error by plotting raw scaling data for K and B_{c2}^* at 4 K *without extrapolation to 0 K*. This is a great simplification. (Note that at temperatures much greater than 4 K, this simple correspondence breaks down Cheggour and Hampshire 1999 and 2002.)

Figure 9 shows similar raw scaling data, this time all at 4.2 K, for a number of different conductors to complement the data for the OST-RRP[®] conductor in figure 8. These include several bronze-process and internal-tin conductors, as well as the same OST-RRP[®] conductor of figure 8 (again shown by purple square symbols). The results consistently show a slope that is steeper than one, especially noticeable at higher compressive strain. For the WST-ITER internal tin conductor (blue circles), the slope is approximately a straight line corresponding to $s = \sim 1.4$, over the *entire* strain range.

From a fundamental standpoint, we can understand why the simple parameterization of $g(\varepsilon)$ in equation (25) holds, from the calculations of Oh and Kim (2006) mentioned at the end of section 4.3. Their numerical solution of the nonlinear Eliashberg equations (Eliashberg 1960) shows that the strain dependence of the critical temperature also applies to the

thermodynamic critical field, the upper critical field, and the Ginzburg–Landau κ_1 , with less than 2% error out to compressive strains of about -0.8% , but with deviations increasing to 7% at -1% strain. From equations (23) and (24), we see that the strain dependence of $g(\varepsilon)$ is comprised of just these terms, plus any strain dependence that the Sommerfeld constant γ and the microstructure term $A^*(\varepsilon)$ might contribute. This justifies in large measure the modeling of $g(\varepsilon)$ in terms of $b_{c2}(\varepsilon)$ alone.

The main point is that the parameterization given by equation (25) applies, *provided the strain exponent s is fitted to each conductor*. In Part 3, we will see that the approximation of equation (25) also provides high accuracy for the global fitting carried out there. The globally fit values of s also agree well with the raw scaling data values of s in figure 9.

4.5. Strain dependence of the upper critical field $b_{c2}(\varepsilon)$ (equation (5)): the curvature parameter u is a scaling constant at moderate strains, with a value $u = 1.7 \pm 0.1$

Several parameterizations have been suggested to model the strain dependence of $b_{c2}(\varepsilon)$. The function $b_{c2}(\varepsilon)$ is defined by the separable strain part of the effective upper critical field given in equation (10c), $B_{c2}^*(T, \varepsilon)/B_{c2}^*(0, 0) \equiv b_{c2}(\varepsilon) b_{c2}(t)$. The strain part, $b_{c2}(\varepsilon)$, is given by measurements of $B_{c2}^*(0, \varepsilon)$ (where $b_{c2}(t) = 1$), or approximately by $B_{c2}^*(4 \text{ K}, \varepsilon)$. Most of the $b_{c2}(\varepsilon)$ parameterizations are empirical in nature. Individual parameterizations of each strain model are given in appendix B, along with a detailed description of each.

Although all of the $b_{c2}(\varepsilon)$ parameterizations fit raw scaling data for $B_{c2}^(0, \varepsilon)$ about equally within experimental error, there are functional and convenience differences between them that affect their practical use.* These are compared in table 5. In this table, the fitting equations representing $b_{c2}(\varepsilon)$ are divided into two groups depending on whether the parameterizations are given in terms of uniaxial longitudinal strain only, or three-dimensional strains. In this article, we have focused on the uniaxial strain data that are available, because of the scarcity of 3D data. Fortunately, the uniaxial expressions serve well for conductor characterization

Table 6. Values of the strain sensitivity index a^- obtained by fitting equation (26) with $u = 1.7$ to raw scaling data for $B_{c2}^*(4\text{ K}, \varepsilon)$ over the moderate strain range ($-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}$). (Values of a^- are given for strains expressed as absolute values, not as percentages.) For all materials, the curvature parameter u was measured from plots similar to figure 10 to be a constant $u = 1.7 \pm 0.1$, independent of the value of a^- (or a^+). The last column shows the variability of the fitting parameter s in the strain parameterization of $g(\varepsilon) = [b_{c2}(\varepsilon)]^s$ (equation (26)). (From table A10.2a in Ekin 2006 and the references therein.)

Superconductor	Crystal structure	$a^-(\varepsilon_0 < 0)$	$a^+(\varepsilon_0 > 0)$	s
Nb ₃ Al(RHQT)	A15	370	—	0.1
V ₃ G	"	450	650	1.4
Nb ₃ Ge	"	500	—	~2
Nb ₃ Sn*	"	900	1250	1
Nb ₃ Sn + 0.6at%Ti*	"	900	1250	1.1
Nb ₃ Sn + 1.85at%Ti*	"	1100	1450	1.2
Nb ₃ Sn + 0.6at%Ta*	"	900	1250	1.0
Nb ₃ Sn + 2.2at%Ta*	"	1350	1800	~1
V ₃ Si	"	3500	—	~1
PbMo ₆ S ₈	Chevrel	—	1900	~2

Table 7. Values and variability of the fitting parameter for different $h(t)$ models obtained from fitting self-field corrected $h(t)$ transport data for the Luvata-ITER and OST-RRP[®] conductors. The difference in variability (third row) is greater than for the uncorrected transport data in table 3.

Conductor	T Range (K)	Hybrid1 ($\mu = 1$)		Hybrid2 ($\mu = \eta/2$)		MM ($\mu = n - 1$)	
		η	$T_c^*(\text{K})$	η	$T_c^*(\text{K})$	η	$T_c^*(\text{K})$
Luvata-ITER Transport I_c	2.26–14	1.92	16.11 ^a	1.91	16.11 ^a	1.91	16.11 ^a
Oxford RRP [®] Transport I_c	2.45–14	2.25	16.40 ^a	2.29	16.40 ^a	2.36	16.40 ^a
Variability		17 %		20 %		24 %	

^a T_c^* determined from raw scaling data for the transport $B_{c2}^*(T, \varepsilon)$

and qualification. For the 3D models (Deviatoric, Invariant Function, and Exponential), the 1D parameter values can also be used to calculate 3D strain performance.

One model (Exponential) has the capability to extrapolate moderate strain data ($-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}$) to large compressive strains ($< -1\%$).

Below we consider the main practical differences.

4.5.1. Strain sensitivity index. Three models (Power Law, Extended Power Law, and Exponential) have a single fitting parameter not comingled with other parameters, which serves as an effective strain sensitivity index. They have the significant advantage of giving a consistent strain metric that does not change with the value of the other parameters in $b_{c2}(\varepsilon)$, or with the strain range being fit.

An example of this is shown in terms of the Power Law model

$$b_{c2}(\varepsilon) = 1 - a |\varepsilon_0|^\mu, \text{ Power-Law uniaxial model,} \\ (-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}), \quad (26)$$

where ε_0 is the intrinsic strain defined as $\varepsilon_0 \equiv \varepsilon - \varepsilon_m$, ε_m is the strain where $I_c(\varepsilon)$ is maximum, and u is a constant $u = 1.7$. The parameter a in equation (26) has two values: a^- for the compressive strain range ($\varepsilon_0 \leq 0$) and a^+ for tensile ($\varepsilon_0 > 0$), with the ratio $a^-/a^+ \approx 0.75$. As illustrated by table 6, the strain sensitivity of different conductors can be

conveniently compared at a glance in terms of the single parameter a^- in the third column. The same parameter also serves to compare strain sensitivities across the A-15 landscape: Nb₃Al has the lowest strain sensitivity, V₃Si the highest, and in Nb₃Sn the sensitivity depends on the dopant material and dopant level.

This feature of a strain sensitivity index is also provided by the single fitting parameter C_1 in the Exponential model (equation (B.11)).

4.5.2. Constant curvature parameter $u = 1.7 \pm 0.1$ at moderate strains ($-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}$) – providing noise rejection and extrapolation capability over the dominant peak strain range. Four models (Power Law, Extended Power Law, Invariant Strain Function, and Exponential) incorporate the moderate-strain curvature constant, either explicitly or implicitly through the second strain invariant J_2 (described below). The strain curvature parameter u in equation (26) has been measured to be a constant at moderate strains for about thirty different Nb₃Sn conductors. The same value is also observed in other A15 superconductors and Chevrel-phase superconductors (table 6). The large range of a^- values covered by these measurements shows that the value of u is independent of the value of a^- .

An example of the determination of u is presented with $B_{c2}^*(4.03\text{ K}, \varepsilon)$ and $B_{c2}^*(0, \varepsilon)$ raw scaling data for the WST

Figure 7. Raw-scaling-data correlation of the strain dependence of ratios $B_{c2}^*(0, \epsilon)/B_{c2}^*(0, 0)$ and $T_c^*(\epsilon)/T_c^*(0)$ for binary and ternary Nb_3Sn conductors. The solid line has a slope of 3, showing that the exponent value $w = 3.0 \pm 0.3$ in equation (9) holds for these Nb_3Sn conductors independent of conductor type and additives over a wide strain range from $-1.8\% < \epsilon_0 < +0.39\%$. The light dashed line has a slope of 2.2 corresponding to the average value of w obtained from global-fit data (Taylor and Hampshire 2005). [Graph is from figure 10.36 of Ekin (2006), using raw scaling data for $B_{c2}^*(0, \epsilon)$ and $T_c^*(\epsilon)$ obtained by analyzing larger $I_c(B, T, \epsilon)$ datasets from the references cited therein.].

Figure 8. Strain exponent s in equation (25). Plotted in this way, the slope corresponds to the exponent s , which parameterizes the strain part $g(\epsilon_0)$ of the USL prefactor $K(T, \epsilon_0)$. The figure shows OST-RRP[®] raw scaling data for $B_{c2}^*(4.02 \text{ K}, \epsilon_0)$ and $K(4.02 \text{ K}, \epsilon_0)$ (square purple symbols), as well as $B_{c2}^*(0, \epsilon_0)$ and $K(0, \epsilon_0)$ data obtained by extrapolating the 4 K raw scaling data to 0 K with equation (4) and different temperature $h(t)$ parameterizations (circle, diamond, and triangle symbols). Nearly the same results were obtained independent of which $h(t)$ model was used to carry out the extrapolation. The data show that the slope deviates from $s = 1$ at compressive strains outside the moderate range. The data also show that $g(\epsilon_0)$ and $b_{c2}(\epsilon_0)$ can be well approximated simply from raw scaling data at $\sim 4 \text{ K}$ (square symbols) without extrapolating to 0 K.

Figure 9. Comparison of raw scaling data at 4.2 K for $g(\epsilon) \approx K(4.2 \text{ K}, \epsilon_0)/K(4.2 \text{ K}, 0)$ and $b_{c2}(\epsilon) \approx B_{c2}^*(4.2 \text{ K}, \epsilon_0)/B_{c2}^*(4.2 \text{ K}, 0)$ in different conductors over a wide intrinsic strain range (indicated approximately along the top scale). Plotted in this way, the slope of the correlation gives the strain exponent s in equation (25). Nb_3Sn conductors are characterized by a value of s typically in the range 1.1–1.4 for all the Nb_3Sn conductors analyzed, including several internal-tin and bronze-process wires, as well as the RRP[®] conductor shown in figure 8. For the internal-tin WST-ITER conductor, s is nearly a constant $s \approx 1.4$ over the entire strain range.

Figure 10. The moderate-strain curvature constant $u = 1.7$, evaluated for the WST conductor. (The slope of this plot of $\ln[1 - b_{c2}(\epsilon_0)]$ versus $\ln|\epsilon_0|$ gives u , and the intercept gives a^- ; see equation (26) and the comments in section 4.5.2.) Data are trimmed to moderate intrinsic strains $\epsilon_0 > \sim -0.6\%$. The value $u = 1.7$ is independent of the value of a^- , as shown by a similar analysis of about thirty different Nb_3Sn conductors. These data also show that u is effectively unchanged by the magnetic self-field correction, as discussed in section 5 (orange square and red diamond symbols). [Note also that the same constant value of u is obtained from $B_{c2}^*(T, \epsilon)$ data extrapolated to $T = 0 \text{ K}$ (blue diamonds and orange squares) or measured at $T = 4 \text{ K}$ (green circles and red diamonds). This is a significant simplification for determining values of u .]

Figure 11. Numerical calculation of the strain dependence of the critical temperature of binary Nb₃Sn from a strong coupling theory (Markiewicz 2004; this reference is not to be confused with the later *empirical* Invariant Strain Function, Markiewicz 2006). The results of this fundamental numerical integration give the curvature scaling constant $u = 1.7$ without any adjustable fitting parameters, because the J_2 strain invariant dominates the other terms at moderate strains. (From figure 10.33 of Ekin 2006.)

Figure 12. The Exponential $b_{c2}(\varepsilon)$ model exhibits the curvature constant $u = 1.7$ at moderate strains, thus making connection with the Power Law curvature constant observed in numerous superconducting wires at moderate strains (Ekin 2006, 2010). The red curve shows a typical Power Law $b_{c2}(\varepsilon)$ strain dependence for constant curvature parameter $u = 1.7$ (with a strain sensitivity index $a^- = 1250$ at compressive strains and $a^+ = 1650$ at tensile strains). The black curve shows the Exponential model with only the fitting parameter C_1 and no adjustable parameters for curvature. (Courtesy of B Bordini, CERN.)

conductor in figure 10. From equation (26) we see that the slope of this plot of $\ln[1 - b_{c2}(\varepsilon)]$ versus $\ln|\varepsilon_0|$ gives $u = 1.7$, and the intercept gives $\ln(a^-)$ [or, if ε_0 is plotted as percent (not absolute), then the intercept is $\ln(a^-) - u \ln(100)$]. This same constant value for u was obtained whether the data were corrected or not for magnetic self field (discussed later in section 5.7).

The fundamental numerical calculation of $T_c^*(\varepsilon)$ by Markiewicz (2004) provides a theoretical basis for the constancy of the curvature of $b_{c2}(\varepsilon)$ near the peak. Figure 11 shows that over the moderate strain range, the dominant J_2 strain invariant gives the curvature constant $u = 1.7$ without any adjustable parameters. Thus, although this feature is explicitly contained in the two Power Law models through the exponent u , the Invariant Strain model and Exponential model also *implicitly* incorporate this scaling constant, since they have the second strain invariant J_2 as their lowest-order strain term (appendixes B.5 and B.6). Figure 12 shows the curvature constant results for the Exponential model. [Note this constant is not part of the Deviatoric model where the lowest-order strain term is the sharp strain cusp given by $J_2^{1/2}$; instead an empirical peak rounding parameter is introduced, appendix equations (B.5)–(B.7)].

The practical significance of the curvature constant is that:

- (1) These models have the ability to smooth noise and detect outlier data points in the dominant I_c regime.
- (2) It also gives these four $b_{c2}(\varepsilon)$ models extrapolation capability over the dominant peak strain range. Since these models have only two fitting parameters over this regime, ε_m and a^- , or ε_{i0} and C_1 (depending on the model), they can be determined by the measurement of just three points in the moderate strain range: one near the peak I_c , and one on either side of the peak.

4.5.3. Extrapolation of moderate strain data to high compressive strains. One model, the Exponential model, has the capability to extrapolate moderate strain data ($-0.5\% < \varepsilon_0 < \varepsilon_{0irr}$) to high compressive strains ($\varepsilon_0 \ll -0.5\%$). Figure 13 shows the results of such an extrapolation for B_{c2}^* raw scaling data measured at 4.02 K. A single parameter fit of C_1 to the six data points nearest the moderate strain peak is used to extrapolate B_{c2}^* to higher compressive strains ($< -1\%$) (red dashed curve). The extrapolation is reasonably accurate, but it is not as precise for this particular dataset because of the lack of any tensile strain data, as well as noise and a coarse measurement grid near the limits of the moderate strain range. Similar extrapolations with the Exponential model for the WST dataset give higher extrapolation accuracy than shown in figure 13.

4.5.4. Preferences for the parameterization of $b_{c2}(\varepsilon)$.

Although there is little difference in fitting accuracy for any of the strain functions, there are significant convenience and functional differences between the $b_{c2}(\varepsilon)$ models. The advantages of two models are noteworthy because of their combination of features:

- *Invariant Strain Function* (appendix equations (B.9) and (B.10))

Figure 13. Strain extrapolation capability of the Exponential $b_{c2}(\epsilon)$ model. The blue solid curve shows the fit to *all* the data points; the red long-dashed curve shows the capability to extrapolate the moderate strain data to high compressive strains, from a fit of only the *six points* nearest the peak (marked by red arrows to the right of the dashed line). Data are for the OST-RRP[®] conductor.

- (1) Implicitly contains the strain curvature constant $u = 1$ in its lowest order term J_2 (figure 11).
 - (2) Fits 3D strain data measured in tape conductors (tenHaken *et al* 1995, Markiewicz 2008a).
 - (3) Provides high interpolation accuracy (but with 3 fitting parameters).
- **3D Exponential model** (appendix equations (B.11) and (B.12)).
- (1) Provides a single fitting parameter C_1 that serves as a *strain sensitivity index* C_1 for comparing conductors and facilitates the use of *default* parameter values when needed.
 - (2) Implicitly contains the moderate-strain curvature constant $u = 1.7$, which gives the model consistency and extrapolation capability over the dominant peak- I_c regime (figure 12).
 - (3) Fits 3D critical current versus transverse stress data in a full multifilamentary strand (Wang *et al* 2014).
 - (4) Extrapolates moderate strain data ($-0.5\% < \epsilon_0 < \epsilon_{0irr}$) to higher compressive strains ($\epsilon_0 < -1\%$) (figure 13).

5. Magnetic self-field correction for transport data; comparison with magnetization data

Section 5 is a systematic reanalysis (following the steps in section 4) of the thousands of critical current data, except this time the source data are corrected for magnetic self field. Although the correction significantly changes the shape of the master scaling curves, the results are remarkably simple (summarized in section 5.8). An initial comparison of

magnetization data and self-field corrected transport data is also made in matching Nb₃Sn samples (sections 5.3 and 5.5).

Magnetic self-field corrections are suggested for magnet applications, because they facilitate comparison of data between short-sample predictions and magnet performance, as well as between data measured on different sample holders in different laboratories. They are also essential for comparison of transport and magnetization measurements of I_c .

The magnetic self-field correction becomes significant when transport current is high, such as at the low temperatures (~ 1.9 K) planned for the LHC high luminosity upgrade (HL-LHC). In general, the contribution of the self-field from transport current in a plane normal to the direction of current flow depends on the peak magnetic field (Garber *et al* 1989) and is proportional to the current in the wire and inversely proportional to the diameter of the outermost ring of superconductor filaments in the conductor. For samples measured on coil sample holders, it also depends on the coil geometry because of the magnetic field generated by the solenoidal sample geometry. The dependence on sample-holder geometry has been recently calculated by finite element analysis for a number of the sample holders used in the ITER benchmarking study (Bordini 2010, Cheggour *et al* 2017).

In the following subsections, we analyze the effect of magnetic self-field corrections applied to the NIST datasets described in this article (OST-RRP[®], WST, and Luvata). Key questions are: (1) whether the self-field corrections affect fundamental unified scaling, and (2) if scaling still holds, whether the values of the scaling constants change. Information was not available for specific sample holder geometries in other laboratories and so these were not corrected, but the results for the NIST datasets should provide a reasonable measure of the effects on the scaling parameters. In these datasets, I_c reached ~ 1000 A, with self-field corrections up to 16% at low temperatures, especially for the Luvata and OST-RRP[®] datasets, which included transport I_c data down to 2.26 and 2.45 K, respectively.

Magnetization: In sections 5.3 and 5.5, we also compare self-field corrected transport I_c with magnetization data for the OST-RRP[®] conductor, utilizing matched samples from the same billet, reacted with the same heat treatment. When transport currents are high, such a comparison with magnetization data necessitates correction for the magnetic self field.

5.1. Effect of the magnetic self-field correction on the master scaling curve

The self-field correction has a relatively large effect on the master pinning-force curve. It adjusts all the magnetic fields and pinning forces ($F_p \equiv I_c B$) to higher values, especially at lower fields and temperatures where currents are highest and the corrections are greatest. An example of the effect of the correction on the master curves is presented in figures 14(a) versus (b) for the Luvata dataset. Similar shifts were also

Figure 14. Master scaling curves showing a shift of data to higher relative magnetic fields and a change in shape when the magnetic self-field correction is applied: (a) Data uncorrected for magnetic self-field; (b) Corrected data. Raw scaling data for $B_{c2}^*(T)$ were obtained for the wide temperature range 2.26–14 K measured with the Luvata ITER-TF Nb₃Sn conductor, billet #NT8404, 0.82 mm diameter. No strain was applied to the sample, other than strain introduced by differential thermal contraction of the Ti sample holder (sample unsoldered except at the ends, Lorenz force directed inward). Similar effects of the magnetic self-field correction were also observed in the OST-RRP[®] and WST master curves, shown in appendix figures A2 and A4, respectively. Luvata data from Goodrich *et al* 2013, measured with the NIST vari-temp apparatus.

obtained for the OST-RRP[®] and WST conductors, as seen in figures A2 and A4 in the appendix.⁷

The significant result, from the standpoint of scaling, is that all the data still scale into a master scaling curve whether or not they are corrected for magnetic self field; this was observed over the entire range of strains and temperatures down to 2.26 K. Thus, the USL holds for both corrected and uncorrected data, although the scaling parameters and constants could change.

Also, the correction changes the *shape parameters* p and q of the master scaling:

- The low-field shape parameter p is reduced by the self-field corrected data from $p = 0.56$ to 0.43 in figure 14, and the high-field shape parameter q is reduced from 1.72 to 1.53. A similar change was observed for fitted values of p and q for the WST data, shown in appendix figure A4.
- The normalized magnetic field $b_{\max}$ at the pinning-force maximum is also somewhat reduced from 0.24 to about 0.22.

Similar changes in the shape of the master scaling curves were found for the OST-RRP[®] conductor, shown in appendix figure A2.

A shift in the pinning force curve was also found by Baumgartner *et al* (2014, 2015) when Nb₃Sn samples were neutron irradiated. The shift was modeled in terms of a

second point-pinning mechanism being introduced by radiation. Although their results show that both a shift and a second pinning mechanism were introduced, the data still temperature scaled into a master scaling curves, similar to the persistence of scaling observed here for the shift from magnetic self-field corrections.

5.1.1. Negligible effect of trim level. The trim level does not significantly change the shape of the pinning force curves for either self-field corrected or uncorrected data. For example, in figure 14(a), a trim level of $F_p < 125$ AT gave shape parameters $p = 0.563$ and $q = 1.721$ for the uncorrected Luvata data. By comparison, a trim level of $F_p < 25$ AT (one fifth the size) applied to the same dataset gave shape parameters for the master curve that were effectively unchanged: $p = 0.564$ and $q = 1.742$.

5.2. Effect of self-field correction on raw scaling data for $B_{c2}^*(T, \varepsilon)$ and the temperature scaling constant ν

Despite the significant changes in the shape parameters p and q , figure 15(a) shows that the temperature part of the corrected $B_{c2}^*(T, \varepsilon)$ is still fit by equation (4) with no effective change in the scaling parameter ν within experimental error ($\nu = 1.504$ corrected versus $\nu = 1.508$ uncorrected). The quality of the fit over the temperature range 2.26–14 K is shown by the small RMSE of 0.58% for the corrected data (with a comparable RMSE of 0.41% for the uncorrected data)⁸. A similar result was obtained for the OST-RRP[®] conductor in figure 15(b)

⁷ For the OST-RRP[®] wire, there was insufficient data below the pinning force peak to fit p . This is a common situation, especially after correcting for the magnetic self field. In this case, it is necessary to fix p to a reasonable value to prevent erroneous fitted values. The transport RMS fitting errors were low for fixing at either $p = 0.4$ or 0.5, although fixing at $p = 0.5$ gave RMS fitting errors that were about 20% lower than at $p = 0.4$ (RMSE = $\sim 0.08\%$ – 0.09% for $p = 0.5$, and RMSE = $\sim 0.10\%$ – 0.11% for $p = 0.4$).

⁸ The RMS error is usually given in this article in terms of *percentages* to facilitate intercomparisons of different types of data. It is obtained by normalizing the RMS error by the zero-temperature value of $B_{c2}^*(T = 0K, \varepsilon)$ or $K(T = 0K, \varepsilon)$.

Figure 15. (a) Self-field corrected versus uncorrected $B_{c2}^*(T, \epsilon)$ data for the Luvata conductor, showing no effective change within experimental error for the temperature scaling parameter in equation (4): $\nu = 1.504$ corrected, versus $\nu = 1.508$ uncorrected; RMSE = 0.58% and 0.41% respectively. (b) Similarly, for the OST-RRP[®] conductor, no effective change in ν was observed with the self-field correction; RMSE = 0.21% for both cases. Data *extrapolated* from above 4 K to lower temperatures with equation (4) [red curve in figure 15(a)] merged with the fit over the full temperature range (green curve) ($\nu = 1.498$ extrapolated versus 1.504 not extrapolated). Magnetization data in figure (b) (gray crosses) for $B_{c2}^*(T, \epsilon)$ for the OST-RRP[®] conductor are also compared with the transport data. The magnetization data for $B_{c2}^*(T, \epsilon)$ have a different, flatter temperature dependence compared with that of the transport data. They are not represented well by equation (4), which fits the transport data. Magnetization data and scaling analysis from Bordini and Richter 2014.

where the RMSE of the fit with equation (4) was only 0.21% for both the corrected and uncorrected data.

Note that the slight difference in magnitude between the corrected and uncorrected $B_{c2}^*(T, \epsilon)$ data shown in these figures does not affect the value of the scaling parameter ν , which describes the *normalized* $B_{c2}^*(T, \epsilon)$ curve, not its *magnitude*. That is, if the corrected and uncorrected $B_{c2}^*(T, \epsilon)$ curves were normalized, they would fall on top of each other.

5.2.1. Extrapolation tests for $B_{c2}^*(T, \epsilon)$. These were also carried out with equation (4), by fitting it to $B_{c2}^*(T, \epsilon)$ data for the Luvata conductor above 4.03 K and then extrapolating to lower temperatures. The two curves (red: extrapolated; green: not extrapolated) essentially merged in figure 15(a). The extrapolated fit predicted the data measured at 2.26 K to within 0.7%. Also, values of ν for the full-temperature fit versus the extrapolated fit were nearly the same ($\nu = 1.498$ versus 1.504).

5.3. Comparison with magnetization data for $B_{c2}^*(T, \epsilon)$

Magnetization raw scaling data for $B_{c2}^*(T, \epsilon)$ are compared with transport data for the OST-RRP[®] conductor in figure 15(b). Both types of measurements were carried out near zero applied strain in matching Ta-doped RRP[®] samples from the same billet, and reacted with the same heat treatment. Values of J_c and the I_c -V curvature parameter n [e.g., section 10.1 of Ekin (2006)] are high where the comparison is made, so the effective electric-field criterion does not play a significant role. Raw scaling data for both the transport and magnetization measurements were determined with $p = 0.5$ and $q = 2.0$ (although it makes no significant difference for

the transport data, as shown in figure 17 below). For magnetization, scaling analysis is from B_{c2}^* determined by the slope change of reversible magnetization data (Bordini and Richter 2014).

Instead of the smooth downward curvature of equation (4), the curvature of the magnetization $B_{c2}^*(T, \epsilon)$ data in figure 15(b) flattens in the mid temperature range. As a result, the parameterization of equation (4) for the temperature part of $B_{c2}^*(T, \epsilon)$ does not represent the magnetization data well.

The difference in shape of the magnetization $B_{c2}^*(T)$ curve may be a result of compositional inhomogeneities in the Ta-doped RRP[®] superconductor. The affected outer regions of each filament could carry inhomogeneous magnetization shielding currents that alter the measured $B_{c2}^*(T, \epsilon)$ dependence from that of transport data (Suenaga 1985). This is a significant consideration in relating magnetization to transport I_c measurements. Energy-dispersive x-ray spectroscopy (Tarantini *et al* 2016) show greater radial compositional variations across the A15 layer for RRP[®] conductors with Ta doping, compared with Ti doping.

5.4. Effect of self-field correction on raw scaling data for $K(T, \epsilon)$: parameterization models for $h(t)$ and extrapolations of data above 4 K to lower temperatures

Figures 16(a) and (b) show graphs of the raw scaling data for the USL prefactor $K(T, \epsilon)$ for the Luvata and OST-RRP[®] conductors, analyzed with the magnetic self-field correction (red diamond symbols), and without correction (green circles). A comparison of the red and green solid curves in each graph shows that *the temperature part $h(t)$ of the prefactor is*

Figure 16. (a) Magnetic self-field corrected versus uncorrected $K(T, \epsilon)$ data for the Luvata-ITER conductor, showing that the dimensionless prefactor scaling function $h(t)$ for the temperature part of $K(T, \epsilon)$ is effectively unchanged between the self-field corrected data (red diamonds) versus the uncorrected data (green circles). The diamond and circle symbols fall on top of each other. (b) A similar result was obtained for the OST-RRP[®] conductor. The Hybrid1 and Hybrid2 models fit both the corrected and uncorrected data for both types of conductors with a low RMSE ($\sim 0.5\%$ Luvata-ITER, $\sim 0.7\%$ OST-RRP[®]). Extrapolation tests of the different $h(t)$ models from above 4 K to lower temperatures (dashed curves) show significant low-temperature errors (red labels) for the Durham model in figure (a), and for the G/ITER and F&W models in figure (b). In contrast, extrapolation errors were remarkably low for both Hybrid models down to the lowest temperature measured, 2.26 K, within about a third of a kelvin of the planned operating temperature for the HL-LHC magnets (indicated by the black vertical tick mark at 1.9 K in both figures). Extrapolation error is $\sim 0.5\%$ at 2.26 K for Luvata, and $\sim 1.3\%$ at 2.45 K for OST-RRP[®]. The extrapolated Hybrid1 and Hybrid2 curves merged with the full-fit (red curves) in both figures (a) and (b). (Trim $F_P = 200$ AT.)

also effectively unchanged by the magnetic self-field correction within the experimental error. That is, the green circles fall on top of the red diamond symbols for both the Luvata-ITER data in figure 16(a) and the OST-RRP[®] data in figure 16(b). The Hybrid1 and Hybrid2 models [equations (19) and (20)] both fit the corrected and uncorrected data with a low RMSE ($\sim 0.5\%$ for Luvata-ITER, $\sim 0.7\%$ for OST-RRP[®]). Similar results were also observed for the WST-ITER conductor (where data were available only above ~ 4 K; not shown).

Note that the temperature parameterization $h(t)$ is dimensionless, and describes the *shape* of the prefactor temperature dependence, not its absolute value; hence the use of normalized plots of $K(T, \epsilon)/K(4, \epsilon)$ for comparing fits of the various $h(t)$ models. A normalization temperature of ~ 4 K was chosen because it is convenient and accessible for most measurements.

5.4.1. Extrapolation tests for $h(t)$. These were also carried out for each $h(t)$ model by fitting only data above 4 K, and then evaluating the accuracy of extrapolations of the fit to temperatures below 4 K. Extrapolations to the low temperature regime have practical importance because of the difficulty to measure transport I_c at very low temperatures due to high-current heating and instabilities. Figure 16(a) for the Luvata conductor shows significant extrapolation errors below 4 K for the Durham model (green dashed curve). Similarly, figure 16(b) for the OST-RRP[®] conductor shows significant extrapolation errors below 4 K for the F&W model (purple dashed curve) and G/ITER model (red dashed curve).

Interpolative error was also high in the case of the G/ITER model.

The Hybrid1, Hybrid2 and MM models had significantly smaller extrapolation errors than the other $h(t)$ models (Durham, F&W, and G/ITER); they fit the low temperature data within experimental error. For the Hybrid1 and Hybrid2 models, RMS extrapolation errors were $<0.3\%$ for the Luvata-ITER dataset, and $<0.8\%$ for OST-RRP[®]. The extrapolated curves for the Hybrid1 and Hybrid2 models effectively merged with the full-fit (red curve in both figures 16(a) and (b)).

As a side note, the extrapolation errors, although small, correlated with noise around 4 K, near the end of the fitted temperature range where the extrapolation started. This suggests that when extrapolations are anticipated, measurements with a finer measurement grid near the start of the extrapolation would improve extrapolation accuracy. (Similarly, a finer grid of strain measurements near the start of the *strain* extrapolation would benefit the accuracy of strain extrapolations with the Exponential $b_{c2}(\epsilon)$ model; see figure 13 in section 4.5.)

5.4.2. Variability. Although the Hybrid 1, Hybrid2 and MM models had nearly the same extrapolation errors for temperature below 4 K, a practical difference between the models is that the fitting parameters in the Hybrid1 and Hybrid2 models are more consistent than for the MM model (or for the Hybrid3 model described near the beginning of section 4.2). The data summary in table 7 shows the variability of these models for fitting self-field corrected

Figure 17. Magnetization $h(t)$ data (gray crosses) compared with transport $h(t)$ data for the OST-RRP[®] conductor. The magnetization data have a similar curvature to the transport data, but a slightly lower temperature slope. Magnetization raw scaling data for the prefactor K were determined with $p = 0.5$ and $q = 2.0$. Transport raw scaling data are essentially unaltered when normalized, whether they were determined with either $p = 0.5$ and $q = 2.0$, or $p = 0.4$ and $q = 2.12$ (shown respectively by blue dots and red diamonds in the figure, which fall on top of each other). The fitted curve for the transport data is calculated with T_c^* determined from B_{c2}^* data. The fitted curve for the magnetization data, on the other hand, is carried out with T_c^* determined from K because of the lack of a good fit of magnetization B_{c2}^* to equation (4). [This is shown in figure 15(b).] Magnetization data and scaling analysis are from Bordini and Richter 2014.

transport data for the Luvata-ITER and OST-RRP[®] conductors. The difference in variability shown in the third row of table 7 is greater than for the uncorrected transport data in table 3.

The practical significance is that the higher variability of the fitting parameter adversely affects the ability to determine default values for fitting applications (described in Part 3). The difference in variability is mathematical and arises from the difference in temperature dependence for kappa compared with that of the upper critical field, as described in footnote 6 in section 4.2.

Note that the values for the effective T_c^* in table 7 are lower than values of the limiting T_c . This is similar to the difference between B_{c2}^* and B_{c2} , where the difference has been ascribed to compositional inhomogeneities in practical conductors (e.g., Godeke *et al* 2005, and sections 10.3.3–4 in Ekin 2006). Fortunately, the regions near the limiting T_c and B_{c2} have negligible effect on scaling in practice, because the effective scaling parameters T_c^* and B_{c2}^* are determined by the overall technological temperature range, not the tail region. This is shown by figure 6, which was truncated at 15 K, and by the stability of the scaling parameters in other tests we have made where the upper limit of the fitting range for the transport OST-RRP[®] $h(t)$ data was progressively

Figure 18. Determination of the cross-link scaling parameter w from raw scaling data for the OST-RRP[®] Nb₃Sn conductor. The slope of the data was effectively unchanged, $w = \sim 3$, for both the magnetic self-field corrected data (red diamonds) and uncorrected data (green circles). A similar result was obtained for the WST-ITER conductor.

truncated from 14 K down to 10 K, which produced a change of less than 0.2% in the values of either T_c^* or η .

5.4.3. Conclusion: parameterization of $h(t)$. Thus, from the standpoint of parameter variability and extrapolation accuracy, the data corrected for magnetic self field show the Hybrid1 and Hybrid2 $h(t)$ models are preferred. Similar to the uncorrected data in section 4.2, the corrected data show that the fitting parameters are more consistent, and have smaller variability. Also, these two models are able to extrapolate I_c data to within $\sim 1\%$ from temperatures above ~ 4 K down to the lowest temperature measured 2.26 K, which is within about one-third kelvin of the planned operating temperature for the HL-LHC magnets. [The short one-third kelvin extrapolation from 2.26 to 1.9 K is indicated by the black vertical tick mark at 1.9 K in both figures 16(a) and (b).] This temperature extrapolation capability is significant, because the regime below 4 K is where transport measurements become difficult because of heating effects and instabilities.

5.5. Magnetization versus transport data for $h(t)$

Figure 17 shows a comparison between magnetization and transport data for the temperature part $h(t)$ of the prefactor $K(T, \epsilon)$ for the OST-RRP[®] conductor. Again, all data are normalized at about 4 K, because the scaling function $h(t)$ is dimensionless and depends only on normalized raw scaling data, not absolute values. Comparison shows that the magnetization data (gray cross symbols) have a similar curvature to the transport data, but a slightly lower temperature slope.

5.6. Cross-link parameter w

The cross-link parameter w is defined by the parameterization: $T_c^*(\epsilon_0)/T_c^*(0) = [B_{c2}^*(0, \epsilon_0)/B_{c2}^*(0, 0)]^{1/w}$, equation (9). Values of $T_c^*(\epsilon_0)$ and $B_{c2}^*(0, \epsilon_0)$ are obtained from the $T_c^*(\epsilon_0)$ and $B_{c2}^*(0, \epsilon_0)$ intercepts of $B_{c2}^*(T, \epsilon_0)$ curves extrapolated with equation (4) (shown in figure 3).

Figure 18 shows a logarithmic plot of normalized values of $T_c^*(\varepsilon_0)$ versus $B_{c2}^*(0, \varepsilon_0)$ for the OST-RRP[®] conductor, for both the magnetic self-field corrected and uncorrected data. The slope of such a log–log plot corresponds to w (similar to figure 7 for uncorrected data). The data show that *the scaling parameter w is also effectively unchanged ($\Delta w < 0.01$) by the magnetic self-field correction*, and reasonably fit by a slope of 3. This was also the case for the magnetic self-field correction for the WST-ITER dataset, where the difference was also negligible ($\Delta w = \sim 0.01$) (although the value of w for both corrected and uncorrected data was slightly less than 3 for the WST-ITER conductor).

A large set of raw scaling data for $B_{c2}^*(T, \varepsilon_0)$ is needed to determine w , which can take weeks of data acquisition to obtain. The value of w cannot be accurately determined from global fitting because of its strong interaction with the other fitting parameters (section 4.3). So, from a practical standpoint, it is indeed fortunate that w remains a stable scaling constant $w = 3.0 \pm 0.3$, independent of the magnetic self-field correction.

5.7. Strain curvature parameter u

The effect of the magnetic self-field correction on the strain curvature parameter u was included in an earlier plot of the strain function $b_{c2}(\varepsilon)$ [$\equiv B_{c2}^*(0, \varepsilon)/B_{c2}^*(0, 0)$] for the WST conductor (figure 10 near the beginning of section 4.5). Like the other scaling constants, the data show that u is also effectively unchanged within experimental error by the correction for magnetic self-field ($\Delta u = \sim 0.02$). A similar result was also obtained for the OST-RRP[®] conductor.

5.8. Summary: magnetic self-field correction and magnetization data

5.8.1. Magnetic self-field correction. Analysis of the effect of the magnetic self-field correction for the transport I_c data shows two main results:

- (1) The magnetic self-field correction significantly changes the shape of the master scaling curve, with reductions in both p and q . Despite this change in shape, values of the scaling constants are remarkably stable. *The value of v increased by less than ~ 0.02 , w increased by ~ 0.01 , and u decreased by only ~ 0.02 .* This stability reflects the intrinsic nature of unified scaling when the basic pinning mechanism is unchanged over the range of fields, temperatures and strains under consideration. A similar persistence of scaling was obtained by Baumgartner *et al* (2014, 2015), even when a second pinning mechanism was introduced.
- (2) Analysis of these data also shows [figures 16(a) and (b)] that temperature extrapolations can be carried out with the Hybrid1 and Hybrid2 $h(t)$ models from data above ~ 4 K down to 2.26 K, with RMS extrapolation errors of only 0.3%–0.8%. Such extrapolations to low temperatures are significant because of the difficulty to measure transport I_c in this regime due to instabilities and heating effects.

5.8.2. Comparison of magnetization with transport data.

Magnetization and transport data were obtained on matched Ta-doped RRP[®] samples, obtained from the same billet and reacted with the same heat treatment. This initial comparison of transport and magnetization measurements shows the following:

- (1) The temperature part $b(t)$ of the *upper critical field* $B_{c2}^*(T, \varepsilon)$ for magnetization data that is different from that for transport data. This may be a result of micrographically observed compositional inhomogeneities in the Ta-doped RRP[®] conductor, which could lead to inhomogeneous shielding currents that alter the magnetization temperature dependence from that of transport data.
- (2) The temperature part $h(t)$ of the *prefactor* $K(T, \varepsilon)$ for the magnetization data corresponds to that of the transport data reasonably well, with a similar curvature but with a slightly lower temperature slope (figure 17).

6. Derivation of the ESE relation, preferences, and summary

6.1. Scaling parameters—which are constants; which are not

The extensive analysis of each separable part of the USL in sections 4 and 5 determines which parameters in the general parameterization of equation-set (10) have the same value for all the Nb₃Sn tested and which are conductor specific. The results show:

Three scaling constants:

$$\begin{aligned} v &= 1.50 \pm 0.04 \\ \text{upper-critical-field temperature parameter} \\ &(\text{equation (4), figures 3 and 15}) \end{aligned} \quad (27a)$$

$$\begin{aligned} w &= 3.0 \pm 0.3 \\ \text{cross-link parameter, which can be precisely} \\ &\text{determined only from raw scaling data} \\ &(\text{equation (9), figures 7 and 18}) \end{aligned} \quad (27b)$$

$$\begin{aligned} u &= 1.7 \pm 0.1 \\ \text{moderate-strain curvature parameter,} \\ &(\text{equation (26), figure 10}) \end{aligned} \quad (27c)$$

Fitting parameters: Equally important, the remainder of the scaling parameters in equation-set (10) need to be fitted for each conductor, rather than fixed as is often the case, especially for the strain exponent s .

6.1.1. Upper-critical-field temperature parameter v . For the first scaling constant, a value of $v = 1.50 \pm 0.04$ is found. A value of $v = 1.52$ has been used as a good approximation of equation (4) to the microscopic Maki-DeGennes expression, but the difference between equation (4) and the Maki-DeGennes expression is only about $\pm 2\%$, which is not significant compared with the experimental error for $B_{c2}^*(T, \varepsilon)$

data. Also, the best fit of equation (4) to $B_{c2}^*(T, \varepsilon)$ is usually with ν slightly *less* than 1.5, rather than more (see table 1). We conclude, based on extensive raw scaling data, that the added complexity of evaluating the Maki–DeGennes relation or the Werthamer theory, is not needed for fitting accuracy in lieu of the simpler parameterization given by equation (4) with $\nu = 1.5$.

6.1.2. Crosslink parameter w . The second constant w is perhaps the most important scaling constant, because it is very difficult to determine from global fitting. It is the only parameter that is fundamentally non-separable, so its determination from scaling analysis requires complete, full matrix datasets covering wide ranges of magnetic field, temperature, and strain. Such data would be challenging to determine on a routine bases for individual conductors (section 4.3).

6.1.3. Moderate-strain curvature parameter u . The third scaling constant, $u = 1.7$, describes the curvature of the strain function $b_{c2}(\varepsilon)$ [aka $S(\varepsilon)$] over the *moderate* strain range ($-0.5\% < \varepsilon_0 < \varepsilon_{0irr}$). It does not appear explicitly in most of the $b_{c2}(\varepsilon)$ models, except for the Power Law and Extended Power Law parameterizations. However, it is inherent in two other $b_{c2}(\varepsilon)$ models, the Exponential model of Bordini *et al* and the Invariant Strain model of Markiewicz, through the second strain invariant J_2 (as shown by figures 11 and 12 in section 4.5).

The practical significance of the curvature constant is that it gives these four models the ability to extrapolate the strain dependence over the peak strain range from just three data points: one near the peak I_c , and one on either side of the maximum. Because the curvature is constant, these models also have the ability to smooth noise and detect outlier data points in the dominant I_c regime.

6.1.4. Stability of the scaling constants with respect to factors in the analysis of the pinning force curves.

- *Magnetic self-field corrections* applied to the transport data have negligible effect ($\Delta < 1\%$) on the values of the scaling constants (section 5). This is consistent with scaling being an intrinsic property of the Nb_3Sn grains.
- The *trim level* utilized in determining the raw scaling data also has little effect. A change in trim level of five times results in less than a 2% change in the values of the scaling constants (section 5.1.1).
- The precise *values of p and q* (including the default values $p = 0.5$ and $q = 2$) do not change the values of the constants within experimental error ($\Delta < \sim 1\%$).

6.2. ESE, the ‘easy’ fit

Combining the above results for the scaling constants and fitting parameters with the general parameterization of the USL (equation-set 10) gives the ESE relation, where the ten

fitting parameters plus $b_{c2}(\varepsilon)$ are now reduced to five plus $b_{c2}(\varepsilon)$:

Extrapolative Scaling Expression (ESE)

$$I_c(B, T, \varepsilon)B = C [b_{c2}(\varepsilon)]^s (1 - t^{1.5})^{\eta-\mu} (1 - t^2)^{\mu} b^p (1 - b)^q \quad (28a)$$

with reduced magnetic field $b \equiv B/B_{c2}^*(T, \varepsilon)$ and reduced temperature $t \equiv T/T_c^*(\varepsilon)$, where:

$$B_{c2}^*(T, \varepsilon) = B_{c2}^*(0, 0)(1 - t^{1.5})b_{c2}(\varepsilon) \quad (28b)$$

$$T_c^*(\varepsilon) = T_c^*(0)[b_{c2}(\varepsilon)]^{1/3} \quad (28c)$$

and five fitting parameters: C , $B_{c2}^*(0, 0)$, $T_c^*(0)$, s , either η or μ (but not both), plus the parameters in $b_{c2}(\varepsilon)$.

The extensive raw scaling data analysis of sections 4 and 5 show that these five scaling parameters, plus those in $b_{c2}(\varepsilon)$, *need to be fitted for general interpolation and extrapolation accuracy*.

The ESE parameterization is flexible to the extent that:

- p and q , the shape parameters of the pinning force curve are preferably also fitted (simultaneously with the other parameters) to minimize errors in the magnetic field dependence, especially over wide ranges of relative magnetic field b . However, when data are insufficient, such as when there is little or no low magnetic field data below the pinning-force peak to determine p (a common situation with magnetic self-field corrections), or not enough high magnetic-field data to determine q , then default values $p = 0.5$ and $q = 2.0$ are needed. Use of these default p and q values gives little loss in fitting accuracy when the range of relative field b is not extensive.
- η or μ , the parameters for the temperature part $h(t)$ of the prefactor can be prescribed according to any of the models summarized in section 4.2, as long as one of the parameters is fitted. The need for a fitted temperature parameter may arise from composition inhomogeneities (figure 6). Although interpolation accuracies are similar for most of the $h(t)$ models, extrapolation differences are significant, as described below in section 6.4.
- $b_{c2}(\varepsilon)$ [aka $S(\varepsilon)$]: the ESE relation can be parameterized effectively with any of the $b_{c2}(\varepsilon)$ strain functions summarized in appendix B. However, functional considerations for $b_{c2}(\varepsilon)$ exist (section 6.4).

6.3. Differences between the ESE relation and other fitting equations

ESE is readily applied by a quick simultaneous fit of its parameters to available data (like the other global-fitting equations). The distinction between ESE and the present interpolative parameterizations of the USL is that, rather than empirically postulating which parameters are fixed and which are constant, the ESE relation is derived by an extensive

analysis of raw scaling data from pinning-force curves in many different Nb₃Sn conductors. The key difference is that a global ESE fit gives scaling parameters that register all the data into a master pinning-force curve, which provides *extrapolation* capability.

The parametric differences between ESE and present non-extrapolative fitting equations are as follows:

Durham fitting equation (Taylor and Hampshire 2005, Lu *et al* 2008):

- The cross-link parameter w is typically fixed in this model at an average value of 2.2, instead of 3.0 for ESE (figure 7).
- The strain parameter s , which relates the strain part of the prefactor to that of B_{c2}^* , $g(\varepsilon) = [b_{c2}(\varepsilon)]^s$, is fixed at $s = 1$, rather than fitted as in the ESE relation.

G/ITER fitting equation (Godeke *et al* 2006, 2009, 2013, Mentink 2008, Bottura and Bordini 2009):

- The temperature parameters are fixed at $\eta = 2$ and $\mu = 1$, instead of one of these parameters being left a free fitting parameter as in the ESE relation. The errors this produces are reasonably small for many (but not all) of the ITER conductors, but RMS errors are up to four times larger in some cases (figures 4(b) and 5).
- The strain parameter is fixed at $s = 1$, whereas raw-scaling data show it needs to be fitted for accuracy.

MAG fitting equation (Mentink 2014):

- This simplified mathematical form results only if the parameter s is fixed at the constant value $s = 1$. The analysis with raw scaling data in section 4.4 shows that the value of s needs to be fitted to accommodate different Nb₃Sn conductors, with typical values of s ranging from about $s = 1.1$ to 1.4 (figure 9).

6.4. Preferences for the temperature parameters η and μ , and the $b_{c2}(\varepsilon)$ strain function

Although the fitting accuracy of different parameterizations of $h(t)$ and $b_{c2}(\varepsilon)$ in ESE are comparable, there are significant differences between these models in practice.

6.4.1. Preferred parameterization for the temperature part $h(t)$ of the prefactor. Overall fitting accuracies and extrapolation tests with raw scaling data show that the Hybrid1 and Hybrid2 models for $h(t)$ have significant extrapolation and consistency advantages compared with the other models:

- (1) *Extrapolation* of data from above 4 K to lower temperatures is an important capability of these models. Extrapolation errors are only ~1% down to the lowest temperature measured, 2.26 K, which approaches the operating temperature of 1.9 K for the LHC high luminosity upgrade [figures 16(a) and (b)].
- (2) *Fitting errors* to raw-scaling data for $h(t)$ over the full temperature range 2.26–14 K give RMS errors less than

0.8%. This low error was obtained for both magnetic self-field corrected and uncorrected data.

- (3) *Variability*: values of the fitting parameter in the Hybrid1 and Hybrid2 models have a variability that is significantly less than that of the MM model (table 7).

Use of the Hybrid1 $h(t)$ parameterization with ESE gives the extrapolative form

ESE, with Hybrid1

$$I_c(B, T, \varepsilon)B = C [b_{c2}(\varepsilon)]^s (1 - t^{1.5})^{\eta-1} (1 - t^2) b^p (1 - b)^q \quad (29)$$

and the Hybrid2 parameterization of ESE gives:

ESE, with Hybrid2

$$I_c(B, T, \varepsilon)B = C [b_{c2}(\varepsilon)]^s (1 - t^{1.5})(1 - t^2)^{\eta/2} b^p (1 - b)^q, \quad (30)$$

where the rest of the equation sets are given, as above, by equations (28b) and (28c). Thus, either of these parameterizations gives high parameter consistency and excellent accuracy for extrapolating transport data from above 4 K to below 4 K. The minimal errors of such extrapolations (~1%) are especially useful for transport measurements because of low-temperature heating effects and instabilities.

6.4.2. Preferred parameterization for the $b_{c2}(\varepsilon)$ strain function. Although all of the strain parameterizations for $b_{c2}(\varepsilon)$ in section 4.5 give good interpolative fitting accuracy to raw scaling data for $B_{c2}^*(T, \varepsilon)$, two $b_{c2}(\varepsilon)$ parameterizations have significant convenience and functional advantages:

- The *Invariant Strain Function* [appendix equations (B.9) and (B.10)] is noteworthy for several reasons: (1) uniaxial strain measurements can be extended to three dimensions, (2) this model intrinsically incorporates the moderate strain curvature parameter $u = 1.7$ in its lowest order term, the second strain invariant J_2 (which gives the model consistency and extrapolation capability over the dominant peak strain range (figure 11); (3) it provides high *interpolation* accuracy of wide ranging strain data (but with 3 fitting parameters).
- The *Exponential* model [appendix equations (B.11) and (B.12)] has a unique combination of features. It: (1) treats three-dimensional strains, (2) provides a single strain-sensitivity index C_1 , (3) implicitly incorporates the moderate strain curvature constant $u = 1.7$ in its lowest order term J_2 (figure 12), and (4) has the ability to extrapolate strain data from moderate strains to high compressive strains (figure 13).

On balance, the *Exponential* model is generally preferred among the different strain models because of its unique capability to predict conductor performance at high compressive strains, and the simplicity of a single fitting parameter C_1 . The Invariant model is useful if extra interpolation accuracy is needed.

6.5. Two types of extrapolation

Equation-set (28) is the starting point for Part 3 of this series of articles. There, we evaluate the accuracy of ESE to extrapolate complete $I_c(B, T, \varepsilon)$ datasets from limited data. Although the accuracy results are comparable for ESE parameterized with different combinations of $h(t)$ and $b_{c2}(\varepsilon)$, parameterizations specifically with the Hybrid1 and Hybrid2 $h(t)$ (equations (29) or (30)), and the Exponential $b_{c2}(\varepsilon)$ (equations (B.11) and (B.12)), provide an additional type of extrapolation capability. The distinction is as follows:

- (1) The first type of extrapolation (diagrammed in figures 1 and 2 at the beginning of this article) is the capability to take $I_c(B)$ curves measured at a subset of T, ε values and extrapolate $I_c(B)$ curves for the rest of the T, ε values, thus filling in the remainder of the $I_c(B, T, \varepsilon)$ dataset. This type of extrapolation considerably shortens data-acquisition time by eliminating the need to carry out measurements at all the T, ε combinations. However, this extrapolation capability does not predict data at temperatures or strains beyond the *range limits* of the given data.
- (2) The second type of extrapolation allows extension of the temperature and strain limits of the supplied data. Figures 16(a) and (b) show this for *temperature* extrapolations with the Hybrid1 and Hybrid2 models, where data from above 4 K are extrapolated to lower temperatures. Figure 13 shows this for *strain* extrapolations with the Exponential $b_{c2}(\varepsilon)$ model, where data at moderate strains are extrapolated to high compressive strains.

Thus, the ESE equation-set (28) can be used with almost any of the $h(t)$ and $b_{c2}(\varepsilon)$ models to give the first of type of extrapolation, whereas parameterization of ESE specifically with the Hybrid $h(t)$ models [equation-sets (29) or (30)] and the Exponential $b_{c2}(\varepsilon)$ model [appendix equation-set (B.12)] provides capability for extrapolations of both types.

In Part 3, we focus mainly on evaluating the accuracy of extrapolations of the first type, but give examples of the second type as well. The exceptional accuracy of ESE extrapolations enables several new extrapolation capabilities, which are illustrated with practical examples. Guidelines for the practical application of ESE are also described.

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Appendix A. Properties of the Nb₃Sn conductors analyzed in this study, master scaling curves, and raw scaling data

Table A1 lists properties for a number of the Nb₃Sn conductors analyzed in this study. *Raw scaling data* were obtained by registering F_p - B pinning-force curves into unified master scaling curves for each wire. Examples of the master curves built up by this procedure are shown in appendices A1–A4. Appendices A1 and A2 also show additional master curves to illustrate the effects of the magnetic self-field corrections described in section 5. An example listing of raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$ for one of the conductors is given in table A2 in appendix A.1. Complete tabulations of raw-scaling data are given for all the conductors as Excel[™] files online in the supplemental website accompanying this article, www.ResearchMeasurements.com. Also included in the online material is an example of an automated process for simultaneously computing raw scaling data for large datasets. The website also provides complete files of the source data for the NIST datasets.

A.1. OST, high- J_c , RRP[®] conductor for HL-LHC accelerator magnets and NMR magnets

Wire characteristics

This Nb₃Sn conductor was manufactured with a RRP[®] process by OST (figure A1). It is a high- J_c strand, 0.7 mm in diameter, and designed for the LHC high luminosity upgrade and for NMR magnets, billet #8781, 54/61 sub-element

Table A1. Properties of the Nb₃Sn conductors analyzed in this study.

	Conductor	Type	J_c	Application	# Sub elements, diffusion barrier	Cu: non- Cu ratio	Doping	Diam. (mm)	Reference
	A1.1 Oxford Superconducting Technology	RRP [®]	High	HL-LHC & NMR	54, Nb distributed	0.855	Ta 7.5 wt%	0.7	Present article + www.ResearchMeasurements.com
	A1.2 Western Superconducting Technology	Internal tin	Moderate	ITER-TF	19, Ta single	1.031	Ti	0.82	Present article + www.ResearchMeasurements.com
	A1.3 Luvata	Internal tin	Moderate	ITER-TF	19, Ta single	1.1	Ti ~2 wt%	0.83	Goodrich (2009), Cheggour <i>et al</i> (2014)
8	A1.4 Vacuumschmelze	Bronze	Moderate	ITER				0.81	Taylor and Hampshire (2005) ^a
	A1.5 Oxford Superconducting Technology	Internal tin	Moderate	ITER-TF	Ta single	1.5	Ti 1.5%	0.81	Goodrich and Stauffer (2005)
	A1.6 Furukawa	Bronze	Moderate	ITER					Taylor and Hampshire (2005) ^a
	A1.7 EM-LMI	Internal tin	Moderate	ITER	Nb distributed		Ta	0.81	Taylor and Hampshire (2005) ^a

^a Source $I_c(B, T, \varepsilon)$ data were obtained online at <http://dur.ac.uk/superconductivity.durham/> from the Durham group (Taylor and Hampshire 2005).

Table A2. Example tabulation of raw scaling data for the effective upper critical field $B_{c2}^*(T, \varepsilon)$ and scaling law prefactor $K(T, \varepsilon)$ used to form the master scaling curves in figure A2 for the OST-RRP[®] conductor. Tabulations of raw scaling data for the other conductors in table A1 are given as Excel[™] files online in the supplemental website. B_{num} gives the number of data points in each F_p versus B curve; B_{min} and B_{max} are the minimum and maximum magnetic fields for each curve.

Index	Strain, ε (%)	Approx. ε_0 (%)	T (K)	$B_{c2}^*(T, \varepsilon)$ (T)	$K(T, \varepsilon)$ (AT)	B_{num}	B_{min} (T)	B_{max} (T)	Self-field corrected	
									$B_{c2}^*(T, \varepsilon)$ (T)	$K(T, \varepsilon)$ (AT)
1	−0.755	−1.016	2.45	16.86	24 951	11	8.0	16	17.07	27 394
2	−0.755	−1.016	3.32	16.18	22 696	10	7.0	16	16.36	25 001
3	−0.755	−1.016	4.07	15.48	20 559	11	6.0	16	15.62	22 762
4	−0.755	−1.016	5.00	14.49	18 353	9	7.0	15	14.66	20 165
5	−0.755	−1.016	6.00	13.37	15 205	8	6.0	13	13.52	16 705
6	−0.755	−1.016	8.00	10.72	9498	7	4.0	10	10.81	10 528
7	−0.755	−1.016	10.00	7.77	4666	7	2.5	7	7.79	5193
8	−0.755	−1.016	12.00	4.73	1436	7	1.0	4	4.55	1658
9	−0.661	−0.927	4.07	16.07	22 192	10	7.0	16	16.26	24 398
10	−0.661	−0.927	5.00	15.11	19 699	9	7.0	15	15.28	21 637
11	−0.661	−0.927	6.00	13.91	16 615	9	6.0	14	14.07	18 276
12	−0.661	−0.927	8.00	11.21	10 603	7	5.0	11	11.34	11 623
13	−0.661	−0.927	10.00	8.24	5360	8	2.5	8	8.25	6010
14	−0.661	−0.927	12.00	5.11	1771	6	1.5	4	5.10	1958
15	−0.566	−0.837	4.07	16.67	23 786	10	7.0	16	16.87	26 144
16	−0.566	−0.837	5.00	15.66	21 359	9	8.0	16	15.86	23 392
17	−0.566	−0.837	6.00	14.52	17 985	9	7.0	15	14.70	19 685
18	−0.566	−0.837	8.00	11.79	11 645	8	5.0	12	11.93	12 790
19	−0.566	−0.837	10.00	8.75	6073	7	3.0	8	8.81	6726
20	−0.566	−0.837	12.00	5.54	2135	7	1.5	5	5.51	2387
22	−0.472	−0.748	4.08	17.44	25 271	10	7.0	16	17.65	27 709
23	−0.472	−0.748	5.00	16.38	22 880	9	8.0	16	16.59	25 014
24	−0.472	−0.748	6.00	15.21	19 399	9	7.0	15	15.40	21 206
25	−0.472	−0.748	8.00	12.48	12 657	8	5.0	12	12.62	13 879
26	−0.472	−0.748	10.00	9.33	6775	8	3.0	9	9.39	7515
27	−0.472	−0.748	12.00	6.06	2572	7	1.5	5	6.01	2900
29	−0.378	−0.659	4.04	18.18	27 465	9	8.0	16	18.42	29 952
30	−0.378	−0.659	5.00	17.16	24 051	36	7.0	16	17.37	26 327
31	−0.378	−0.659	6.00	15.93	21 066	10	7.0	16	16.14	22 992
32	−0.378	−0.659	8.00	13.08	14 215	8	6.0	13	13.25	15 503
33	−0.378	−0.659	10.00	10.02	7655	7	3.5	9	10.13	8391
34	−0.378	−0.659	12.00	6.61	3088	7	2.0	6	6.61	3408
36	−0.190	−0.480	4.07	20.08	30 432	9	8.0	16	20.38	32 849
37	−0.190	−0.480	5.00	18.87	28 252	8	9.0	16	19.16	30 539
38	−0.190	−0.480	6.00	17.60	24 753	8	9.0	16	17.85	26 845
40	−0.190	−0.480	8.00	14.82	16 533	9	6.0	14	15.03	17 874
41	−0.190	−0.480	10.00	11.40	9850	7	5.0	11	11.57	10 628
42	−0.190	−0.480	12.00	7.88	4340	6	3.0	7	8.00	4665
43	−0.190	−0.480	14.00	4.27	1053	5	1.0	3	4.15	1173
45	0.000	−0.302	4.10	22.19	33 531	8	9.0	16	22.58	35 681
46	0.000	−0.302	5.00	20.93	31 324	7	10.0	16	21.30	33 387
47	0.000	−0.302	6.00	19.57	27 402	8	9.0	16	19.90	29 267
48	0.000	−0.302	8.00	16.60	19 125	10	7.0	16	16.87	20 448
49	0.000	−0.302	10.00	13.12	11 651	8	5.0	12	13.32	12 456
50	0.000	−0.302	12.01	9.30	5697	6	3.5	8	9.46	6074
51	0.000	−0.302	14.00	5.51	1709	6	1.5	4	5.52	1836
53	0.035	−0.267	2.45	24.15	39 961	5	12.0	16	24.62	42 313
54	0.035	−0.267	3.32	23.24	37 307	6	11.0	16	23.68	39 581
55	0.035	−0.267	4.07	22.46	34 872	9	10.0	16	22.88	37 034
56	0.035	−0.267	5.00	21.18	32 178	7	10.0	16	21.56	34 315
57	0.035	−0.267	6.00	19.86	28 129	8	9.0	16	20.21	30 029
58	0.035	−0.267	8.00	16.90	19 667	10	7.0	16	17.18	21 001
59	0.035	−0.267	10.00	13.45	11 871	9	5.0	13	13.67	12 640
60	0.035	−0.267	12.00	9.59	5853	8	3.0	9	9.72	6278
61	0.035	−0.267	14.00	5.67	1834	7	1.0	4	5.54	2072
63	0.091	−0.214	4.10	22.86	35 334	9	10.0	16	23.30	37 450
64	0.091	−0.214	5.00	21.65	33 094	6	11.0	16	22.06	35 174
65	0.091	−0.214	6.00	20.24	29 230	7	10.0	16	20.60	31 170

Table A2. (Continued.)

Index	Strain, ε (%)	Approx. ε_0 (%)	T (K)	$B_{c2}^*(T, \varepsilon)$ (T)	$K(T, \varepsilon)$ (AT)	B_{num}	B_{min} (T)	B_{max} (T)	Self-field corrected	
									$B_{c2}^*(T, \varepsilon)$ (T)	$K(T, \varepsilon)$ (AT)
66	0.091	-0.214	8.00	17.26	20 475	10	8.0	16	17.56	21 818
67	0.091	-0.214	10.00	13.79	12 498	8	6.0	13	14.03	13 280
68	0.091	-0.214	12.00	9.95	6185	7	3.5	9	10.12	6572
69	0.091	-0.214	14.00	5.98	2027	7	1.5	5	5.98	2192
71	0.183	-0.127	4.07	23.67	36 152	7	10.0	16	24.13	38 164
72	0.183	-0.127	5.00	22.27	34 107	6	11.0	16	22.70	36 161
73	0.183	-0.127	6.00	20.89	30 043	7	10.0	16	21.28	31 903
74	0.183	-0.127	8.00	17.86	21 347	9	8.0	16	18.16	22 701
75	0.183	-0.127	10.00	14.27	13 302	8	6.0	13	14.52	14 111
76	0.183	-0.127	12.00	10.42	6638	7	3.5	9	10.60	7041
77	0.183	-0.127	14.01	6.27	2305	7	1.5	5	6.28	2510
79	0.257	-0.057	4.08	23.98	36 598	8	10.0	16	24.46	38 578
80	0.257	-0.057	5.00	22.64	34 322	6	11.0	16	23.09	36 288
81	0.257	-0.057	6.00	21.19	30 474	7	10.0	16	21.59	32 310
82	0.257	-0.057	8.00	18.14	21 552	9	8.0	16	18.45	22 866
83	0.257	-0.057	10.00	14.57	13 412	9	6.0	14	14.83	14 176
84	0.257	-0.057	12.00	10.61	6810	8	3.5	10	10.78	7226
85	0.257	-0.057	14.00	6.42	2432	7	1.5	5	6.38	2671
86	0.257	-0.057	15.00	4.35	1047	6	1.0	3.5	4.23	1160
87	0.313	-0.004	4.08	24.16	36 557	7	10.0	16	24.65	38 481
88	0.313	-0.004	5.00	22.75	34 326	6	11.0	16	23.20	36 250
89	0.313	-0.004	6.00	21.37	30 183	7	10.0	16	21.78	31 897
90	0.313	-0.004	8.00	18.24	21 605	9	8.0	16	18.55	22 905
91	0.313	-0.004	10.00	14.69	13 331	9	6.0	14	14.97	14 051
92	0.313	-0.004	12.00	10.73	6780	8	3.5	10	10.91	7173
93	0.313	-0.004	14.00	6.64	2392	7	1.5	5	6.64	2583
94	0.313	-0.004	15.00	4.48	1052	6	1.0	3.5	4.36	1155

Note: Specific factors used in determining these raw scaling data did not significantly affect the ratios of raw scaling data used in the analysis of the scaling parameters ($\Delta < \sim 1\%$). The factors are as follows. Data uncorrected for self-field: fixed $p = 0.5$ and $q = 1.997$ (q was obtained from a simultaneous fit to all the data). Data corrected for self field: fixed $p = 0.4$ and $q = 2.12$ (q was obtained from a simultaneous fit to all the data). (Self-field corrected raw scaling data for $p = 0.5$ and $q = 2.0$ are given in the online supplementary data.) Data are trimmed below $F_p < 200$ AT and removed where only 2 or 3 magnetic field points were left ($B_{\text{num}} \leq 3$). Approximate intrinsic strain ε_0 is calculated assuming $\varepsilon_m = 0.3\%$.

configuration, and given a final heat treatment at 640 °C for 48 h. A micrograph of the conductor cross-section is shown in figure A1. Critical-current measurements were made at NIST by Goodrich and Cheggour. The complete $I_c(B, T, \varepsilon)$ dataset is tabulated online in the supplemental website previously cited.

Master scaling curves for this conductor are shown in figure A2, and the raw scaling data used to register these curves are tabulated in table A2.

A.2. WST, moderate- J_c , internal-tin conductor for ITER TF magnets

Wire characteristics

This WST Nb₃Sn conductor was manufactured with an internal-tin process (Li *et al* 2010). It is a moderate- J_c strand, 0.82 mm in diameter, and designed for the ITER TF magnets, billet #01CW0014A01. It consists of a stack of 19 sub-elements containing Cu-clad Nb filaments placed around Sn cores, as well as a single Ta diffusion barrier surrounding the stack for protecting the outer stabilizing Cu sheath. As shown in figure A3, Sn-Ti cores supplied the Sn and Ti during the

Figure A1. Micrograph of the OST high- J_c , RRP[®] conductor, designed for HL-LHC accelerator magnets and NMR magnets (micrograph courtesy of Peter Lee, Florida State University).

Figure A2. Master scaling curves $f(b)$ for the OST-RRP[®] dataset, which provided raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$. [Values of $F_{Pmax}(T, \varepsilon)$ used in the figures are related to $K(T, \varepsilon)$ by a proportionality constant; see footnote in section 2.1] Values of $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$ are tabulated in appendix table A2 and online in the supplemental website. (a) Data uncorrected for magnetic self-field effects: here p was fixed at the default value $p = 0.5$ because there were insufficient data below the pinning-force peak to determine p . (b) Self-field corrected data: p was fixed at $p = 0.4$. Data were trimmed below $F_P < 200$ AT. The precise values used for the trim level, and for fixing p had only a $\sim 1\%$ effect on the scaling parameter values calculated from these raw scaling data (section 5.1). Critical-current measurements were made at NIST by Cheggour and Goodrich.

final reaction heat treatment at 650 °C for 100 h, ITER heat cycle B.

Critical-current measurements were made at NIST by Cheggour and Goodrich. Master scaling curves, both with and without magnetic self field corrections, are shown in figure A4. The raw scaling data used to register the F_P - B curves into the master curves are tabulated in the supplemental information. Source $I_c(B, T, \varepsilon)$ data are also tabulated online.

A.3. Luvata, moderate- J_c , internal-tin conductor for ITER TF magnets

Wire characteristics

The Nb₃Sn conductor is an internal-tin Nb₃Sn pre-production ITER strand, billet #NT8404, 0.82 mm diameter, designed for the TF magnets, and fabricated by Luvata (Pyon *et al* 2007). As shown in figure A5, it consists of a stack of 19 sub-elements containing Cu-clad Nb filaments placed around Sn cores, as well as a single Ta diffusion barrier surrounding the stack for protecting the outer stabilizing copper. The strand also has Sn-core spacers inserted between the outermost and middle rows of Nb sub-elements, in addition to the Sn cores located at the center of each sub-element. Copper to non-copper ratio was 1.1. The conductor was given a reaction heat treatment of 650 °C for 100 h. Temperature measurements $I_c(B, T)$ were conducted by Goodrich *et al* (2013), and strain measurements $I_c(\varepsilon)$ at 12 T were made by Cheggour *et al* (2014) in a separate apparatus. Both datasets are given online as Excel[™] files in the supplemental website, along with raw scaling data. Since the strain measurements were conducted at a fixed magnetic field, the master scaling curve shown in figure A6 was generated with temperature data only.

Figure A3. Micrograph of the WST moderate- J_c , internal-tin conductor, designed for ITER toroidal-field magnets (micrograph courtesy of Peter Lee and Charlie Sanabria, Florida State University).

The effect of magnetic self field corrections on the master scaling curve was shown earlier for this conductor in figure 14. A comparison of figure A6 with figure 14 illustrates the relatively small effect of the trim level on the master scaling curve; earlier data were obtained for a trim level of $F_P < 125$ AT, whereas in figure A6 a trim level of only $F_P < 25$ AT was used, one fifth the size. Despite the large difference in relative trim level, the fitted curve shape parameters are little changed: $p = 0.564$ and $q = 1.742$ for

Master scaling curves, uncorrected and corrected for magnetic self field

Figure A4. Master scaling curves $f(b)$ for the WST-ITER dataset, which provided raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$. (a) Data uncorrected for magnetic self field. (b) Self-field corrected data. The raw scaling data for both master curves were determined with p and q fitted (since for this dataset sufficient data below the peak in the master scaling curve were available to determine p). Data below $F_p < 125$ AT were trimmed for both curves. Raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$ obtained from these 84 scaled curves are tabulated in the supplemental website, along with the source $I_c(B, T, \varepsilon)$ data. Measurements by Cheggour and Goodrich.

Figure A5. Micrograph of the Luvata internal-tin, Nb₃Sn pre-production ITER strand, designed for the ITER toroidal-field magnets (micrograph courtesy of T Pyon, Luvata Waterbury Inc.).

Figure A6. Master scaling curve $f(b)$ for the Luvata ITER-TF Nb₃Sn conductor, variable temperature data only, with fitted $p = 0.56$ and $q = 1.74$ (for this dataset, there were sufficient magnetic-field data below the peak in the master scaling curve to fit p). Pinning-force data were trimmed below $F_p < 25$ AT. Raw scaling data obtained from this master curve are tabulated online in the supplemental website, along with the source $I_c(B, T)$ dataset, which was measured by Goodrich.

$F_p < 25$ AT in figure A6, versus $p = 0.563$ and $q = 1.721$ for $F_p < 125$ AT in figure 14.

A.4. Vacuumschmelze, moderate- J_c , bronze-process conductor

Wire characteristics

The Nb₃Sn conductor is a multifilamentary, bronze-process wire, 0.81 mm in diameter, manufactured by Vacuumschmelze.

It was given a heat treatment that terminated at 650 °C for 175–200 h (Taylor and Hampshire 2003, 2005). Source $I_c(B, T, \varepsilon)$ data were obtained from <http://dur.ac.uk/superconductivity.durham/>, measured by the Durham group (Taylor and Hampshire 2005). Raw scaling data are tabulated in the supplemental data website accompanying this article.

Figure A7 is a good illustration of how higher temperature data (blue square symbols) usually provide more data on the low-field side of the pinning force peak, sufficient to determine the low-field curve shape parameter p . In contrast,

Figure A7. Master scaling curve $f(b)$ for the Vacuumschmelze dataset, which provided raw scaling data for $B_{c2}^*(T, \varepsilon)$ and $K(T, \varepsilon)$. Values of p and q were fitted, $p = 0.58$ and $q = 1.74$. Note that the 4.2 K data (the red symbols near the middle of the curve) occupy only a limited section on the high-field side of the peak, and thus the higher temperature 12 K (blue square symbols) were essential for determining the low-field shape parameter p . Data below $F_p < 25$ AT were trimmed. Raw scaling data obtained from this master curve and are tabulated online at www.ResearchMeasurements.com. (Measurements by Taylor and Hampshire 2005.)

the 4.2 K data (the red star symbols near the middle of the curve) occupy only a limited section on the high-field side of the peak.

Appendix B. Parameterization models for the upper critical field strain function $b_{c2}(\varepsilon)$ (equation (3b) and section 4.5)

A number of parameterizations have been suggested to model the strain function $b_{c2}(\varepsilon)$. The strain function is defined by the separable parts of the effective upper critical field given in equation (10c), $B_{c2}^*(T, \varepsilon)/B_{c2}^*(0, 0) \equiv b_{c2}(\varepsilon) b_{c2}(t)$. The strain part, $b_{c2}(\varepsilon)$, is thus given by measurements of $B_{c2}^*(0, \varepsilon)$ (since at $T = 0$, $b_{c2}(t) = 1$), or approximately by $B_{c2}^*(4 \text{ K}, \varepsilon)$. As noted earlier, $b_{c2}(\varepsilon)$ is also denoted $S(\varepsilon)$ in the literature; the nomenclature $b_{c2}(\varepsilon)$ is used here to emphasize its *physical* meaning—it is not an arbitrary strain parameterization, but a fit to actual measured raw scaling data for the strain part of $B_{c2}^*(T, \varepsilon)$.

This appendix gives the specific parameterizations of each of the $b_{c2}(\varepsilon)$ models considered in the summary comparisons of section 4.5, along with a discussion of the advantages and disadvantages of each.

B.1. Power Law model

The Power Law fitting equation was originally proposed to model $b_{c2}(\varepsilon)$ at *moderate* strains ($-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}$),

$$\text{Power Law uniaxial model (Ekin 1980)} \quad (B.1)$$

$$b_{c2}(\varepsilon) = 1 - a |\varepsilon_0|^u \quad (-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}),$$

where ε_0 is the intrinsic strain defined as $\varepsilon_0 \equiv \varepsilon - \varepsilon_m$, ε_m is the strain at the $I_c(\varepsilon)$ maximum, and u is a constant $u = 1.7$. The parameter a in equation (B.1) and has two values: a^- for the compressive strain range ($\varepsilon_0 \leq 0$) and a^+ for tensile ($\varepsilon_0 > 0$), with the ratio $a^-/a^+ \approx 0.75$.

Advantages

- One fitting parameter at compressive strains, a^- , which serves as a *single index of strain sensitivity*.
- Utilizes the curvature parameter $u = 1.7$, a scaling constant at moderate strains ($-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}$).
- Extrapolation capability over the moderate strain range.
- Simplest parameterization for many magnet designs at *moderate* strains.

Disadvantages

- Not generally applicable: the Power Law becomes inaccurate for fitting $b_{c2}(\varepsilon)$ at higher compressive strains and is restricted to moderate strains of about $-0.5\% < \varepsilon_0 < \varepsilon_{0\text{irr}}$. Compressive strains outside this range become relevant for large magnets if they are fabricated with structural materials having high thermal contraction compared to that of the conductor. However, the parameterization still serves as the simplest expression for the many magnets designed to operate within about 0.5% of the strain peak.
- One-dimensional: the Power Law treats only longitudinal strains along the conductor axis, although Markiewicz (2008a) has considered an extension of the Power Law to three dimensional strains by substituting the deviatoric strain for the longitudinal strain in equation (B.1).

B.2. Extended Power Law model

The Extended Power Law parameterization of $b_{c2}(\varepsilon)$ was formulated to retain the advantages of the simple Power Law over the moderate strain range, as well as provide for fitting $b_{c2}(\varepsilon)$ at high compressive strains ($\varepsilon_0 \ll -0.5\%$)

Extended Power Law uniaxial model (Ekin 2006)

$$b_{c2}(\varepsilon) = 1 - a |\varepsilon_0|^{1.7} + a_1 |\varepsilon_0 - 0.5\%|^{a_2} H(-0.5\% - \varepsilon_0) \quad (B.2)$$

Again, ε_0 is the intrinsic strain defined as $\varepsilon_0 \equiv \varepsilon - \varepsilon_m$, and ε_m is the applied strain at the $I_c(\varepsilon)$ maximum. As before, the parameter a has two values: a^- for the compressive strain range ($\varepsilon_0 \leq 0$) and a^+ for tensile ($\varepsilon_0 > 0$), with the ratio $a^-/a^+ \approx 0.75$. $H(x)$ is the heavyside function ($=0$ for $x < 0$ and $=1$ for $x > 0$), which restricts the additional term to compressive strains beyond $\varepsilon_0 < -0.5\%$. Two parameters, a_1 and a_2 , are added to characterize the high compressive strain range.

Advantages

- Retains a single strain sensitivity index a^- over the moderate strain range, which serves as a strain sensitivity index.

- Utilizes the curvature scaling constant u at moderate strains. Thus, this model continues to provide accurate extrapolations and noise rejection over the peak strain range, because the two extra parameters, a_1 and a_2 , are restricted by the Heavyside function to fit only high compressive strain data ($\varepsilon_0 < -0.5\%$).

Disadvantage

- Parameter values change with the strain range at high compressive strains: the two additional fitting parameters, a_1 and a_2 , are interdependent. So they *change with the strain range being fit*. Thus, parameter consistency is lost at high compression strains, although the parameter a^- characterizing the moderate strains remains the same as the strain range changes.
- One-dimensional: similar to the Power Law, it treats only longitudinal strains along the conductor axis.

B.3. Polynomial model

A polynomial fitting equation was proposed by the Durham group to represent $b_{c2}(\varepsilon)$:

Polynomial uniaxial model (Taylor and Hampshire 2005)

$$b_{c2}(\varepsilon) = 1 + c_2\varepsilon_0^2 + c_3\varepsilon_0^3 + c_4\varepsilon_0^4 \quad (\text{B.3})$$

where ε_0 is the intrinsic strain, $\varepsilon_0 \equiv \varepsilon - \varepsilon_m$. This model utilizes three parameters: c_2 , c_3 , and c_4 , which are simultaneously fit over the entire available strain range.

Advantages

- Simplicity: the fitting equation utilizes a standard fourth order polynomial. From both the standpoint of programming ease and conceptual simplicity, this model is appealing.
- With 3 fitting parameters (c_2 , c_3 , and c_4), the interpolation accuracy is high.

Disadvantages

- Parameter values change with the fitted strain range, because of their interdependence.
- No single strain sensitivity index.
- Interpolation capability only: the interdependence of the three parameters precludes extrapolation and noise rejection over the dominant peak regime.

B.4. Deviatoric model

Following Testardi (1973), Welch originally proposed the dependence of the critical temperature on three-dimensional *hydrostatic* and *deviatoric* strains

Deviatoric 3D model (Welch 1980)

$$T_c = T_c(0) + \Gamma_1 \varepsilon_{\text{hyd}} + 1/2 \Delta_v (\varepsilon_{\text{hyd}})^2 + 2/5 \Delta_t (\varepsilon_{\text{dev}})^2 \quad (\text{B.4a})$$

where ε_{hyd} is the hydrostatic strain, given in terms of principal strains, $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$, by

$$\varepsilon_{\text{hyd}} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad (\text{B.4b})$$

and ε_{dev} is the deviatoric strain, given by

$$\varepsilon_{\text{dev}} = 2/3 [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2]^{1/2}. \quad (\text{B.4c})$$

From analysis of ultrasonic attenuation, Welch evaluated the fitting constants Γ_1 , Δ_v , and Δ_t in equation (B.4a) and concluded, in a key finding, that the effects of the deviatoric component Δ_t greatly outweigh the effects of the hydrostatic components Γ_1 and Δ_v for strains greater than about $\pm 0.1\%$. The effects of deviatoric strain were also modeled by Scanlan *et al* (1980) for different superconductor geometries.

Note that in Welch's description, T_c (and thus B_{c2}^*) varies as the *square* of the deviatoric strain, $\varepsilon_{\text{dev}}^2$. In contrast to this, ten Haken (1994) later proposed that B_{c2}^* varies as the *square root of the square* of the deviatoric strain $(\varepsilon_{\text{dev}}^2)^{1/2}$. This later form gives a triangular-shaped dependence with a sharp point at $\varepsilon_{\text{dev}} = 0$. In this parameterization, the sharp cusp is removed by adding an *empirical* parameter $\varepsilon_{0,a}$ to provide peak rounding ($\varepsilon_{0,a}$ is not to be confused with the intrinsic strain parameter ε_0 and its embedded parameter ε_m). The rounding parameter $\varepsilon_{0,a}$ was interpreted as arising from extrinsic (non-constant) off-axis residual strains. This later Deviatoric model is given for the uniaxial strain case by

$$\begin{aligned} &\text{Deviatoric uniaxial model (tenHaken 1994)} \\ B_{c2}^*(\varepsilon) &= B_{c2}^*(0) - C_a [\varepsilon_0^2 + \varepsilon_{0,a}^2]^{1/2}, \end{aligned} \quad (\text{B.5})$$

where the intrinsic strain is defined as usual, $\varepsilon_0 \equiv \varepsilon - \varepsilon_m$.

Subsequently, a rotation of this relation was added to account for the asymmetric dependence on tensile and compressive strains (Godeke *et al* 2006). Finally, following the strain invariant proposals of Welch (1980) and Markiewicz (2004, 2006), Arbelaez *et al* (2009) reformulated the model in terms of strain invariants and added a cubic term in strain to better fit $b_{c2}(\varepsilon)$ data at high compressive strains, which gives the present three-dimensional Deviatoric model

$$\begin{aligned} &\text{Deviatoric 3D model (Arbelaez et al 2009)} \\ b_{c2}(\varepsilon) &= (1 - C_h I_1) (1 - C_{d,1} J_2^{1/2} - C_{d,2} J_3). \end{aligned} \quad (\text{B.6a})$$

Similar to equation-set (B.4a)–(B.4c), I_1 is the hydrostatic strain invariant, and J_2 and J_3 are the second and third deviatoric strain invariants:

$$I_1 = \varepsilon_1 + \varepsilon_2 + \varepsilon_3, \quad (\text{B.6b})$$

$$J_2 = 1/6 [(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2], \quad (\text{B.6c})$$

$$J_3 = (\varepsilon_1 - I_1/3)(\varepsilon_2 - I_1/3)(\varepsilon_3 - I_1/3). \quad (\text{B.6d})$$

However, the ten Haken/Godeke Deviatoric model is fundamentally different from the Welch model, in that the dominant term is not J_2 , but $J_2'^2$. The anisotropic dependence of equation-set (B.6a) is contained in the J_3 term. For the uniaxial case, equation (B.6a) becomes

Deviatoric uniaxial model (Arbelaez *et al* 2009)

$$b_{c2}(\varepsilon) = \frac{1 - C_{a,1} \sqrt{\varepsilon_0^2 + \varepsilon_{0,a}^2} - C_{a,2} (\varepsilon_0^3 - 3\varepsilon_{0,a}^2 \varepsilon_0)}{1 - C_{a,1} \varepsilon_{0,a}} \quad (\text{B.7})$$

where, again, ε_0 is the intrinsic strain, $\varepsilon_0 \equiv \varepsilon - \varepsilon_m$.

In practical usage, the hydrostatic parameter C_h in equation (B.6a) is set equal to zero. Thus, not counting the peak strain constant ε_m , this model utilizes three fitting parameters ($\varepsilon_{0,a}$, $C_{d,1}$, and $C_{d,2}$), or ($\varepsilon_{0,a}$, $C_{a,1}$, and $C_{a,2}$) for the uniaxial case. Bottura and Bordini (2009) have proposed simplifying the modified Deviatoric fitting function for uniaxial strain, equation (B.7), with the following relation to calculate $C_{a,2}$

Reduction in parameters (Bottura and Bordini 2009)

$$C_{a,2} = 1.034 \times 10^3 C_{a,1}, \quad (\text{B.8})$$

which reduces the number of fitting parameters to two ($\varepsilon_{0,a}$ and $C_{a,1}$).

Advantages

- 3D fitting equation: the model gives an expression for all three components of strain, enabling three dimensional strain modeling to be calculated from one-dimensional measurements.
- 3D strain testing: in a significant finding, the model has been shown to fit the 3D $B_{c2}^*(\varepsilon)$ measurements of tenHaken *et al* (1995) for a Nb₃Sn tapes mounted on brass and steel sample holders.

Disadvantages

- Constant curvature not incorporated over the peak region. Because the lowest-order term in this model is $J_2^{1/2}$ rather than J_2 , the intrinsic peak region becomes a sharp triangular point, which is then fitted with an *extrinsic* peak-rounding parameter, $\varepsilon_{0,a}$.
- Interdependent fitting parameters ($C_{a,1}$, $\varepsilon_{0,a}$ and $C_{a,2}$) that change with the fitted strain range. No single strain sensitivity index.
- Mathematically inconsistent expansion in strain invariants J_i . The mixture of terms in $J_2^{1/2}$ and J_3 in this model (equation (B.6a)) is mathematically inconsistent with a Taylor expansion in $J_2, J_3, \dots, J_i$, and thus in fundamental disagreement with the treatments of Welch (1980) and Markiewicz (2004).

B.5. Invariant Strain Function

Markiewicz (2004) was able to calculate all the strain features of $T_c^*(\varepsilon)$, and thus $B_{c2}^*(\varepsilon)$, in terms of *strain invariants*, by numerically integrating the change in the phonon spectrum as a function of strain, coupled to a change in T_c^* through the Eliashberg relations. This was a milestone in the recent modeling of $b_{c2}(\varepsilon)$. His numerical integration was the first recent analysis (after Welch 1980) of $T_c^*(\varepsilon)$ and $B_{c2}^*(\varepsilon)$ to utilize the 3D strain invariants: the I_1 (hydrostatic strain), J_2 (square of the deviatoric strain), and J_3 (cubic strain term).

The curvature constant $u = 1.7$ at moderate intrinsic strains is an *inherent feature of the lowest order term J_2 alone*. This is seen in figure 11, which shows that his numerical integration result for $T_c^*(\varepsilon)$ is in agreement with the

curvature constant $u = 1.7$ in the Power Law over the moderate strain range (where the contributions of the higher order invariants are small compared to J_2). Thus, this numerical analysis gives a fundamental basis for the observation of the curvature constant at *moderate* strains in terms of J_2 , without the use of any adjustable parameters for higher order strain invariants.

From a practical standpoint, however, Markiewicz's numerical integration is difficult to implement. So, Markiewicz (2006) also proposed an empirical Invariant Strain Function that is straightforward to use. However, like most of the other $b_{c2}(\varepsilon)$ fitting equations, it is just that—an interpolative fitting equation. It has three adjustable parameters to sum the contributions of the lowest order strain invariants

Invariant strain 3D model (Markiewicz 2006)

$$b_{c2}(\varepsilon) = (1 - a_1 I_1)^{-1} (1 + a_2 J_2 + a_3 J_3 + a_4 J_2^2)^{-1} \quad (\text{B.9})$$

again, with I_1 , J_2 , and J_3 defined by equations (B.6b)–(B.6d). If the hydrostatic parameter a_1 is set equal to zero, this model utilizes three fitting parameters (a_2 , a_3 , and a_4). For uniaxial strain (without the hydrostatic parameter a_1), the fitting equation then becomes the inverse of the Durham polynomial model, that is:

Invariant Strain uniaxial model (Markiewicz 2006)

$$b_{c2}(\varepsilon) = [1 + c_2 \varepsilon_0^2 + c_3 \varepsilon_0^3 + c_4 \varepsilon_0^4]^{-1} \quad (\text{B.10})$$

However, this model is different from the Durham polynomial model in that: (1) It treats three-dimensional strains. (2) The lowest order term, J_2 , does not cause the function to extrapolate to negative values at high strains (as it does in the polynomial), but instead asymptotically approaches zero. (3) The J_2 strain invariant inherently contains the curvature constant $u = 1.7$, which gives this model consistency and noise filtering over the peak I_c region (figure 11).

Advantages

- 3D fitting equation: the Invariant Strain Function was suggested by Markiewicz's more fundamental numerical analysis. A uniaxial determination of the c_i parameters in the uniaxial equation (B.10) gives the parameters a_i in equation (B.9) needed for 3D applications, as detailed in Markiewicz (2006).
- 3D strain testing: like the Deviatoric model, Markiewicz (2008a) has shown that the 3D model equation accurately fits the 3D $B_{c2}^*(\varepsilon)$ measurements of tenHaken *et al* (1995) for Nb₃Sn tapes mounted on brass and steel sample holders. The Invariant Strain Function also gives more realistic fitting parameter values than those of the Deviatoric description (Markiewicz 2008a).
- Fundamental support for the curvature constant $u = 1.7$ over the peak region (figure 11).

Disadvantages

- Parameter values change with the fitted strain range.
- No single strain sensitivity index. Again, the lack of a strain sensitivity index is a result of the interdependence of the three fitting parameters (a_{2-4} or c_{2-4}).

B.6. Exponential model

The Exponential model proposed by Bordini *et al* (2013) recalls the McMillan formula of the critical temperature (McMillan 1968) developed within the framework of the Midgal–Eliashberg theory:

Exponential 3D model (Bordini *et al* 2013)

$$b_{c2}(\varepsilon) = \frac{e^{-C_1 \frac{J_2+3}{J_2+1} J_2} + e^{-C_1 \frac{I_1^2+3}{I_1^2+1} I_1^2}}{2}. \quad (\text{B.11})$$

For the case of strain applied along the axis of a single round wire, the two strain invariants in equation (B.11) (given by equations (B.6b) and (B.6c)) reduce to

Exponential uniaxial model (Bordini *et al* 2013)

$$I_1 = (1 - 2\nu)\varepsilon + \varepsilon_{l0} + 2\varepsilon_{t0} \quad (\text{B.12a})$$

$$J_2 = \frac{1}{3}(\varepsilon_{l0} - \varepsilon_{t0} + (1 + \nu)\varepsilon)^2 \quad (\text{B.12b})$$

with the empirical relationship

$$\varepsilon_{t0} = -\nu \varepsilon_{l0} + 0.1. \quad (\text{B.12c})$$

Here, ε is the axial applied strain, ν is the ‘effective’ Poisson’s ratio, and ε_{l0} and ε_{t0} are the longitudinal and transverse residual strains (expressed as percent) due to differential thermal contraction between the Nb₃Sn and the other materials of the composite wire. From high-energy synchrotron x-ray diffraction measurements on three different types of Nb₃Sn wires (bronze route, RRP[®] internal tin, and powder-in-tube strands) placed under axial tensile stress at 4.2 K, the effective Poisson’s ratio of Nb₃Sn was measured to be about $\nu = 0.36$ (Scheuerlein 2012). These data also showed that the longitudinal residual strain $\varepsilon_{l0} \approx -\varepsilon_m$. Thus, the Exponential model effectively has only *one* fitting parameter C_1 (because ε_{l0} is independently fixed by the peak in the $I_c(\varepsilon)$ curve, similar to ε_m for the other $b_{c2}(\varepsilon)$ models).

Advantages

- Theoretical relationship between the proposed Exponential strain function and the elastic strain energy; analogy drawn with the exponential form of the McMillan equation.
- Inherently contains the strain curvature constant through the lowest order strain invariant J_2 (figure 12); this provides consistency and noise rejection over the dominant peak regime.
- Stable parameter values: one fitting parameter, C_1 (not counting $\varepsilon_{l0} \approx -\varepsilon_m$, as with the other models), which serves as a single strain sensitivity index.
- 3D strain testing: coupled with finite element analysis, the model fits 3D critical current versus transverse stress data in a full multifilamentary strand (Wang *et al* 2014). The study validates the 3D modeling capability and methodology of this model for treating a filamentary composite geometry.
- Extrapolation capability over the extended strain range: the model can extrapolate moderate strain data ($-0.5\% < \varepsilon_0 < \varepsilon_{0irr}$) to high compressive strains ($\varepsilon_0 < -1\%$).

Figure 13 shows this for the OST-RRP dataset; similar extrapolations with the WST dataset give higher extrapolation accuracy than shown in figure 13.

Disadvantages

- Interpolations are not as precise: the extra fitting parameters of other strain models give them increased interpolation precision, but the fitting errors of the Exponential model are reasonably small (blue curve in figure 13) considering the use of only one interpolation parameter C_1 . The single fitting parameter is also the basis for the stability and extrapolation capability of this model.

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