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An Open Cosmological Model in String Theory

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Abstract

We present an open cosmological model as a solution to the $\alpha'()$ string theory equations of motion.

String theory¹⁾ is the first promising candidate for a viable theory of everything, a long-standing goal of theoretical physics. It is a theoretical construct potentially unifying gravity with the other forces of nature. A variety of different types of constructions²⁾ for string solutions have been pursued, many showing encouraging and realistic features.

Most of the solutions so far uncovered take flat four dimensional Minkowski spacetime as the background coordinate, but as superstring theory contains (super) gravity it is clearly of interest to look for solutions on more general coordinate backgrounds. One can search for these solutions by attempting the direct construction of a conformally invariant, and modular invariant world-sheet sigma model with the background spacetime as the target space^{1,2)}. Alternatively one may extract an effective field theory³⁾ describing the interaction of the light modes of the string spectrum and search for nontrivial classical background solutions of the theory.

Solutions⁴⁻¹²⁾ of this theory have been searched for in the hope that they can help us in two ways: first, and most obvious, they will incorporate effects produced by non-minimal couplings, which can be interpreted as manifestations of quantum gravity, and second, we can use these semiclassical solutions to construct the conformal field theories at the string world-sheet level, which may be useful in choosing the string theory vacuum.

The effective field theory has been analysed as an alternative theory of massless excitations in the cosmological⁴⁻¹²⁾ limit and in the physics of black holes¹²⁻¹⁸⁾. A majority of these solutions come about as a direct consequence of the non-minimal couplings, which typically induce contributions to the energy-momentum tensor in the gravitational equations of motion that violate the energy conditions, as well as non-trivial source terms in the matter equations. In this letter we will reinvestigate the

cosmological sector of the theory. We will, however, retain in our action the terms of order of the string tension α' , i.e. the dilaton-graviton coupling via the Gauss-Bonnet term^{3,7,19}. This term modifies significantly the structure of the solutions as will be shown below. When Robertson-Walker cosmology is considered, the Gauss-Bonnet term leads to a variety of admissible solutions as it supports open RW universes, which are absent otherwise. We will argue that these solutions are naturally expected in the light of the recent advances in conformal field theory on non-compact manifolds. Apparently, in this framework our open Robertson-Walker solutions appear to correspond to a conformal field theory of the Wess-Zumino-Witten type constructed on $SO(2,3)/SO(1,3)$ manifold.

We start with the action for the background field theory defined in 4 dimensions by^{3,19},

$$S = \int d^4x \sqrt{g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{2} (\partial_\mu \phi)^2 - \frac{\alpha'}{16\kappa^2} e^{-\sqrt{2}\phi} \tilde{R}^2 - e^{-2\sqrt{2}\phi} H_{\mu\nu\lambda}^2 - W(\phi) \right\} \quad (1)$$

Where $H_{\mu\nu\lambda}$ is the Kalb-Ramond axion field strength, ϕ is the dilaton and $\tilde{R}^2$ is the Gauss-Bonnet term defined in terms of the curvature tensor by

$$\tilde{R}^2 = R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} - 4 R_{\mu\nu} R^{\mu\nu} + R^2 \quad (2)$$

$W(\phi)$ is a dilaton potential, which may arise typically from supersymmetry breaking¹⁾, or as a consequence of the central charge deficit at the string world-sheet level^{4,5}. We follow the conventions of the Ref. [19]. Here we ignore the Yang-Mills gauge fields as well as the Chern-Simons terms typically

included in the definition of the Kalb-Ramond field strength, as we work in the RW background geometry,

$$ds^2 = -dt^2 + R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right) \quad (3)$$

and the Chern-Simons terms vanish trivially in this case¹⁹.

In 4D, the antisymmetric rank 3 tensor $H_{\mu\nu\lambda}$ is dual to a vector, and the Bianchi identity which holds for it further asserts that the vector is derivative of a scalar. Hence, introducing the scalar axion field a by

$$H_{\mu\nu\lambda} = \sqrt{g} e^{-2\sqrt{2}\phi} \epsilon_{\mu\nu\lambda\rho} \partial^\rho a \quad (4)$$

and noting that the Gauss-Bonnet form for the Robertson-Walker geometry is

$$\tilde{R}^2 = 24 (\dot{H} + H^2) \left(H^2 + \frac{k}{R^2} \right) \quad (5)$$

where H is the Hubble parameter, we can write the equations of motion as follows (in the units of the Planck mass):

$$\dot{a} = e^{-2\sqrt{2}\phi} \frac{c}{R^3} \quad (6)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{3\alpha'}{\sqrt{2}} e^{-\sqrt{2}\phi} \left(H^2 + \frac{k}{R^2} \right) (\dot{H} + H^2) + \frac{\partial W}{\partial \phi} - 12\sqrt{2} e^{-2\sqrt{2}\phi} \frac{c^2}{R^6} = 0 \quad (7)$$

$$3H^2 + 3\frac{k}{R^2} = \frac{1}{2} \dot{\phi}^2 + W(\phi) + 6e^{-2\sqrt{2}\phi} \frac{c^2}{R^6} + \frac{3\alpha'}{\sqrt{2}} e^{-\sqrt{2}\phi} \left(H^2 + \frac{k}{R^2} \right) H\dot{\phi} \quad (8)$$

$$(2 - \sqrt{2}\alpha'e^{-\sqrt{2}\phi}H\dot{\phi})\dot{H} + 3H^2 + \frac{k}{R^2} - \frac{1}{\sqrt{2}}\alpha'e^{-\sqrt{2}\phi}(H^2 + \frac{k}{R^2})(\dot{\phi} - \sqrt{2}\dot{\phi}^2) - \sqrt{2}\alpha'e^{-\sqrt{2}\phi}\dot{\phi}H^3 + \frac{1}{2}\dot{\phi}^2 + 6e^{-2\sqrt{2}\phi}\frac{c^2}{R^6} - W(\phi) = 0 \quad (9)$$

The axion equation of motion, $\ddot{a} + 3H\dot{a} + 2\sqrt{2}\dot{a}\dot{\phi} = 0$ was solved to yield Eq. [6] where c is a constant of integration. The solution [6] has already been substituted into the remaining equations of motion [7-9] corresponding to the dilaton, R_{00} and R_{ij} equations respectively. Of course only two of the three equations [7-9] are independent, e.g. Eq. [9] can be derived from [7] and [8].

To proceed with the analysis we need to fix the functional form of the potential $W(\phi)$. We choose to follow the Ansatz of Ref. [5], where the potential arises from the central charge deficit in the string action, and after the conformal transformation which brings the action in the form of Eq. [1], becomes

$$W(\phi) = V_0 e^{\sqrt{2}\phi} \quad (10)$$

Where V_0 is the constant measuring the central charge deficit. Also, we note that the form of the equations of motion [7-9] suggests they admit solutions representing power law evolution of the scale factor, with logarithmically varying dilaton. Hence we look for the solutions of the form

$$R = R_0 p^p \quad (11a)$$

$$\phi = \phi_0 + q \ln t \quad (11b)$$

where the parameters p and q are to be determined from the equations of motion. A quick look at Eqs. [7-9] then fixes p and q . Namely, since $H \sim t^{-1}$ the ansatz [11a-11b] solves the equations of motion only if all the terms then have the same time dependence. This is true only if $q = -\sqrt{2}$.

The discussion of the solutions to [7-9] will be simplified by defining the parameter constants

$$\xi = \alpha'e^{-\sqrt{2}\phi_0} \quad (12a)$$

$$\eta = V_0 e^{-\sqrt{2}\phi_0} \quad (12b)$$

$$\delta = ce^{-\sqrt{2}\phi_0} \quad (12c)$$

To start with let us first consider cases in which the 3-space curvature $k=0$ and the axion field is constant, $\dot{a}=c=\delta=0$. Combining Eqs. [7] and [8] yields

$$\frac{3}{2}\xi p^4 + \frac{3}{2}\xi p^3 + 3p^2 - 3p = 0 \quad (13a)$$

$$\eta = 3p^2 - 1 + 3\xi p^3 \quad (13b)$$

A trivial solution to [13] has $p=0$ in which case, the Hubble parameter, $H=0$ and $\eta=-1$. More general solutions will be given by the cubic equation for p in [13a].

It is interesting to note that when $\xi=0$, we recover the "linearly"

expanding solution⁽⁴⁻⁶⁾ with $p=1$ and $\eta=2$. Of course in this special case many other solutions also exist, e.g. $\xi=9$ gives $p=\frac{1}{3}$ and $\eta=\frac{1}{3}$.

More generally when $k \neq 0$ and $\delta \neq 0$, exact solutions are only possible if $p=1$ (a solution with $p=0$ and $q=0$ is also possible). After some simple algebra, the equations of motion reduce to the following two independent equations:

$$\eta = 2 + 12 \frac{\delta^2}{R_0^6} \quad (14a)$$

$$(1 + \xi) \left(1 + \frac{k}{R_0^2}\right) = 1 + 6 \frac{\delta^2}{R_0^6} \quad (14b)$$

Note that in the limit that $\xi \rightarrow 0$ ($\alpha' \rightarrow 0$) Eq. [14b] becomes $k = \frac{6\delta^2}{R_0^4}$ which would indicate the existence of only closed or spatially flat cosmologies. However, one can immediately observe the novelty manifested by this solution, compared to the previous work. If we rewrite the Eq. [14b] as

$$\frac{k}{R_0^2} = (1 + \xi)^{-1} \left(6 \frac{\delta^2}{R_0^6} - \xi\right) \quad (15)$$

it is clear that negative values of k are allowed, for sufficiently large ratio $\frac{\alpha' R_0^6}{6c^2} e^{\sqrt{2} \phi_0}$. Hence we see that the α' order terms enrich the structure of solutions of the theory by allowing extra topological structure to appear as an

admissible solution. Furthermore, it is clear that these solutions are perturbative in the limit $\alpha' \rightarrow 0$. The fact that the open solutions exist for any non-zero value of α' but are absent when $\alpha'=0$, is similar to the Schwarzschild geometry in the limit that $m \rightarrow 0$.

In terms of the associated conformal field theory, we would identify these solutions as representing Wess-Zumino-Witten (WZW) models, with target space the non-compact coset $SO(2,3)/SO(1,3)^{(10,11)}$, which has the same topological structure as spatial slices of the $k=-1$ Robertson-Walker universe. The central charge associated with a level x WZW theory on this coset is

$$c = \frac{10x}{(x+3)} - \frac{6x}{(x+2)} = 4 - 6 \frac{(3x+4)}{(x+3)(x+2)} \quad (16)$$

exactly as for the Wick-rotated theory on the $SO(4,1)/SO(3,1)$ coset discussed in Ref. [5]. (An alternative construction exists, in which the world-sheet theory is a Wess-Zumino-Witten theory on an $SO(1,3)/SO(3)$ coset tensored with a free time coordinate with background charge⁽²⁰⁾.) While the construction of this solution as a WZW model on the $SO(2,3)/SO(1,3)$ coset guarantees world-sheet conformal invariance (i.e. that the solution satisfies the classical string equations of motion to all orders in the α' expansion), the issues of unitarity and modular invariance of the quantum theory are at present under active study (see Ref. [10,11]) and we will consider them elsewhere.

Finally, we note that because the open solutions found here are structurally similar to the closed and flat solutions found previously, ($p=1$ and $q=\sqrt{2}$) the linear expansion of these solutions may not immediately be

interpreted as physical expansion^{21,22,23}). For rulers determined by the Compton wavelength of a field with a dilaton dependent mass $m \sim e^{\phi/\sqrt{2}}$, there is no real expansion (this can also be seen by making a conformal transformation on the action [1]), and for rulers whose scale is determined by transdimensional mutation, the Universe may appear to contract exponentially²²). These are however only vacuum solutions. Solutions with matter and/or radiation, will have a more concrete interpretation.

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