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QUARK GLUON SCATTERING AT HIGHER ORDERS IN QCD

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Abstract : The infrared evolution equation which sums up leading logarithmic terms is solved for the partonic reaction $qg \rightarrow qX$ in the elastic case i.e. $M_X \rightarrow 0$. The results of the exact $\mathcal{O}(\alpha_s^3)$ calculation are reproduced.

September 1991

CPT-91/P.2594

Unité Propre de Recherche 7061

1. Introduction

Main features of hard and semihard processes have given clear evidence for the parton substructure of the proton and can be understood in terms of perturbative QCD [1]. If the calculations of two body partonic cross sections at Born level are rather a simple procedure, evaluation of higher orders is tedious and involves a lot of Feynman graphs. Sometimes higher order terms are crucial to bring QCD in agreement with experiment. This is for example what happens for inclusive jet production from hadronic collisions since the Born cross section, which does not depend on the jet algorithm and on the jet cone size, cannot be viewed as a jet cross section [2].

Basically there are two ways to attack the problem. The first one consists into an exact calculation of Feynman graphs to a given order in perturbative QCD. This has been done by Ellis and Sexton [3] for α_s^3 matrix elements for all $(2 \rightarrow 2)$ and $(2 \rightarrow 3)$ parton subprocesses in n dimensions. Next step consists in performing the phase space integration of real and virtual processes as done for inclusive hadron production in ref. [4]. One is then left with mass singularities which have to be absorbed into structure and fragmentation functions evolved beyond leading approximation in order to be ready to perform a phenomenological comparison with data. This method usually involves considerable technical difficulties and is therefore only tractable up to one or two loops. This evaluation may sometimes not lead to good results especially for processes dominated by regions of phase space where perturbative theory breaks down. This happens for evaluation of the transverse momentum P_T dependence of lepton pairs in Drell-Yan processes [5] where resummation to all orders in the low P_T region is crucial.

The second method is based on an evaluation of Feynman diagrams to all orders in the leading logarithmic approximation (hereafter denoted as LLA) for Regge kinematical regions. This approach is based on the calculation of all discontinuities of the scattering amplitude and on the obtention of an explicit expression using dispersion relations and unitarity conditions [6]. It has been applied to the calculation of multi Regge amplitudes for forward $2 \rightarrow 2 + n$ reactions [7]. After performing the phase space integration on momenta of $n + 1$ omitted particles, one gets inclusive cross sections in LLA [8]. Another approach was proposed [9] for the calculation of scattering amplitudes. It is based on the introduction of an ultraviolet cutoff in impact parameter space, which corresponds to an infrared cutoff μ for virtual particles in transverse momentum space. Therefore the scattering amplitudes become dependent on an infrared regulator μ . Infrared evolution equations for scattering amplitudes involving partons have been built and first solved for elastic qq and $q\bar{q}$ amplitudes in DLA (double logarithmic approximation) [10]. Then taking into account coherence effects for soft gluon emission it was possible to obtain infrared evolution equations (hereafter denoted as IRE) also for the inelastic case [11]. An extension of the techniques previously developed to single gluon production from $q\bar{q}$ collisions in order to obtain an IRE for inclusive cross sections has recently been performed [12].

The aim of this paper is to compare the results of exact α_s^3 calculation to analytical ones. For this purpose we will study the pedagogical process $q\bar{q} \rightarrow q + X$ in the backward and forward directions for which elastic scattering amplitude is large. At present stage the results of ref. [12] deal with mass singularities which have been removed in the first approach of ref. [3]. Therefore in order to allow comparison with DLA results one cannot

use directly the expressions for the inclusive cross section given in [3] but start from expressions in which mass singularities have not been removed. This work has to be viewed as a preliminary step towards an identification of large logarithms which can be resummed. It has to be noticed that a confrontation to experimental data will need an evaluation of structure functions and fragmentation functions beyond leading order in the infrared method formalism, which has not yet been performed.

Thanks to the results of ref. [12] a comparison for the crossed reaction :

$$q\bar{q} \rightarrow g + X \quad (1)$$

can be performed for the elastic contribution, i.e. for the term proportional to $\delta(s+t+u)$. In the kinematical region $|t| < s \sim |u|$ one gets from ref. [3], after replacing $-\frac{1}{s}$ by $\log\left(\frac{|u|}{\mu^2}\right)$,

$$\sigma_{el}^{1-loop} = \frac{\alpha_s}{2\pi} C_F \left[-2 \ln\left(\frac{s}{-t}\right) \ln\left(\frac{-t}{\mu^2}\right) + \text{NLO terms} \right] \cdot \sigma^{Born} \quad (2)$$

to be compared with all loops

$$\sigma_{el} = \sigma^{Born} \exp \left\{ -\frac{\alpha_s}{\pi} \left[C_F \ln\left(\frac{s}{-t}\right) \ln\left(\frac{-t}{\mu^2}\right) \right] \right\} \quad (3)$$

exhibiting a perfect agreement for double logarithms.

We will now provide in section 2 the basic techniques of IRE method used to calculate in DLA the partonic cross section $q\bar{q} \rightarrow q\bar{q}$ in the backward direction and then compare it to the α_s^3 evaluation for the elastic case. Section 3 is devoted to the forward direction. Conclusions will be given in section 4.

2. Elastic $q\bar{q} \rightarrow q\bar{q}$ scattering in the backward direction

Let us consider the reaction :

$$q_i(P_1) + g_a(K_1) \rightarrow q_j(P_2) + g_b(K_2) \quad (4)$$

where the indices i, j, a, b label colour states.

The scattering amplitude M_{ijab} can be expressed in terms of Mandelstam variables :

$$s = (P_1 + K_1)^2, \quad t = (P_1 - P_2)^2 \quad \text{and} \quad u = (P_1 + K_2)^2. \quad (5)$$

We are interested into the kinematical region :

$$|u| < s, |t| \quad (6)$$

which is of Regge type. It corresponds to backward scattering i.e. to a scattering angle close to π in the center of mass frame. The diagram we have to compute is shown in fig. 1.

The blob in fig. 1 indicates that radiative corrections are taken into account. The momenta of particles leading to small invariant u lie on the left and the right from the blob whereas

momenta corresponding to large invariant s lie on the bottom and the top from the blob. In principle calculation of the blob involves Feynman graphs exhibiting ultraviolet and infrared singularities. The ultraviolet ones disappear after renormalisation. Concerning infrared singularities (present since gluons are massless) we should introduce the infrared cutoff μ according to the prescription of ref. [11] :

$$\mu < K'_{vt} \quad (7)$$

where K'_v is the momentum of a virtual particle v (which is either a quark or a gluon) and the index t means a minimal value for transverse K'_v components with respect to the planes built from any possible pair of external momenta. Since such a regularisation is invariant under Lorentz boosts in longitudinal space it corresponds to a compactification of impact parameter space and preserves gauge invariance. Moreover eq. (7) allows to treat μ as the smallest available transverse momentum i.e. as a new mass scale.

The amplitude $M_{ij,ab}$ depends now not only on Mandelstam variables but also on μ . It gives opportunity to develop an evolution equation for $M_{ij,ab}$ with respect to μ , i.e. to get the IRE. To build it we will use the method of factorisation of gluons with minimal K'_{vt} . In DLA all K'_{vt} are strongly ordered. Therefore it is possible to find a virtual particle such that :

$$\min \{K'_{vt}\} = K'_t \quad (8)$$

It has been proved, both for elastic and for inelastic cases, that for scattering amplitudes involving less than four external quarks the softest particle should be a gluon. Such a gluon can be factorized in the kinematical region :

$$\mu < K'_t < \sqrt{-u} \quad (9)$$

K'_t acts as an infrared regulator for the other virtual particles. According to ref. [11] we obtain for the softest gluon is the Feynman gauge the IRE drawn in fig. 2.

Letters inside each blob correspond to the infrared regulator. Crosses on external legs indicate that the particles are on shell. Therefore K'_t acts like a mass and there is no dependence on longitudinal components of K' .

Notice that IRE is gauge invariant. It is easy to see that a solution of this equation should be of exponential form. Indeed, the left hand side is $\frac{\partial}{\partial \mu} M_{ij,ab}$ whereas on the right hand side the only quantity which is μ dependent is the lower bound of integration over K'_t (see eq. (9)), the blob being μ independent.

Instead of using ordinary Feynman rules to rewrite IRE in an analytical form, it is more convenient to introduce invariant amplitudes $M_T(s, u, \mu^2)$ corresponding to irreducible representations with dimension T of $SU(3)$ color group in u channel :

$$M_{ij,ab} = \sum_T (P_T)_{j\bar{b}}^{i\bar{a}} M_T \quad (10)$$

In this channel, since both initial and final states contain a quark and a gluon, they belong to the 3×8 representation of $SU(3)$. According to

$$3 \times 8 = 3 + 15 + \bar{6} \quad (11)$$

the only viable representation is $T = 3$ from quark and gluon colored states. For $T = 3$ the projection operator reads :

$$(P)_{j\bar{b}}^{i\bar{a}} = K(t_a t_b)_{j\bar{b}}^{i\bar{a}} \quad (12)$$

where K is fixed by condition $P^2 = P$. One gets :

$$K = 1/C_F \text{ with } C_F = \frac{N^2 - 1}{2N} \quad (N = 3) \quad (13)$$

The invariant amplitude M_T is the product of the Born scattering amplitude by some function which depends only logarithmically on s and u once radiative corrections in DLA have been taken into account. We get :

$$M_T \equiv M_T^{Born} B \left(Y \equiv \ln \frac{s}{\mu^2}, Y \equiv \ln \frac{-u}{\mu^2} \right) \quad (14)$$

Therefore, dropping M_T^{Born} , we get for the left hand side of IRE :

$$\mu^2 \frac{\partial}{\partial \mu^2} B = - \left(\frac{\partial B}{\partial Y} + \frac{\partial B}{\partial \bar{Y}} \right) \quad (15)$$

The first graph of right hand side of IRE reads :

$$\frac{\alpha_s}{4\pi} K(t_a t_b)_{j\bar{b}}^{i\bar{a}} f_{aac}(t_c)_{i\bar{i}}' \int_{\mu^2}^{-u} \frac{dK'_t}{K'_t} \ln \frac{s}{K'_t} B \left(\frac{s}{K'_t}, \frac{u}{K'_t} \right) \quad (16)$$

One can easily write down the expressions for the other terms of IRE. One finally gets :

$$\frac{\partial B}{\partial X} + \frac{\partial B}{\partial \bar{Y}} = -2\lambda_1 X B - 2\lambda_2 Y B \quad (17)$$

where $2\lambda_1 = \frac{d^2}{dX^2} C_F$ and $2\lambda_2 = \frac{d^2}{dY^2} N$

Since DLA contributions are absent in the region (9) when $-u \sim \mu^2$ we will solve eq. (17) thanks to the initial condition :

$$B|_{Y=0} = 1 \quad (18)$$

One gets :

$$B = \exp(-2\lambda_1 X Y + (\lambda_1 - \lambda_2) Y^2) \quad (19)$$

which can be rewritten as :

$$B = \left(\frac{s}{-u} \right)^{-2\lambda_1 Y} \exp(-(\lambda_1 + \lambda_2) Y^2) \quad (20)$$

which exhibits the expected Regge behaviour.
One finally obtains

$$\sigma_{el}^{all\ loops} = \sigma_B \exp \left(-\frac{g^2}{4\pi^2} C_F \ln \left(\frac{s}{-u} \right) \ln \left(\frac{-u}{\mu^2} \right) \right) \quad (21)$$

where the Born cross section reads :

$$\sigma_B = \pi \left(\frac{g^2}{4\pi} \right)^2 \frac{1}{s^2} \left(\frac{C_F}{N} \right) \frac{(s^2 + u^2)}{|u|s} \quad (22)$$

This result has been compared with $\alpha(\alpha_s^3)$ evaluation of the elastic term (i.e. proportional to $\delta(s+t+u)$) for $q\bar{q} \rightarrow q+Y$ after having performed $t \leftrightarrow u$ crossing :

$$\sigma_{el}^{1\ loop} = \sigma_B \left[1 - \frac{\alpha_s}{\pi} C_F \ln \left(\frac{s}{-u} \right) \ln \left(\frac{-u}{\mu^2} \right) \right] \quad (23)$$

3. Forward $q\bar{q} \rightarrow q\bar{q}$ scattering

In terms of Mandelstam variables introduced in the previous section forward scattering corresponds to the kinematical region :

$$|t| < s \sim |u| \quad (24)$$

The cross section in the Born approximation reads (see fig. 3) :

$$\bar{\sigma}_B = \pi \left(\frac{g^2}{4\pi} \right)^2 \frac{1}{s^2} \left(\frac{s^2 + u^2}{t^2} \right) \quad (25)$$

The corresponding amplitude satisfies the Regge behaviour :

$$\tilde{M}_T^{Born} \sim g^2 \left(\frac{s}{t} \right) \quad (26)$$

When we take into account radiative corrections to the Born amplitude we have to compute the trajectory of a Regge pole with gluon quantum numbers. The IRE can be obtained from fig. 4 using the techniques developed in the previous section.
One gets :

$$\tilde{M}_T = \tilde{M}_T^{Born} \left(\frac{s}{-t} \right)^{-\frac{3}{2}\alpha_s} N \ln \left(-\frac{s}{\mu^2} \right) \quad (27)$$

leading to the cross section :

$$\bar{\sigma}_{el}^{all\ loops} = \bar{\sigma}_B \exp \left(-\frac{\alpha_s}{\pi} N \ln \left(-\frac{t}{\mu^2} \right) \ln \left(\frac{s}{-t} \right) \right) \quad (28)$$

The formal solution of IRE for forward scattering contains additional exponential factors compared to backward case but since they depend on infrared regularisation they vanish when emission of soft gluons is taken into account.

Equation (28) is in agreement with $\alpha(\alpha_s^3)$ calculation of the process $q\bar{q} \rightarrow q+Y$ in the kinematical limit $|t| < s \sim |u|$ since the elastic term reads :

$$\bar{\sigma}_{el}^{1\ loop} = \bar{\sigma}_B \left(1 - \frac{\alpha_s}{\pi} N \ln \left(-\frac{t}{\mu^2} \right) \ln \left(\frac{s}{-t} \right) + \dots \right) \quad (29)$$

4. Conclusion

We have checked that the results for $q\bar{q} \rightarrow g\bar{g}$ and $q\bar{q} \rightarrow q\bar{q}$ reactions obtained in DLA using IRE method, which can be easily solved, are consistent with one loop calculations for elastic terms. This comparison is a first promising step towards a resummation of large logarithmic terms originating from Regge kinematics. The inelastic terms corresponding to large M_X , which can be obtained using the optical theorem, are more complicated and will be studied later.

Acknowledgments

We are greatly indebted to L.N. Lipatov for enlightening discussions. P.C. would like to thank Academy of Sciences of USSR for kind hospitality in Leningrad and Protvino during completion of this work. B.E. would like to thank the Centre de Physique Théorique in Marseille for kind hospitality.

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Figure captions

- Fig. 1 Kinematics for backward $qg \rightarrow qg$ scattering.
- Fig. 2 IRE for the elastic backward amplitude. Crosses on the inner lines mean that the corresponding particles are on shell.
- Fig. 3 Kinematics for forward $qg \rightarrow qg$ scattering at Born level.
- Fig. 4 IRE for the elastic forward amplitude.

Fig. 1

Fig. 2

Fig. 3

Fig. 4