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PERSISTENT CURRENTS IN LHC MAGNETS

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Persistent Currents in LHC Magnets

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Abstract - The field errors due to persistent currents in LHC dipoles, quadrupoles, and correction windings are discussed. The errors due to variations in the magnetization are shown to be significant. The paper discusses the influence of magnetic cycling, the contribution of the different parts of the dipole winding, and simple scaling with coil radius and field.

I. INTRODUCTION

The Large Hadron Collider (LHC) is a project under consideration by CERN for a proton-proton collider to be built on top of the existing 27 km-long LEP electron-positron collider. The LHC magnet system will consist of a large number of superconducting dipoles, quadrupoles, and correction windings operating in superfluid helium at a temperature of 1.8 K, a detailed description of which can be found in [1]. At injection the magnetic field is only about 5% of maximum field. This means that considerable field errors are generated by the persistent currents in the superconductors even when using the proposed filament diameter of $d = 5 \mu\text{m}$. The LHC dipoles and quadrupoles are really two magnets contained within one iron shield; in most cases however they can be treated as independent magnets with a circular iron shield. A simplified cross section of part of the coils of the main dipole and quadrupole, together with field lines is shown in Fig. 1.

Fig. 1 LHC Dipole and Quadrupole with field lines. The coil consists of two layers of a superconducting cable.

Fig. 2 Scaled magnetization (M) as function of the field (B) for a filament diameter of $5 \mu\text{m}$ at 1.8 K

II. MAGNETIZATION MODEL

The first task for calculating the effect of persistent currents in the magnet aperture is to establish a reasonable model of the magnetization due to the persistent currents. The magnetization model used is an analytical approximation to a measured magnetization-field relationship for a wire with a filament diameter of $4.7 \mu\text{m}$ at $T = 4.2 \text{ K}$. Since it is necessary to scale for filament diameter, critical current, filling factor of the cables and operating temperature, it is assumed that the magnetization can be scaled like the classical Bean model for the flat plate [2], assuming a critical current for fields around B_{p1} and B_{p2} (see Fig. 2) which depends on the field like $J_c = B^{-m}$. Therefore the measured values of B_{p1} and B_{p2} are scaled by $(J_c(B_p, T) * d)^{1/(1+m)}$. The value of m was taken to be 0.52 from measurements by [3]. Scaling of J_c with T is done with the formula given in [4]: $T_c(B) = 9.2 (1 - B/14.5)^{0.59}$ and $J_c(B, T) = J_c(B, 4.2) (T_c(B) - T) / (T_c(B) - 4.2)$. An example for a wire with $d = 5 \mu\text{m}$ at $T = 1.8 \text{ K}$ is given in Fig. 2.

III. FIELD ERROR CALCULATION

A. Method

To calculate the field errors, the field due to the magnetization currents is summed over the windings of the magnet. Since the magnetization depends on the local field, a field map of the windings, usually produced by a field computation program like POISSON [5], is necessary. Only in special cases (i.e.

concentric correction windings) where the field inside the windings is especially simple, can an analytical approximation be made. For the LHC magnets the influence of the iron shield can usually be neglected.

A program, REM, was written to perform the above calculations. It can take into account different magnetic cycles and scale the magnetization to different filament diameters and temperature. From the minimum sector of the magnet cross section needed for the calculation (i.e. a 45° sector or half a 'pole', for a quadrupole) the program can generate the contribution to the field error of each pole of the complete magnet, thus making it possible to calculate asymmetry errors, for example those due to different magnetization in the poles.

B. Definition of field error

The field generated by the magnetization currents is written as $B_y + iB_x = B_0 \sum_n (b_n + ia_n) (z/x_0)^{n-1}$.

- $B_y + iB_x$ is the field vector in complex number representation.
- $z = x + iy$ is the position vector in the complex plane.
- n is the multipole number, $n = 1$ is a dipole, $n = 2$ is a quadrupole etc. and $n = N$ is the multipole number of the main component B_0 as generated at $z = x_0$ by the transport currents. In all tables below $x_0 = 0.01$ m.
- b_n and a_n represent the normal and skew field errors relative to the main field B_0 at $z = x_0$.

IV. RESULTS OF CALCULATION

A. Parameters for scaling the magnetization

When scaling the measured magnetization to 1.8 K we suppose that the LHC cables have a critical current density of 3000 A/mm² in the filaments at $B = 5$ T and $T = 4.2$ K. The filament diameter is taken as 5 μ m.

B. Dipole

Two geometries have been calculated. They differ slightly in the geometry of the coils and the iron. This gives an idea of the possibility of change of the errors due to optimization of the final design. Table 1 gives the errors at a central field of 0.5 T, which is supposed to be near the LHC injection field.

The small dipole error in Geometry 1 is due to large errors cancelling out in the windings (see V.A). It is therefore not surprising that the change in b_n is large in Geometry 2 for $n = 1$.

The small $n = 2$ term is due to the two-in-one design of the magnet.

C. Quadrupole

The field errors given in Table 2 were calculated for two different geometries with coil inner diameter of $D = 0.050$ m and 0.056 m, which gives an idea of how the higher multipoles can

TABLE 1
SYMMETRIC ERRORS.

RELATIVE FIELD ERROR $b_n (a_n = 0)$ IN LHC DIPOLE AT $B_0 = 0.5$ T
AT $X = 0.01$ m (INNER COIL DIAMETER $D = 0.050$ m).
FIELD CYCLE: 0 - 10 - 0 - 0.5 T ERRORS IN UNITS OF $1.E - 4$.

	Geometry 1	Geometry 2
n	b_n	b_n
1	-0.51	-4.10
2	---	0.12
3	-3.01	-3.95
5	0.59	0.45
7	0.02	0.00

TABLE 2
SYMMETRIC ERRORS.

RELATIVE FIELD ERROR $b_n (a_n = 0)$ IN LHC QUADRUPOLE AT A
CENTRAL FIELD OF 0.15 T AT $X = 0.01$ m (GRADIENT = 15 T/m).
FIELD CYCLE (GRADIENT): 0 - 150 - 0 - 15 T/m
ERRORS IN UNITS OF $1.E - 4$.

	Coil inner Diameter = 0.056 m	Coil inner Diameter = 0.050 m
n	b_n	b_n
2	-9.70	-0.97
6	-0.52	-1.03
10	0.00	0.04
14	0.00	0.00

diminish when the coil radius increases, while maintaining an optimum design.

The $n = 6$ error is reduced by a factor of 2 compared to the LHC quadrupole with inner diameter of 50 mm.

D. Correction elements

A number of correction magnets in LHC have been designed with concentric coils. It can be shown from symmetry consideration that a 2P-pole inside a 2N-pole field generates multipoles with $n = 2kP - N$ while a 2P-pole outside a 2N-pole field generates multipoles with $n = 2kP + N$ (k integer), provided full penetration of the filaments is assumed and the 2P-pole carries no transport current. Detailed calculations for some of the LHC correctors are given in [6] and [7].

V. ANALYSIS OF RESULTS

A. Contribution of the the individual winding regions of the dipole

The contribution to the total field error of the individual winding regions of Geometry 1 is shown in Fig. 3. Note for instance how the multipole contributions change sign in the first layer, the almost complete compensation of the $n = 1$ term, and a reasonable compensation of the $n = 3$ term. The $n = 5$ and $n = 7$ contribution come mostly from the first layer. For an

Fig. 3: Contribution of the dipole coil regions to the relative field error (in units of 10^{-4} at $B_0 = 0.5$ T and $x = 0.01$ m). The values in the tables are for $n = 1, 3, 5$ and 7

From Fig. 3 it is easy to estimate how the field error will vary if the magnetization varies in a region. For example if the magnetization is higher by 10% in the outer layer this increases its contribution to the field error by approximately 10%.

B. Errors when the poles have different magnetization

The strands for the fabrication of the superconducting cables will come from different manufacturers and different billets. It would therefore be possible that the poles in a magnet have different magnetization, leading to asymmetric field errors in the aperture. How large these magnetization differences could be is not well known. An estimation has been made by assuming differences of $\pm 5\%$ in the poles. For differences of other magnitude linear scaling is quite reasonable.

Dipole - Supposing that the upper and lower pole have different magnetization we get the result of table 3.

The skew quadrupole, which could be of random nature, is not negligible.

TABLE 3
ASYMMETRIC ERRORS. RELATIVE FIELD ERROR b_n AND a_n IN
LHC DIPOLE AT $B = 0.5$ T AT $X = 0.01$ m.
ERRORS IN UNITS OF $1.E - 4$.
UPPER POLE MAGNETIZATION UP BY 5%,
LOWER POLE DOWN BY 5%.

n	b_n	a_n
2	0.	-1.24
4	0.	-0.02

TABLE 4
ASYMMETRIC ERRORS. RELATIVE FIELD (b_n AND a_n) ERRORS AT
A CENTRAL FIELD OF 0.15 T AT $X = 0.01$ m (GRADIENT = 15 T/m).
ERRORS IN UNITS OF $1.E - 4$.

Relative magnetization in poles								
Case 1		Case 2		Case 3		Case 4		
1.05	1.05	0.95	1.05	0.95	1.05	1.00	1.10	
0.95	0.95	1.05	0.95	0.95	1.05	1.00	1.00	
n	b_n	a_n	b_n	a_n	b_n	a_n	b_n	a_n
1	0.	-7.54	0.	0.	7.54	0.	-3.77	-3.77
3	0.	-8.83	0.	0.	0.83	0.	0.42	-0.42
4	0.	0.	0.	-0.63	0.	0.	0.	-0.32
5	0.	-0.13	0.	0.	0.13	0.	-0.06	-0.06

Quadrupole - In table 4 a study is shown for two poles with stronger and two poles with weaker magnetization (Case 1, 2 and 3). The symmetric errors ($n = 2, 6, 10, \dots$) remain the same. Then Case 4 is studied where one pole has stronger magnetization than the others. No symmetric skews are generated.

C. Scaling with field

For injection fields not exactly equal to 0.5 T, field errors can be estimated by assuming that they vary with field like the magnetization. The relative field error will then vary as $B_0^{-(1+m)} = B_0^{-1.52}$, assuming a rising field.

D. Scaling with inner radius of the coils

Sometimes one needs a simple estimation of how the field errors could diminish when the inner radius (r_1) of the coil increases, while keeping B_0 constant. Even an approximate estimation is not always possible since the field error depends also on the coil geometry. It is clear from Fig. 3 that in this particular case the $n=1$ term is almost certain to rise since there is almost perfect cancellation in the windings. One is reduced to using scaling rules from some limiting cases.

- Infinitely thick one-layer dipole with sector coils, sector angles constant, b_n scales as: $r_1^{-(n-1)}$ for $n > 1$.
- Infinitely thick one-layer quadrupole with sector coils, sector angles constant, b_n scales with $r_1^{-(n-1+m)}$, i.e. $r_1^{-1.52}$ for $n=6$.

Although strictly valid only for very thick coils (see Appendix), this is probably a good approximation for thin coils also.

E. Influence of the minimum field during cycling

The minimum field used for cycling has an influence on the field errors at injection. From the calculations made for the dipole (Table 5), it appears that for a filament diameter of $5 \mu\text{m}$ a minimum cycle field below 0.2 T is recommendable to minimize the change of errors when starting acceleration. The maximum cycle field is assumed to be so high that a negligible part of the magnet windings has a field below B_{p2} when the central field for the dipole is at maximum. (At least 3 T.)

TABLE 5
RELATIVE FIELD ERRORS IN LHC DIPOLE AT X=0.01M AT B=0.5
TESLA WITH MINIMUM CYCLING FIELD VARYING BETWEEN 0
AND 0.4 TESLA.

n	-Minimum cycle field (T)-				
	0.0	0.1	0.2	0.3	0.4
1	-0.59	0.14	0.99	2.29	2.78
3	-3.01	-2.89	-2.71	-2.44	-2.12
5	0.59	0.60	0.61	0.62	0.62
7	0.02	0.02	0.02	0.02	.015

F. Time dependence of the persistent currents

Time dependence has been observed in the first 1m-long models of the LHC dipoles [8] which like in the case of HERA [9] seems to depend on the magnetic cycling. It was too early, however, to define a magnetic cycle to minimize this effect.

VI. CONCLUSION

At injection in LHC the $n = 3$ and $n = 5$ due to persistent currents in the dipole are the most important contributions to the total error. The partial cancellation of the sextupole and decapole in the different winding regions indicates that an effort should be made to study other coil geometries. The calculations made show the importance of using magnet poles with the same magnetization.

APPENDIX

SCALING OF FIELD ERRORS FOR SPECIAL COIL CONFIGURATIONS

A. General expression for the multipole coefficient

Putting $F = B_y + iB_x = \sum_n c_n Z^{n-1}$ (like in III.B), we get by a method similar to [10] for a 2N-pole magnet, neglecting the iron shield:

$$c_n = 2 i \frac{\mu_0}{\pi} \frac{nN}{r_1^{n-1}} \int_{\phi_1}^{\phi_2} \int_{\rho} M e^{i(n-1)\phi} \rho^n d\phi d\rho \dots \quad (1)$$

with $\rho = r/r_1$ the reduced radius

$n = (2k + 1)N$, k integer

$M e^{i\alpha}$ is the magnetization vector, either parallel or antiparallel to the field in the windings. M depends on the external field B in the windings.

B. Maximum value of c_n

Note that for $n > 1$, c_n is generated in a region of the coil near the aperture even when the coil becomes very large. This can be seen by considering a coil sector with angle $< \frac{\pi}{2N}$ and radius between r_1 and r_2 :

$$|c_n| < \mu_0 M_{\max} \frac{n}{n-1} \left\{ \frac{1}{r_1^{n-1}} - \frac{1}{r_2^{n-1}} \right\} \leq \mu_0 M_{\max} \frac{n}{n-1} \frac{1}{r_1^{n-1}} \dots \quad (2)$$

Here $M_{\max}$ is the maximum magnetization anywhere in the coil.

C. Approximation for very thick coils

The field inside the coils very near to the aperture is approximately equal to the field at the limit of the aperture. For $N \leq 2$, (dipole and quadrupole) this region can be made larger by increasing the outer radius of the coil. Let us now assume that the outer radius is so large that most of the contribution to c_n comes from this region. We then find approximately for a sector coil with angles ϕ_1 and ϕ_2 :

$$c_n \approx -2 \frac{\mu_0}{\pi} \frac{NM(r_1)}{r_1^{n-1}} \frac{n}{n-1} \left\{ \frac{\sin(N+n)\phi_2 - \sin(N+n)\phi_1}{N+n} \right\}$$

where $M(r_1)$ is the magnetization for the field at $x = r_1$.

This approximate solution also explains the changing of sign (see Fig. 3) of the multipoles in the first layer of the dipole.

For a quadrupole with constant gradient G and $M \sim \frac{1}{B^m}$:

$$M(r_1) \sim \frac{1}{B(r_1)^m} = \frac{1}{(Gr_1)^m} \sim \frac{1}{r_1^m}. \text{ So } c_n \sim \frac{1}{r_1^{n-1+m}}.$$

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