Experimental Limits on M_t and M_H – Present and Future.¹

D. Schaile

Universität Freiburg

Abstract

Existing data from precision tests of the electroweak Standard Model at LEP, the $p\bar{p}$ -colliders and νN -scattering experiments constrain the mass of the top quark to $M_t = 144^{+23}_{-26} {}^{+19}_{-21}$ GeV, where the central value and first error refer to a Higgs mass value of 300 GeV. The second error shows the variation of M_t for Higgs mass values spanning the range $50 < M_H(\text{GeV}) < 1000$. The present apparent sensitivity of the data to the value of the Higgs mass is investigated. Future data from LEP will significantly tighten the bounds on M_t . Limits on M_H however will remain of order $\Delta M_H/M_H \approx 2$ without an independent measurement of M_t . The Higgs mass limits would profit substantially from a high precision measurement of M_t with $\Delta M_t = 0.5$ GeV, which could be performed at a next generation e^+e^- linear collider. The study of future bounds on M_t and M_H is complemented by a pocket calculator approximation which allows a scaling of results with assumed experimental errors.

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1 Introduction

Up to now all experimental data successfully confirm the Standard Model of electroweak interactions [1]. The observed particle spectrum however still reveals two important gaps. No direct experimental evidence has been found for the top quark, the isospin partner of the b quark, nor for the Higgs boson, originating from the mechanism of spontaneous symmetry breaking, which is needed to explain the masses of the fundamental constituents in a theory with local gauge invariance.

The experimental precision today allows the testing of quantum effects. In the electroweak theory these are also sensitive to high mass particles which cannot be produced directly at present energies. In particular stringent constraints can be imposed already for the mass of the top quark M_t within the framework of the Minimal Standard Model. As the bounds on M_t depend on the mass of the Higgs boson M_H , the question arises, whether present or future measurements would allow for a separate determination of M_t and M_H . A more modest attitude is to rely on the top quark to be found in the near future. The full accuracy of the precision tests of the electroweak Standard Model could then be exploited to set limits on M_H .

In section 2 existing bounds on M_t and M_H from LEP data will be discussed. These limits will then be compared and combined with other sources of precision tests of the electroweak theory and it will be shown that present Higgs mass limits are still rather questionable. However LEP has just started and in a few years from now the situation will likely have changed. The approved physics goals of the LEP program are the scan of the Z^0 line-shape until the systematic limits are reached, the running at the peak to give statistics of $O(10^7)$ Z^0 s per experiment, and the increase of the center of mass energy above the W-pair production threshold, which will provide a determination of the W-mass to about 100 MeV by each LEP experiment. Possible additions to the LEP program are the running with longitudinal polarisation and a high luminosity phase, based on a multibunch "Pretzel" scheme.

The measurements at LEP which constrain or will constrain the Standard Model are

- The Z^0 line-shape ($M_Z, \Gamma_Z, \Gamma_{\text{had}}, \Gamma_{\text{lept}}$).
- The lepton forward-backward asymmetries $A_{\text{FB}}^{\text{lept}}$.
- The tau-polarisation $A^{\text{pol}}(\tau)$.
- The $b\bar{b}$ sector providing $\Gamma_{b\bar{b}}$ and $A_{\text{FB}}^{b\bar{b}}$.
- The direct measurement of M_W at LEP 200.
- The left-right polarisation asymmetry A_{LR} , if longitudinal polarisation can be made available.

Section 3 is devoted to the discussion of the future accuracies which might be reached for the above quantities. Based on these accuracies we will then derive limits on M_t and M_H in section 4. Special consideration will be given to the case that M_t is known from the Tevatron or a next generation e^+e^- Linear Collider. In section 5 we will discuss how the limits given depend on the accuracies assumed and provide an approximate formula, which allows some scaling of results.

2 Present constraints on M_t and M_H

Recent LEP results of precision tests of the Standard Model have been presented at the at the Joint International Lepton-Photon Symposium and Europhysics Conference on High Energy Physics, Geneva, July - August 1991 [2]. The input data used to discuss present constraints from LEP are summarised in Table 1. As theoretical parametrisation for these input data we use the full one-loop Standard Model calculation, with leading $O(\alpha^2 M_t^4)$ and $O(\alpha\alpha_s)$ terms, provided by the program ZFITTER [3]. The parameters M_Z , M_t , M_H and $\Lambda_{\overline{MS}}$ are determined by a χ^2 minimisation taking into account the correlation matrix for the input data from the line-shape and lepton forward-backward asymmetries (M_Z , Γ_{tot} , Γ_{had} , $(g_1^a)^2$ and $(g_1^v)^2$). Studies have shown that the resulting parameters are fairly insensitive to the detailed choice of the correlation matrix, which could most accurately be obtained from a combined fit to all LEP data or a refined averaging procedure based on the parameters and correlation matrices for the parameters of each individual LEP experiment. These being not available for all the data presented we simply use the correlation matrix obtained from a fit to the OPAL data. The result of the fit to the LEP data is given in Table 2. For the presentation of fit results for the parameter M_t in Table 2 we first fix the value of the Higgs mass to $M_H = 300 \text{ GeV}$ to obtain the central value and the 1st error quoted. The 2nd error then shows the variation of the central value of M_t for Higgs mass values spanning the interval $50 < M_H (\text{GeV}) < 1000$. The most conservative approach is to add both errors linearly. Figure 1 a) shows the χ^2 curve of the above fit as function of M_t for 3 different Higgs masses.

Complementary information on M_t can be obtained from the measurement of M_W at the $p\bar{p}$ colliders and the measurement of $\sin^2\theta_W$ in νN -scattering in conjunction with the precise measurement of M_Z at LEP. The input data from these experiments which enter the analysis are given in Table 1 b). The χ^2 curves as function of M_t , which result from a combination of the LEP value of M_Z with the collider measurement of M_W or $\sin^2\theta_W$ from νN -scattering, are shown in Figure 1 b) and c), respectively. The comparison of the steepness of the χ^2 curves in Figure 1 a), b) and c) shows, that the dominant contribution to top mass limits today results from the LEP measurements alone. From Figure 1 and the numerical results in Table 2 it can be seen that all results are in good agreement and can therefore be combined. The combined result is

$$M_t = 144_{-26}^{+23+19}_{-21} \text{ GeV}$$

where the central value and first error refer to a Higgs mass value of 300 GeV and the second error indicates the variation of M_t for Higgs mass values spanning the range $50 < M_H (\text{GeV}) < 1000$. The resulting χ^2 curves as function of M_t are shown in Figure 2.

In view of the χ^2 differences between the minima for various Higgs masses in Figure 2, it is very tempting to relax the most conservative approach pursued above or in Table 2, and quote as error for M_t the value obtained from the envelope for all values of M_H in the range $50 < M_H (\text{GeV}) < 1000$ or equivalently from a fit with M_H varied freely within the above bounds. This procedure results in $M_t = 124_{-28}^{+25} \text{ GeV}$ and $M_H = 50_{-...}^{+192} \text{ GeV}$. In this fit the central value of M_H is at the lower boundary imposed. Figure 3 shows the χ^2 curve for the combined data set as function of M_H . Radiative corrections being sensitive to terms proportional to $\log(M_H/M_Z)$, we expect a quadratic behavior of this χ^2 curve for a logarithmic Higgs mass scale. The observed dependence is however linear in $\log M_H$. To investigate the origin of this behavior, we have studied the partial contribution χ_i^2 of each observable to χ^2 which are displayed in Figure 4 for two fits with $M_H=50 \text{ GeV}$ and $M_H=1000 \text{ GeV}$,

²Note that we evaluated χ^2 using a covariance matrix with off-diagonal elements for the data to account for correlations of observables from the Z^0 line-shape measurements. The analysis of contributions of individual observables to χ^2 therefore is an approximation, which is however adequate for the investigation performed.

respectively. The sensitivity of each observable to M_H is given by the increase in χ^2 when varying M_H from 50 to 1000 GeV. This difference in partial contributions $\Delta\chi_i^2$ can be written as

$$\Delta\chi_i^2 = \frac{[x_i - x_i^{SM}]^2 - [x_i - (x_i^{SM} + \epsilon_i)]^2}{\sigma_i^2(x_i)} = \frac{2\epsilon_i(x_i - x_i^{SM}) + \epsilon_i^2}{\sigma_i^2(x_i)}$$

In this expression x_i refers to the measured value of observable i , x_i^{SM} denotes the Standard Model prediction for $M_H=50$ GeV with all other parameters adjusted to minimise χ^2 and ϵ_i indicates the variation of this prediction when setting M_H to 1000 GeV and again minimising χ^2 with respect to all other parameters. Figure 4 shows that the dominant contribution to the increase in χ^2 from $M_H=50$ GeV to $M_H=1000$ GeV arises from $(g_1^a)^2$ and $A_{FB}^{b\bar{b}}$, two quantities which have a non-negligible χ_i^2 anyhow. Higgs mass effects being small and washed out by correlations with the unknown top mass, we are still in a situation, where the true Higgs mass indicator $\epsilon_i/\sigma_i(x_i)$ of any single observable is smaller than one. The linear dependence of the χ^2 curve on $\log M_H$ suggests that the increase in χ^2 is dominated by the terms proportional to $\epsilon_i(x_i - x_i^{SM})$ rather than by the true effect proportional to ϵ_i^2 . Therefore we consider it premature to derive statements on M_H from current LEP data. We also hesitate to claim that the result for M_t from the envelope of all χ^2 curves for M_H is more meaningful than the conservative treatment of errors which is implied by the presentation of results in Table 2.

3 Estimates of future accuracies of LEP precision measurements

Given the anticipated LEP performance of 10^7 Z^0 s per experiment, the accuracy for all observables described above will be dominated by systematics. So nobody really can make a prediction, unless we have the data. Furthermore we are here concerned with averages of four LEP experiments. In many cases therefore, what really counts is the fraction of the systematic error, which is common to all four experiments. An estimate of this common systematic error however also requires an evaluation of future progress in the evolution of detailed theoretical calculations. We will first give a discussion of the accuracies used in the fit. They should however be taken with some reservation and some imagination. To give a more complete picture, the interplay between the assumed measurement errors and the accuracy of the Standard Model parameters determined will be discussed in a subsequent section.

In the study of future bounds on M_t and M_H we consider the following input data and accuracies:

- M_Z with $\Delta M_Z=5$ MeV.

The accuracy for M_Z is already now dominated by the accuracy of the LEP center of mass energy determination. At present the center of mass energy is calibrated using flux-loop measurements, a flip coil in a reference magnet in series with all magnets in the ring, and protons circulating on the same orbit as the positrons. The accuracy of the current determination of the LEP center of mass energy is 20 MeV. Further progress, which may result in an accuracy of 5 MeV or better, is expected from the method of depolarising resonances. The feasibility of this method has been demonstrated successfully at the time of writing.

- Γ_{tot} with $\Delta\Gamma_{tot}=5$ MeV.

At present the error on Γ_{tot} averaged over the four LEP experiments is $\Delta\Gamma_{tot}=10$ MeV including a common systematic error of 5 MeV resulting from a point to point uncertainty of 10 MeV on the center of mass energy. The final accuracy chosen assumes an improvement of the present experimental error on Γ_{tot} by a factor of two and a reduction of the point to point uncertainty on the center of mass energy from 10 MeV at present to 5 MeV.

- Γ_{had} with $\Delta\Gamma_{\text{had}}=2$ MeV.

At present the dominant systematic error of each individual experiment to Γ_{had} arises from the experimental systematics of the luminosity measurement ($\approx 0.7\%$). In an average of the four LEP experiments, the experimental systematics can be treated as largely uncorrelated and therefore the theoretical uncertainty of the luminosity ($\approx 0.3\%$) already today contributes significantly. For the 0.1% error assumed we probably need a complete second order calculation for small angle Bhabha scattering and an upgrade of luminosity detectors. Other sources of LEP common systematic errors are fragmentation uncertainties (at present estimated to $<0.1\%$ and therefore neglected) and the point to point LEP energy uncertainty which enters Γ_{had} via its correlation with Γ_{tot} .

- Γ_{lept} with $\Delta\Gamma_{\text{lept}}=0.1$ MeV.

The discussion of systematic errors for Γ_{lept} is similar to that of Γ_{had} .

- $A_{\text{FB}}^{\text{lept}}$ with $\Delta A_{\text{FB}}^{\text{lept}}=0.002$.

Given the present accuracy of $\Delta A_{\text{FB}}^{\text{lept}}=0.004$ (without common systematic error) our estimate is conservative. At present the experimental systematic error of an individual LEP experiment is typically $\Delta A_{\text{FB}}^{\text{lept}}=0.003$, the theoretical uncertainty of order $\Delta A_{\text{FB}}^{\text{lept}}=0.002$ and the effect of the 20 MeV LEP center of mass uncertainty is $\Delta A_{\text{FB}}^{\text{lept}}=0.0016$. Given improvements we might well approach $\Delta A_{\text{FB}}^{\text{lept}}=0.001$ with 10^7 Z^0 s per experiment.

- $A^{\text{pol}}(\tau)$ with $\Delta A^{\text{pol}}(\tau)=0.004$.

The present LEP error is $\Delta A^{\text{pol}}(\tau)=0.035$. Improvements are expected both in statistics and systematics. A study performed in [9] quotes for the $\tau \rightarrow \pi\nu$ channel a statistical uncertainty of 0.01 for $3 \cdot 10^6$ Z^0 s and a systematic uncertainty of 0.006. Given $O(10^7)$ Z^0 s per experiment and that already now other τ decay channels have been exploited successfully the statistical error might be reduced by a factor of 5. The systematic error being dominated by the particle identification capability of each detector might also go down by a factor of 2 in a combined average of all channels and experiments.

- $\Gamma_{b\bar{b}}$ with $\Delta\Gamma_{b\bar{b}}=3$ MeV and $A_{\text{FB}}^{b\bar{b}}$ with $\Delta A_{\text{FB}}^{b\bar{b}}=0.003$.

For the $b\bar{b}$ sector new detection techniques using Microvertex detectors have just started. We tend to give some credit to ongoing developments by attributing an accuracy to $\Gamma_{b\bar{b}}$ and $A_{\text{FB}}^{b\bar{b}}$ which at present might be judged as very optimistic.

- M_W with $\Delta M_W=60$ MeV.

Two methods could be used to derive M_W at LEP200: The threshold scan and the reconstruction of the mass from the W decay products. Both methods can result in an accuracy of about 100 MeV per experiment [8]. The threshold scan which is less model dependent than the decay product analysis, requires however running below and at the W threshold, whereas other physics topics favor the highest energy possible. The combined error for 4 experiments of 60 MeV may turn out to be conservative.

- A_{LR} with $\Delta A_{\text{LR}}=0.003$.

The error on A_{LR} error is based on a study extensively described in [10].

To reach the accuracies above, it is absolutely mandatory to have parametrisations of the Z^0 lineshape, lepton, b-quark and polarisation asymmetries which have an accuracy of better than 0.1%. This as mentioned in the context of partial widths also holds for small angle Bhabha scattering.

We have not included in our considerations the hadronic uncertainty in the calculation of the QED vacuum polarisation. The present uncertainty of $\Delta\alpha(M_Z^2)=0.0009$ [11] does not affect the partial

widths and contributes to the theoretical prediction for M_W with $\Delta M_W=17$ MeV and to the left right polarisation asymmetry A_{LR} with $\Delta A_{LR}=0.002$. A substantial fraction of the present error on $\Delta\alpha(M_Z^2)$ is due to experimental errors in the measurement of the cross-section $e^+e^- \rightarrow \text{hadrons}$ below $\sqrt{s}=10$ GeV, and might be reduced in the future.

4 Future constraints on M_t and M_H

In estimating future bounds on M_t and M_H we will use as simulated input data the SM prediction for all observables given in the previous section with $M_Z=91.175$ GeV, $\alpha_s(s=M_Z^2)=0.118$ and M_t and M_H as given in the context of the specific study performed. As in the study of present bounds we will use a χ^2 minimisation to determine the parameters M_Z , M_t , M_H and $\Lambda_{\overline{MS}}$. Note that in this study we will no longer constrain $\Lambda_{\overline{MS}}$ from independent measurements. The accuracy on $\Lambda_{\overline{MS}}$ or α_s is given by the measurement error on Γ_{had} and Γ_{tot} and amounts to $\Delta\Lambda_{\overline{MS}}=40-50$ MeV or $\Delta\alpha_s(M_Z^2)=0.003-0.004$. This accuracy is superior to the present theoretical uncertainties involved in deriving $\Lambda_{\overline{MS}}$ from the topology of hadronic events. We also perform a slight change of input parameters using Γ_{lept} and $A_{\text{FB}}^{\text{lept}}(s=M_Z^2)$ instead of $(g_1^a)^2$ and $(g_1^v)^2$, to circumvent any definition ambiguities.³ The correlations between the parameters M_Z , Γ_{tot} , Γ_{had} and Γ_{lept} derived from the Z^0 line-shape have been taken into account.

In a first step we investigate the precision on M_t and M_H during various stages of LEP running. The input data correspond to the Standard Model predictions for $M_t=150$ GeV and $M_H=100$ GeV. We perform 3 fits with the following input data

- i) M_Z , Γ_{tot} , Γ_{had} , Γ_{lept} , $A_{\text{FB}}^{\text{lept}}$, $A^{\text{pol}}(\tau)$, $\Gamma_{b\bar{b}}$, $A_{\text{FB}}^{b\bar{b}}$ (referred to as 'LEP I' in the following)
- ii) LEP I + M_W
- iii) LEP I + M_W + A_{LR}

Figure 5 a)-c) shows χ^2 curves as function of M_t for these 3 stages of LEP operation. The comparison with Figure 1 shows that LEP is capable of tightening significantly its constraints on the Minimal Standard Model. Table 3 displays the results of these fits. From Figure 5 we observe, that the minima of the χ^2 depend on the Higgs mass assumed. The minima of the χ^2 curves for $M_H=100$ GeV and $M_H=1000$ GeV in Figure 5 differ by $\Delta\chi^2=2.3$. This difference is only slightly increased when we add a measurement of M_W to $\Delta\chi^2=2.4$. If however longitudinal polarisation could be made available one could obtain $\Delta\chi^2=3.7$. The χ^2 curves as function of M_H for these fits are displayed in Figure 6. Here we do observe the expected quadratic behavior as function of $\log M_H$. Note that the χ^2 difference over the full range of M_H for the fit to the 'LEP I' observables with future accuracies is comparable to that observed with present data (c.f. Figure 3). This is also an argument to treat present Higgs mass bounds with suspicion.

In a second step we investigate the benefit of an independent top mass measurement. Figure 7 shows χ^2 curves as function of M_H for three values of the top mass resolution $\Delta M_t=0.5$, 5.0 and 10.0 GeV, respectively. The input data have been simulated to correspond to $M_t=150$ GeV and M_H as indicated on the figures. Numerical results for ΔM_t and ΔM_H from these fits are given in Table 4 column 2 and 3. Figure 8 shows the 1- σ contours in the M_t - M_H plane for $M_t = M_H=150$ GeV for the three different top mass resolutions above and without an independent constraint on M_t . The

³At present the definition of g_1^a and g_1^v used by the LEP experiments happens to be comparable.

gain in accuracy on M_H by the decorrelating effect of an independent top mass measurement is clearly visible. Figure 8 illustrates that matching discovery limits for M_H from direct searches with precision measurements at LEP in combination with an independent top mass determination may reveal interesting aspects.

We have also investigated the top quark mass dependence of these results varying the central value of M_t in the range of 120-180 GeV. There is no difference in the χ^2 curves as function of M_H for $\Delta M_t = 0.5$ GeV. We observe a negligible increase of the widths of the χ^2 curves with increasing M_t for $\Delta M_t = 5.0$ GeV and 10.0 GeV respectively.

The fits have been repeated omitting A_{LR} from the input data, results are given in Table 4 column 4. By comparison with the previous results we see that reducing the errors of a direct measurement of M_t from 5 GeV to 0.5 GeV has a comparable impact on the accuracy of M_H as the measurement of the polarisation asymmetry with $\Delta A_{LR} = 0.003$.

5 How do the results depend on the measurement accuracy ?

To investigate how the results obtained in the previous section depend on the accuracies for the observables included in the fit, we perform an analysis of the partial contributions of each observable to the increase in χ^2 when varying the Higgs mass away from its value m_0 where χ^2 is minimised. These partial contributions are shown in the diagrams in Figure 9 for different accuracies of an independent measurement of M_t . For these diagrams the Higgs mass has been varied from $m_0 = 300$ GeV by $\Delta m = 150$ GeV to 450 GeV minimising the remaining parameters M_Z , M_t and $\Lambda_{\overline{MS}}$. The first diagram in Figure 9 shows the partial contributions to $\Delta\chi^2$ for the case of no independent measurement of M_t . As has been shown in the previous section, we cannot expect an upper $1-\sigma$ limit on M_H below 1 TeV for a Higgs mass of 300 GeV without independent measurement of M_t . Therefore the partial contributions to $\Delta\chi^2$ for a variation of M_H by +150 GeV are tiny. The following diagrams in Figure 9 then demonstrate that the significance of a bound on M_H increases steadily with the accuracy for an independent top mass measurement.

For all diagrams in Figure 9 the contribution to $\Delta\chi^2$ of Γ_{tot} and Γ_{had} are negligible, because they determine α_s . Also the lepton asymmetry does not contribute very much. The measurement of $\Gamma_{b\bar{b}}$, which in principle can be used to separate Higgs and top dependent virtual corrections, needs more than a factor of two higher accuracy than the 3 MeV used in the fits, to become important. The partial contribution to $\Delta\chi^2$ from M_t shows a peculiar behavior. Varying M_H off its value which minimises χ^2 , correlations between M_H and M_t introduce a significant shift in M_t . The correlation coefficient for the parameters M_t and M_H varies between 0.95 for no independent top mass measurement to 0.63 for $\Delta M_t = 5$ GeV and 0.10 for $\Delta M_t = 0.5$ GeV, respectively. The shift in units of ΔM_t is maximal for $\Delta M_t = 5$ GeV. For $\Delta M_t < 5$ GeV the decorrelating strength of the independent top mass measurement dominates, resulting in an increased sensitivity of the LEP measurements to M_H .

To allow some scaling of errors we propose the following procedure: For a given set of observables with errors σ_i the increase in χ^2 when varying M_H by Δm from its value m_0 which minimises χ^2 can be written as

$$\Delta\chi^2(\Delta m, \sigma_i) = \sum_i \left(\frac{x_i - \bar{x}_i(m_0, \Delta m)}{\sigma_i} \right)^2 = \sum_i a_i \left(\log \frac{m_0 + \Delta m}{m_0} \right)^2$$

resulting in

$$a_i = \left(\frac{x_i - \bar{x}_i(m_0, \Delta m)}{\sigma_i} \right)^2 \frac{1}{\left(\log \frac{m_0 + \Delta m}{m_0} \right)^2}$$

The coefficients a_i , derived for $M_H = m_0 = 300$ GeV and $\Delta m = 150$ GeV, are given in Table 5 for $\Delta M_t = \infty, 10, 5$ and 0.5 GeV respectively. In the approximation that the parameter set that minimises χ^2 does not depend on the accuracies for the observables, we obtain for a different set of errors δ_i

$$\Delta\chi^2(\Delta m, \delta_i) = \sum_i \left(\frac{x_i - \bar{x}_i(m_0, \Delta m)}{\delta_i} \right)^2 = \sum_i b_i \left(\log \frac{m_0 + \Delta m}{m_0} \right)^2$$

with

$$b_i = a_i \frac{\sigma_i^2}{\delta_i^2}.$$

The error on M_H is defined as the variation Δm_1 in M_H needed to cause an increase in χ^2 by one. We therefore have

$$\Delta\chi^2(\Delta m_1, \delta_i) = 1 = \sum_i b_i \left(\log \frac{m_0 + \Delta m_1}{m_0} \right)^2$$

or

$$\pm \frac{\Delta M_H}{M_H} = \pm \frac{\Delta m_1}{m_0} = \exp \left(\pm \sqrt{\frac{1}{\sum_i b_i}} \right) - 1 \quad (1)$$

where the + and - signs refer to the positive and negative error of M_H respectively.

In view of the strong dependence of correlations between M_t and M_H on the error for an independent measurement of M_t , it is evident, that the expression derived from the coefficients for one error of M_t cannot be used to extrapolate with ΔM_t too far from the value for which they were derived.

Figure 10 illustrates scope and limitations of the approximation. In Figure 10 a) we use the coefficients derived for $\Delta M_t = 0.5, 5$ and 10 GeV respectively to compute the positive and negative errors on M_H as function of M_H using formula (1) with $b_i = a_i$ as given in Table 5. The lines in Figure 10 refer to the approximation, the symbols give the results of the full minimisation procedure (c.f. Table 4 column 3). Good agreement is observed, the approximation however has a slightly steeper dependence on M_H as the full minimisation procedure, which has to be attributed to parameter correlations. A similar comparison is shown in Figure 10 b), but here the coefficient b_i related to the observable A_{LR} is set to zero. The symbols show the results of the full minimisation procedure applied to all observables excluding the measurement of A_{LR} (c.f. Table 4 column 4). It is seen that the approximation is a reasonable tool to predict the change in ΔM_H when varying the accuracy of the input data. Note that in reducing the accuracy of the input data the difference in the energy dependence of the full minimisation procedure and the approximation increases. Therefore results of the approximation should be treated with some reservation in the low Higgs mass range, where the difference of $O(20 \text{ GeV})$ in the absolute Higgs mass error amounts to a significant fraction of the overall error.

Summarising this section we can state, that the crucial measurements at LEP, which will allow us to set limits on M_H are Γ_{l+l-} , $A_{FB}^{b\bar{b}}$, $A^{pol}(\tau)$, M_W and A_{LR} . A reduction in the accuracy of only a single observable is not expected to change significantly the bounds obtained. To obtain interesting bounds on M_H an independent measurement of M_t with good accuracy is absolutely vital. An analysis of future bounds on M_H has also been presented recently in [12]. Our work differs from this study, by taking into account not only Γ_{l+l-} but the full harvest of results expected from LEP I, and by treating with special emphasis the case, that an independent measurement of M_t with a precision of $\Delta M_t = 0.5$ GeV can be made available by a next generation e^+e^- linear collider. In contrast to [12] we

find that LEP data together with an independent measurement of M_t can provide meaningful bounds on M_H , even if the ultimate option of a polarised Pretzel LEP would not be feasible. From Figure 9 it is however evident, that if the advocated measurement of A_{LR} with a precision of $\Delta A_{LR}=0.001$ could be made available, it would dominate the bounds.

6 Conclusions

Within the framework of the Minimal Standard Model precision measurements today from LEP, the $p\bar{p}$ -colliders and νN -scattering result in a bound on M_t of

$$M_t = 144^{+23}_{-26} {}^{+19}_{-21} \text{ GeV}$$

where the central value and the first error refer to a Higgs mass value of 300 GeV, the second error shows the variation of M_t for Higgs mass values spanning the range $50 < M_H(\text{GeV}) < 1000$. We have shown that the dominant contribution to this bound arises from LEP data alone. Present data numerically favor a low Higgs mass. We have given arguments that at the present level of precision this effect is likely to be a pure coincidence.

Future measurements at LEP may constrain M_t with an error $\Delta M_t=6-8$ GeV for fixed Higgs mass. Higgs mass bounds from future measurements at LEP alone will still be rather modest, reaching an accuracy of order $\Delta M_H/M_H \approx 2$. Direct measurements of M_t from $p\bar{p}$ - or pp -colliders are expected to determine M_t with a precision of $\Delta M_t=5-10$ GeV, independent of M_H . The Higgs mass limits would profit substantially from a measurement of M_t with $\Delta M_t=0.5$ GeV, which can be performed at a next generation e^+e^- linear collider. Reducing the errors of a direct measurement of M_t from 5 GeV to 0.5 GeV has a comparable impact on the accuracy of M_H as the measurement of the polarisation asymmetry with $\Delta A_{LR}=0.003$. With an independent measurement of M_t reasonable bounds of order $\Delta M_H/M_H = 0.3 - 0.5$ can be reached. Several LEP observables Γ_{l+l-} , $A_{FB}^{b\bar{b}}$, $A^{pol}(\tau)$, M_W and A_{LR} give comparable contributions to these limits.

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measurement	result
a) <u>LEP</u>	
line-shape and lepton asymmetries:	
M_Z	$91.175 \pm 0.021 \text{ GeV}$
Γ_{tot}	$2.487 \pm 0.010 \text{ GeV}$
Γ_{had}	$1740 \pm 9 \text{ MeV}$
$(g_1^a)^2$	0.2942 ± 0.0012
$(g_1^v)^2$	0.0012 ± 0.0003
+ OPAL correlation matrix	
τ -polarisation:	
$A^{\text{pol}}(\tau)$	-0.134 ± 0.035
$A_{\text{FB}}^{\text{pol}}(\tau)$	-0.20 ± 0.08
$b\bar{b}$ -asymmetry:	
$A_{\text{FB}}^{b\bar{b}}$	0.126 ± 0.022
$q\bar{q}$ -asymmetry:	
$\sin^2\bar{\theta}_W$ from $A_{\text{FB}}^{q\bar{q}}$	0.2310 ± 0.0035
topology of hadronic events:	
α_s [4]	0.118 ± 0.008
b) <u>pp and νN</u>	
M_W/M_Z [5]	$0.8813 \pm 0.0036 \pm 0.0019$
M_W [6]	$79.91 \pm 0.39 \text{ GeV}$
$\sin^2\theta_W(\nu N)$ [7]	0.2300 ± 0.0064

Table 1: Precision measurements of Standard Model parameters included in this analysis. Section a) summarises LEP averages [2], section b) results from hadron colliders and νN -scattering.

Input data	M_t (GeV)	χ^2/NDOF
a) <u>LEP alone</u>		
line-shape + $A_{\text{FB}}^{\text{lept}}$	$123^{+37}_{-47} {}^{+22}_{-23}$	1.8/(6 - 3)
+ $A^{\text{pol}}(\tau)$, $A_{\text{FB}}^{\text{pol}}(\tau)$	$129^{+35}_{-44} {}^{+22}_{-22}$	3.2/(8 - 3)
+ $A_{\text{FB}}^{\text{b}\bar{\text{b}}}$	$146^{+31}_{-38} {}^{+21}_{-22}$	6.7/(9 - 3)
+ $A_{\text{FB}}^{\text{q}\bar{\text{q}}}$	$150^{+29}_{-34} {}^{+20}_{-22}$	6.9/(10 - 3)
b) <u>LEP M_Z only + UA2, CDF, CHARM, CDHS</u>		
$M_Z + M_W$	$146^{+43}_{-50} {}^{+16}_{-19}$	0.7/(3 - 2)
$M_Z + \sin^2\theta_W(\nu\text{N})$	$123^{+55}_{-56} {}^{+17}_{-20}$	0.0/(2 - 2)
c) <u>LEP all measurements + UA2, CDF, CHARM, CDHS</u>		
$LEP + M_W$	$149^{+24}_{-28} {}^{+19}_{-21}$	7.5/(12 - 3)
$LEP + M_W$ + $\sin^2\theta_W(\nu\text{N})$	<div style="border: 1px solid black; padding: 2px;">$144^{+23}_{-26} {}^{+19}_{-21}$</div>	7.7/(13 - 3)

Table 2: Results of top mass fits to data from LEP, UA2, CDF, CHARM and CDHS. The central value and the 1st error refer to $M_H = 300 \text{ GeV}$, fixed. The 2nd error gives the variation of the central value of M_t for Higgs mass values spanning the interval $50 < m_H(\text{GeV}) < 1000$. The value of α_s has been constrained in these fits to $\alpha_s = 0.118 \pm 0.008$. Section a) summarises top mass limits derived from LEP data alone. Section b) refers to limits which only use the M_Z measurement from LEP and data from the hadron colliders or νN -scattering. Contrary in section c) **all** LEP observables which contribute to a bound on M_t are included and then combined with M_W from UA2 and CDF, and finally (enclosed by a box) also with $\sin^2\theta_W$ from the νN -scattering experiments CHARM and CDHS.

Input data	M_t $M_H=100$ GeV fixed (GeV)	M_t $50 < M_H(\text{GeV}) < 1000$ (GeV)	M_H (GeV)
LEP I	$150^{+8.2}_{-7.8}$	150^{+21}_{-13}	$100^{+342}_{-...}$
+ M_W	$150^{+6.4}_{-6.2}$	150^{+19}_{-11}	$100^{+324}_{-...}$
+ A_{LR}	$150^{+5.8}_{-5.7}$	150^{+16}_{-11}	$100^{+224}_{-...}$

Table 3: Accuracies on M_t and M_H for various stages of LEP running. The input data are the SM predictions for $M_t=150$ GeV and $M_H=100$ GeV. Free parameters of the fits are M_Z , M_t , M_H and $\Lambda_{\overline{MS}}$. The second column shows the result for M_t with M_H fixed to 100 GeV. The 3rd and 4th columns give results from a fit with M_H treated as free parameter, allowed to vary in the range $50 < M_H(\text{GeV}) < 1000$. The dots for the negative error for M_H indicate that the error exceeds the bounds from direct searches.

Input data	M_t (GeV)	M_H (GeV)	M_H no A_{LR} (GeV)
LEP I + M_W + A_{LR}	150^{+16}_{-11}	$100^{+224}_{-...}$	
+ $\Delta M_t = 10$ GeV	$150^{+8.4}_{-7.7}$	$100^{102}_{-...}$	$100^{+125}_{-...}$
+ $\Delta M_t = 5$ GeV	150 ± 4.7	100^{+69}_{-43}	$100^{+86}_{-...}$
+ $\Delta M_t = 0.5$ GeV	150 ± 0.5	100^{+52}_{-37}	100^{+67}_{-45}
LEP I + M_W + A_{LR}	150^{+21}_{-18}	$300^{+...}_{-210}$	
+ $\Delta M_t = 10$ GeV	$150^{+9.1}_{-8.9}$	300^{+269}_{-147}	300^{+312}_{-161}
+ $\Delta M_t = 5$ GeV	150 ± 4.9	300^{+173}_{-114}	300^{+203}_{-128}
+ $\Delta M_t = 0.5$ GeV	150 ± 0.5	300^{+128}_{-93}	300^{+156}_{-109}
LEP I + M_W + A_{LR}	150^{+14}_{-22}	$600^{+...}_{-440}$	
+ $\Delta M_t = 10$ GeV	150 ± 9.2	$600^{+...}_{-280}$	$600^{+...}_{-304}$
+ $\Delta M_t = 5$ GeV	150 ± 4.9	600^{+320}_{-214}	600^{+375}_{-239}
+ $\Delta M_t = 0.5$ GeV	150 ± 0.5	600^{+237}_{-174}	600^{+288}_{-200}

Table 4: Accuracies on M_t and M_H for different accuracies of an independent measurement of M_t . The input data are the SM predictions for $M_t=150$ GeV and $M_H=100, 300$ and 600 GeV, respectively. Free parameters of the fits are M_Z , M_t , M_H and $\Lambda_{\overline{MS}}$, with M_H being allowed to vary in the range $50 < M_H(\text{GeV}) < 1000$. The comparison between the 3rd and 4th column shows the effect of the measurement of A_{LR} with a precision of $\Delta A_{LR}=0.003$. Errors given as dots either exceed the lower bounds on M_H from direct searches or reach beyond $M_H=1$ TeV.

observable	calculated with error σ_i	coefficient a_i for $\Delta M_t = \infty$	coefficient a_i for $\Delta M_t = 10 \text{ GeV}$	coefficient a_i for $\Delta M_t = 5 \text{ GeV}$	coefficient a_i for $\Delta M_t = 0.5 \text{ GeV}$
M_Z (GeV)	0.0050	0.0079	0.0164	0.1746	0.6308
Γ_{had} (GeV)	0.0020	0.0067	0.0006	0.0255	0.1125
Γ_{l+l-} (GeV)	0.0001	0.0973	0.0000	0.1502	0.8345
Γ_{tot} (GeV)	0.0050	0.0079	0.0000	0.0146	0.0754
$\Gamma_{b\bar{b}}$ (GeV)	0.0030	0.0523	0.0280	0.0073	0.0006
$A_{\text{FB}}^{\text{lept}}$	0.0020	0.0030	0.0213	0.0693	0.1752
$A_{\text{FB}}^{b\bar{b}}$	0.0030	0.0949	0.2603	0.6058	1.2865
$A^{\text{pol}}(\tau)$	0.0040	0.0852	0.2561	0.6259	1.3662
M_W (GeV)	0.0600	0.0675	0.0109	0.3479	1.5219
A_{LR}	0.0030	0.1217	0.4021	1.0243	2.2895
M_t (GeV)	varied	0.0000	1.4246	1.8941	0.0554

Table 5: The coefficients $a_i = \left(\frac{x_i - \bar{x}_i(m_0, \Delta m)}{\sigma_i} \right)^2 \frac{1}{\left(\log \frac{m_0 + \Delta m}{m_0} \right)^2}$ needed to apply the approximate formulae for ΔM_H given in the text. The units in the column 'observable' apply to the error σ_i . As discussed in the text, these coefficients depend in a non trivial way on the accuracy of an independent determination of M_t and are therefore tabulated for several values of ΔM_t . The size of these coefficients is a direct measure of the relative importance of each observable in constraining M_H .

Figure 1: χ^2 curves as function of M_t for the Standard Model fit to

a) the LEP data alone (c.f. Table 2 a))

b) the combination of the LEP value of M_Z with the ratio M_W/M_Z from UA2 and M_W from CDF (c.f. Table 2 b)).

c) the combination of the LEP value of M_Z with $\sin^2\theta_W$ from the νN -scattering experiments CHARM and CDHS (c.f. Table 2 b)).

Figure 2: χ^2 curves as function of M_t for the combined fit to LEP data, M_W/M_Z from UA2, M_W from CDF and $\sin^2\theta_W$ from the νN -scattering experiments CHARM and CDHS (c.f. Table 2 c)).

Figure 3: χ^2 curve as function of M_H for the combined fit to LEP data, M_W/M_Z from UA2, M_W from CDF and $\sin^2\theta_W$ from the νN -scattering experiments CHARM and CDHS.

Figure 4: The partial contributions χ_i^2 of each observable i to χ^2 for the combined fit to LEP data, $M_W^r = M_W/M_Z$ from UA2, M_W from CDF and $s_w^2 = \sin^2\theta_W = 1 - (M_W/M_Z)^2$ from the νN -scattering experiments CHARM and CDHS. The Higgs mass has been set to 50 GeV and 1000 GeV, respectively, minimising χ^2 w.r.t. the remaining parameters M_Z , M_t and $\Lambda_{\overline{MS}}$.

Figure 5: χ^2 curves as function of M_t for Higgs mass values in the range $50 < M_H(\text{GeV}) < 1000$ GeV for various stages of LEP running. Input data are Standard Model predictions for $M_t = 150$ GeV and $M_H = 100$ GeV with errors given in the text. The stages considered are

- a) $M_Z, \Gamma_{\text{tot}}, \Gamma_{\text{had}}, \Gamma_{\text{lept}}, A_{\text{FB}}^{\text{lept}}, A^{\text{pol}}(\tau), \Gamma_{b\bar{b}}, A_{\text{FB}}^{b\bar{b}}$ (referred to as 'LEP I' in the following)
 - b) LEP I + M_W
 - c) LEP I + $M_W + A_{\text{LR}}$
- The fits are based on the measurement errors discussed in the text.

Figure 6: χ^2 curves as function of M_H for various stages of LEP running. The input data to the fit are the Standard Model predictions for $M_t = 150$ GeV and $M_H = 100$ GeV with errors given in the text. No independent constraint on M_t is imposed.

Figure 7: χ^2 curves as function of M_H for different accuracies of an independent measurement of M_t . The input data to the fit are the Standard Model predictions for the full set of observables from LEP I, M_W and A_{LR} generated with $M_t = 150$ GeV and $M_H = M_H^{\text{in}}$ as indicated in the figure. The fits are based on the measurement errors discussed in the text.

Figure 8: $1\text{-}\sigma$ contours in the $M_t - M_H$ plane for different accuracies of an independent measurement of M_t . The input data to the fit are the Standard Model predictions for the full set of observables from LEP I, M_W and A_{LR} with $M_t = M_H = 150 \text{ GeV}$.

Figure 9: The partial contributions $\Delta\chi_i^2$ of each observable i to the increase in χ^2 when varying the Higgs mass away from its value m_0 which minimises χ^2 . The Higgs mass has been varied from $m_0=300 \text{ GeV}$ by $\Delta m=150 \text{ GeV}$ to 450 GeV minimising χ^2 w.r.t the remaining parameters M_Z , M_t and $\Lambda_{\overline{MS}}$. The subdiagrams refer to different accuracies of an independent measurement of M_t .

Figure 10: The error ΔM_H as function of M_H . The symbols show the result of the full minimisation procedure, the lines are calculated using an approximation described in the text.

a) The energy dependence of the approximation with coefficients derived from the measurements of LEP I, M_W and A_{LR} is tested against the full minimisation procedure.

b) The approximation is used to predict the accuracy of M_H without the measurement of A_{LR} . The approximation is based on the same coefficients as for a), the coefficient related to A_{LR} however has been set to zero.