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Multiplicity Distributions of Charged Hadrons in νp and $\bar{\nu} p$ Charged Current Interactions.

*Birmingham-CERN-Imperial College-München (MPI)-Oxford-
University College London Collaboration (WA21)*

G.T. JONES, R.W.L. JONES ^{†)}, B.W. KENNEDY ^{‡)}
University of Birmingham, Edgbaston, Birmingham, B15 2TT, UK

D.R.O. MORRISON
CERN, European Laboratory for Particle Physics, CH-1211 Geneva 23, Switzerland

M.M. MOBAYYEN, S. WAINSTEIN
Department of Physics, Imperial College London, London, SW7 2BZ, UK

M. ADERHOLZ, D. HANTKE, U.F. KATZ, J. KERN, N. SCHMITZ, W. WITTEK
Max-Planck-Institut für Physik, Werner-Heisenberg-Institut, W-8000 München 40, Germany

H. P. BORNER, G. MYATT, D. RADOJICIC
Department of Nuclear Physics, University of Oxford, Oxford, OX1 3RH, UK

S. BURKE ^{*})
Department of Physics and Astronomy, University College London, London, WC1E 6BT, UK

^{†)} Now at Queen Mary and Westfield College London, London, E1 4NS, UK

^{‡)} Now at University College London, London WC1E 6BT, UK

^{*}) Now at Oliver Lodge Laboratory, University of Liverpool, Liverpool L69 3BX, UK

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Department of Nuclear Physics, University of Oxford, Oxford, OX1 3RH, UK

S. BURKE ^{*})

Department of Physics and Astronomy, University College London, London, WC1E 6BT, UK

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ABSTRACT

Using data on νp and $\bar{\nu} p$ charged current interactions from a bubble chamber experiment with BEBC at CERN, the multiplicity distributions of charged hadrons are investigated. The analysis is based on $\sim 20\,000$ events with incident ν and $\sim 10\,000$ events with incident $\bar{\nu}$. The invariant mass W of the total hadronic system ranges from 3 GeV to ~ 14 GeV. The experimental multiplicity distributions are fitted by the binomial function (for different intervals of W and in different intervals of the rapidity y), by the Levy function and the lognormal function. All three parametrizations give acceptable values for χ^2/NDF . For fixed W , forward and backward multiplicities are found to be uncorrelated. The normalized moments of the charged multiplicity distributions are measured as a function of W . They show a violation of KNO scaling.

^{†)} Now at Queen Mary and Westfield College London, London, E1 4NS, UK

^{‡)} Now at University College London, London WC1E 6BT, UK

^{*}) Now at Oliver Lodge Laboratory, University of Liverpool, Liverpool L69 3BX, UK

1. Introduction

The multiplicity distributions of charged hadrons provide tests of any theory of hadron production and represent a useful tool to look for similarities and differences in various reactions. In this paper, using the final statistics of the WA21 experiment on νp and $\bar{\nu} p$ charged current reactions

$$\nu p \rightarrow \mu^- + \text{hadrons} \quad (1)$$

$$\bar{\nu} p \rightarrow \mu^+ + \text{hadrons}, \quad (2)$$

the multiplicity distributions of charged hadrons are investigated in detail. Results based on part of the statistics and using a different method to correct for systematic effects are published in [1]. The results presented here supercede those of that publication.

In a previous paper [2] the average charged multiplicities were measured as a function of the total hadronic mass W and of the four-momentum transfer squared Q^2 to the target. In the present paper the charged multiplicity distributions are studied as a function of W in terms of the binomial function [3], the Levy function [4] and the lognormal function [5]. The parameters of the binomial function are also determined for the charged multiplicity distributions in certain regions of the cms rapidity y

$$y = \frac{1}{2} \ln \frac{E + p_l}{E - p_l} \quad (3)$$

for various W intervals. Correlations between forward and backward multiplicities are investigated. Finally the normalized moments of the charged multiplicity distributions are measured as a function of W .

2. Experimental procedure

a) The data sample

The data were obtained using the bubble chamber BEBC, filled with liquid hydrogen. The chamber was exposed to the horn-focussed ν and $\bar{\nu}$ wide band beams at the CERN SPS. It was equipped with an External Muon Identifier (EMI) [6] to allow the identification of muons. During the last part of the experiment the bubble chamber was surrounded by a cylindrical array of single wire proportional tubes, the Internal Picket Fence (IPF) [7], which recorded position and time information for most of the charged particles entering or leaving the bubble chamber body. A total of 458 400 pictures with incident ν and 307 400 pictures with incident $\bar{\nu}$ have been taken in several runs during the years 1977 to 1983. All data have been analyzed using the HYDRA program chain of CERN.

All film was double-scanned for events with at least three charged particles in the final state. The scanning efficiency was 95% for 3-prong events and about 99% for higher multiplicities. In the $\bar{\nu}p$ interaction also 1-prong events occur, i.e. events in which the muon is the only charged particle in the final state. For these events the scanning efficiency is very low and the $\bar{\nu}$ energy cannot be estimated reliably. The contribution of 1-prong events in the various W intervals was therefore determined from an experimental estimate [8] of the 1-prong rate as a function of the $\bar{\nu}$ energy (see ref. [2]). Muons were identified using the EMI and IPF.

A fiducial volume of 19 m^3 was defined to ensure good measurement conditions. The muon momentum was required to be greater than $3 \text{ GeV}/c$ and the invariant mass W of the total hadronic system to be between 3 and 14 GeV. The resulting sample consists of 13 167 νp and 6 408 $\bar{\nu} p$ charged current events.

Nearly all neutrons and π^0 mesons are undetected. Therefore the energy E_ν of the incident ν ($\bar{\nu}$) cannot be directly inferred from the visible energy. Instead, E_ν is estimated from the measured momenta of the muon and of the observed hadrons by a method based on transverse momentum balance [9].

b) Monte Carlo simulations and corrections

Extensive Monte Carlo (MC) calculations have been performed to study and correct for experimental errors and deficiencies, and for the cuts applied. Two different MC programs have been used in order to obtain an estimate of the systematic error of the corrections.

The first set of 150 000 MC events (for νp and $\bar{\nu} p$ each) was generated using the LUND program [10] (version JETSET 5.2 / LEPTO 4.3) with parameters tuned to describe the main features of this experiment [11]. A second set of events with equal statistics was generated using a longitudinal phase space model (LPS) [12, 13, 14]. These sets of generated events represent samples of so called “initial” events, where the kinematic quantities (particle mass, momentum and angles) of the primary and of all secondary particles are known.

The quantities in the “initial” events were then modified to simulate the experimental situation. In the simulation the scanning efficiency, the measurement errors and the particle misidentification were taken into account, E_ν was estimated as for the data, and the same experimental cuts were applied as to the data. Special care has been taken in simulating neutral particles (γ , K^0 , Λ) which decay or convert into charged particles close to the primary vertex. The tracks from these particles were partly assigned to the primary vertex, thus leading to an apparent charged multiplicity which is higher than the genuine one. The events modified in this way represent the so called “modified” event samples.

Both MC versions were carefully tuned to the data. Each quantity d to be measured (for example the average charged multiplicity for a given W interval) was determined in parallel for the data, for the samples of initial and for the samples of modified MC events. The corrected quantity d_{corr} was then obtained according to

$$d_{\text{corr}} = d_{\text{data}} \cdot \frac{d_{\text{initial}}^{\text{MC}}}{d_{\text{modified}}^{\text{MC}}} \quad (4)$$

The differences between the final results, when correcting with the two MC versions (LUND and LPS), were found to be of the same order of magnitude as the statistical errors or less. In section 3 only the results corrected with the LUND MC are presented, and the errors quoted include statistical and systematic errors, added in quadrature. The systematic error is taken to be 50% of the correction applied,

where the value of 50% is estimated from the difference between the LUND and LPS corrected results. In order to avoid large statistical fluctuations data points were only accepted, if there were at least 10 entries in the corresponding bin for each of the three event samples. For the analysis in different rapidity intervals at least 5 entries per bin were required.

3. Results

a) Global charged multiplicities

The multiplicity distribution of charged hadrons is defined as

$$P(n) = \frac{N(n)}{\sum_{\lambda} N(\lambda)} \quad (5)$$

where $N(n)$ is the number of events with n charged hadrons, and the sum extends over the values $\lambda = 2, 4, \dots$ in νp and $\lambda = 0, 2, 4, \dots$ in $\bar{\nu} p$ reactions. $P(n)$ was determined for the W intervals listed in Table 1, and corrected as described in the previous section. Table 1 also gives the total number of events found and the average hadronic mass $\langle W \rangle$ in the selected W intervals.

A theoretical investigation of fragmentation processes like cascading and/or stimulated emission leads to multiplicity distributions according to negative binomial distributions [3]. For each W interval the corrected multiplicity distribution $P(n)$ was fitted by a binomial function according to

$$P(n; k, \bar{n}) = \frac{k(k+1)\dots(k+n-1)}{n!} \left[\frac{\bar{n}}{\bar{n}+k} \right]^n \left[\frac{k}{\bar{n}+k} \right]^k \quad (6)$$

with $1/k$, $\bar{n}$ and a normalization factor as free parameters. The parameter $\bar{n}$ is the theoretical mean multiplicity. The parameters $1/k$ and $\bar{n}$ determine the width D_n of the distribution

$$D_n^2 = \bar{n} + \frac{\bar{n}^2}{k}. \quad (7)$$

In the limit $\frac{1}{k} \rightarrow 0$ the distribution turns into a Poissonian, while for negative integer values of k it is the ordinary binomial distribution. Following ref. [3] for negative binomial distributions the parameter $1/k$ is the average fraction of particles

already present stimulating the emission of an additional particle. The physical interpretation of negative values of $1/k$ is not clear. Fig. 1 shows the experimental points for $P(n)$ together with the fitted distributions (solid lines). The numerical values of the fitted parameters and of the width D of the distribution of $z = n/\bar{n}$,

$$D^2 = \left(\frac{Dn}{\bar{n}} \right)^2 = \frac{1}{\bar{n}} + \frac{1}{k}, \quad (8)$$

are listed in Table 2, together with the χ^2/NDF values of the fits. In $\bar{\nu}p$ reactions the 1-prong contribution $N(0)$ was taken into account and $P(0)$ was included in the fits. However it turns out that the parameter values and χ^2/NDF change only little when $N(0)$ is set to zero and $P(0)$ is not included in the fits.

The fitted values of $1/k$ and $\bar{n}$ are displayed as a function of W^2 in Fig. 2. In the whole W range considered ($3 \text{ GeV} < W \lesssim 14 \text{ GeV}$) the parameter $1/k$ turns out to be negative, i.e. the multiplicity distribution is narrower than a Poissonian. The sign change of $1/k$ in the region $W \approx 10 - 14 \text{ GeV}$ which is seen in lepton-nucleon data [15], extending to higher W [16], is compatible with our data. An extrapolation from non-diffractive pp and $\bar{p}p$ collisions [17] suggests a sign change in a similar energy range ($\sqrt{s} \sim 10 \text{ GeV}$).

KNO scaling implies that all moments of the KNO-distribution $\Psi(z) = \bar{n}P(n)$ with $z = n/\bar{n}$ should be independent of W . Using the results from the fits to the multiplicity distribution $P(n)$, the W dependence of the width D of the corresponding KNO-distribution can be tested using eq. (8). As is seen from Table 2, D decreases with increasing W , indicating a violation of KNO scaling (see also section c).

The results presented in Table 2 and Fig. 2 are consistent with those obtained in an $\bar{\nu}d$ experiment [15], although the restriction in [15] to multiplicities $n > 2$ leads to small differences, especially in the values for $\bar{n}$. The results for νp reactions are very similar to those obtained in a μp experiment [16].

To investigate the mutual dependence of the parameters $1/k$, $\bar{n}$ and W various fits have been performed, the results of which are given in Table 3. The results for the parametrization

$$\frac{1}{k} = a_4 + b_4 \cdot \ln \bar{n} \quad (9)$$

are drawn as straight lines in Figs. 2c and 2d, together with the experimental data. The values obtained for a_4 and b_4 in νp scattering are similar to those reported for

pp and $\bar{p}p$ collisions: $a_4 = -0.245 \pm 0.004$ and $b_4 = 0.149 \pm 0.002$ [18]. The slope b_2 of the straight line in the parametrization $1/k = a_2 + b_2 \cdot \ln s$ fitted for $\bar{p}p$ [19], ($a_2 = -0.098 \pm 0.008$ and $b_2 = 0.0282 \pm 0.0009$), is close to the corresponding values for νp and $\bar{\nu}p$ reactions.

b) Charged multiplicities in limited rapidity intervals.

The multiplicity distributions of charged hadrons were also determined in the limited cms rapidity intervals $|y| < y_{cut}$, with $y_{cut} = 0.5, 1.0, 1.5, 2.0$ and 2.5 . While the global charged multiplicity in an event has to be even because of charge conservation, the multiplicity n can now acquire also odd values, which leads to even/odd fluctuations in the multiplicity distribution. The population at odd values dies away as the rapidity window approaches the full rapidity range. Therefore in those cases, where both (even and odd) multiplicities were fitted simultaneously, two normalization parameters were used, one for the even and one for the odd values of the distribution, in addition to the common parameters $1/k$ and $\bar{n}$.

For νp reactions the odd multiplicities were ignored, and the charged multiplicity $n = 0$, which is absent if no y - cut is applied, was accepted in the smallest y bin ($|y| < 0.5$) only. For $\bar{\nu}p$ reactions odd multiplicities were included in the fit if the odd value population exceeded 50% of the even value population, i.e. for all W intervals in the cases $|y| < 0.5$ and $|y| < 1.0$, and for $W > 7$ GeV also in the case $|y| < 1.5$.

The results for $1/k$ and $\bar{n}$ from fits of the form eq. (6) to the data are listed in Table 4 and drawn in Fig. 3 as a function of W and in Fig. 4 as a function of y_{cut} (for $\bar{\nu}p$ reactions the values for $|y| < 0.5$ are not meaningful due to large fluctuations and are therefore not shown). As expected from other experiments [15, 16, 19, 20] the binomial function distribution represents the data well in almost all W and y intervals.

The main observations are:

- For fixed W , the value of $\bar{n}$ increases with increasing y_{cut} as long as the y interval is completely contained in the y -range accessible at the selected W (Figs. 4b,d). This increase of $\bar{n}$ is due to an extension of phase space and is accompanied by a decrease of $1/k$ (Figs. 4a,c).

- For fixed W , the width D of the z -distribution decreases with increasing width of the y interval, as was observed in pp [20] and $\bar{\nu}d$ reactions [15] as well (Tables 4a,b).
- For a fixed y interval, $\bar{n}$ and $1/k$ become rather independent of W as soon as the selected y interval is completely contained in the y range accessible at the particular value of W (Figs. 3a,b,c,d). This is equivalent to an approximate independence of W of
 - the height of the central rapidity distribution and
 - the width of the multiplicity distribution in the central rapidity region.
- The increase of $\bar{n}$ and $1/k$ with increasing W in those cases where the selected y interval exceeds the y range accessible at a particular value of W corresponds to that already discussed in section a) (Figs. 3a,b,c,d).

The fit results for $1/k$ and $\bar{n}$ of the charged multiplicity distributions in the forward ($y > 0$) and backward ($y < 0$) hemispheres are given in Table 5 and are displayed as a function of W^2 in Fig. 5. In νp reactions $1/k$ and $\bar{n}$ (with the exception of $\bar{n}$ in the highest W interval) are similar in the forward and backward region, while an increasing difference between the forward and backward regions with increasing W is observed in $\bar{\nu}p$ reactions. The width D of the z -distribution has a tendency to decrease with increasing W in both hemispheres.

In terms of the quark parton model and neglecting scattering of sea quarks, the data show that the average hadron multiplicity is similar for u- and d-quark fragmentation, while uu fragmentation leads to higher average hadron multiplicities than ud fragmentation.

The results presented in this section are qualitatively similar to those from a μ^+p [16] and a $\bar{\nu}d$ experiment [15].

c) Further fits to the global charged multiplicity distributions.

In order to test KNO scaling [21] in the form proposed by [22] (KNO-G scaling) the following expression $Q(n)$ is fitted to the experimental corrected multiplicity distribution $P(n)$:

$$Q(n) = \int_{z_{n-1}}^{z_{n+1}} \Psi_i(z) dz \quad (10)$$

where $z_n = \frac{n+\gamma}{\langle n \rangle + \gamma}$, with $\gamma = -1$ and $+1$ for νp and $\bar{\nu} p$ reactions respectively and $\langle n \rangle$ is the experimental expectation value of n (in contrast to the previously defined $\bar{n}$ which is the theoretical mean multiplicity) [23, 24]. The value of γ is chosen, that the integral in eq. (10) starts at zero for the lowest physical multiplicity. For $\Psi(z)$ one of the following functions has been chosen

$$\Psi_1(z) = \alpha_1 \cdot \frac{c_1^{(c_1 z + 1)}}{\Gamma(c_1 z + 1)} \cdot e^{-c_1} \quad (11)$$

(Levy function [4]) and

$$\Psi_2(z) = \frac{\alpha_2}{\sqrt{2\pi}\sigma} \cdot \frac{1}{z + c_2} \cdot e^{-\frac{(\ln(z+c_2)-\mu)^2}{2\sigma^2}} \quad (12)$$

(lognormal function [5, 24, 25]).

In the derivation by Levy [4] the parameter c_1 represents the average number of fireballs in a reaction. The number of fireballs itself is assumed to be distributed according to a Poisson law. Apart from a normalization parameter α_1 the free parameter of the Levy function is c_1 , which is related to the width $D = \sqrt{\langle z^2 \rangle - \langle z \rangle^2}$ by $D = \frac{1}{\sqrt{c_1}}$.

A lognormal distribution results if a scale invariant branching process is assumed as the multiparticle production mechanism [5]. In addition to the normalization parameter α_2 the lognormal function has two free parameters, which were chosen to be c_2 and the width $D = \sqrt{e^{\sigma^2} - 1} \cdot e^{(\mu + \frac{\sigma^2}{2})}$. The parameters α_2 , c_2 , μ and σ are related via the normalization conditions for $\Psi_2(z)$ [24].

Fits were performed separately for the various W intervals and for the whole W region ($W > 3$ GeV). As can be seen from Tables 6 and 7, the Levy function and the lognormal function yield very similar results for the widths. Fig. 6 shows the data together with the fitted Levy function $\Psi_1(z)$ for the interval $5 < W < 7$ GeV (solid line).

The width D of the distributions shows a clear trend to decrease with increasing W , indicating a violation of KNO scaling, both for νp and $\bar{\nu} p$ scattering. However,

the global fits for $W > 3$ GeV, in which KNO scaling is implicitly assumed, yield acceptable values of χ^2/NDF in both cases.

It should be noted that the absolute value of D and the strength of its W dependence depend on the value of γ chosen. For the global fit of the Levy function, with $W > 3$ GeV and using $\gamma = 0$, D is found to be 0.361 ± 0.005 for νp and 0.449 ± 0.008 for $\bar{\nu} p$ reactions. Thus, with $\gamma = 0$ the KNO distribution is narrower for νp than for $\bar{\nu} p$ scattering, as was previously observed [1].

In μp reactions [26, 27] a value $D = \frac{1}{\sqrt{c_1}} = 0.375 \pm 0.003$ was obtained with $\gamma = 0$, similar to the result for the νp reaction.

Fits of the lognormal distribution to data from e^+e^- and pp collisions have been published in [28]. In Table 8 the resulting parameters are shown together with those of this experiment. The values for νp and $\bar{\nu} p$ interactions lie in between those from e^+e^- and pp collisions. A compilation of results on the lognormal function for e^+e^- collisions at higher energies can be found in [29].

d) Forward – backward multiplicity correlations.

The average backward ($y < 0$) multiplicity $\langle n_b \rangle$ as a function of the forward ($y > 0$) multiplicity n_f is plotted in Fig. 7 for various W intervals. The oscillations of $\langle n_b \rangle$ at low values of n_f are due to charge conservation, which requires $n_f + n_b$ to be even. Therefore the value for $n_f = 0$ has not been included in a fit to the experimental data of the form:

$$\langle n_b \rangle = a + b \cdot n_f \quad (13)$$

Since b and therefore the correlation between n_f and $\langle n_b \rangle$ was found to be consistent with zero for all W intervals, b was set to zero in the fits to the data for $n_f \geq 1$. The results are listed in Table 9 and shown together with the data in Fig. 7. This result is found in μp reactions as well [26], whilst for $\bar{p} p$ collisions at high W a strong positive correlation is seen [19, 27, 30].

The fact that there is no correlation between n_f and $\langle n_b \rangle$ indicates, in the framework of the quark parton model, that the quarks and diquarks fragment relatively independently of each other.

e) Normalized moments of the global charged multiplicity distributions.

The normalized moments C_q are defined as

$$C_q = \frac{\langle (n + \gamma)^q \rangle}{\langle n + \gamma \rangle^q} \quad (14)$$

with $\gamma = -1$ and $+1$ for νp and $\bar{\nu} p$ reactions respectively [24]. The resulting C_q for $q = 2, 3, 4, 5$ are listed in Table 10 and plotted as a function of W in Fig. 8. The value of C_2 does not give any new information as C_2 is equal to $(1 + D^2)$, where D is the width of the z -distribution. The decrease of D with increasing W , observed in section c), is reflected in a corresponding decrease of the moments C_q , most pronounced in the higher moments. According to Fig. 8, νp and $\bar{\nu} p$ reactions seem to behave quite differently, the W -dependence of the C_q being much stronger for νp scattering. However, it should be noted, that the strength of the W -dependence is very sensitive to the choice of γ . So one finds for example very similar behaviour of the C_q for νp with $\gamma = 0$ and $\bar{\nu} p$ with $\gamma = +1$, and also for νp with $\gamma = -1$ and $\bar{\nu} p$ with $\gamma = 0$. In all cases however, the moments decrease with increasing W , confirming the violation of KNO scaling.

4. Conclusions

The results on the distributions of charged hadron multiplicities in νp and $\bar{\nu} p$ scattering can be summarized as follows:

- The charged multiplicity distribution $P(n)$ can be well fitted by the binomial function in each of several W intervals between 3 GeV and 14 GeV.
- In the W range considered $P(n)$ is narrower than a Poissonian, approaching a Poissonian at the upper end of the W range.
- The parameter $1/k$ rises linearly with the parameter $\bar{n}$.
- The distribution of the charged multiplicity in central rapidity intervals can be well fitted by the binomial function for each W interval. The parameter $1/k$ and the width D of the z -distribution ($z = n/\bar{n}$) decrease with increasing width of the rapidity bin. The height of the rapidity distribution and D in the central rapidity region are approximately independent of W .
- The z -distributions, with $z = \frac{n+\gamma}{\langle n \rangle + \gamma}$, ($\gamma = -1$ for νp and $\gamma = +1$ for $\bar{\nu} p$) in the various W intervals are equally well fitted by the Levy and the log-normal function. Although the quantitative results on the width D of the z -distribution depend on the value of γ chosen, there is a clear dependence of D on W for a fixed value of γ , implying violation of KNO scaling.
- For fixed W , the forward and backward charged multiplicities are uncorrelated.
- The normalized moments C_q decrease with increasing W , confirming the violation of KNO scaling. The strength of the W -dependence rises with increasing order q .

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Table 1

Total number of events found N and average hadronic mass $\langle W \rangle$
in the selected W intervals

a) νp reactions

W [GeV]	3 – 4	4 – 5	5 – 7	7 – 10	10 – 14
N	3414	2931	3504	2373	945
$\langle W \rangle$ [GeV]	3.5	4.5	5.9	8.2	11.5

b) $\bar{\nu} p$ reactions

W [GeV]	3 – 4	4 – 5	5 – 7	7 – 10	10 – 14
N	2414	1584	1582	672	156
$\langle W \rangle$ [GeV]	3.5	4.5	5.8	8.1	11.3

Table 2

Parameters of the binomial distributions fitted to the total charged multiplicity distributions in different W intervals

a) νp reactions

$W[\text{ GeV}]$	$\frac{1}{k}$	$\bar{n}$	$D = \sqrt{\frac{1}{\bar{n}} + \frac{1}{k}}$	χ^2/NDF
3 – 4	-0.107 ± 0.007	3.54 ± 0.04	0.419 ± 0.009	6.3/1
4 – 5	-0.083 ± 0.007	4.09 ± 0.06	0.402 ± 0.010	0.2/1
5 – 7	-0.051 ± 0.007	4.87 ± 0.07	0.392 ± 0.010	2.2/2
7 – 10	-0.035 ± 0.009	5.66 ± 0.08	0.377 ± 0.012	1.0/3
10 – 14	-0.036 ± 0.011	6.48 ± 0.14	0.345 ± 0.017	0.9/3

b) $\bar{\nu} p$ reactions

$W[\text{ GeV}]$	$\frac{1}{k}$	$\bar{n}$	$D = \sqrt{\frac{1}{\bar{n}} + \frac{1}{k}}$	χ^2/NDF
3 – 4	-0.068 ± 0.019	2.83 ± 0.07	0.534 ± 0.020	0.2/1
4 – 5	-0.061 ± 0.015	3.41 ± 0.07	0.482 ± 0.016	1.5/2
5 – 7	-0.029 ± 0.011	4.22 ± 0.07	0.456 ± 0.013	0.3/3
7 – 10	-0.012 ± 0.021	4.96 ± 0.12	0.436 ± 0.024	2.0/2
10 – 14	-0.022 ± 0.025	5.76 ± 0.22	0.390 ± 0.033	1.2/2

Table 3

Fit results concerning the mutual dependence of $1/k$, $\bar{n}$ and W

a) νp reactions

Parametrization	a_i	b_i	χ^2/NDF
$\bar{n} = a_1 + b_1 \cdot \ln W^2$	0.40 ± 0.13	1.25 ± 0.04	1.6/3
$\frac{1}{k} = a_2 + b_2 \cdot \ln W^2$	-0.187 ± 0.016	0.035 ± 0.005	7.6/3
$\frac{1}{k} = a_3 + b_3 \cdot \bar{n}$	-0.200 ± 0.018	0.028 ± 0.004	6.4/3
$\frac{1}{k} = a_4 + b_4 \cdot \ln \bar{n}$	-0.284 ± 0.031	0.142 ± 0.020	2.2/3

b) $\bar{\nu} p$ reactions

Parametrization	a_i	b_i	χ^2/NDF
$\bar{n} = a_1 + b_1 \cdot \ln W^2$	-0.44 ± 0.20	1.30 ± 0.06	2.7/3
$\frac{1}{k} = a_2 + b_2 \cdot \ln W^2$	-0.132 ± 0.040	0.027 ± 0.011	1.9/3
$\frac{1}{k} = a_3 + b_3 \cdot \bar{n}$	-0.128 ± 0.037	0.022 ± 0.009	1.4/3
$\frac{1}{k} = a_4 + b_4 \cdot \ln \bar{n}$	-0.168 ± 0.049	0.093 ± 0.036	0.9/3

Table 4a

Parameters of the binomial distribution fitted to the charged multiplicity distributions in different y and W intervals, for νp reactions.

y_{cut}	$\frac{1}{k}$	$\bar{n}$	$D = \sqrt{\frac{1}{\bar{n}} + \frac{1}{k}}$	χ^2/NDF
3 GeV < W < 4 GeV				
0.5	-0.019 ± 0.036	1.75 ± 0.07	0.743 ± 0.029	0.7/1
1.0	-0.073 ± 0.018	3.09 ± 0.08	0.501 ± 0.020	0.1/1
1.5	-0.095 ± 0.010	3.47 ± 0.06	0.440 ± 0.012	1.6/1
2.0	-0.106 ± 0.007	3.55 ± 0.04	0.419 ± 0.010	5.7/1
2.5	-0.108 ± 0.007	3.55 ± 0.04	0.418 ± 0.009	6.3/1
4 GeV < W < 5 GeV				
0.5	0.074 ± 0.036	1.68 ± 0.05	0.818 ± 0.025	0.0/1
1.0	-0.007 ± 0.032	3.14 ± 0.10	0.558 ± 0.030	0.0/1
1.5	-0.074 ± 0.010	4.00 ± 0.08	0.419 ± 0.013	1.7/2
2.0	-0.078 ± 0.008	4.08 ± 0.07	0.409 ± 0.011	2.6/2
2.5	-0.080 ± 0.007	4.09 ± 0.06	0.406 ± 0.010	2.7/2
5 GeV < W < 7 GeV				
0.5	0.065 ± 0.038	1.65 ± 0.04	0.819 ± 0.025	2.9/1
1.0	0.018 ± 0.025	3.23 ± 0.09	0.573 ± 0.023	0.1/2
1.5	-0.012 ± 0.013	4.56 ± 0.08	0.455 ± 0.015	0.2/2
2.0	-0.042 ± 0.009	4.88 ± 0.09	0.404 ± 0.012	1.1/3
2.5	-0.046 ± 0.008	4.87 ± 0.07	0.399 ± 0.010	3.5/3
7 GeV < W < 10 GeV				
0.5	0.166 ± 0.054	1.53 ± 0.05	0.905 ± 0.032	4.3/1
1.0	0.030 ± 0.032	3.16 ± 0.11	0.589 ± 0.029	0.2/1
1.5	0.045 ± 0.023	4.49 ± 0.10	0.517 ± 0.023	1.7/2
2.0	-0.026 ± 0.010	5.48 ± 0.09	0.396 ± 0.013	0.7/3
2.5	-0.033 ± 0.009	5.66 ± 0.10	0.380 ± 0.013	0.8/4
10 GeV < W < 14 GeV				
0.5	0.224 ± 0.108	1.35 ± 0.08	0.982 ± 0.125	0.8/1
1.0	-0.028 ± 0.063	2.89 ± 0.19	0.563 ± 0.059	0.6/1
1.5	-0.019 ± 0.024	4.38 ± 0.14	0.457 ± 0.027	0.1/2
2.0	-0.017 ± 0.014	5.97 ± 0.13	0.388 ± 0.019	0.9/3
2.5	-0.032 ± 0.012	6.46 ± 0.15	0.351 ± 0.018	0.8/3

Table 4b

Parameters of the binomial distribution fitted to the charged multiplicity distributions in different y and W intervals, for the $\bar{\nu}p$ reaction.

y_{cut}	$\frac{1}{k}$	$\bar{n}$	$D = \sqrt{\frac{1}{\bar{n}} + \frac{1}{k}}$	χ^2/NDF
3 GeV < W < 4 GeV				
0.5	0.111 ± 0.054	1.32 ± 0.04	0.931 ± 0.032	0.4/3
1.0	0.024 ± 0.032	2.23 ± 0.06	0.687 ± 0.025	3.6/3
1.5	-0.087 ± 0.011	2.96 ± 0.10	0.501 ± 0.016	3.9/2
2.0	-0.083 ± 0.012	2.91 ± 0.09	0.511 ± 0.016	0.2/1
2.5	-0.085 ± 0.012	2.89 ± 0.08	0.511 ± 0.015	0.2/1
4 GeV < W < 5 GeV				
0.5	0.212 ± 0.067	1.37 ± 0.05	0.970 ± 0.037	1.6/3
1.0	0.030 ± 0.027	2.46 ± 0.06	0.661 ± 0.022	10.7/5
1.5	-0.062 ± 0.020	3.38 ± 0.12	0.483 ± 0.023	2.2/2
2.0	-0.070 ± 0.019	3.46 ± 0.09	0.468 ± 0.022	0.7/1
2.5	-0.071 ± 0.016	3.48 ± 0.08	0.465 ± 0.019	0.7/1
5 GeV < W < 7 GeV				
0.5	0.185 ± 0.075	1.38 ± 0.05	0.954 ± 0.042	2.4/2
1.0	0.128 ± 0.033	2.67 ± 0.07	0.709 ± 0.024	4.2/5
1.5	0.002 ± 0.024	3.85 ± 0.15	0.511 ± 0.025	4.9/3
2.0	-0.026 ± 0.016	4.25 ± 0.11	0.457 ± 0.019	1.9/3
2.5	-0.031 ± 0.014	4.24 ± 0.10	0.452 ± 0.016	0.4/2
7 GeV < W < 10 GeV				
0.5	0.104 ± 0.078	1.46 ± 0.06	0.889 ± 0.047	0.3/2
1.0	0.161 ± 0.042	2.77 ± 0.09	0.722 ± 0.030	6.9/5
1.5	0.070 ± 0.028	3.94 ± 0.10	0.569 ± 0.025	5.0/7
2.0	0.002 ± 0.028	4.87 ± 0.15	0.455 ± 0.032	2.5/3
2.5	-0.012 ± 0.022	5.00 ± 0.13	0.433 ± 0.027	1.9/2
10 GeV < W < 14 GeV				
0.5	0.740 ± 0.520	1.33 ± 0.26	1.222 ± 0.221	0.5/1
1.0	0.149 ± 0.121	2.50 ± 0.22	0.742 ± 0.085	5.6/3
1.5	0.103 ± 0.125	4.27 ± 0.51	0.581 ± 0.110	8.0/4
2.0	-0.021 ± 0.036	5.54 ± 0.29	0.399 ± 0.047	0.9/2
2.5	-0.035 ± 0.024	5.87 ± 0.24	0.368 ± 0.033	1.5/2

Table 5

Parameters of the binomial distribution fitted to the charged multiplicity distributions for $y > 0$ and $y < 0$ in different W intervals

a) νp reactions.

W [GeV]		$\frac{1}{k}$	$\bar{n}$	$D = \sqrt{\frac{1}{\bar{n}} + \frac{1}{k}}$	χ^2/NDF
3 – 4	$y > 0$	-0.182 ± 0.035	1.66 ± 0.10	0.649 ± 0.039	0.0/1
	$y < 0$	-0.164 ± 0.037	1.73 ± 0.10	0.642 ± 0.039	0.0/1
4 – 5	$y > 0$	-0.126 ± 0.034	1.94 ± 0.11	0.625 ± 0.036	0.1/2
	$y < 0$	-0.100 ± 0.052	1.94 ± 0.10	0.645 ± 0.046	2.3/2
5 – 7	$y > 0$	-0.085 ± 0.033	2.40 ± 0.11	0.575 ± 0.033	0.5/3
	$y < 0$	-0.068 ± 0.030	2.35 ± 0.08	0.597 ± 0.028	1.0/3
7 – 10	$y > 0$	-0.068 ± 0.025	2.83 ± 0.11	0.534 ± 0.027	0.2/4
	$y < 0$	-0.050 ± 0.017	2.72 ± 0.06	0.563 ± 0.017	1.1/4
10 – 14	$y > 0$	-0.064 ± 0.023	3.33 ± 0.14	0.486 ± 0.027	1.2/4
	$y < 0$	-0.045 ± 0.028	3.01 ± 0.08	0.535 ± 0.027	0.7/3

b) $\bar{\nu} p$ reactions.

W [GeV]		$\frac{1}{k}$	$\bar{n}$	$D = \sqrt{\frac{1}{\bar{n}} + \frac{1}{k}}$	χ^2/NDF
3 – 4	$y > 0$	-0.152 ± 0.051	1.46 ± 0.11	0.730 ± 0.049	0.4/2
	$y < 0$	-0.108 ± 0.032	1.33 ± 0.05	0.802 ± 0.026	1.1/2
4 – 5	$y > 0$	-0.153 ± 0.030	1.87 ± 0.09	0.619 ± 0.032	1.1/3
	$y < 0$	-0.070 ± 0.036	1.58 ± 0.05	0.749 ± 0.028	0.3/2
5 – 7	$y > 0$	-0.073 ± 0.029	2.28 ± 0.10	0.604 ± 0.029	0.6/4
	$y < 0$	-0.038 ± 0.040	1.92 ± 0.07	0.694 ± 0.032	0.4/2
7 – 10	$y > 0$	-0.047 ± 0.037	2.74 ± 0.12	0.564 ± 0.036	1.4/4
	$y < 0$	0.072 ± 0.074	2.19 ± 0.16	0.728 ± 0.056	3.8/2
10 – 14	$y > 0$	-0.032 ± 0.079	3.26 ± 0.22	0.524 ± 0.078	0.6/2
	$y < 0$	0.115 ± 0.125	2.57 ± 0.34	0.710 ± 0.095	0.7/2

Table 6

Parameter c_1 and dispersion D of the Levy function, fitted to the charged multiplicity distributions in different W intervals

a) νp reactions ($\gamma = -1$)

W [GeV]	c_1	$D = \frac{1}{\sqrt{c_1}}$	χ^2/NDF
3 – 4	4.10 ± 0.19	0.494 ± 0.011	0.5/2
4 – 5	4.47 ± 0.22	0.473 ± 0.012	1.4/2
5 – 7	4.92 ± 0.25	0.451 ± 0.012	0.4/3
7 – 10	5.35 ± 0.35	0.432 ± 0.014	0.6/4
10 – 14	6.22 ± 0.61	0.401 ± 0.020	2.1/4
> 3	4.69 ± 0.12	0.462 ± 0.006	26.9/19

b) $\bar{\nu} p$ reactions ($\gamma = +1$)

W [GeV]	c_1	$D = \frac{1}{\sqrt{c_1}}$	χ^2/NDF
3 – 4	7.02 ± 0.55	0.377 ± 0.015	0.1/2
4 – 5	8.04 ± 0.56	0.353 ± 0.012	0.5/3
5 – 7	8.17 ± 0.45	0.350 ± 0.010	1.7/4
7 – 10	8.19 ± 1.05	0.349 ± 0.022	2.8/3
10 – 14	9.42 ± 1.63	0.326 ± 0.028	1.4/3
> 3	7.94 ± 0.29	0.355 ± 0.006	10.1/19

Table 7

Parameter c_2 and width D of the lognormal function fitted to the charged multiplicity distributions in different W intervals

a) νp reactions ($\gamma = -1$)

W [GeV]	c_2	D	χ^2/NDF
3 – 4	2.14 ± 1.32	0.502 ± 0.016	0.0/1
4 – 5	6.11 ± 5.20	0.495 ± 0.019	0.0/1
5 – 7	2.00 ± 1.05	0.457 ± 0.013	0.1/2
7 – 10	2.42 ± 1.35	0.438 ± 0.015	0.4/3
10 – 14	19.77 ± 31.67	0.410 ± 0.022	0.6/3
> 3	2.41 ± 0.69	0.470 ± 0.007	26.9/18

b) $\bar{\nu} p$ reactions ($\gamma = +1$).

W [GeV]	c_2	D	χ^2/NDF
3 – 4	2.01 ± 1.62	0.380 ± 0.016	0.2/1
4 – 5	2.34 ± 1.72	0.354 ± 0.012	0.2/2
5 – 7	1.13 ± 0.50	0.352 ± 0.010	0.4/3
7 – 10	0.42 ± 0.60	0.351 ± 0.022	0.8/2
10 – 14	2.14 ± 6.53	0.329 ± 0.030	1.4/2
> 3	1.37 ± 0.42	0.357 ± 0.007	8.4/18

Table 8

Parameter c_2 and width D of the lognormal function for e^+e^- and pp collisions and the νp , $\bar{\nu}p$ reactions of this experiment.

reaction	range [GeV]	c_2	D	χ^2/NDF	γ	Ref.
e^+e^-	$7 < \sqrt{s} < 91$	0.56 ± 0.03	0.277 ± 0.001	209/282	+1	[29]
$\bar{\nu}p$	$3 < W \lesssim 12$	1.37 ± 0.42	0.357 ± 0.007	8.4/18	+1	this exp.
νp	$3 < W \lesssim 14$	2.41 ± 0.69	0.470 ± 0.007	26.9/18	-1	this exp.
pp	$3 < \sqrt{s} < 62$	4.25 ± 0.20	0.629 ± 0.003	458/316	-1	[28]

Table 9

Parameter a from the forward-backward correlation fit ($b = 0$).

	νp		$\bar{\nu}p$	
W [GeV]	a	χ^2/NDF	a	χ^2/NDF
3 - 4	1.81 ± 0.06	3.2/4	1.40 ± 0.03	3.5/4
4 - 5	2.10 ± 0.06	7.8/5	1.63 ± 0.05	4.8/4
5 - 7	2.48 ± 0.06	7.5/6	1.96 ± 0.06	2.9/5
7 - 10	2.87 ± 0.06	5.4/6	2.13 ± 0.08	3.4/5
10 - 14	3.12 ± 0.07	5.7/6	2.57 ± 0.22	0.6/3

Table 10

Normalized moments of the global charged multiplicity distributions

a) νp reactions ($\gamma = -1$)

W [GeV]	C_2	C_3	C_4	C_5
3 – 4	1.28 ± 0.01	1.91 ± 0.04	3.19 ± 0.11	5.83 ± 0.30
4 – 5	1.25 ± 0.02	1.77 ± 0.06	2.78 ± 0.14	4.70 ± 0.34
5 – 7	1.22 ± 0.01	1.72 ± 0.05	2.70 ± 0.12	4.63 ± 0.31
7 – 10	1.20 ± 0.01	1.62 ± 0.04	2.41 ± 0.11	3.88 ± 0.27
10 – 14	1.16 ± 0.02	1.50 ± 0.05	2.09 ± 0.13	3.09 ± 0.27

b) $\bar{\nu} p$ reactions ($\gamma = +1$)

W [GeV]	C_2	C_3	C_4	C_5
3 – 4	1.16 ± 0.01	1.48 ± 0.04	2.05 ± 0.09	3.01 ± 0.18
4 – 5	1.14 ± 0.01	1.43 ± 0.03	1.95 ± 0.07	2.83 ± 0.16
5 – 7	1.13 ± 0.01	1.42 ± 0.03	1.93 ± 0.07	2.82 ± 0.15
7 – 10	1.11 ± 0.01	1.36 ± 0.03	1.79 ± 0.06	2.50 ± 0.14
10 – 14	1.10 ± 0.01	1.31 ± 0.04	1.65 ± 0.09	2.18 ± 0.17

Figure captions

Fig. 1: Normalized multiplicity distributions of charged hadrons produced in a) νp and b) $\bar{\nu} p$ reactions for different intervals of W . Each consecutive distribution is shifted down by an order of magnitude. The solid lines represent binomial functions fitted to the experimental points.

Fig. 2: Parameters obtained in fits of the binomial function to the experimental multiplicity distributions:

- a) $\frac{1}{k}$ as a function of W^2 .
- b) $\bar{n}$ as a function of W^2 .
- c) $\frac{1}{k}$ as a function of $\bar{n}$ for νp reactions together with a fitted straight line.
- d) $\frac{1}{k}$ as a function of $\bar{n}$ for $\bar{\nu} p$ reactions together with a fitted straight line.

Fig. 3: Parameters $\frac{1}{k}$ and $\bar{n}$ as functions of W obtained in fits of the binomial function in different rapidity intervals $|y| < y_{cut}$

- a) and b) for νp and c) and d) for $\bar{\nu} p$ reactions.

Fig. 4: Parameters $\frac{1}{k}$ and $\bar{n}$ as functions of the rapidity cut for different W intervals, obtained in fits of the binomial function

- a) and b) for νp and c) and d) for $\bar{\nu} p$ reactions.

Fig. 5: Parameters $\frac{1}{k}$ and $\bar{n}$ as functions of W^2 for the forward ($y > 0$) and backward ($y < 0$) hemispheres, obtained in fits of the binomial function

- a) and b) for νp and c) and d) for $\bar{\nu} p$ reactions.

Fig. 6: z -distributions for different W intervals

- a) for νp , ($z_n = \frac{n-1}{\langle n \rangle - 1}$)
- b) for $\bar{\nu} p$ reactions ($z_n = \frac{n+1}{\langle n \rangle + 1}$).

The solid lines represent the expression $(\langle n \rangle + \gamma)Q(n)$ with $Q(n)$ given by eq. (10) with the Levy function $\Psi_1(z)$ as kernel and for the fitted parameters in the interval $5 < W < 7$ GeV (n is assumed to be continuous).

Fig. 7: Average backward multiplicity $\langle n_b \rangle$ as a function of the forward multiplicity n_f for different W intervals ; a) for νp and b) for $\bar{\nu} p$ reactions.

Fig. 8: Normalized central moments C_q ($q = 2, 3, 4, 5$) of the multiplicity distributions for different W intervals: a) for νp and b) for $\bar{\nu} p$ reactions. The open symbols in b) represent the values of C_q when the 1-prong contributions are ignored.

Fig. 1

Fig. 2

Fig. 3

Fig.4

Fig. 5

Fig. 6

Fig. 7a

Fig. 7b

Fig. 8