CERN-TH-6332/91
UMN-TH-1014/91
December 1991

Classical Hair for Kerr-Newman Black Holes in String Gravity

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Abstract

We compute the external field strength hair present for black holes in string gravity. We retain all couplings in the effective Lagrangian for string gravity through first order in α' , the string tension. We work in general black hole backgrounds, both rotating and charged, and compute both dilaton and axion hair to that order, thus expanding and systematizing previous treatments.

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The advent of string theory¹⁾ has given us a consistent quantum theory of gravity, unified with the other fundamental interactions. The nature, and predictions, of string gravity are thus of considerable interest. Does the fundamental theory leave distinctive signatures on the macroscopic force it produces? Are there testable predictions we can compare with astrophysical data? As an extension of Einstein gravity we may hope that it will lead to new features, present in the solutions of string gravity, which are not present in General Relativity.

One instance where string gravity leads to new and unexpected features is in the structure of black holes. In Einstein gravity, minimally coupled to other fields, the most general black hole solution is described by the Kerr-Newman family of rotating charged black hole solutions. The only external field strengths present are those required by gauge invariance. The "no hair" property has been proven in considerable generality²⁾, and encapsulates our understanding of classical black holes in Einstein gravity.

It is the purpose of this paper to examine the general structure of the classical black hole solutions (including rotation and charge) in string gravity, when we include the next order corrections in α' , the string tension, beyond Einstein gravity^{3,4)}. We will consistently compute the external field strength hair for the axion and dilaton fields that appear in the string effective action, and consider corrections to the space-time metric, to leading order in α' , the expansion parameter for the string corrections. The presence of exterior field strength hair, in the classical black hole solutions to the low energy effective action describing string gravity, is a qualitatively new feature of string gravity.

This work extends and systematizes previous treatments of black holes in string gravity⁵⁻¹¹. For non-rotating, uncharged, black holes in four dimensions Callan, Myers and Perry⁶ found that retaining the $O(\alpha')$ higher order curvature terms resulted (after a conformal transformation) in a black hole with Schwarzschild metric, but with an exterior dilaton field which locally deviated from its asymptotic value at infinity (in effect, dilaton "hair"). In a similar manner, the $O(\alpha')$ gauge kinetic terms in string theory could set up dilaton hair around charged black holes, as computed by Gibbons and Maeda⁵ and Garfinkle, Horowitz and Strominger⁸. For rotating black holes, the present authors, with M. Duncan⁷, found that the non-minimal gravitational coupling of the Kalb-Ramond axion, due to the $O(\alpha')$ Lorentz Chern-Simons contribution to its field strength, meant that there was a dipole axion source proportional to the angular momentum of the black hole, which resulted in long range dipole axion hair for rotating black holes. In a similar manner, the gauge Chern-Simons contributions to the axion field strength would result in a monopole axion source term for dyonic black holes^{9,10} (should they exist). Finally Shapere, Trivedi and Wilczek¹¹ have related the dilaton and axion hair for nonrotating black holes with electric and magnetic charge, via the Gaillard-Zumino¹² duality rotations.

In this paper, we for the first time consider the solutions to the full gravitational multiplet (axion, dilaton and graviton) in string gravity consistently to $O(\alpha')$. We will show how the most general metric (to $O(\alpha')$) reduces to the Kerr-Newman metric, and we will compare this result with the interesting solutions of refs. 5,8, and 11. For the most part, our results will be completely general, i.e. we will derive the full source terms for the axion and dilaton and employ the full Kerr Green's function to obtain our

solution. Only as a final step will we resort to the slow rotation limit in order to obtain closed form analytical solution.

The exterior field strength hair exhibited in these solutions is classical "secondary" hair. It is secondary¹³, in that it is generated by a non-minimal coupling to a primary field strength (gravitational or electromagnetic) that is forced to be present by gauge invariance. As such it does not represent a new quantum number associated with the black hole. On the other hand it is a classical, coherent, macroscopic, field strength, which might have detectable, macroscopic effects in astrophysical situations. And it is created by couplings which are model independent consequences of any four-dimensional string theory. As such it represents a real, model-independent, potentially experimentally testable prediction of string theory, and so is clearly of considerable interest.

Finally, as we have pointed out previously⁹, it is possible for standard model particles to pick up non-minimal higher dimensional effective interactions from quantum corrections, which are of such a form that these fields appear as secondary hair on black holes. We gave the example of the neutral pion, whose one-loop anomalous coupling to $F\tilde{F}$ (responsible for its experimentally observed $\gamma\gamma$ decay mode), and $R\tilde{R}$, mean that external field strength pion hair will be produced around dyonic, or rotating, black holes. This secondary hair is semiclassical, as the coupling inducing it appears as an order $\hbar$ radiative correction. One expects¹³ that the zoo of higher dimension operators induced by quantum corrections to the effective action for standard model fields, will result in many varieties of semiclassical, secondary, standard model "hair".

Our starting point for the calculation of axion and dilaton hair is the low-energy string Lagrangian⁴)

$$S = \int d^4x \sqrt{g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{6} e^{-2\sqrt{2}\kappa\phi} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{1}{2} (\partial_\mu \phi)^2 \right. \\ \left. + \frac{\alpha'}{16\kappa^2} e^{-\sqrt{2}\kappa\phi} (\hat{R}^2 - F_{\mu\nu} F^{\mu\nu}) \right\} \quad (1)$$

where the 2-point action (1) has been expanded in the string tension to $O(\alpha')$. $H_{\mu\nu\lambda}$ are the components of the 3-form

$$H_{\mu\nu\lambda} = \partial_\lambda B_{\mu\nu} + \frac{\alpha'}{8\kappa} (\Omega_{3L\mu\nu\lambda} - \Omega_{3Y\mu\nu\lambda}) \quad (2)$$

where $B_{\mu\nu}$ is the antisymmetric field in the gravitational multiplet and the gauge and Lorentz Chern-Simons terms are given by

$$\Omega_{3Y\mu\nu\lambda} = \frac{1}{2} \text{Tr}(A_\lambda F_{\mu\nu}) - \frac{2}{3} A_{[\mu} A_\nu A_{\lambda]} \quad (3a)$$

$$\Omega_{3L\mu\nu\lambda} = \frac{1}{2} \text{Tr}(\omega_\lambda R_{\mu\nu}) - \frac{2}{3} \omega_{[\mu} \omega_\nu \omega_{\lambda]} \quad (3b)$$

where the trace is taken over the (suppressed) gauge and Lorentz indices and the spin connection $\omega^{ab}_\mu = -e^b_\alpha e^a_{\beta\mu}$ with vierbein e^a_μ . Note that the H^2 term in (1) already contains terms of $O(\alpha')$. In the action (1) ϕ is the dilaton and the Gauss-Bonnet combination is given by $\hat{R}^2 = R^{abcd} R_{abcd} - 4 R_{ab} R^{ab} + R^2$. The Kerr-Newmann solution is

$$ds^2 = \left(\frac{r^2 + A^2 \cos^2 \theta - 2GMr + Gq^2}{r^2 + A^2 \cos^2 \theta} \right) dt^2 - \left(\frac{r^2 + A^2 \cos^2 \theta}{r^2 + A^2 - 2GMr + Gq^2} \right) dr^2 \\ - \left(\frac{[(r^2 + A^2)^2 - A^2 \sin^2 \theta (r^2 + A^2 - 2GMr + Gq^2)] \sin^2 \theta}{r^2 + A^2 \cos^2 \theta} \right) d\phi^2 \\ - \left(\frac{2A \sin^2 \theta (2GMr - Gq^2)}{r^2 + A^2 \cos^2 \theta} \right) dt d\phi - (r^2 + A^2 \cos^2 \theta) d\theta^2 \quad (4a)$$

and

$$A_\mu = \left\{ \left(\frac{Q_e R}{r^2 + A^2 \cos^2 \theta} \right), 0, 0, -\left(\frac{Q_e R A \sin^2 \theta}{r^2 + A^2 \cos^2 \theta} \right) \right\} \quad (4b)$$

which represents a background spacetime which satisfies the equations of motion to leading order in α' and about which we will do our expansion in $O(\alpha')$ corrections to determine the leading string modifications to the structure of black holes. In the metric (4), $q^2 = q_e^2 + q_m^2$ representing both electric and possible magnetic charges and A is angular momentum per unit mass. The vector potential A_μ has been shown only in terms of the electric charge, constructing $F_{\mu\nu}$ for both electric and magnetic charges is done in the standard way.

Note that because the gauge kinetic term in Eq. (1) already contains a factor α' , the physical charge q^2 in the metric should also be scaled as α' ; $q^2 = (\alpha'/4\kappa^2) Q^2$, where Q^2 is the charge which will define $F_{\mu\nu}$. It is clear therefore that in the limit $\alpha' \rightarrow 0$, with the charge Q fixed, $O(\alpha'^0)$ expansion), we are left only with the Kerr solution, ($q^2=0$). In this sense, the consistent $O(\alpha')$ expansion of the Kerr-Newmann solution in string theory can be viewed as an $O(\alpha')$ correction to Kerr. Similarly in the absence of rotation, the Reiser-Nordstrom solution should be viewed as an $O(\alpha')$ correction to Schwarzschild.

As we will see below, the $O(\alpha')$ solutions for the axion, a , and dilaton, ϕ , will be determined by sources which are necessarily at least $O(\alpha')$ so any

$O(\alpha')$ corrections to $g_{\mu\nu}$ will not affect these solutions. Therefore we will be justified in using the Kerr metric for our computations of axion and dilaton hair to leading order. Furthermore, it will be possible to show that the only $O(\alpha')$ corrections to $g_{\mu\nu}$ are those involving q^2 , making the metric in Eq. (2) the complete metric through $O(\alpha')$ in the string expansion.

Let us first consider the equation for the 3-form. Actually, it will be advantageous to re-express the action in terms of the dual of H , $Y = *H$

$$Y_\sigma = \sqrt{g} \epsilon_{\mu\nu\rho\sigma} H^{\mu\nu\rho} \quad (5)$$

From Eq. (3), we have the Bianchi identity

$$dH = \frac{\alpha'}{8\kappa} \text{Tr}(R \wedge R - F \wedge F) \quad (6)$$

which we can write as

$$(\sqrt{g} Y^\rho)_{,\rho} = -\left(\frac{\alpha'}{8\kappa}\right) \frac{\epsilon^{\mu\nu\lambda\sigma}}{4} (R_{\alpha\beta\mu\nu} R_{\lambda\sigma}{}^{\alpha\beta} + F_{\mu\nu} F_{\lambda\sigma}) \quad (7)$$

Using the constraint (6), we can rewrite the action in terms of the truly dynamical axion mode of the theory as

$$S = \int d^4x \sqrt{g} \left\{ \frac{1}{2\kappa^2} R - \frac{1}{4} e^2 \sqrt{2} \kappa \phi (\partial_\mu b) (\partial^\mu b) - \frac{1}{2} (\partial_\mu \phi) (\partial^\mu \phi) \right. \\ \left. + \frac{\alpha'}{16\kappa^2} \phi \sqrt{2} \kappa (\hat{R}^2 - F_{\mu\nu} F^{\mu\nu}) - b \left(\frac{\alpha'}{8\kappa} \right) \frac{\epsilon^{\mu\nu\rho\lambda}}{4! \sqrt{g}} (R_{\alpha\beta\mu\nu} R_{\rho\lambda}{}^{\alpha\beta} + F_{\mu\nu} F_{\rho\lambda}) \right\} \quad (8)$$

where $Y^\rho = -3e^2 \sqrt{2} \kappa \phi (\partial^\rho b)$. This leads to the axion equation of motion

$$\square a = \left(\frac{\alpha'}{8\kappa} \right) \frac{\sqrt{2} \epsilon^{\mu\nu\rho\lambda}}{4! \sqrt{g}} (R_{\alpha\beta\mu\nu} R_{\rho\lambda}{}^{\alpha\beta} + F_{\mu\nu} F_{\rho\lambda}) \quad (9)$$

where a is the rescaled axion which approximates the one with canonical kinetic term, $\partial P_a = e^2 \sqrt{2} \kappa \phi (\partial^\rho b) / \sqrt{2}$. (Because we are anticipating that the dilaton $\phi \propto \alpha'$ we neglect the exponentials in writing Eq. (9).)

The dilaton equation of motion is now rather easily obtained from the action (8) and the axion field redefinition

$$\square \phi = \frac{\alpha' \sqrt{2}}{16\kappa} \phi \sqrt{2} \kappa (\hat{R}^2 - F_{\mu\nu} F^{\mu\nu}) + \sqrt{2} \kappa e^{-2\sqrt{2} \kappa \phi} (\partial_\mu a) (\partial^\mu a) \quad (10)$$

Noting that the solution for a is already of order α' , and $(\partial_\mu a) (\partial^\mu a) \propto \alpha'^2$ thus this term may be dropped, and expanding the exponential in the dilaton as before and keeping terms of order α' we arrive at

$$\square \phi = \frac{\alpha' \sqrt{2}}{16\kappa} (\hat{R}^2 - F_{\mu\nu} F^{\mu\nu}) \quad (11)$$

Note that both the axion and dilaton are first non-trivial at order α' .

We can now examine the graviton equation of motion dropping all terms which are manifestly of higher order in α' , i.e. terms such as $\alpha' (\partial_\mu \phi)$, $\alpha' (\partial_\mu a)$, $(\partial_\mu \phi) (\partial^\mu \phi)$ or $(\partial_\mu a) (\partial^\mu a)$. The field equations then become

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R = \frac{\alpha'}{8} \left[\frac{1}{2} g^{\mu\nu} \hat{R}^2 - 2R^{\mu\nu} - 2R^{\lambda\rho\sigma} R^{\nu}_{\lambda\rho\sigma} + 4R^{\mu\sigma} R^{\nu}_{\sigma} + 4R_{\alpha\beta} R^{\mu\alpha\nu\beta} \right] + \frac{\alpha'}{4} (F^{\mu\lambda} F^{\nu\lambda} - \frac{1}{4} g^{\mu\nu} F_{\alpha\beta} F^{\alpha\beta}) \quad (12)$$

The first term (in brackets) on the right-hand side vanishes identically in four dimensions (because the Gauss-Bonnet combination in $D=4$ appears only as a boundary term). Now one can see clearly that the only corrections to $g_{\mu\nu}$ at $O(\alpha')$ are those in the gauge sector thus justifying our earlier claims. The gauge sector contributions appear as $O(\alpha')$ contributions to the gravitational equations. But we recall that the q^2 charge explicitly appearing in the Kerr-Newman metric is $q^2 = (\alpha'/4\kappa^2) \times e^{-\sqrt{2}\kappa\phi} Q^2$ where Q^2 is the charge defining the gauge kinetic term. So for fixed Q^2 (with κ and the asymptotic value of ϕ held fixed), in the limit of α' small where we know that our expansion is reliable, we have that q^2 must also be small, i.e. to ensure that the $O(\alpha')$ expansion is reliable we must be working in the small physical charge limit of the Kerr-Newman solution. Since we are consistently working perturbatively in the physical charge, our solutions do not extend to the limit of extreme black hole charge, where the solutions of refs. (5 and 8) exhibit new features, but where they are subject¹⁴ to large corrections from higher order terms in the string effective action.

Refs. (5 and 8) exhibit a family of exact charged black hole solutions to a Lagrangian of the form of Eq. (1) (but without the Gauss-Bonnet terms), which in the small charge limit reduce to Reissner-Nordstrom, in agreement with our considerations. Their solutions show important deviations from the Reissner-Nordstrom metric for large charge (the extreme charge limit) but in this region there is in string theory no longer

any justification for dropping higher terms in the α' expansion in the effective action¹⁴. So it is not clear to what degree the interesting metric^{5,8} and thermodynamic¹⁴ properties of their solutions, in that limit, represent the true nature of string theory black holes with large charges.

With some effort it is straightforward to calculate the source terms for the axion and dilaton. For the axion source,

$$\frac{g_{\mu\nu\rho\lambda}}{4\sqrt{g}} (R_{ab\mu\nu} R^{\lambda ba} + F_{\mu\nu} F_{\rho\lambda}) = \frac{8G^2 M^2 r A \cos\theta (3r^2 - A^2 \cos^2\theta)(r^2 - 3A^2 \cos^2\theta)}{(r^2 + A^2 \cos^2\theta)^6} + \frac{Q_e Q_m ((r^2 - A^2 \cos^2\theta)^2 - 4r^2 A^2 \cos^2\theta)}{3(r^2 + A^2 \cos^2\theta)^4} + \frac{2(Q_e^2 - Q_m^2) r A (r^2 - A^2 \cos^2\theta) \cos\theta}{3(r^2 + A^2 \cos^2\theta)^4} \quad (13)$$

while the dilaton source is given by

$$\begin{aligned} (\hat{R}^2 - F_{\mu\nu} F^{\mu\nu}) = & \frac{48G^2 M^2 (r^2 - A^2 \cos^2\theta)((r^2 + A^2 \cos^2\theta)^2 - 16r^2 A^2 \cos^2\theta)}{(r^2 + A^2 \cos^2\theta)^6} \\ & + \frac{2(Q_e^2 - Q_m^2)((r^2 - A^2 \cos^2\theta)^2 - 4r^2 A^2 \cos^2\theta)}{(r^2 + A^2 \cos^2\theta)^4} \\ & - \frac{16Q_e Q_m r A (r^2 - A^2 \cos^2\theta) \cos\theta}{(r^2 + A^2 \cos^2\theta)^4} \end{aligned} \quad (14)$$

To solve these equations of motion, we construct the static Green's function satisfying

$$\nabla^2 G(x,y) = - \frac{\delta^3(x-y)}{\sqrt{g(y)}} \quad (15)$$

Then

$$a,\phi(x) = - \int d^3y \sqrt{g(y)} G(x,y) J_{a,\phi}(y) \quad (16)$$

The solution to (15) has been given in Ref. 15.

$$\begin{aligned} & \frac{1}{(r^2 + A^2 \cos^2 \theta) \partial r} ((r^2 - 2GMr + A^2) \frac{\partial G}{\partial r} + \frac{1}{(r^2 + A^2 \cos^2 \theta) \sin \theta} \frac{\partial}{\partial \theta} (\sin \theta \frac{\partial G}{\partial \theta})) \\ & = - \frac{\delta(r-b) \delta(\cos \theta_0 - 1) \delta(\phi_0)}{(r^2 + A^2 \cos^2 \theta)} \quad (17) \end{aligned}$$

We have previously solved this system to $O(A)$, $q^2=0$ for the axion equation of motion.⁷⁾ But since the first non-trivial contribution to the dilaton source is of $O(A^2)$ we must modify slightly our previous procedure. (At this point we have made no approximation such as small A or small q , other than the expansion in α' .)

Solutions to the homogeneous equation ∇^2 can be written in terms of spherical harmonics

$$G(r,\theta) = \sum R_l(r) P_l(\cos \theta) \quad (18)$$

where

$$\frac{\partial}{\partial r} ((r^2 - 2GMr + A^2) \frac{\partial R_l}{\partial r}) - l(l+1)R_l = 0 \quad (19)$$

With a suitable change of variables ($z = (r - GM)/\sqrt{G^2 M^2 - A^2}$), one can recognize Eq.(24) as Legendre's equation, so that solutions to Eq.(15) become

$$\begin{aligned} G(r,\theta) &= \sum C_l P_l((b-GM)/\sqrt{G^2 M^2 - A^2}) Q_l(z) P_l(\cos \theta) \quad (r > b) \\ &= \sum C_l Q_l((b-GM)/\sqrt{G^2 M^2 - A^2}) P_l(z) P_l(\cos \theta) \quad (r < b) \quad (20) \end{aligned}$$

with $C_l = (2l+1)/4\pi(\sqrt{G^2 M^2 - A^2})$.

In terms of this Green's function, the classical external axion and dilaton field is

$$\begin{aligned} a,\phi &= - \int_{r_H}^{\infty} db \int_{\theta_0}^{\pi} d\phi_0 (b^2 + A^2 \cos^2 \theta_0) \sin \theta_0 \\ &\quad \times G(r,\theta,\phi,b,\theta_0,\phi_0) I(b,\theta_0,\phi_0) \quad (21) \end{aligned}$$

where $r_H = GM + \sqrt{G^2 M^2 - A^2}$ and $G(r,\theta,\phi,b,\theta_0,\phi_0)$ differs from Eq.(20) only in that $P_l(\cos \theta)$ becomes $P_l(\lambda)$ with $\lambda(\theta,\phi) = \cos \theta \cos \theta_0 + \sin \theta \sin \theta_0 \cos(\phi - \phi_0)$.

To facilitate the computation (the integration of the Green's function given below over the source), we expand Eqs. (13) and (14) to $O(A^2)$ which gives

$$a = \frac{\alpha' \sqrt{2}}{8\kappa} \left[\frac{24G^2 M^2 A \cos \theta}{r^7} + \frac{Q_e Q_m (r^2 - 10A^2 \cos^2 \theta)}{3r^6} + \frac{2(Q_e^2 - Q_m^2) A \cos \theta}{3r^5} \right] \quad (22)$$

$$\phi = \frac{\alpha' \sqrt{2}}{16\kappa} \left[\frac{48G^2 M^2 (r^2 - 21A^2 \cos^2 \theta)}{r^8} + \frac{2(Q_e^2 - Q_m^2) (r^2 - 10A^2 \cos^2 \theta)}{r^6} - \frac{16Q_e Q_m A \cos \theta}{r^5} \right] \quad (23)$$

(We have corrected here a factor of $\sqrt{2}$ error in previous work^{7,9)} in the axion source as well as included the factor $(\alpha'/8\kappa)$ as the coupling of the Chern-Simons term; the factor $\sqrt{2}$ is due to our definition of a .)

Integrating the source terms (13) and (14) over the Green's functions defined above, we find the following solutions for classical axion and dilaton hair around a slowly rotating, weakly charged, string black hole:

$$a(r, \theta) = \frac{\alpha'\sqrt{2}}{8\kappa} \left\{ \left[-\frac{Q_c Q_m}{6GMr} \left(1 - \frac{A^2}{3r^2} + \dots \right) \right] P_0(\cos\theta) + \left[\frac{A}{12GMr^2} \left(-\frac{5}{2} \left(1 + \frac{2GM}{r} + \frac{18G^2M^2}{5r^2} \right) - (Q_c^2 - Q_m^2) \right) + \dots \right] P_1(\cos\theta) + \left[\frac{Q_c Q_m A^2}{9GMr^3} + \dots \right] P_2(\cos\theta) + \dots \right\} \quad (24)$$

$$\phi(r, \theta) = \frac{\alpha'\sqrt{2}}{16\kappa} \left\{ \left[\frac{-1}{GMr} \left(2 \left(1 + \frac{GM}{r} + \frac{4G^2M^2}{3r^2} \right) + (Q_c^2 - Q_m^2) \right) + A^2 \left(\frac{1}{2G^3M^3r} \left(1 + \frac{GM}{r} + \frac{8G^2M^2}{3r^2} + \frac{6G^3M^3}{r^3} + \frac{64G^4M^4}{5r^4} \right) + \frac{Q_c^2 - Q_m^2}{3GMr^3} \right) + \dots \right] P_0(\cos\theta) + \left[\frac{2Q_c Q_m A}{GMr^2} + \dots \right] P_1(\cos\theta) + \left[\frac{2A^2}{3GMr^3} \left(\frac{14}{5} \left(1 + \frac{3GM}{r} + \frac{48G^2M^2}{7r^2} \right) + (Q_c^2 - Q_m^2) \right) + \dots \right] P_2(\cos\theta) + \dots \right\} \quad (25)$$

and to the order in α' to which we work the metric background solution is (weakly charged) Kerr-Newman.

The form of the axion and dilaton hair seen in the solutions reproduces (in the appropriate limits) the behaviors seen previously in special cases: axion hair with a dipole piece from rotation,⁷⁾ and a monopole piece as the product of electric and magnetic charges,^{9,10)} dilaton monopole hair from the Gauss-Bonnet⁶⁾ and gauge kinetic terms.^{5,8)} The general solutions have new structure as well, including cross term contributions in partial waves that had not previously supported field strengths, and which only appear in the complete $O(\alpha')$ calculation.

The form of the solutions that we give holds for massless axions and dilatons of the string (tree level) effective action. In the full quantum string theory it is likely that non-perturbative quantum dynamics generates a mass for both the axion and dilaton^{1).} For black holes (and distances from the black holes) small compared to the Compton wavelengths of the axion and dilaton, our solutions should remain a good approximation to the form of the hair produced. On the other hand, for black holes large on the scale of the Compton wavelength of the axion and dilaton (so their propagation is effectively local) the exterior field strength for these fields will be proportional to the local source strength Eqs. (13) and (14) at a given location.

In summary, we have given the consistent $O(\alpha')$ computation of the exterior field strengths, including the metric, for (weakly) charged rotating black holes in string theory. These results generalize all previous treatments as both dilaton and axion solutions are derived in a charged and rotating background. We set up the general solutions for arbitrary rotation, but give analytic results in the slow rotating limit. The existence, and structure, of the axion and dilaton hair produced represents a model independent prediction of string gravity, which was not present in Einstein

gravity. Hopefully, our knowledge of astrophysics will soon progress to the point that these string predictions may be put to experimental test!

The work of BAC was supported in part by the Natural Sciences and Engineering Research Council of Canada. The work of NK and KAO was supported in part by DOE grant DE-AC02-83ER-40105. In addition the work of KAO was also supported in part by a Presidential Young Investigator Award. The work of NK was supported in part by the University of Minnesota Doctoral Dissertation Fellowship.

References

- 1) For reviews, see M. Green, J. Schwarz and E. Witten, *Superstring Theory*, (Cambridge U.P., Cambridge, 1987); D. Lust and S. Theisen, *Lectures on String Theory*, (Springer, Berlin, 1989).
- 2) W. Israel, *Phys. Rev.* **164** (1967) 1776; B. Carter, *Phys. Rev. Lett.* **26** (1971) 331; R. Price, *Phys. Rev. D* **5** (1972) 2419 and 2439; D.C. Robinson, *Phys. Rev. Lett.* **34** (1975) 905; for a review, see: B. Carter in *General Relativity, an Einstein Centenary Survey*, ed. S. W. Hawking and W. Israel (Cambridge Univ. Press, Cambridge 1979); W. Israel, *Comm. Math. Phys.* **8** (1968) 245; R.M. Wald, *Phys. Rev. Lett.* **26** (1971) 1653; E.D. Fackerell and J.R. Ipser, *Phys. Rev. D* **5** (1972) 2455.
- 3) C. Lovelace, *Phys. Lett.* **B135** (1984) 75; E.S. Fradkin and A.A. Tseytlin, *Phys. Lett.* **B158** (1985) 316; C.G. Callan, D. Friedan, E.J. Martinec and M. J. Perry, *Nucl. Phys.* **B262** (1985) 593; A. Sen, *Phys. Rev. D* **32** (1985) 2102; and *Phys. Rev. Lett.* **55** (1985) 1846; C.G. Callan, I.R. Klebanov and M.J. Perry, *Nucl. Phys.* **B278** (1986) 78; S. Deser and N. Redlich, *Phys. Lett.* **B176** (1986) 350; D. Lust, S. Theisen and G. Zoupanos, *Nucl. Phys.* **B296** (1988) 800; J. Lauer, D. Lust and S. Theisen, *Nucl. Phys.* **B304** (1988) 236.
- 4) D. Gross and J. H. Sloan, *Nucl. Phys.* **B221** (1987) 41.
- 5) G.W. Gibbons, *Nucl. Phys.* **B207** (1982) 337; G.W. Gibbons and K. Maeda, *Nucl. Phys.* **B298** (1988) 741.
- 6) C.G. Callan, R.C. Myers and M.J. Perry, *Nucl. Phys.* **B311** (1988/89) 673.
- 7) B. Campbell, M. Duncan, N. Kaloper and K.A. Olive, *Phys. Lett.* **B251** (1990) 34.
- 8) D. Garfinkle, G.T. Horowitz and A. Strominger, *Phys. Rev. D* **43** (1991) 3140.
- 9) B. Campbell, N. Kaloper and K.A. Olive, *Phys. Lett.* **B263** (1991) 364.
- 10) K.-Y. Lee and E. Weinberg, Columbia preprint CU-TP-515, 1991.
- 11) A. Shapere, S. Trivedi and F. Wilczek, *Mod. Phys. Lett.* **A6** (1991) 2677.

- 12) M. K. Gailard and B. Zumino, Nucl. Phys. **B123** (1981) 221.
- 13) S. Coleman, J. Preskill, and F. Wilczek, Mod. Phys. Lett. **A6** (1991) 1631.
- 14) J. Preskill, P. Schwarz, A. Shapere, S. Trivedi and F. Wilczek, Mod. Phys. Lett. **A6** (1991) 2353.
- 15) D. R. Brill, P. L. Chrzanowski, C. M. Pereira, E. D. Fackerell, and J. R. Ipser, Phys. Rev. **D5** (1972) 1913.