Universal Seesaw and Radiative Quark Mass Hierarchy

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A new approach to the fermion mass problem is suggested. The mass hierarchy is first radiatively generated in a sector of heavy isosinglet fermions and then projected to the visible ones by means of a universal seesaw. In this way flavour-changing phenomena are naturally suppressed. A quantitatively correct picture of quark masses and mixing is obtained. In particular, in contrast to previous approaches, the correct value of the Cabibbo angle can be accommodated without troubles for the perturbation expansion. By fitting the values of the masses of the five known quarks, we find that the natural value of m_t is in the 100 GeV range.

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1. The observed hierarchy of fermion masses is about 1–2 orders of magnitude between neighbouring families [1]. This feature renders attractive the idea that radiative corrections may be responsible for mass generation [2–4]. This means that the masses of the light fermions arise as a radiative effect of the tree-level masses of heavy fermions. In theories with spontaneously broken gauge symmetry, the vanishing of certain mass terms at the tree-level can occur as a consequence of the representation content of the fields in the theory [3]. These mass terms then appear in the effective action through operators of dimension > 4 (i.e. involving more than 1 scalar field) and are thereby finite. In Ref. [5] a scheme was proposed which gave a qualitative picture of the mass hierarchy among the quark families. The model of Ref. [5] had a field content such that:

- i) only one family (namely the third) becomes massive at the tree level;
- ii) the chiral symmetries of the 1st and 2nd generations are broken by Yukawa couplings to coloured scalar fields. The loop effects due to these couplings are responsible for the “propagation of mass” to the lighter families. The 2nd becomes massive at the 1-loop level and the 1st only at 2 loops. Then the mass hierarchy between families is given by the successive powers of a perturbative expansion parameter $\epsilon \simeq 10^{-1}$ – 10^{-2} , and is expected to be $\epsilon^2 : \epsilon : 1$.

Within this scheme, however, it is rather difficult to obtain a quantitatively correct picture. Depending on the weak-isospin properties of the coloured scalar involved in mass generation, different problems arise. If the scalar field is an isosinglet [5,6], then mass generation follows an isospin flip mechanism: at 1 loop the c quark mass m_c arises out of a graph with a b quark while m_s arises from a graph with the top quark. Then in general m_c is proportional to m_b and m_s to m_t , in contradiction with experimental values (see the discussion in Ref. [7]). When the coloured scalar is an isotriplet, a model can be built [7] in which the mass of each quark is given by a graph with the heavier quark of the same charge, according to the chain: $m_t \rightarrow m_c \rightarrow m_u$, $m_b \rightarrow m_s \rightarrow m_d$. In this way, a realistic mass pattern can be obtained in a natural way. The weak interactions of the triplet scalar, however, give rise to flavour-changing couplings of the Z^0 which are in general incompatible with experimental bounds. The arising of FCNCs in the model of Ref. [7] is essentially related to the fact that the effective low-energy theory, resulting from integrating out the “flavour” interactions, is not quite the standard model. In this effective theory, at the electroweak scale, fermion masses are determined by $d \geq 5$ operators involving scalars in non trivial representations of $SU(2)_L \otimes U(1)_Y$: in order to obtain finite mass terms the physics of flavour should not decouple totally. Thus FCNCs are unavoidable.

In addition a general difficulty of the class of models sharing properties (i) and (ii) should be stressed. As was noticed in Ref. [8], these models lead generically to too small a Cabibbo angle θ_C . The correct value of θ_C implies a choice of parameters which puts perturbation theory (and the idea of radiative mass generation itself) in serious troubles.

In the present paper we propose a new approach to the problem of radiative

mass generation which avoids the difficulties discussed above. We suggest that the mass hierarchy is radiatively generated in a hidden heavy fermion sector and then projected, by means of a universal seesaw mechanism [9,10], to the visible quark sector. In such a way the effective low-energy theory, after integrating out the heavy sector, is simply the standard model with definite Yukawa couplings. Flavour changing phenomena are thus kept under control [11]. The simplest way to realize this idea is to suppose the existence of heavy isosinglet fermion families in a one-to-one correspondence with the light ones. This means that the mass of any observable fermion will be proportional to the inverse mass of its heavy partner [9]. This feature is also attractive for the following reason. We experimentally observe that the mass splitting within the lightest family is small: $m_d/m_u \simeq 1.5\text{--}2$. We further observe an increasing splitting from family to family, with the up-type quark masses growing faster: $m_s/m_c \simeq 0.1\text{--}0.2$, $m_b(m_t)/m_t < 0.03$. This pattern could find a natural explanation in the inverted hierarchy model we suggest. The lightest quarks u and d are the light partners of the two heavy fermions U and D which have tree-level masses. The *small* splitting among these two can be related to the spontaneous breakdown of an up-down symmetry⁽¹⁾. After that, radiative mass generation proceeds, following the chain $M_U \rightarrow M_C \rightarrow M_T$, $M_D \rightarrow M_S \rightarrow M_B$ (as in Ref. [7]), where $(M_T, M_B, \dots)$ denote the masses of the heavy partners of usual quarks $(m_t, m_b, \dots)$. Owing to the different perturbation expansion parameters ϵ_u and ϵ_d in the up and down sector of heavy fermions, the splitting within each family becomes larger and larger. At the end (at 2-loop order) we have the following “product rule”

$$\frac{M_B/M_T}{M_S/M_C} = \frac{M_S/M_C}{M_D/M_U} = \frac{\epsilon_u}{\epsilon_d}. \quad (1)$$

Because of the seesaw mechanism, the pattern of eq. (1) will be reproduced in general by the light-quark masses (with some corrections that we shall explain below).

Moreover, this approach can accommodate the correct Cabibbo angle without affecting the validity of perturbation theory.

This paper is organized as follows. In section 2 we discuss rather generally the mass matrices arising from universal seesaw and radiative corrections. In section 3 we sketch the building of a simple model which implements the above ideas. Finally in section 4 we draw our conclusions.

2. Let us consider, following the tradition [5–7,9], a model based on the gauge group $G_{LR} = SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [12]. We take the quarks in the

⁽¹⁾ In models with direct hierarchy [5,6] a tree-level *large* splitting between top and bottom is in general introduced by hand.

following representations of the gauge group⁽²⁾

$$\begin{aligned}
q_{Li} &= \begin{pmatrix} u_i \\ d_i \end{pmatrix}_L = (1/2, 0, 1/3) & q_{Ri} &= \begin{pmatrix} u_i \\ d_i \end{pmatrix}_R = (0, 1/2, 1/3) \\
U_{Li} &= (0, 0, 4/3) & U_{Ri} &= (0, 0, 4/3) \\
D_{Li} &= (0, 0, -2/3) & D_{Ri} &= (0, 0, -2/3)
\end{aligned} \tag{2}$$

where i is the family index (the indices of colour $SU(3)$ are omitted); $q_{L,R}$ are the usual quarks, while $Q = U, D$ represent their vectorlike heavy partners [9] (in what follows we call them Q -fermions). In the Higgs sector we introduce a $SU(2)_L$ doublet $H_L = (1/2, 0, 1)$ and a $SU(2)_R$ doublet $H_R = (0, 1/2, 1)$. The most general Yukawa and mass terms are then

$$\begin{aligned}
\mathcal{L}_{\text{mass}} &= \Gamma_{u,L}^{ij} \bar{q}_{Li} \tilde{H}_L U_{Rj} + \Gamma_{u,R}^{ij} \bar{q}_{Ri} \tilde{H}_R U_{Lj} + M_u^{ij} \bar{U}_{Li} U_{Rj} + \text{h.c.} \\
&+ \Gamma_{d,L}^{ij} \bar{q}_{Li} H_L D_{Rj} + \Gamma_{d,R}^{ij} \bar{q}_{Ri} H_R D_{Lj} + M_d^{ij} \bar{D}_{Li} D_{Rj} + \text{h.c.}
\end{aligned} \tag{3}$$

where $\tilde{H} = \epsilon H^*$. The Γ 's are Yukawa couplings and the M 's are bare masses. Because of the Higgs field content there are no direct $\bar{q}q\phi$ Yukawa terms. After spontaneous symmetry breaking

$$H_L = (0, v_L) \quad H_R = (0, v_R) \quad v_R \gg v_L \tag{4}$$

the mass term out of eq. (3) is written in block form (for up-type quarks)

$$(\bar{u}, \bar{U})_L \begin{pmatrix} 0 & v_L \Gamma_{u,L} \\ v_R \Gamma_{u,R}^\dagger & M_u \end{pmatrix} \begin{pmatrix} u \\ U \end{pmatrix}_R \tag{5}$$

and analogously for the down-type quarks. When $M_u \gg v_L, v_R$, the resulting mass matrix for the light states is given by the seesaw formula

$$\mathcal{M}_{\text{light}}^u = v_L v_R \Gamma_{u,L} M_u^{-1} \Gamma_{u,R}^\dagger. \tag{6}$$

In what follows we do not make any particular assumption on the structure of the Yukawa coupling matrices $\Gamma_{u,L}$ and $\Gamma_{u,R}$. We only suppose that their typical order of magnitude is 1, as much as a gauge coupling. Notice that the magnitude of the Γ_L^{ij} sets the upper limit of the light-quark masses. Is is obvious that, for generic Γ 's, when M_u has a hierarchical spectrum, an analogous hierarchy will appear in the eigenvalues of $\mathcal{M}_{\text{light}}^u$. In order to be definite let us suppose that, for some reason, the Yukawa coupling matrices are the same in the up- and down-quark sector:

$$\Gamma_{u,L}^{ij} = \Gamma_{d,L}^{ij} = \Gamma_L^{ij} \quad \Gamma_{u,R}^{ij} = \Gamma_{d,R}^{ij} = \Gamma_R^{ij} \tag{7}$$

⁽²⁾ The inclusion of leptons is straightforward.

Let us also suppose that the heavy fermion matrices M_u and M_d are generated by radiative effects with a mechanism like that of Ref. [7], and are thus “aligned” in flavour space:

$$M^u = M_U (\hat{P}_1 + \epsilon_u \lambda^\dagger_L \hat{P}_1 \lambda_R + \beta_u \lambda^\dagger_L \lambda_L \hat{P}_1 \lambda^\dagger_R \lambda_R + \dots) \quad (8a)$$

$$M^d = M_D (\hat{P}_1 + \epsilon_d \lambda^\dagger_L \hat{P}_1 \lambda_R + \beta_d \lambda^\dagger_L \lambda_L \hat{P}_1 \lambda^\dagger_R \lambda_R + \dots) \quad (8b)$$

where $\hat{P}_1$ is a 1-dimensional projector, $M_{U,D}$ are the tree-level masses of the heaviest generation, and the λ 's are Yukawa coupling matrices of the fermions to the scalars involved in mass generation [7]. The $\epsilon_{u,d}$ are expansion parameters and $\beta_{u,d} \simeq C \epsilon_{u,d}^2/2$, with C a number close to 1. The mass eigenvalues of matrices (8) verify the product rule of eq. (1) up to $O(\epsilon)$ corrections [7]. The light-fermion masses are easily evaluated by considering the inverse mass matrix (6)

$$(\mathcal{M}_{\text{light}}^u)^{-1} = \frac{M_U}{v_L v_R} (\Gamma_R^\dagger)^{-1} (\hat{P}_1 + \epsilon_u \lambda^\dagger_L \hat{P}_1 \lambda_R + \beta_u \lambda^\dagger_L \lambda_L \hat{P}_1 \lambda^\dagger_R \lambda_R + \dots) (\Gamma_L)^{-1} \quad (9)$$

which has the same form as (8). Thus the product rule (1) is verified at the lowest order in ϵ by the inverse light masses m_i^{-1} , i.e. by the masses themselves:

$$\frac{m_t}{m_b} \simeq \left(\frac{m_c}{m_s} \right)^2 \frac{m_d}{m_u} = R. \quad (10)$$

Here the masses are taken at the typical scale Λ_F at which the heavy fields are integrated out, $\Lambda_F \gg v_L$ ⁽³⁾.

The mixing angles arising from matrix (9), and from its analogue for d quarks, are expected to be of the order of mass ratios [5–7]. The Cabibbo–Kobayashi–Maskawa mixings are essentially determined by the down-quark mass matrix since $\epsilon_d \gg \epsilon_u$. Thus one has roughly $V_{us} \sim m_d/m_s$, $V_{cb} \sim m_s/m_b$ and $V_{ub} \sim m_d/m_b$. Notice that for V_{cb} and V_{ub} the naïve expectations quoted above are consistent with their experimental values. On the other hand the measured value of the Cabibbo angle ($\theta_C \simeq V_{us} = 0.22$) is a bit larger than its “radiative” value $V_{us} \sim m_d/m_s$. In fact, it was shown in Ref. [8] that by considering an ansatz similar to eq. (8) for light-quark mass matrices, the correct θ_C implies for some entry $|\epsilon_d \lambda_{ij}|^2 \simeq 1$, causing serious difficulties to perturbation theory and to the idea of radiative mass generation itself. In our case, however, the situation is rather different: the large size of θ_C can be attributed to the “relative” fluctuations (indeed not large) of the entries in $\Gamma_{L,R}$ in eq. (9), without affecting the perturbative expansion in λ . In order to see this, let us consider for simplicity the left–right symmetric case: $\lambda_L = \lambda_R = \lambda$, $\Gamma = \Gamma_R = \Gamma_L$ ^(*). Let us also suppose $\lambda_{ij} = \lambda_{ji}$. By a suitable

⁽³⁾ The experimental data on flavour-changing phenomena ($K^0\text{--}\overline{K}^0$ mixing etc.) yield the generic bound $\Lambda_F > 10\text{--}100$ TeV.

^(*) In this case, as was noticed in Ref. [13], the strong CP problem can be solved without axion at the price of P-parity.

redefinition of the fields, we can always take $\hat{P}_1 = \text{diag}(1, 0, 0)$, $\lambda_{13} = 0$, and $\Gamma_{ij} = 0$ for $i < j$. As a further simplification let us take Γ real⁽⁴⁾. Then by observing that $\Gamma^{\dagger-1} \hat{P}_1 \Gamma^{-1} = (\Gamma_{11})^{-2} \hat{P}_1$ and by redefining $\bar{\lambda} = \Gamma \lambda \Gamma^{-1}$ we write the inverse mass matrix for d quarks as

$$(\mathcal{M}_{\text{light}}^d)^{-1} = \frac{M_D}{(\Gamma_{11}^2 v_L v_R)} \left(\hat{P}_1 + \epsilon_d \bar{\lambda}^\dagger \hat{P}_1 \bar{\lambda} + \beta_d (\bar{\lambda}^* \bar{\lambda})^\dagger \hat{P}_1 \bar{\lambda}^* \bar{\lambda} + \dots \right). \quad (11)$$

Notice that in general $\bar{\lambda}_{ij} \neq \bar{\lambda}_{ji}$. Let us take $\epsilon_d > 0$. We know [8] that the correct $\sin \theta_c$ implies $\epsilon_d |\bar{\lambda}_{11}|^2 \sim 1$. The diagonalization of (11) yields, in lowest order,

$$\begin{aligned} \frac{m_d}{m_s} &\simeq \epsilon_d |\bar{\lambda}_{12}|^2 (1 + \epsilon_d |\bar{\lambda}_{11}|^2)^{-2} & \frac{m_s}{m_b} &\simeq \frac{1}{2} C \epsilon_d |\bar{\lambda}_{23}|^2 (1 + \epsilon_d |\bar{\lambda}_{11}|^2) \\ V_{us} &\simeq \epsilon_d \bar{\lambda}_{11} \bar{\lambda}_{12}^* (1 + \epsilon_d |\bar{\lambda}_{11}|^2)^{-1} & V_{cb} &\simeq \frac{1}{2} C \epsilon_d \bar{\lambda}_{23}^* T_1 & V_{ub} &\simeq \frac{1}{2} C \epsilon_d \bar{\lambda}_{12}^* \bar{\lambda}_{23}^* T_2 \\ T_1 &= \bar{\lambda}_{22} + \bar{\lambda}_{11}^* \bar{\lambda}_{12} / \bar{\lambda}_{12}^* & T_2 &= \bar{\lambda}_{11} \bar{\lambda}_{22} - \bar{\lambda}_{12} \bar{\lambda}_{21} \end{aligned} \quad (12)$$

where terms proportional to $\epsilon_d |\bar{\lambda}_{11}|^2$ have been kept. Notice that, due to the smallness of ϵ_u , the enhancement of $\epsilon_u |\bar{\lambda}_{11}|^2$ does not essentially affect the corresponding mass ratios in the up-quark sector. By comparing (12) with the experimental values of masses and mixing angles one has

$$\begin{aligned} \epsilon_d |\bar{\lambda}_{11}|^2 &\simeq 0.7-1.0 & \epsilon_d |\bar{\lambda}_{12}|^2 &\simeq 0.15-0.2 & \epsilon_d |\bar{\lambda}_{23}|^2 &\simeq 0.05-0.07 \\ \epsilon_d |T_1|^2 &\simeq 0.15-0.25 & \epsilon_d |T_2|^2 &\simeq 0.1. \end{aligned} \quad (13)$$

Apart from $\bar{\lambda}_{11}$, the above parameters are all *small*. This feature implies that eq. (12) is indeed a good approximation. Higher-order terms affect it by at most 10–15%. It is curious to observe that, while V_{us} is proportional to the entry $\bar{\lambda}_{11}$, V_{bc} and V_{bu} are respectively proportional to the “invariants” T_1 and T_2 . When Γ mixes only the first and the second families ($\Gamma_{31} = \Gamma_{32} = 0$) $\bar{\lambda}_{11}$ may well be enhanced. However T_1 and T_2 are invariant⁽⁵⁾, so that they remain *small* as long as the λ_{ij} are *small*. Notice that, by starting with all $|\lambda_{ij}|$ nearly equal, the spread in eq. (13) only needs a fluctuation of the order of 3–5 in the magnitude of the various Γ_{ij} . This is perfectly acceptable.

The $O(\epsilon_d |\bar{\lambda}_{11}|^2)$ corrections to masses in eq. (12) affect the mass formula (10). In the general case, with a complex⁽⁶⁾ ϵ_d , we have $m_t/m_b \simeq R |1 + \epsilon_d |\bar{\lambda}_{11}|^2|^3 = \tilde{R}$. The ratio $\tilde{R}/R$ depends on the phase $\arg(\epsilon_d)$. For $-2\pi/3 \lesssim \arg(\epsilon_d) \lesssim 2\pi/3$ it is $\tilde{R}/R > 1$, while in the remaining range it is $\tilde{R}/R < 1$. The case $\arg(\epsilon_d) = 0$ is the

⁽⁴⁾ In the case of three families, Γ has six entries since it is triangular. By a suitable redefinition of the fields phases, five entries can be made real.

⁽⁵⁾ When λ is a 2×2 matrix, T_1 is invariant under $\lambda \rightarrow B^{-1} \lambda B$ if B is real and triangular: $B_{12} = 0$. T_2 is the determinant.

⁽⁶⁾ Details on this case and also on the case $\lambda_L \neq \lambda_R$ will be given elsewhere.

limiting one: $\tilde{R} \simeq 5R$. Thus, in a wide range of $\arg(\epsilon_d)$, we have $m_t/m_b = \tilde{R} \gtrsim R = 50\text{--}200$. Of course the relation $m_t/m_b = \tilde{R}$ is exact only in the seesaw limit of eq. (6). This relation implies $m_t \sim v_L$, hence we can expect that terms of order v_R/M_T are non negligible when the matrix in eq. (6) is diagonalized. Considering these terms, we get [14]

$$\frac{m_t}{m_b}(\Lambda_F) = \tilde{R} \left[1 + \left(\frac{\tilde{R}m_b(\Lambda_F)}{\Gamma_{33}^L v_L} \right)^2 \right]^{-1/2} \leq \tilde{R}. \quad (14)$$

Notice, as already stated, that the coupling Γ_{33}^L sets the upper bound of m_t . The exact seesaw is recovered when Γ_{33}^L is very large. However, as long as Γ_{33}^L is $O(1)$, the ratio $m_t/m_b(\Lambda_F)$ remains of order $\tilde{R} \gtrsim 50\text{--}100$. The evolution of masses to low energy is determined not only by QCD but also by the effective top Yukawa coupling λ_t [15]. In particular the ratio m_t/m_b does not stay constant and decreases with the scale μ according to

$$16\pi^2 \mu \frac{d}{d\mu} \ln(m_t/m_b) = 3\lambda_t^2 \quad (15)$$

while the mass ratios involving only the other four quarks stay constant with good approximation [15]. For instance, by taking true the relation of eq. (10) at the cut-off scale $\Lambda_F = 10^{10}$ GeV and letting $5 \leq (m_c/m_s) \leq 10$ (so that R varies by a factor 4), we find that the physical top mass lies in the range $140 \leq m_t \leq 240$ GeV. Here the upper bound is determined by the infrared “quasi-fixed” point in the coupled equations of λ_{top} and α_{strong} [16], and is a simple consequence of the fact that λ_{top} is large at the cut-off scale.

3. In the previous section we illustrated our basic ideas in the frame of a $G_{LR} = SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ model. However, we imposed some essential constraints by hand. Namely, we assumed that

- A. the mass matrices of heavy fermions have a “radiative” structure;
- B. there is a parallelism between Yukawa couplings in the up and in the down sectors ($\Gamma_u = \Gamma_d$ and equal λ ’s in equations (8a) and (8b)).

On the other hand, in the context of G_{LR} symmetry

- $\cancel{A}$. all Q -quarks are allowed to have tree level bare masses;
- $\cancel{B}$. owing to the different $B - L$ of U_i and D_i , it is impossible to impose an up-down symmetry that realizes B.

Here we show how to implement conditions A and B by simply adding two extra $U(1)$ factors to the gauge group. We stress that there is no need to impose any horizontal structure. Let us consider the gauge group $G'_{LR} = [SU(2)_L \otimes U(1)_L] \otimes [SU(2)_R \otimes U(1)_R] \otimes U(1)_{\overline{B}-\overline{L}}$ with the quark fields of eq. (2) accommodated in the following representations

$$\begin{aligned} q_{Li} &= (1/2, 0, 0, 0, 1/3) & q_{Ri} &= (0, 0, 1/2, 0, 1/3) \\ U_{Li} &= (0, 1, 0, 0, 1/3) & U_{Ri} &= (0, 0, 0, 1, 1/3) \\ D_{Li} &= (0, -1, 0, 0, 1/3) & D_{Ri} &= (0, 0, 0, -1, 1/3) \end{aligned} \quad (16)$$

The inclusion of leptons in this set is straightforward (in fact $U(1)_{\overline{B}-\overline{L}}$ can be unified with colour $SU(3)$ within Pati–Salam $SU(4)$ [12]). Notice that the theory is then free of gauge anomalies. We also introduce one family of vector-like quarks

$$p_{L,R} = (0, 1/2, 0, 1/2, 1/3) \quad n_{L,R} = (0, -1/2, 0, -1/2, 1/3) \quad (17)$$

where p is up-type and n is down-type^(*). In the scalar sector we introduce

$$\begin{aligned} H_L &= (1/2, 0, 0, 1, 0) & H_R &= (0, 1, 1/2, 0, 0) \\ T_{uL} &= (0, -2, 0, 0, -2/3) & T_{uR} &= (0, 0, 0, -2, -2/3) \\ T_{dL} &= (0, 2, 0, 0, -2/3) & T_{dR} &= (0, 0, 0, 2, -2/3) \\ \phi &= (0, 1/2, 0, -1/2, 0) & \Phi &= (0, 2, 0, -2, 0) & \Omega &= (0, 1/2, 0, 1/2, -1) \end{aligned} \quad (18)$$

where T -scalars are supposed to be colour $SU(3)$ triplets or antisextets. The above field content is such that it is natural to impose both left–right P_{LR} and up–down C_{UD} discrete symmetries. P_{LR} , which is essentially parity, acts in the usual way [12]. We invoke it only to simplify the notation, but it is not essential to our discussion on fermion masses. The C_{UD} symmetry is defined by

$$\begin{aligned} U_{L,R} &\leftrightarrow D_{L,R} & p_{L,R} &\leftrightarrow n_{L,R} & A_{L,R}^\mu &\rightarrow -A_{L,R}^\mu \\ H_{L,R} &\leftrightarrow \tilde{H}_{L,R} & T_{L,R}^u &\leftrightarrow T_{L,R}^d & \phi &\leftrightarrow \phi^* & \Phi &\leftrightarrow \Phi^* \end{aligned} \quad (19)$$

where $A_{L,R}^\mu$ are the gauge bosons of $U(1)_{L,R}$. In order to spontaneously violate C_{UD} -parity we introduce two real G'_{LR} singlet scalars η_+ and η_- , which are respectively C_{UD} -even and C_{UD} -odd. C_{UD} is essential. It implies the parallelism of Yukawa couplings in the up and down sectors. For the spontaneous breakdown of P_{LR} we introduce a G'_{LR} singlet P_{LR} -odd real scalar η_{LR} . The most general Yukawa interactions consistent with gauge invariance, P_{LR} and C_{UD} are

$$\begin{aligned} \mathcal{L}_1 &= \Gamma^{ij} \{ \bar{q}_{Li} U_{Rj} \tilde{H}_L + \bar{q}_{Li} D_{Rj} H_L + (L \leftrightarrow R) \} + \text{h.c.} \\ \mathcal{L}_2 &= \frac{1}{2} \lambda_{ij} \{ T_{uL} U_{Li} C U_{Lj} + T_{dL} D_{Li} C D_{Lj} + (L \leftrightarrow R) \} + \text{h.c.} \\ \mathcal{L}_3 &= h^i \{ \phi \bar{U}_{Li} p_R + \phi^* \bar{D}_{Li} n_R + (L \leftrightarrow R) \} + \text{h.c.} \\ \mathcal{L}_4 &= g_+ \eta_+ (\bar{p}_L p_R + \bar{n}_L n_R) + g_- \eta_- (\bar{p}_L p_R - \bar{n}_L n_R) + g_{LR} \eta_{LR} (\bar{p} \gamma_5 p + \bar{n} \gamma_5 n) + \text{h.c.} \end{aligned} \quad (20)$$

where Γ^{ij} , λ_{ij} , h^i , $g_\pm$, g_{LR} are coupling constants⁽⁷⁾ and C is the charge conjugation matrix. Notice that λ_{ij} is symmetric (antisymmetric) when the T -scalars are colour antisextets (triplets).

^(*) As will be clear from below, the mass ratio u/d for u and d quarks is essentially determined by p/n . Notice also that the formula $u/d \sim p/n$ is invariant under a 180° rotation.

⁽⁷⁾ In eq. (20) we did not introduce the bare mass term $\bar{p}p + \bar{n}n$. This is done by imposing a discrete symmetry R under which all fermions change as $\psi_L \rightarrow i\psi_L$, $\psi_R \rightarrow -i\psi_R$ while all scalars change sign. R also forbids trilinear scalar terms.

Let us suppose that the v.e.v.'s $\langle \eta_{\pm} \rangle = v_{\pm}$, $v_+ \sim v_-$ determine the highest scale of our theory. At this scale, C_{UD} and R are broken, and p and n get a mass $M_p \neq M_n$. At a lower scale, $\langle \phi \rangle = v_{\phi}$ breaks $U(1)_L \otimes U(1)_R$ down to $U(1)_{L+R}$. The v.e.v. of Ω then breaks $U(1)_{L+R} \otimes U(1)_{\overline{B}-\overline{L}}$ to the usual $U(1)_{B-L}$: $B-L = Y_L + Y_R + \overline{B} - \overline{L}$. Due to $\langle \phi \rangle$ one combination of Q -fermions ($U \propto h^i U_i$, $D \propto h^i D_i$) becomes massive via seesaw mixing with p and n [interactions $\mathcal{L}_3$ in eq. (20), see Fig. 1]. By a suitable redefinition of the Q -fermion basis we can always take $h^1 = h$, $h^{2,3} = 0$. Then, in the limit $v_{\phi} \ll v_{\pm}$, we have $M_{U,D} \simeq h^2 v_{\phi}^2 / M_{p,n}$. At the breakdown of $U(1)_L \otimes U(1)_R \rightarrow U(1)_{L+R}$ also Φ gets a v.e.v. Thus $T_{uL}-T_{uR}$ and $T_{dL}-T_{dR}$ get mixed via the interaction terms

$$\mathcal{L}_{T_L T_R} = \lambda_+ \eta_+ (T_{uL} T_{uR}^* \Phi + T_{dL} T_{dR}^* \Phi^*) + \lambda_- \eta_- (T_{uL} T_{uR}^* \Phi - T_{dL} T_{dR}^* \Phi^*) + \text{h.c.} \quad (21)$$

Notice that the most general operators involving two T -fields and two η -fields (η_+ , η_- , or η_{LR}) yield, after insertion of the η 's v.e.v.'s, different contributions to the masses of all four scalars T_{uL} , T_{dL} , T_{uR} , and T_{dR} . At this point the loop corrections [5–7] due to $\mathcal{L}_2$ in eq. (20) generate mass matrices for Q -fermions (see Fig. 2). These matrices are equal to those in eq. (10), where $\hat{P}_1 = \text{diag}(1, 0, 0)$ and λ is the Yukawa coupling matrix of eq. (20) ($\lambda_L = \lambda_R = \lambda$ due to P_{LR} symmetry). The expansion parameters $\epsilon_{u,d}$ are [7]

$$\epsilon_u = \frac{3}{32\pi^2} \sin 2\xi_u \ln r_u \quad \beta_u = \frac{\epsilon_u^2}{2} (1 + g(r_u)) \quad r_u = \left(\frac{M_+^u}{M_-^u} \right)^2 \quad (22)$$

and analogously for ϵ_d and β_d . In eq. (22) $\xi_{u,d}$ and $M_+^{u,d}$, $M_-^{u,d}$ are respectively the mixing angle and the eigenvalues of the mass matrix of the scalars $T_{u,dL}$ and $T_{u,dR}$. The function $g(r) = g(1/r)$, given in Ref. [7], goes to zero when $r \rightarrow \infty$. However, in a “reasonable” range ($1 < r < 10$) $g(r)$ is constant and close to 0.3 with good accuracy. It should be noticed that eq. (22) is valid in the regime $M_U \ll M_-^u$, $M_+^u \ll M_p$ and $M_D \ll M_-^d$, $M_+^d \ll M_n$ ($M_{p,n}$ acts as a cut-off of $M_{U,D}$ in the diagram of Fig. 2 [7]). The above regime has been chosen for calculational convenience. However, at least for the down sector, it is better to have $M_{\pm}^d$ not smaller than M_n , otherwise ϵ_d becomes too small [7]. The smallness of ϵ_d would require too large ($> O(1)$) λ 's [see eqs. (12-13)]. From eq. (12) and the analogue for up quarks we have $\epsilon_d/\epsilon_u \simeq (1 + \epsilon_d |\bar{\lambda}_{11}|^2)^2 m_c m_d / m_u m_s$. By use of eq. (13) and the values of masses, we get $\epsilon_d/\epsilon_u \simeq 30$ –60. In the general case, i.e. with complex ϵ_d or without left-right symmetry, it could also be $\epsilon_d/\epsilon_u \sim m_c m_d / m_u m_s \simeq 10$ –20. The smallness of ϵ_u could be due to a small splitting $r_u \simeq 1$ or to a small mixing angle ξ_u in eq. (22), as compared with the analogue quantities for down quarks. If we take into account all allowed contributions to the T -scalars mass matrices when $\eta_{\pm}$, η_{LR} and Φ get a v.e.v., it is well possible to realize one of the above conditions⁽⁸⁾. Of course some parameters must be tuned in the proper way (in

⁽⁸⁾ Notice that if we neglect the effects of $\langle \eta_{LR} \rangle$, for instance $\langle \eta_{LR} \rangle \ll \langle \eta_{\pm} \rangle$, we have maximal mixing $\xi_{u,d} = 45^\circ$.

fact, in the general case $\epsilon_d/\epsilon_u \sim 10$, only a one-order of magnitude cancellation is needed). A perhaps more interesting way to obtain a small ϵ_u is to avoid the condition $M_U \ll M_-^u, M_+^u \ll M_p$ and consider $M_\pm^u$ somewhat larger than M_p . In this case the diagram in Fig. 2a is suppressed by $(M_p/M_\pm^u)^2$. For instance by taking $\xi_u = \xi_d$ and the following values for masses

$$\begin{aligned} (M_+^d)^2 + (M_-^d)^2 &= M_n^2 & (M_+^u)^2 + (M_-^u)^2 &= 4M_n^2 \\ (M_+^d)^2 - (M_-^d)^2 &= (M_+^u)^2 - (M_-^u)^2 = \frac{1}{2}M_n^2 \end{aligned} \quad (23)$$

we get $\epsilon_d/\epsilon_u \simeq 40$. Notice that the splitting between corresponding quantities in the up and down sector in eq. (23) is not larger than a factor $M_n/M_p = (m_d/m_u)(1 + \epsilon_d|\bar{\lambda}_{11}|^2) \simeq 4$. Of course, in this range of parameters, both ϵ_u and β_u depend also on M_p ; in addition they are not related to each other as in eq.(22).

4. We have shown that the idea of radiative fermion mass generation can be made consistent with the observed values of quark masses and mixings. In order to do this we have supposed the existence of heavy isosinglet quarks (Q -fermions) in one-to-one correspondence with the visible ones. The mass hierarchy, first radiatively generated in the heavy sector, is projected to the visible one by means of a universal seesaw mechanism. We have considered a model based on the gauge group $G'_{LR} = [SU(2)_L \otimes U(1)_L] \otimes [SU(2)_R \otimes U(1)_R] \otimes U(1)_{\bar{B}-\bar{L}}$ and possessing a discrete up-down symmetry C_{UD} . At a high scale, $U(1)_L \otimes U(1)_R \otimes U(1)_{\bar{B}-\bar{L}}$ is broken to $U(1)_{B-L}$. At this stage only one family (U and D) of Q -fermions becomes massive. At a lower scale v_R , $SU(2)_R \otimes U(1)_{B-L}$ is broken to $U(1)_Y$. The seesaw masses $m_{u,d} = O(v_L v_R/M_{U,D})$ set the ratio between the above scales $v_R/M_{U,D} = O(10^{-4})$. The radiative masses of the other Q -fermions lay between v_R and $M_{U,D}$.

Many problems of previous approaches to radiative mass generation are solved in our scheme. By fitting the values of the masses of the five known quarks, we find that the natural value of m_t is in the 100 GeV range. FCNCs are avoided by taking the Q -fermions sufficiently heavy. In addition the correct value of the Cabibbo angle can be obtained without troubles for perturbation theory.

A more detailed exposition of the above scheme, including also leptons and in the context of grand unification, will be given elsewhere.

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Figure captions

- Fig.1 Tree-level diagram generating the mass of the heaviest up-type Q -fermion U .
 An analogous diagram gives mass to D .
- Fig.2a-b One- and two-loop diagrams generating the hierarchical mass matrix of Q -fermions.

Fig. 1

Fig. 2