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POLARIZED STRUCTURE FUNCTIONS AT HERA: INTRODUCTION AND OVERVIEW

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POLARIZED STRUCTURE FUNCTIONS AT HERA: INTRODUCTION AND OVERVIEW

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1. Introduction

In the following I will give a general overview of the status and the importance of the study of polarized structure functions¹ together with a listing of the activities of our Working Group. I will try to make it clear that the experimental program on polarized structure functions is a very important part of the HERA physics program, so that every effort should be made in order to implement a sufficient electron polarization and to perform the measurements.

As is well known, there are two polarized structure functions in a nucleon. At present there are only data [1, 2] on $g_1^p(x, Q^2)$, the first polarized structure function of the proton, while its neutron counterpart $g_1^n(x, Q^2)$, as well as $g_2^p(x, Q^2)$ and $g_2^n(x, Q^2)$, are totally unmeasured. I will first discuss the interesting aspects of g_1 in the next section and I will then consider g_2 in section 3. Finally our conclusions will be summarized in section 4. More detailed reports on the activities of our Working Group can be found in separate articles in these Proceedings. The main items studied at the Workshop were a) one-particle inclusive π and K asymmetries (Düren, Mankiewicz, Rith, Schäfer, Veltri); b) the development of the PEPSI event generator for polarized deep inelastic scattering (Mankiewicz, Rith, Schäfer, Veltri); c) large- p_T jets and heavy-quark production at HERA and SMC (Guillet, Vogelsang); d) the structure function g_2 and its physical interpretation (Mankiewicz, Schäfer); e) a proposal to shoot the HERA p beam on a polarized p target in order to measure the gluon polarized structure function (Glück, Vogelsang); f) experimental problems concerning g_1 and g_2 (Düren, Rith).

2. The structure function g_1 and the spin of the nucleon

A wave of excitement and a flurry of theoretical papers were generated in 1988–89 by the data on deep inelastic scattering of polarized muons on polarized protons by the EMC Collaboration at CERN [1]. When taken together with previous data from SLAC [2], these results lead to a reasonably accurate determination of $g_1^p(x, Q^2)$ for $0.01 \leq x \leq 0.7$, with an average Q^2 given as $\langle Q^2 \rangle = 11 \text{ GeV}^2$ from the EMC and 5 GeV^2 at SLAC. These data have suscituated a particular interest because they appear to imply that the total helicity carried by quarks and antiquarks in the proton is compatible with zero [3]. More precisely the combined SLAC and EMC data lead to a rather small value for the first moment of $g_1^p(x, Q^2)$ (in the stated Q^2 range):

$$\int_0^1 g_1^p(x, Q^2) dx = 0.126 \pm 0.018 \quad (2.1)$$

¹ The Polarized Structure Function Working Group: G. Altarelli, M. Düren, M. Glück, J.Ph. Guillet, L. Mankiewicz, K. Rith, A. Schäfer, M. Veltri, W. Vogelsang.

Indeed this result, interpreted in the naïve parton model, assuming the validity of the Bjorken sum rule [4],

$$\int_0^1 (g_1^p(x, Q^2) - g_1^n(x, Q^2)) dx = \frac{1}{6} g_A/g_V \quad (2.2)$$

and of the SU(3) analysis of hyperon decays [5] (which determines the D/F ratio of octet axial currents) leads to the following values for the polarized quark moments:

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s \approx 0.12 \pm 0.17 \quad (2.3)$$

and

$$\Delta u \approx 0.78 \pm 0.06; \quad \Delta d \approx -0.47 \pm 0.06; \quad \Delta s \approx -0.19 \pm 0.06 \quad (2.4)$$

where Δq ($q = u, d, s$) is the total helicity carried by quarks and antiquarks of type q . Note that, given the helicity sum rule

$$\frac{1}{2}\Delta\Sigma + \Delta g + \Delta L_z = \frac{1}{2} \quad (2.5)$$

the experimental value of $\Delta\Sigma$ in eq. (2.3) implies, if correct, that either $\Delta g + \Delta L_z$ is large or there is something wrong in the extrapolation down to $x = 0$ from the region $x \geq 0.02$, where data exist.

The puzzling aspect is the large difference between constituent and parton quarks that emerges from these results [6]. For comparison, in the naïve SU(6) constituent model one would have $\Delta\Sigma = 1$, $\Delta u = 4/3$, $\Delta d = -1/3$, and $\Delta s = 0$. One expects that, for conserved quantities, constituent and parton results should coincide. For example, for unpolarized densities, the charge moments and the momentum sum rule integral have the same value for constituents and for partons. Now Δu , Δd , and Δs are conserved by first-order QCD evolution [7,8]. In spite of this, already in the non-singlet sector there is a difference of roughly 30% between constituents and partons. In fact $\Delta u - \Delta d = 1.25$ (Bjorken sum rule) and $\Delta u + \Delta d - 2\Delta s = 0.68$, while SU(6) predicts $5/3$ and 1 , respectively. This difference is attributed to helicity non-conservation induced at low energy scales by quark-mass effects which break chirality. This symmetry breaking in the non-perturbative region creates a difference in the initial values for the perturbative QCD evolution. However the huge difference observed in the singlet sector ($\Delta\Sigma = 0.12 \pm 0.17$ vs $\Delta\Sigma = 1$) appears too large to be attributed to mass terms and demands a special explanation.

I think that it is fairly clear by now that the reason why the singlet moment is special is the presence in that channel of the axial anomaly. The singlet moment measures the forward matrix element of the singlet axial current whose divergence is different from zero even in the massless limit due to the Adler-Bell-Jackiw anomaly [9]. This anomaly is responsible for the extra non-conservation in the singlet channel. There is a minority alternative proposal that invokes the Skyrme model for an explanation. In this model one argues [10] that $\Delta\Sigma$ is suppressed by $1/N_c$, while Δu , Δd , and Δs are of order 1. This explanation is even questionable [11] within the Skyrme model. Also, to me it is unclear what precise relation there is between the Skyrme model and QCD. While there is a large consensus on ascribing the effect to the anomaly [12–18], there is still an open debate on the detailed underlying mechanism.

In one explanation [13–15] the effect of the anomaly is entirely described within the QCD-improved parton model and is attributed to a large gluon helicity component, which can be measured in suitable hard processes. More precisely one has:

$$\Delta\Sigma \approx 0.12 \pm 0.17 = \Delta\Sigma' - N_f \frac{\alpha_s(Q)}{2\pi} \Delta g(Q) \quad (2.6)$$

The meaning of this formula is as follows: $N_f = 3$ is the number of excited flavours; Δg is the first moment of the polarized gluon density; formally, it appears as a non-leading correction at order α_s . However, it can be shown from the one-loop QCD evolution equations [7, 8] that $\Delta g(Q) \sim [\alpha_s(Q)]^{-1}$, so that the gluon contribution is actually constant in Q^2 in the leading approximation. The splitting of $\Delta\Sigma$ into a modified quark and a gluon term depends on the definition of the quark, which is formally the leading term. The partition in eq. (2.6) corresponds to $\Delta\Sigma'$ defined in such a way as to be conserved, not only at one-loop but also at two-loop accuracy in the massless theory [16]. If it is true that for conserved quantities partons and constituents coincide, then $\Delta\Sigma'$ will be constituent-like, say within 30%, which is the breaking expected from mass terms. Thus we expect $\Delta\Sigma' \approx 0.7$. It can be shown that the presence of the gluon contribution and its peculiar behaviour with Q^2 are due to the anomaly, and $\Delta g(Q)$ can be measured in suitable hard processes (large- p_T jets in deep inelastic scattering [15], large- p_T photons produced at hadron colliders [19], heavy-pion photoproduction [20], ...).

In the previous mechanism the effect of the anomaly is completely described within the context of the QCD-improved parton model: Δg can be measured in hard processes and once the anomalous gluon term is removed $\Delta\Sigma'$ approaches its naive value. In this sense the mechanism is "perturbative", although Δg can certainly not be computed in perturbation theory. The splitting of $\Delta\Sigma$ into a quark-like plus a gluon-like terms, both of them gauge- and isospin-invariant, with the correct behaviour under the renormalization group, has been discussed in very general terms in refs. [21]. One can imagine that further non-perturbative contributions also due to the anomaly (i.e. instantonic terms) could be present in the conserved quark term making it different from the constituent limit even after subtraction of the anomalous gluon component. In fact model calculations [17, 18] show that in general soft, non-perturbative quark-like terms could be generated by instantons:

$$\Delta\Sigma = [\Delta\Sigma' - N_f\Omega] - N_f \frac{\alpha_s(Q)}{2\pi} \Delta g(Q). \quad (2.7)$$

The term $-N_f\Omega$ can be seen as a sea polarization induced by instantonic effects.

One can wonder if the anomaly effect can be large enough to agree with the observed difference between $\Delta\Sigma$ and $\Delta\Sigma'$. Recently it was claimed in ref. [22] that, by evaluating the matrix element $\langle p, s | k_\mu | p, s \rangle$ on the lattice, one obtains the bound

$$3f \frac{\alpha_s}{2\pi} \Delta g + \Omega \leq 0.05. \quad (2.8)$$

The operator k_μ is defined from the divergence of the singlet axial current:

$$\partial_\mu \bar{\psi} \gamma_5 \psi = \text{Tr}(\bar{q} \gamma_5 q) + \partial_\mu k^\mu. \quad (2.9)$$

The forward matrix element of k_μ is gauge-invariant, but for topologically non-trivial (instanton) transformations. Naively it corresponds to the left-hand side of eq. (2.8), but this identification is not obvious. There are subtleties involved in the procedure of lattice calculation in this particularly delicate case (chiral symmetry, the definition on the lattice of the topological charge, the transition from the discrete to the continuum limit, instantons, ...). Of course, if demonstrated to be true, the result in eq. (2.8) would invalidate every explanation based on the anomaly, were it perturbative or not. But in order for the evaluation to be reliable a number of consistency checks should be passed. For example one should demonstrate that the crucial property of the logarithmic increase of Δg with Q^2 is realized. Also the detailed implementation of the helicity sum rule should be clarified. Finally I mention that claims that Δg is small, based on experiments on asymmetries in pion production from polarized hadron-hadron scattering [23], have been recently criticized [24].

Going back to eq. (2.5), which gives the total helicity of the proton, we observe that in perturbative QCD the logarithmically increasing term in Δg is cancelled by an analogous term in ΔL_z according to a mechanism that was elucidated in ref. [25]. The sum $\Delta g + \Delta L_z$ thus has to be regarded as a single unit, since it is difficult to disentangle what really is gluon and what is angular momentum. This sum, which is different if the quark term is identified with $\Delta\Sigma$ or $\Delta\Sigma'$, completes the proton helicity. A possibility which cannot be excluded is that there may be contributions to the sum rule arising from very small values of x (in particular terms that reduce to $\delta(x)$ contributions in the limit $Q^2 \rightarrow \infty$). In this case, the measured values at finite x values would indicate an apparent violation of the spin sum rule. I think that the Ω contribution in eq. (2.7), if non-negligible, is a likely candidate for such a term. In fact this quantity arises from instantonic effects related to the special properties of the operator k_μ and are peculiar of the first moment [it is clear that $\delta(x)$ terms only affect the first moment]. In this case Ω would not be directly relevant to the explanation of the EMC result but it would apparently invalidate the spin sum rule [eq. (2.5)].

In conclusion it is evident that new input from experiment is extremely important. First, the EMC result should be checked and the precision on $\Delta\Sigma$ improved. Note that the skyrmion explanation demands $\Delta\Sigma < \Delta s$ while the one with Δg large prefers $\Delta\Sigma > \Delta s$. Secondly, it is crucial to verify the validity of the Bjorken sum rule. This is a fundamental test of the QCD-improved parton model (which specifies the perturbative corrections to the sum rule). There are non-QCD models that contemplate the violation of this sum rule [26]. Finally it would be great to directly measure Δg from hard processes.

At HERA the proposed HERMES experiment could successfully address the first two items. The first moment of g_1^p can be measured at $(Q^2) \approx \text{few GeV}^2$ with $\sim 6\%$ precision (compared with $\sim 10\%$ of the SMC, the ongoing CERN experiment [27], and $\sim 19\%$ of the EMC). The structure function g_1 for the neutron can also be measured as a function of x and the Bjorken sum rule tested at the 5–8% level, a better figure than those envisaged for the SMC and the SLAC experiment E142 [28]. On the other hand, HERMES is too limited in energy for a meaningful determination of Δg . An interesting alternative suggestion for measuring Δg at HERA has been proposed in the Working Group [29]. It is envisaged to scatter the unpolarized p beam of HERA on a polarized p target and measure the polarization asymmetry in the process:

$$p \text{ (pol)} \rightarrow \gamma^* \text{ (large } p_T) X \rightarrow \mu^+ \mu^- X \quad (2.10)$$

by using the lepton angular distribution as a polarimeter for the virtual photon. With Ebeam ≈ 1 TeV one obtains $\sqrt{s} \approx 43$ GeV. The details of this proposal are discussed in the accompanying article by W. Vogelsang [29].

3. The structure function g_2

The Fourier transform of the forward matrix element between polarized nucleons of the product of two electromagnetic currents leads to the sum of a symmetric (S) and an antisymmetric (A) tensor [8, 30]:

$$W_{\mu\nu} = \int d^4x e^{iqx} \langle p, s | J_\mu(x) J_\nu(0) | p, s \rangle = W_S^{\mu\nu} + iW_A^{\mu\nu} \quad (3.1)$$

The symmetric part is independent of the polarization and contains the unpolarized structure functions. The antisymmetric part, linear in the polarization vector s_μ , can be written in terms of two structure functions g_1 and g_2 :

$$iW_A^{\mu\nu} = i\epsilon^{\mu\nu\lambda\sigma} q^\lambda \left\{ \frac{s^\sigma}{M\nu} g_1(x, Q^2) + \frac{M\nu s^\sigma - (qs) p^\sigma}{(M\nu)^2} g_2(x, Q^2) \right\}, \quad (3.2)$$

where M , p are the nucleon mass and four-momentum, respectively, q the virtual photon four-momentum, $M\nu = (p q)$, and x is the Bjorken variable $x = Q^2/2M\nu$. It is simple to show [8] that

for longitudinal polarization what matters is the combination $[g_1 - (2Mx/Q^2)g_2]$ so that the g_2 contribution is negligible at large Q^2 . Instead, for transverse polarization, $iW_{\mu\nu}^A$ is proportional to $2x/Q [g_1 + g_2]$, so that g_1 and g_2 enter with equal coefficients but the whole contribution is of order $1/Q$.

The fact that g_2 is related to the transverse polarization already indicates that a partonic interpretation may not be so easy to find, because a massless parton can only have longitudinal polarization. Moreover it is simple to compute $iW_{\mu\nu}^A$ for a free quark, no matter if massive or massless. In either case it is found that g_2 vanishes (while g_1 is non-vanishing and finite even for $m = 0$). Thus g_2 cannot be expressed as a sum of incoherent contributions over free on-shell partons. The partons must be interacting and/or virtual in order to contribute to g_2 .

It is not terribly important if it is not clear what the parton interpretation of g_2 is. After all, in totally inclusive deep inelastic scattering the light-cone operator expansion is a much more powerful and general method. In the light-cone expansion of $iW_{\mu\nu}^A$, two classes of operators occur [30]. For the simplest case of non-singlet channels, one class is represented by the operator of twist 2 given by:

$$\bar{q}\gamma_5\gamma^\mu(\sigma D_\mu)_1 D_2 \dots D_n \lambda_{\mu_1} q, \quad n \geq 0, \quad (3.3)$$

where λ_i is a flavour matrix. To the second class belong operators that need at least two indices (we call them twist-3 operators), given by:

$$\bar{q}\gamma_5\gamma^\mu(\sigma D_\mu)_1 D_2 \dots D_n \lambda_{\mu_1} q, \quad n > 0, \quad (3.4)$$

(antisymmetric in σ and μ_1 and symmetric in all μ_i 's). The last operator is proportional to the quark mass matrix m . The moments of g_1 and g_2 are given by (only even moments are accessible to the light-cone method):

$$\int_0^1 dx x^n g_1(x, Q^2) = a_n, \quad n = 0, 2, 4, \dots \quad (3.5)$$

$$\int_0^1 dx x^n g_2(x, Q^2) = \frac{n}{n+1} (d_n - a_n), \quad n = 2, 4, \dots \quad (3.6)$$

where a_n and d_n are the matrix elements of the n th operators of twist 2 and 3 respectively. We see that g_1 is only determined by the twist-2 operator sequence, while the second class of operators only enters in the expression of g_2 .

The first important issue about g_2 is whether or not the operators of twist 3 are important. Some people argue that they are not, but I would say that they probably are. Note that if the twist-3 operators are negligible, i.e. all $d_n = 0$, then g_2 is completely determined in terms of g_1 by the so-called Wandzura-Wilczek relation (WW) [31]:

$$g_2 = g_2^{WW} = -g_1 + \int_x^1 \frac{dx'}{x} g_1(x', Q^2) \quad (3.7)$$

while in general

$$g_2 = g_2^{WW} + \bar{g}_2 \quad (3.8)$$

with $\bar{g}_2$ being the contribution from the d_n sequence. From the existing data the shape of $\bar{g}_2$ can be derived, as shown in fig. 1, which is taken from ref. [32]. Note that for a free quark the WW sum rule is violated. In fact for a free quark g_2^{WW} is exactly cancelled by $\bar{g}_2$ because we have said that g_2 is zero for a free-quark target. In this case one can show that the contribution to $\bar{g}_2$ is entirely given by the mass operator in eq. (3.4) (even for massless quarks one must approach the limit from $m \neq 0$, in order to give sense to the transverse polarization). The expectation that the twist-3 contribution should be small is based on the fact that it vanishes for ultra-relativistic on-shell quarks. In fact in that case s^0 is proportional to p^0 and there are not enough four-vectors to form an antisymmetric combination. For a quark in the proton, non-relativistic corrections are expected to be of order m/M or m/Λ , which are small for light quarks (and the contribution of s-quarks from the sea is itself small). But this argument is not solid because for off-shell quarks with virtuality k^2 the quantity k^2/Λ^2 is not small. The virtuality is built up because the quark is interacting. Indeed, by the equations of motion, the first of the twist-3 operators can be shown [33] to be identical to an operator of order $\sqrt{\alpha_s}$ involving the gluon field. For example, in the chiral bag model the quarks inside the bag are much off-shell and, as a consequence, the departures from the WW prediction of g_2 are quite large, as seen from fig. 2 (taken from ref. [34]). In conclusion, I expect that the WW prediction will be violated, and measuring $\bar{g}_2$ is certainly highly interesting because it represents a subtle non-partonic effect produced in a non-trivial way by QCD interactions.

A second important issue is the question of the validity of the Burkhardt-Cottingham (BC) sum rule [35]. Going back to eq. (3.6) we see that the light-cone approach does not predict the first moment of g_2 . However, the presence of the factor of n in the r.h.s. of eq. (3.6) suggests the validity of the (BC) sum rule:

$$\int_0^1 dx g_2(x, Q^2) = 0, \quad (3.9)$$

In a partonic approach or, more generally, from a field theory point of view, based on, say, Feynman diagrams to all orders, the primary quantities are not moments but x -dependent structure functions. If we imagine one well-defined x density which generates all moments, the simplest thing is to continue the dependence of moments on n to all values of n where the resulting dependence is non-singular. Then one way for the BC sum rule not to hold would be that $d_n \sim 1/n$ for $n \rightarrow 0$ (we know that a_0 is non-singular because the first moment of g_1 is finite). Alternatively one can think of contributions which, for $Q^2 \rightarrow \infty$, become proportional to $\delta(x)$; this would make the first moment discontinuous with respect to the extrapolation in n . However, in this case, the corresponding violation of the BC sum rule would not show up from the data at finite x . The behaviour of d_n near $n = 0$ and the validity of the BC sum rule can be related by dispersion relations to the Regge asymptotics for g_2 near $x = 0$ [30, 36]. In presence of conventional Regge poles [i.e. with negative intercept $\alpha(0)$ at $t = 0$] the BC sum rule would be valid. For Regge cuts that could exist even with non-negative branch point the BC sum rule is valid if the discontinuity falls fast enough in Q^2 , as expected [37]. Similarly a fixed pole at $J = 0$ could invalidate the BC sum rule but only by terms of order at most $1/Q^2$ [38]. My conclusion on the BC sum rule is that it is most probably true. However, it is important to check its validity by experiment: it would be very interesting to find that it was not true, because this would imply a non-conventional behaviour of the twist-3 operators. Note however that the sum rule could be valid only up to terms of order $1/Q^2$, so that accurate measurements at not too small values of $\langle Q^2 \rangle$ are needed. As discussed by Rith [39], the HERMES collaboration expects to measure the first moment of g_2 with an absolute error of ± 0.06 at $\langle Q^2 \rangle \sim \text{few GeV}^2$.

Some original work on g_2 has been produced by members of our working group.

Mankiewicz and Ryzak [40] have shown that, while g_1 is related to $|\psi(p \pm \frac{1}{2})|^2$, g_2 is proportional to an overlap integral of the form $\langle \psi(p, +\frac{1}{2}) | O_3 | \psi(p, -\frac{1}{2}) \rangle$, where $\psi(p, s)$ is the wave function in the light-cone formalism and O_3 is the relevant twist-3 operator. This representation is useful for a physical interpretation of g_2 . Using it, Mankiewicz and Schäfer [41] argue that the violation of the BC sum rule would imply the existence of a large soft sea polarization. In a separate contribution [42] the same authors show that g_2 , because of its sensitivity to higher-twist effects, is more vulnerable to nuclear effects; they discuss the implications of disentangling g_2 from data on nuclei.

Finally, I mention that the prediction of the Q^2 evolution of $g_2(x, Q^2)$ is an exceptionally intricate theoretical problem. When the order n of moments increases, more and more independent operators enter in its Q^2 evolution [43] because of mixings of the operators in eqs (3.3) and (3.4) with operators involving gluons. In a recent paper [44] it is shown that the evolution becomes simpler in the limit of large N_c (the number of colours) and in the limit of large n (the order of the moment) if non-leading terms in the corresponding expansion parameters are neglected in the anomalous-dimension matrices. These results can perhaps be used to construct practical approximations to the Q^2 dependence. The issue of the Q^2 evolution of g_2 will eventually become important, but the experimental study of this problem appears to be out of reach for first-generation experiments.

In conclusion the theory of g_2 is difficult and rich of intriguing aspects. No measurements of g_2 exist so far. It is thus quite important to obtain the first data on g_2 for protons and neutrons. The main goals of the first-generation experiments are to check the BC sum rule and to establish the existence of deviations from the WW prescription (both in the singlet and non-singlet channels). HERMES could provide the first precise answers to these important problems.

4. Conclusion

Clearly the search for new physics deserves first priority in the experimental program at HERA. However new physics is not guaranteed. But the promise of HERA is in any case based on a solid platform of important specific questions in the Standard Model that can be addressed (specific to e-p physics as compared with LEP or hadron collider experiments). These items have been reviewed, improved and updated at the present Workshop: structure functions, measurement of α_s , QCD at small x , photoproduction, heavy quarks and so on. I think that it is completely fair to say that in this domain of experiments the study of polarized structure functions indeed would represent a substantial component of the program. There are several really outstanding pieces of work that can be done; like the precise check of the Bjorken sum rule, the accurate measurement of g_2 for protons and neutrons, and the experimental test of both the BC sum rule and the WW relation. The program of HERA in this field is competitive and in many respects superior to what is being realized or planned elsewhere. I feel that the importance of these perspective achievements is by far worth the comparatively moderate cost of implementing and controlling the polarization and of making the experiment.

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Fig. 1 Expectations for $xg_2(x)$ and $x[g_1(x) + g_2(x)]$ based on taking a parametrization of the measured asymmetry and computing xg_2 from the WW relation (from ref. [32])

Fig. 2 Evaluation of $g_2(x)$ in the bag model (from ref. [34]). The lines represent: dotted, twist 2 (i.e. satisfying the WW relation); dashed, twist 3; solid, the sum of the two

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