THE BUBBLE FREE ENERGY IN THE QCD PHASE TRANSITION¹

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Abstract

We study the free energy of the spherical hadron and quark bubbles at $T \approx T_c$ in SU(3) gauge theory. We find that in addition to the contributions proportional to the volume and area of the bubble, there is a large curvature term proportional to the radius. The value of this term agrees with the one computed by Mardor and Svetitsky from the MIT bag model. This work has been done in collaboration with K. Kajantie and L. Kärkkäinen.

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First-order phase transitions, like the finite temperature transition in SU(3) gauge theory, begin with supercooling or superheating followed by the nucleation of the bubbles of the thermodynamically stable phase. The rate of the phase transition is determined by the free energy of the bubbles as a function of the radius r [1]. Expanding the free energy around $r = \infty$, we obtain

$$F(r) = \Delta p \frac{4}{3} \pi r^3 + \alpha 4 \pi r^2 + \gamma r + \dots \quad (1)$$

where Δp is the bulk pressure difference between the two phases and α is the surface tension. In the standard thin wall approximation only two leading terms of eq. 1 are used; this is assumed to be valid when r is large enough, at least much larger than the interface thickness. However, Mardor and Svetitsky [2] found by using the MIT bag model that the *curvature term* γr can be much more important than the surface term for the bubble nucleation processes. This is mainly due to the small value of the surface tension. The $N_\tau = 2$ pure SU(3) gauge lattice Monte Carlo simulations [3,4] give $\alpha \approx 0.1 T_c^3$, whereas the predicted curvature term for the pure gluon gas [2] is

$$\gamma = \pm 16 \frac{2\pi}{9} T^2 \approx \pm 11.2 T^2. \quad (2)$$

The sign is positive (negative) for plasma (hadron) bubbles inside hadron (plasma) bulk phase. For compactness, we use the names hadron and plasma phase even for the pure gauge system. The curvature contribution dominates over the surface tension up to $r \sim 10/T$.

The curvature term lowers the free energy of the hadronic bubbles and makes the plasma $\rightarrow$ hadron transition *less* metastable; correspondingly the hadron $\rightarrow$ plasma transition becomes *more* metastable. In fact, when r is small enough the curvature term always dominates over the volume and surface terms. This seemingly makes the plasma phase unstable against the formation of the hadronic bubbles at *all* temperatures [2].

In order to measure the bubble free energy the bubble has to be enforced on the lattice. This is achieved by using a radius-dependent coupling constant:

$$\beta = \beta_{\text{in}} + \frac{1}{2}(\beta_{\text{out}} - \beta_{\text{in}})(\tanh \frac{|r| - r}{w} + 1) \quad (3)$$

where r is the bubble radius, $|r|$ is the radial coordinate from the centre of the lattice, and the wall thickness parameter $w = 1/2$. Now it is easy to calculate

$$f'(r) \equiv \frac{1}{T} \frac{dF}{dr} = -\left\langle \frac{dS}{dr} \right\rangle = c_1 + c_2 r + c_3 r^2 \quad (4)$$

Fig. 3. Same as fig. 1 for the quark bubbles.

against different definitions of the radius. Let us make a reparametrization $r = \rho + a + b\rho^{-1} + \mathcal{O}(\rho^{-2})$; using eqs. 1,4 the free energy in terms of ρ becomes

$$f(\rho) = \frac{1}{3}c_3\rho^3 + \frac{1}{2}(c_2 + 2c_3a)\rho^2 + [c_1 + c_2a + c_3(a^2 + b)]\rho + \mathcal{O}(\rho^0) \quad (5)$$

When $\Delta\beta \rightarrow 0$ also $c_3 \rightarrow 0$, so that the curvature term $\mathcal{O}(\rho)$ becomes $c_1 + c_2a$. Now $|c_1| \approx 5.5$ and $|c_2| \approx 0.6$, so that even when $|a| \sim 1$, which corresponds to one lattice unit shift in the bubble radius, only $\sim 10\%$ variation in the curvature term is produced. This is well within errors.

We conclude that the MIT bag model prediction [2] for the curvature contribution to the bubble free energy $\gamma = \pm 32\pi/9T^2$ is confirmed by our lattice calculations. We emphasize that the $N_\tau = 2$ lattice used by our simulation is still in the strong coupling regime, and that the deviation from the pseudocritical coupling ($\Delta\beta$) is rather large, and it is possible that the actual value of γ will turn out to be different. However, we find the agreement between the numerical and analytical results rather striking. To determine the stability of the quark phase against

Fig. 4. Same as fig. 2 for the quark bubbles.

bubble nucleation requires still further analysis [5,6].

The numerical simulations were performed with the Cray X-MP at the Centre for Scientific Computation, Helsinki, Finland.

References

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